# Rocket Propulsion

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## Lecture No # 20

## Introduction to Solid Propellant Rockets

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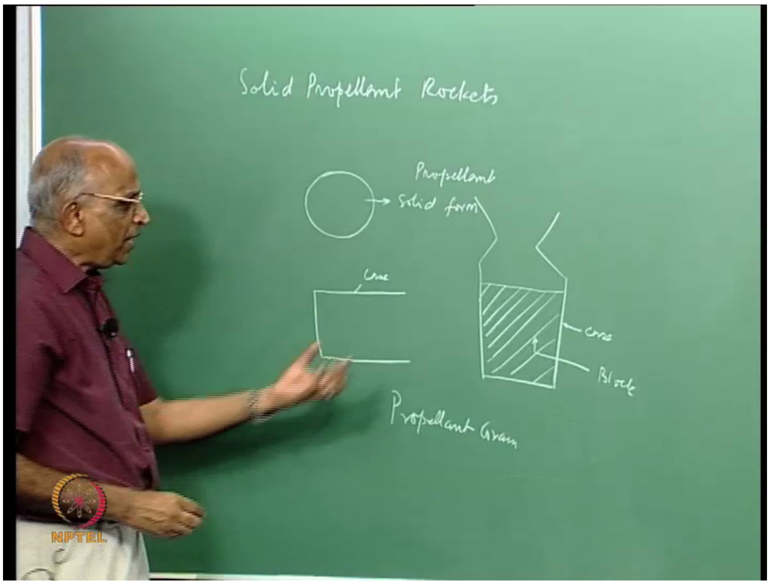

Today we will start with solid propellant rockets and this will be an extremely simple topic. We will take a look at what are the elements of a solid propellant rocket, but primarily, we start from the propellant point of view, because that is what we have covered so far. We told that solid propellants could be double base, may be composite, may be composite modified double base, and also nitramine propellants. Whenever these propellants are made, you cure the substances and make it into something like a solid form, and this solid is what constitutes let us say a solid propellant. We would like to use it to generate thrust.

Therefore, what is it we do? We take, let us say a case, which could be cylindrical. We put the solid propellant inside the case. How do you put it inside the cylinder or a case? We keep the case vertically, we put all the ingredients into it; we make the propellant mixture into a sort of a liquid or a slurry, then we cure it and we get something like a solid which is formed within the cylindrical case. We then attach a nozzle to it and this becomes a solid propellant rocket.

Therefore, in a solid propellant rocket we have a structural member which is a case and which supports the propellant. Either we pour the slurry of the mixture may be a composite slurry consisting of the binder, the ammonium per chlorate and aluminum in into in this. Then we take it into a furnace and cure it at a temperature of between 70 and 200$^{o}$C depending on the composition. We form a block may be a composite solid propellant block which we call as a propellant grain. And when we cure the slurry or the mixture in it and make it as a solid inside a case the rocket or solid rocket is known as a case bonded rocket.

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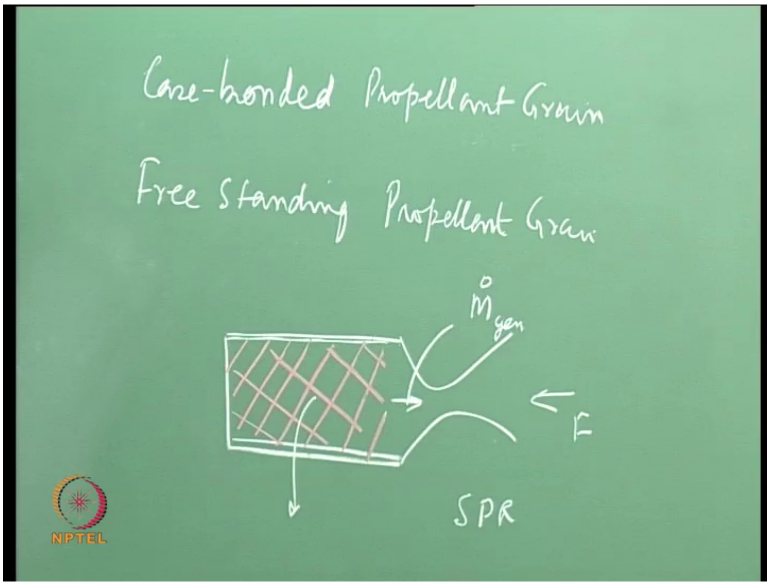

It is also possible to make the propellant in the form of a paste and extrude it out just like we squish out the tooth paste from its container. We could thereafter cure this squished out paste using in a furnace and enclose it in within a covering. Thereafter we could introduce it viz., charge it into the case. In this case, it is known as a free standing propellant grain. And what is free standing? The grain propellant inside it is freely standing it is known as a free standing propellant grain. It is known as a case bonded propellant grain when the propellant is cast in the case. Therefore, without any qualification, we have just introduced the word propellant grain. What does grain mean? The solid propellant block of a particular configuration in a case is what we call as a grain. This grain or propellant grain could either be directly cured in the case itself in which case it is known as case bonded.

And when it is cured outside and we machine it and then push it into the case the propellant grain is known as free-standing. We have both free standing and case bonded grains being used, but when we talk of large solid propellant rockets invariably we cast it in the case itself. But there are other things required.  As the propellant burns, the hardware or the metal case or whatever be the case will get heated and therefore it is necessary for us to protect the case with a liner. This liner could be an insulator on the inner walls of the case and then we put the propellant into it and then we attach a nozzle onto the case.

Therefore, we have a case, may be an insulator or something which will which will make sure that when something burns the hot gases generated by the solid propellant do not come and melt the motor case and then we have the nozzle and this what constitutes the solid propellant rocket. But what is it we want a rocket to do? We want the rocket essentially to generate some thrust.  The thrust of the rocket is equal to $m^\circ \times V_J$ or $m^\circ \times$ specific impulse Isp. We learnt how to calculate the specific impulse and supposing we want to generate a large force, we need to generate more mass flow from the burning solid propellant.

Therefore, in today’s class we will first examine how you design the solid propellant grain, which could either be case bonded or free standing. If we ignite it and start burning it, how do we ensure that we have sufficient amount of mass getting generated from the propellant. We have just studied the chemistry of the propellants, but we have not studied anything about how it will burn. We want to get some feel for how a propellant surface will burn. How do we calculate the mass generation rate and what should be the surface area such that we get the concerned thrust and this is what we will be discussing today.

Let us again look at the problem in a slightly different scenario. However, before that to repeat if we have a composite propellant like we have a polybutadiene which is a resin and the resin, which is in liquid form, is added with the solid crystals of AP and aluminum powder. The slurry so formed is a mixture of all these three constituents and we heat it to a high temperature to form a solid block. This solid block is covered with a liner of insulation and kept inside the case. And what is this resin? It is something, which when heated sets, and this resin is known as thermosetting resin. This is because when heated it becomes hard. Resin is a polymer polybutadiene; like glue.

I will show you some examples. But if you have plastic which is also a resin and if we heat it, then it softens therefore we cannot use such resins for composite propellants and the same thing is true when I use nitrocellulose and nitroglycerine or the insulation. We make it as a liquid, cure it into a solid and make it as a block and this block is known as a propellant grain. Therefore, we are interested in finding the burn rate; the rate at which it will burn or the rate at which mass gets generated from a grain. Is it clear? We must differentiate between a propellant block that is the solid propellant which is obtained after curing.

A propellant grain gives us the necessary m dot of gases generated for providing the thrust. And this propellant grain could either be directly bonded in a case or the hardware of a rocket or it could be prepared elsewhere in which case we just introduce the grain inside the case. The later is known as free standing while the former is known as case bonded grain.

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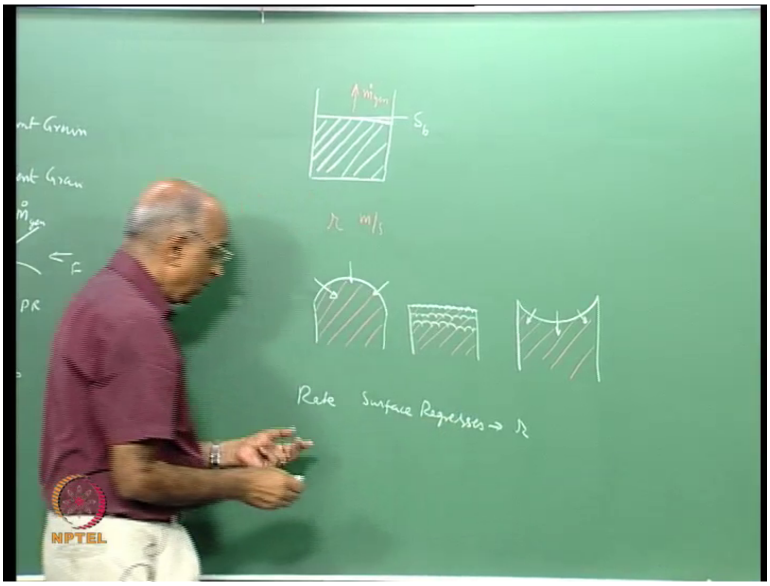

This is all about the terminologies. Let us get back and ask ourselves if we have a case over here in which we have the propellant block inside it and let us say we have no nozzle. We want to generate hot gases from propellant burning. May be this is the exposed surface area of the propellant. It is this surface, which burns and we call it as burning surface area and denote it as $S_b$. Supposing this surface is ignited. We would like to know the rate at which we are generating the hot gases from it. And if the surface burns we would like to have something like r so many meters per second is the rate at which a propellant burns. What is this rate?

Supposing, I have a surface, which is like this, let us say a convex surface and this is the propellant. We can take a flat surface of a propellant, which is the propellant surface here. Or the other extreme is we take a concave surface something like this and this is the propellant here. We could have different configurations of the surface. Or we could have hole right through. Let me consider these cases of flat, concave, convex surfaces of propellant burning.. At the next instant after burning commences, the flame advances by a given distance as shown.

In other words the burning progresses into the propellant. The rate at which the surface regresses is what we call as the burning rate. Rate at which surface regresses, but it must be normal to the surface and that is why we made these different configurations over here. The surface regresses in this direction over here; the surface regresses in this direction normal to the surface in this direction. In all cases the surface evolves normal to it and this constitutes the burning. Therefore, we call burning rate as normal to the surface.

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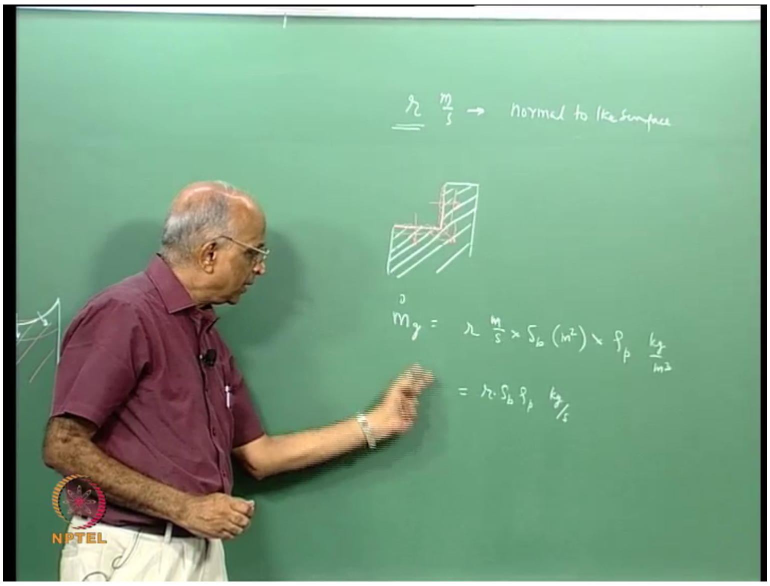

When we say a propellant burns at the rate r meters per second, we mean that the burning is normal to the surface. Let us spend some more time on this. Supposing we have a propellant let us say a block something like this at a corner. The shaded part is the propellant. We start the burning over the exposed surface. How will the burning take place? The burning will take place normal to this vertical surface along X axis. The burning will take place normal to this horizontal surface along the Y axis. And what happens to this contact point? Well! In other words over here since it is normal to this point it will go as a circle (quadrant of a circle) over here. This is how normal to the surfaces that the regression proceeds. May be we will have to look at this in some detail when we come to different configurations of the propellant grain.

Therefore, we define a burning rate in terms of meter per second, which is normal to the surface. The gas generated viz., rate of mass of gas which is generated by the burning in kg/s is equal to r, the burn rate so, many meters per second × the burning surface area so, many meter square; this gives us meter per second into meter square which is meter cube per second that is the volume rate at which the burning of the solid propellant. Knowing the volume rate at which the regression takes place we multiply it by the density of the propellant, which is so much kilogram per meter cube. We get meter cube and meter cube gets cancelled and we get kg/s. Hence $m^\circ = r \times$ burning surface area $\times \rho_{\rho p}$ (so, much kilograms per second).

The rate at which mass gets generated from a surface can be written in terms of the linear regression rate known as burn rate. In terms of a linear dimension into the burning surface area in this case the volume of burning.

The burning surface area is this surface initially and at the next instant of time may be it is over here this is the burning surface area and you have burning surface area into the burn rate into the propellant density as the rate at which mass of gas is generated. And burn rate is the rate of regression of the surface. Therefore, we say mass generated by burning is equal to the burn rate in meters per second ×surface area in meter square × propellant density in kilogram per meter cube. Let us illustrate it through an example, because we also you will recall we have been talking of black powder in the course on explosion also. We said that black powder consists of an oxidizer like $KNO_3$ and it also consisted of something like charcoal you say carbon and some sulfur.

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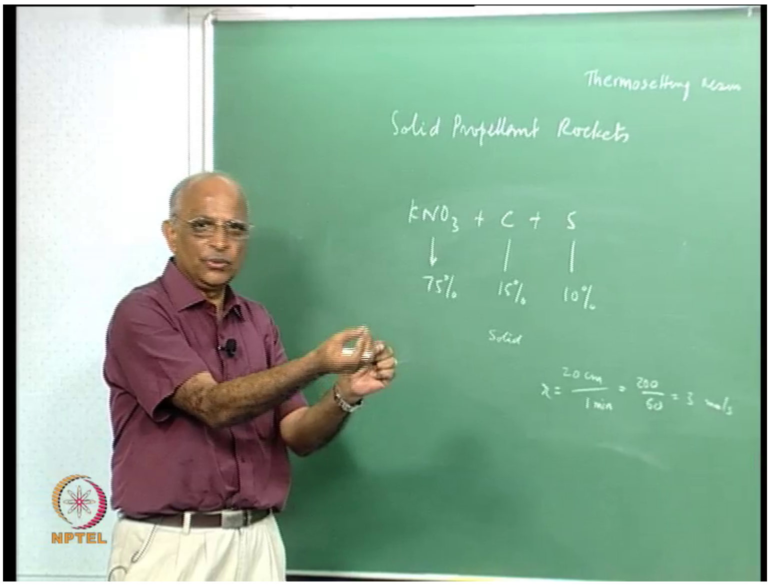

We had said that it consists of something like 75 percent potassium nitrate maybe 15 percent of carbon and sulfur is the remaining 10 percent. And how do you form these crackers or sparklers, which use this black powder. You have these solids this is an oxidizer and fuel you grind them together and make a good mixture of it may be add some glue so that becomes wet. You sort of cure it make it as a solid and that is what constitutes a composition. Well this is also a sort of composite propellant.

Therefore, let us take this example just to understand about burn rate. I brought a sparkler with me. We will see the burn rate of this composition. I will just ignite it I brought two sparklers, because I thought the effect of aluminum must also be clear to us. Let us just ignite it and see what happens? Well! This is ignition right. You light a candle. Well, this is a sparkler. It consists of as I said the composition, and the composition is again carbon, you have sulfur and you have $KNO_3$ and this it will burn by itself without the support of air. It is a propellant and instead of using polybutadiene it has resin that is the glue which is used.

Well, you ignite it at the surface tip. How does burning take place? This surface, which gets ignited and this ignited surface burns the next surface next adjacent to it and so on. We find that this surface is regressing at a particular rate. Well, in this case the whole surface is exposed, but because there is air over here the flame expanded slightly here. The burning surface area is perpendicular to this plane and the burning progresses axially. This is the case of this composition without metal in it. Let us see what is the effect when we have a metal in this sparkler, the other composition being about the same.

You see the distinct effect of the metal. The intensity is very much higher. See the metal having so much energy you find the heat, which is much higher. Metal has a higher heat of combustion and that is why metals are included in solid propellant. Again we are talking of the same surface regressing as it is going on and we find that the propellant regresses; i.e., it burns. And what is happening is gas is getting generated the rate at which the surface is regressing is the rate at which the composition is getting consumed.

The energetics is such that you know the sparks fall on me I feel little warm and is harmless but with metals in it I get burnt. Therefore, you find that metals have a definite role to play and I think this is all what I wanted to illustrate. We are talking of a burning rate, which is linear regression rate of the propellant. We will come back to our sparklers when we talk of combustion instability. We will see how even this can make a cavity unstable. But, when burnt a propellant regresses at a given rate and as it regresses mass gets generated. This is all about burning rate.

What will be the typical burning rate of the sparkler? We had a length something like almost twenty centimeters, it burnt over a period of one minute. Therefore, the mean burn rate r is equal to 200 millimeters divided by 60 seconds, which is about 3 mm per second or 3.3 mm per second. And this is how burning rate is calculated; you have a solid propellant strand; you ignite it and measure the burning rate. Therefore, with this background I think it is time to go ahead and examine the burn rate of let us say the double base propellants and may be the composite propellants. We will try to get into some details and we will try to make some sense of how the composition influences the burn rate.

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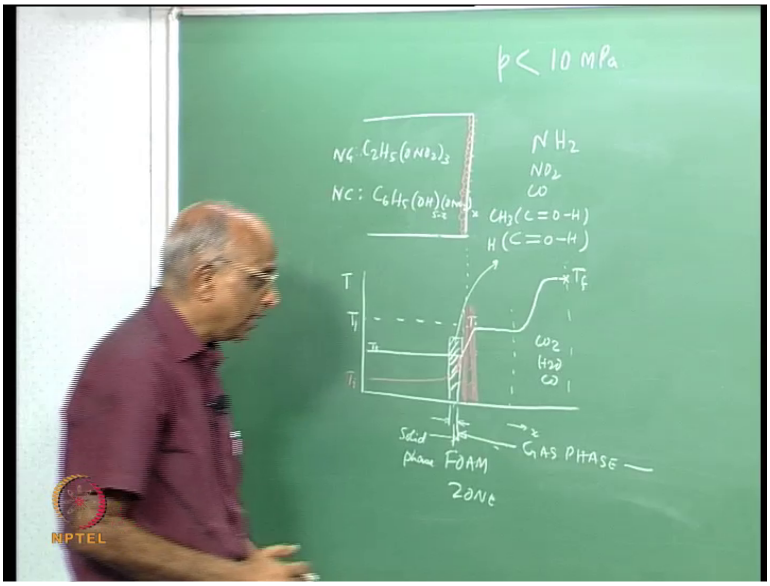

Let us say that we have a double based propellant here, and just the same way we ignited this sparkler we place a fire over here or a heat source over here start the burning. What was the composition of double base propellant? We told that it consist of nitroglycerine which is $C_3H_5(ONO_2)_3$ and nitrocellulose $C_6H_5(OH)$ and $ONO_2$. We had something like five of this OH in cellulose we put x of $ONO_2$ we removed x of OH and put $ONO_2$ there we had 5 minus x over here giving $C_6H_5(OH)_{5-x}(ONO_2)_x$.

Now, we ignite the double base propellant and what happens. The region very near to the surface gets heated, because all these propellants are insulators. Heat does not conduct deeply into it. And therefore, at the other end, if we consider this as the cordinate x, the temperature will be almost the ambient value. Only a small thickness of the propellant near the surface gets heated. And therefore, a plot of the temperature in the propellant with the temperature shown on the y axis and this is my distance along will be something like this. Let us show it in red; the depth of the propellant is really not heated. We have the temperature at the surface as Ts and this is the original ambient temperature.

Let us say that the initial temperature of the propellant Ti, all what we have done is just like we ignited the sparkler we ignite this particular surface of the propellant. And therefore, the surface temperature increases heat gets conducted over a certain thickness while at the end the temperature has the original value. These are insulators. We could have held the sparkler along the charge because of its poor thermal conductivity.

Now, what happens when we heat this particular zone to a higher temperature say Ts. This particular zone begins to foam as it were begins and begins to evolve some hot gases. And the gases, which are evolved are essentially aldehydes which are COH; could be formaldehyde could be acetaldehyde $CH_3COH$. We could get some CO, we could also get some NO. These are the gases, which are evolved. Because something is getting heated these gases tend to come out, and when the gases come out they react further; that means, I have a zone adjacent to the surface which is now at temperature Ts which forms hot gases and the temperature continues to increase in the gas region, in this particular zone.

Adjacent to this particular zone of propellant, which is getting heated which foams and forms hot gases; this zone is known as foam zone and is the solid zone, which is heated. And while it is getting heated it decomposes and generates these gases like aldehydes may be $NO_2$ may be NO may be CO. These gases are coming out and they react and increase the temperature from the surface temperature Ts to a higher value of temperature which we call as $T_1$, the temperature $T_1$. These gases do not completely burn and when the pressure over here is less than, let us say something like 10 MPa that is about 100 bar.

What happens is the gases are not at very high pressure and the process of intermediate products being formed continues without final products being formed. The gases relax for some time they begin to mix you know it takes some time for the gases to burn and therefore the temperatures remain at $T_1$ level for quite some time. That means, hot gases are generated, it reacts increases the temperature from the surface temperature Ts to $T_1$. The gases are still reacting and what are the reactions which could take place? The reactions, which could take place are CO which is formed reacting with aldehydes; even $NH_2$ is formed reacting with CO plus $NH_2$, but these are not very exothermic reactions. And therefore, the temperature remains fairly constant till the time final recombination reactions take place. This constant temperature zone is a precursor to the final reaction. After the reaction takes place, the final flame temperature $T_f$ is reached.

We have seen in the last few classes how to calculate the flame temperature. We can calculate this flame temperature $T_f$. But this flame temperature when the ambient pressure is less than about ten MPa goes in steps initially it goes to $T_1$, then it reacts, but the reactions are not very exothermic and thereafter it increases this in this zone.

Actually, $CO_2$, $H_2O$ which are complete combustion products are formed and you get the maximum temperature $T_f$. Of course, you have CO, because these are all fuel rich substances and therefore, you have something like a two step reaction process in the gas medium or gas phase.

This was the within the solid region; this was outside. We should extend the gas phase right up to the boundary over here, this is the gas phase and this is the solid. I will make it clear in the next diagram.

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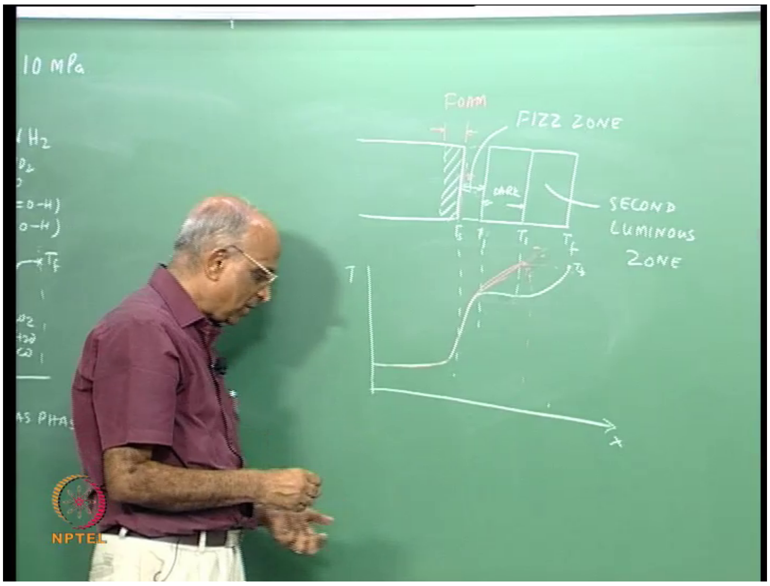

What is it we are talking? We consider a propellant solid block something like this; Initially the surface gets heated. And there after some chemical reactions are occurring in this zone and the surface temperature increases to Ts. In the gas phase, the temperature increases from the surface temperature Ts to $T_1$ over here. Then I have another zone over here in which no increase in temperature we said $T_1$ remains constant over a certain zone and there after the final chemical reactions take place it leads to temperature $T_f$. Let us plot the temperature distribution again over here as a function of x.

We said at the surface this is the temperature starts from the initial value in this zone it increases, then it increases still further then, we have a zone in which the temperature is more or less constant and then the temperature shoots to the $T_f$ over here. This is the way a reaction takes place and temperature evolves. This is all the gas part of the combustion. This is the solid part of it. This is the solid reaction zone; we already called it as foam zone. The zone in which the initial temperature increases in the gas phase is known as the fizz zone, because you are having some reactions zone it is known as fizz zone. This is a luminescent zone, because chemical reactions are happening just like we saw in this sparkler, some luminosity. This luminous zone is known as the hot zone or second luminous zone.

The fizz zone is followed by a dark zone. Why it is dark? There is no further increase in temperature. It looks dark at the low temperature. There after you have this zone which is the second luminous zone, because again high temperature are achieved giving a second luminous zone. This is how a double base propellant burns to give you the high temperature $T_f$. If we have very high pressures like exceeding 100 atmospheres that is 10 MPa, the chemical reaction continues in the dark zone or heat release continues here.

And I could have the temperature going straight away to $T_f$ at high values of pressures. That means, the dark zone will be absent when you have high pressure. We will have this luminous zone viz., the first luminous zone followed by the second luminous zone going to the temperature final temperature, which is $T_f$. That means, the temperature increases like this goes to the surface and then you have this zone and the temperature increases over here. This is at very high pressures whereas at moderate pressures up to 100 atmospheres well you have this dark zone and after which the temperature increases to the final value of $T_f$.

Well, this picture should be sufficient for us to be able to calculate the burn rate. I have a high temperature $T_1$ over here. The surface temperature is lower at Ts and heat gets conducted from the surface. It is this heat, which further heats this propellant surface and causes it to gasify and foam. Therefore, I am interested in events in the fizz zone wherein hot gases or heat from this zone gets conducted over to the surface.

What is the model or what is my mental picture with which I can solve for the burn rate? It is necessary to first consider the physical problem. Like we say in the case of the sparkler. We looked at how regression took place.

The heat is generated here. The temperature in the fizz zone is $T_1$ and the heat gets conducted from $T_1$ to the surface, which is at temperature Ts; and if we can calculate this heat, which is conducted then, we can write the equation at the surface and then get the burn rate. This is my mental picture of the problem. The mental picture might be wrong.

If my mental picture is wrong, the burn rate what I get would be wrong, But the picture seems to be reasonable and has been experimentally verified. But this thickness is what we are talking of. Even though I show it like this, the total thickness of the fizz zone followed by dark zone followed by the second luminous zone is something like fraction of a mm. That means, it is all over here just like what happened in the sparkler.

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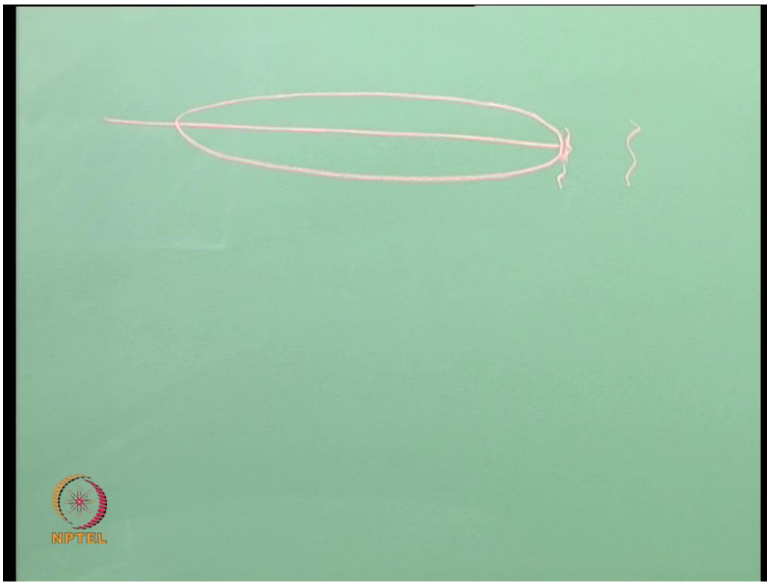

You had a rod. You coated the propellant over it. And what was this propellant? $KNO_3$ plus carbon plus sulfur all bonded together. We found the reaction distances are all within a fraction of a mm. We are talking of distances which are fraction of a millimeter thick.

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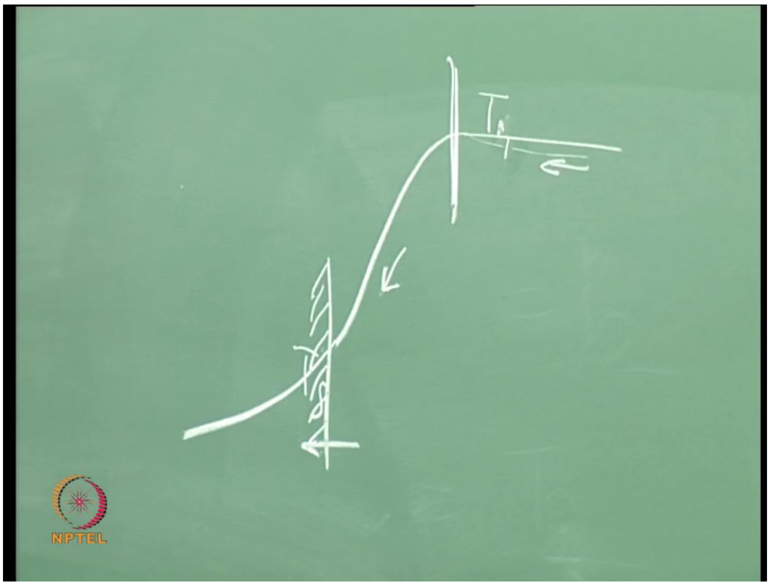

Therefore, let us write an equation for it. It is not difficult at all. We are going to be interested in the fizz zone, which is the zone wherein temperature increases form Ts to $T_1$. Why are we not considering the dark zone? See the temperature is constant here. If temperature is constant here nothing is going to come over here into the fizz zone from the hot zone. If we were say that the temperature at the edge of the fizz zone is $T_1$ and at the surface the temperature is Ts, we have heat conduction from $T_1$ to Ts coming here and still getting conducted in the solid.

We know that heat is getting conducted along in the fizz zone to the surface of the propellant; but in the dark zone since the temperature is constant no heat is getting conducted. Therefore, my condition here is $T_1$ over here, and the heat gets conducted from $T_1$ as the temperature falls to Ts at the surface. We would like to write the equation for the heat conduction.

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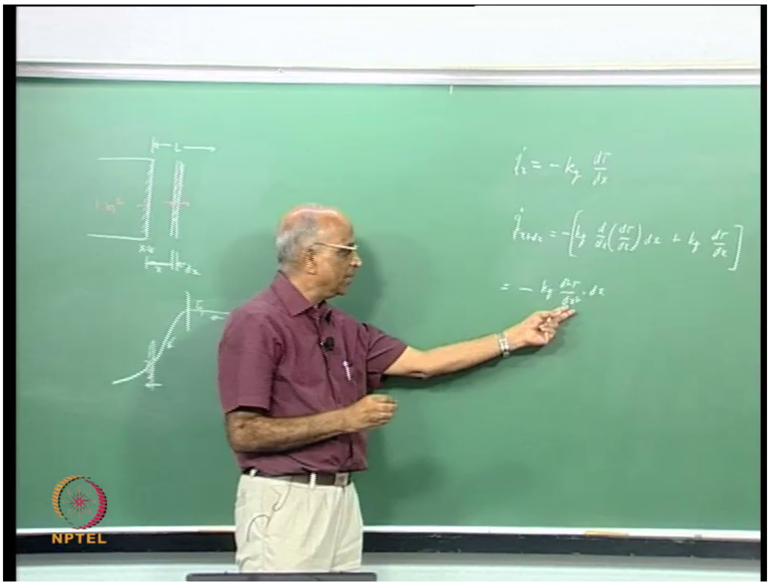

This is the mental picture or the model that we have constructed. Well, we are interested only in the fizz zone; let us forget about the dark zone and the second luminous zone. We now say that let the length of the fizz zone be capital L. We recognize that we are talking of dimensions which are fraction of a mm thick. Let us say we have the surface at x is equal to 0; this is the surface over here. We are interested in let us say a small element over here within the fizz zone at a distance x from the surface and of width dx.

We would like to write an equation for this element and then integrate it to find out how the burning progresses. And this is how we do any heat transfer problem. Therefore, we are interested in finding out for the surface area over here equal to unity - 1 meter square - such that I do not need to carry the value of surface area as I am doing the problem. Let us say there is some heat conduction q° here. What is the value of heat conduction? Rate of heat conduction at x; q° = − thermal conductivity of the gas (kg) × dT/dx by theFourier law. At x plus dx, the rate q° = the small increment over dx which is d/dx of q° + dx plus q°. The net heat which is leaving this is equal to minus kg× $d^2T/dx^2$ × dx and is the net difference between the heat which is entering and leaving. Heat which is entering is equal to −kg dT/dx; heat which is leaving is – [kg dT/dx + a small increment, which is d/dx into kg × dT/dx × dx. Therefore, the net accumulation of heat in a control volume of width dx is equal to minus kg $d^2T/dx^2$ × dx.

We should learn to write these equations at any time for any problem. We will again be coming to such formulations when wish to calculate how much ignition energy is required to ignite a propellant. When burning takes place at the surface, heat from the burning of gases supplies energy to the propellant. We will erase this part in the fizz zone and assume the rate of chemical reaction heat release in the reactions to be equal to $q^\circ_{chem}$. So much let us say joules per volume. This is joules per meter cube and is the rate at which heat is getting generated. And therefore, the amount of heat which is getting generated in this fizz volume is going to be $q^\circ_{chem}$ × the volume of this element. And the volume of this element is unit area into dx giving heat generated as $q^\circ_{chem} \times dx$. This is the rate at which heat is getting generated in this volume of thickness dx.

Is there anything else I have to consider in the heat balance relation? When the propellant burns, we considered the surface to be the frame of reference. With respect to this surface propellant, which is regressing or burning at a value r, hot gases are getting generated. The gases generated at the surface are moving out at a velocity $u_g$ and this $u_g$ will be different from r. This is because r is for a solid while $u_g$ is for a gas. Let the mean density of the gases be $\rho_g$. Therefore, the mass flow rate at which the gases are entering into this control volume is equal to $m^\circ_g$ per unit area. This is equal to $\rho_g$ into $u_g$ that means, I have so much kilogram per meter cube into meter per second therefore, I have so much kilogram per meter square second. This is the mass flux.

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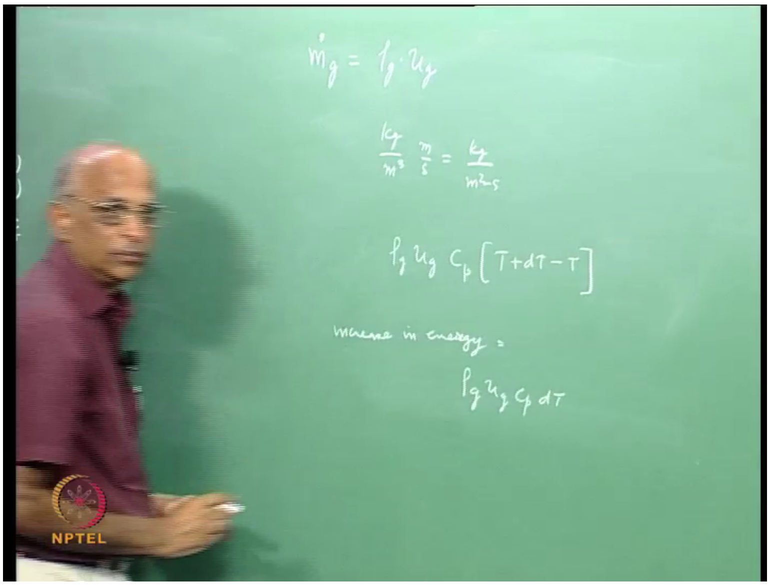

Since we are considering unit area, the mass rate at which gas is moving or the mass flux is equal to $\rho_g$ into $u_g$. And what is the enthalpy that is gained as it moves through the elemental volume.

The gas is moving at this particular x where temperature is T the temperature and at the next x+dx is equal to T + d T, because there is a small increase in temperature. And therefore, the increase in enthalpy of the gases is going to be $\rho_g \times u_g \times$into specific heat let us say at constant pressure into the temperature at exit which is T +dT minus temperature T at x. Because for this element at the left hand side the temperature is T it has increased in temperature over a small distance dx by dT to T plus dT. Therefore, the increase in energy or increase in enthalpy of the gases is equal to $\rho_g \times u_g \times Cp \times d\ T$.

Where does this energy come from? The energy comes from the chemical reactions which generate so much of heat so much joules per meter cube and and also some heat which is getting conducted.

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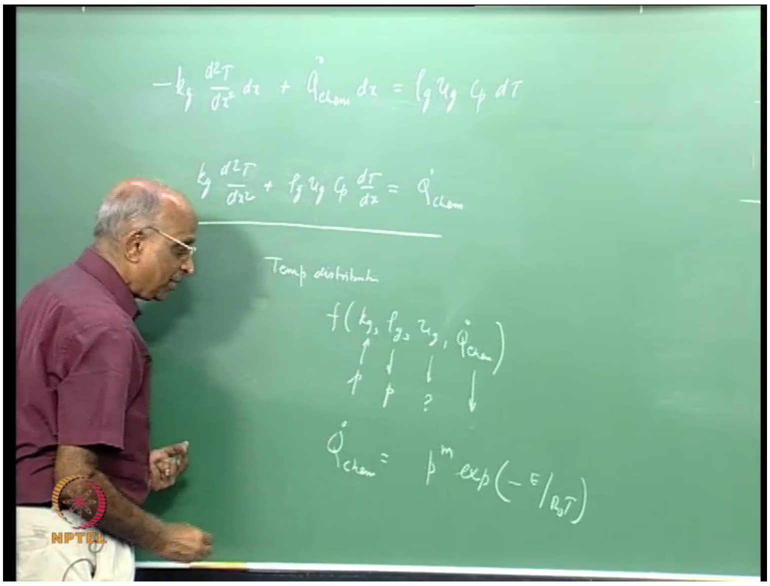

The enthalpy gained plus the heat transferred from the element must be the net heat that is generated. This is expressed by the the equation as $-k_g \times d^2T/dx^2 \times d\ x$, which is the heat transfer + heat which is getting generated $q^{\circ}_{chem} \times d\ x$, which is the net heat accumulation. And this heat supplies the enthalpy = $\rho_g \times u_g \times Cp \times d\ T$. This becomes the energy balance equation and we must be able to write such energy balance for any problem. We can write this equation as $k_g \times d^2T/dx^2$, we take it on the right hand side +

$\rho_g \times u_g \times Cp \times dT/dx.\ dx = q°_{chem}$, where $q°_{chem}$ is the energy release per unit volume in the fizz zone so much joules per meter cube.

Solving this equation, we get the temperature distribution, in the fizz zone. If we know the value of $\rho_g$ into $u_g$ and $q°_{chem}$, then only I can solve for temperature distribution. Once temperature distribution is known, I can find out the temperature at the surface Ts. We can also get the value of dT/dx at x = 0 viz., the surface. And then find out the rate of heat transfer. Therefore solution will give us the temperature distribution, but this temperature distribution is going to be a function of the value of the thermal conductivity of the gas in the fizz zone, the density in the fizz zone, $u_g$ in the fizz zone and the value of $q°_{chem}$.

We examine these quantities. The thermal conductivity of the gas kg should be a function of pressure of the gas. As the pressure increases the conductivity of the gas will go up, because we have more number of molecules in it. The density of the gas $\rho_g$ is again a function of pressure. $u_g$ has to be related it to the burn rate at the surface and if we know it we will get the value of the burn rate r. For a given $u_g$ we can solve this problem for a given $q°_{chem}$. But what is this rate of heat release? The rate of heat release $q°_{che}$ in the gas depends on the pressure $p^m \times e^{-E/(R0 \times T)}$ where E is the activation energy, $R_0$ is the universal gas constant and T is the temperature. We have a steric factor depending on the molecules here.

We therefore find that $q°_{chem}$ is a strong function of pressure., $\rho_g$ is a function of pressure. The thermal conductivity is also a function of pressure. Therefore, we can say that $u_g$, that we wanted to find out should also be a function of pressure.

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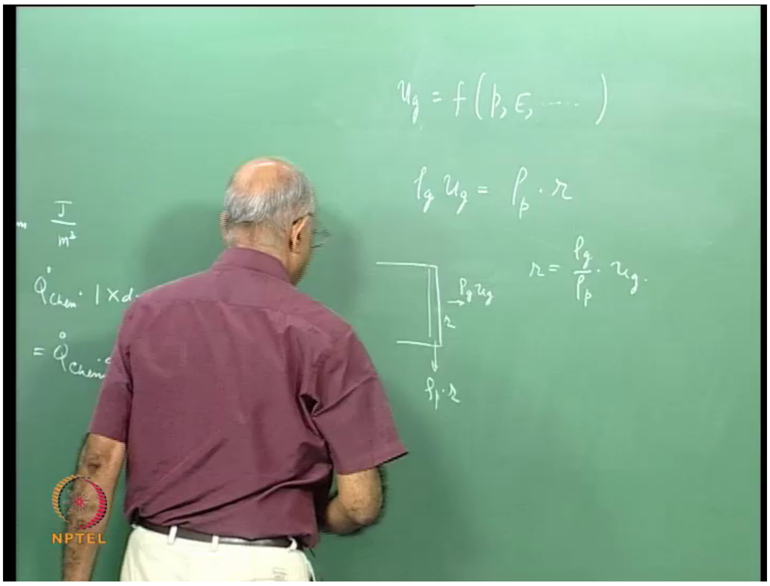

In addition to pressure, we find that $u_g$ should be a function of activation energy and this activation energy should be a function of the composition. May be the density could be other parameters which are there. But what we get is that the temperature distribution and the velocity of the gases $u_g$ would be a function of pressure.

What is the value of $u_g$? The gas velocity $u_g$ in the fizz zone of density rho g equals the mass, which is moving out and must be equal to the density of propellant into the linera burning rate of the propellant.

We have a solid, which regresses at a rate r and which the propellant burn rate r. The mass which is getting generated is equal to $\rho_p \times r$, because we have unit surface area and the rate at which gas is getting generated is equal to $\rho_g \times u_g$ per unit surface area. Therefore, the value of the burn rate $r = (\rho_g / \rho_p) \times u_g$.

But $u_g$ is seen to be a strong function of pressure and therefore the burn rate is going to be again a strong function of pressure and the effect of the other constituents like the composition, activation energy etc.. And therefore, it is possible for us to write the burning rate r in let us say meters per second as equal to a constant × pressure to the power n, where this constant will take care of activation energy, the composition and other features.. It is possible for us to write the burn rate for the double base propellant r = constant × pressure$^n$.

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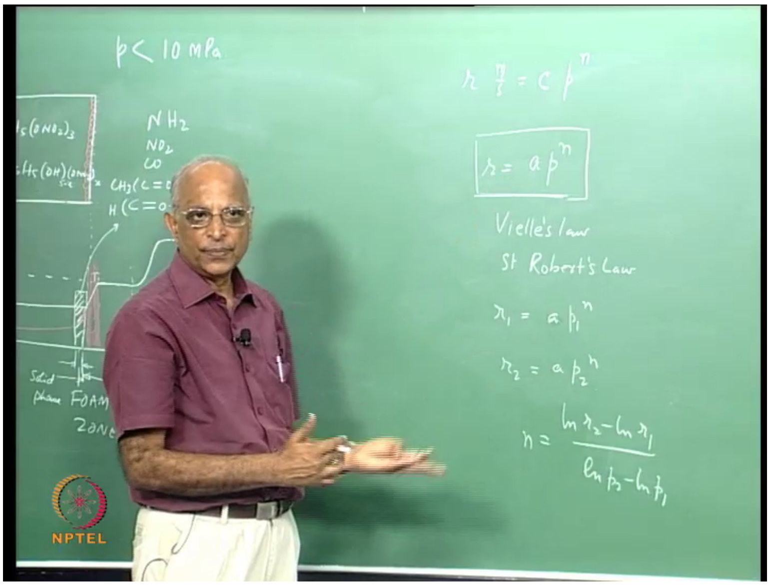

And this form of burn rate is what is known as Vielle's law. It is also known as Saint Robert's law and is $r = a p^n$.

Therefore, what is it we have done so far? We said that the surface regression rate of a double base propellant could be written in terms of the ambient pressure p at which it burns in the form r is equal to a $p^n$ a will have the compositional features may be may be the initial temperature may be the final temperature. Maybe we should have written here $u_g$ is going to be a function of the initial temperature, may be the temperature $T_1$ and so on.

Therefore, we put all the effects other than pressure into this a factor 'a', that is the pre exponent and I have the burn rate law during $r = a p^n$. It becomes much easier to use a law like this rather than keep trying to solve this equation which is a little bit more difficult. And in practice we use this law. How do we determine the value of n? We do an experiment at pressure $p_1$ and measure the burn rate $r_1$ which can be written as a $p_1^n$ . We then measure the burn rate $r_2$ at a pressure which is little bit away at $p_2$. We get $r_2$ as equal to a $p_2^n$. We use these two equations to find out the value of n. We get $n = (\ln r_2 - \ln r_1) / (\ln p_2 - \ln p_1)$.

Once we get the value of n, we determine the value of a and this is how we determine the burn rate law as $r = a p^n$. Just like the sparkler experiment, may be at an ambient pressure, we put a strand of propellant in a vessel and do the experiment. We measure the time taken to regress a particular distance, measure the burn rate, then determine the value of n and a.

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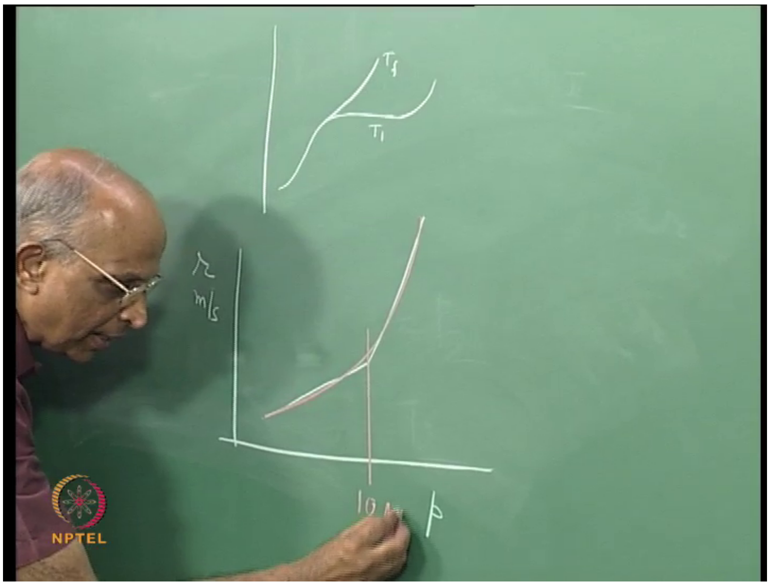

If we have the ambient pressure in excess of let us say 10 MPa, then we had said that there is no such thing as a intermediate $T_1$ zone. The temperature directly goes to something like $T_f$ over here and more heat is conducted. And therefore, if we have to plot the burn rate r let us say meters per second, as a function of pressure at low pressures we have n which is small like this: when pressure goes up it also increases like this exponentially as shown in this plot.

This is how the burn rate varies with pressure when the pressure corresponds to within where the dark zone prevails. However, when the dark zone is absent at higher values of pressure, something greater than about 10 MPa, the heat transfer increase rapidly and the burn rate increases much faster. A plot of burn rate as a function of pressure would steepen as shown. But when we add additives to the propellant, then the dependence changes.

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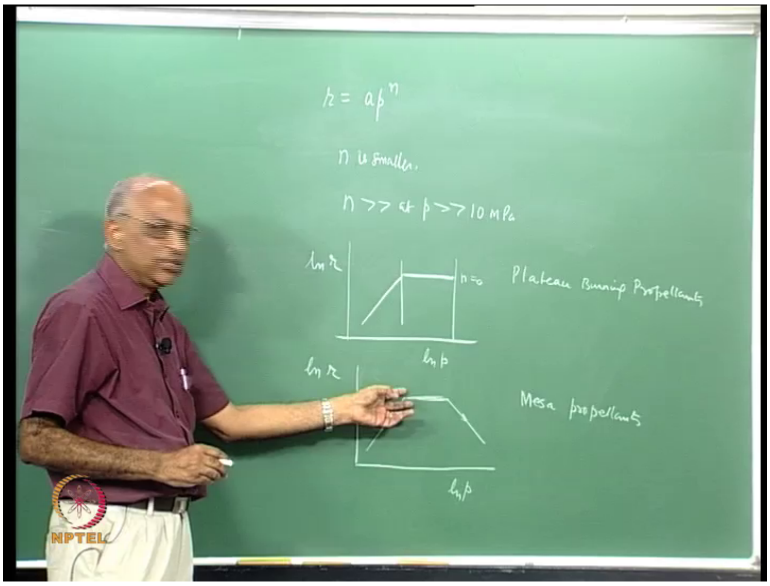

Let me again repeat this point. We found that the burn rate of a double base propellant can be written as r is equal to a $p^n$, where a is a factor which considers all factors other than pressure like temperature of the gases $T_1$, the surface temperature Ts, the activation energy etc.. And pressure is directly taken to a power n. Then we said that at low pressure the value of n is smaller while at high pressures for which there is no dark zone and you have higher temperatures driving the heat conduction the value of n becomes greater. n is larger when p is greater than about 10 MPa.

But we could also add some salts, like lead salts to the double base propellant. Then what happens is instead of getting a curve like the one earlier, we get a different pattern. As a function of pressure, the burn rate initially picks up and there after it does not change with pressure that means, you have a plateau over here. In other words, we have burn rate increasing up to a pressure and it becomes a plateau. Such type of propellants which exhibit this feature of n = 0 over a certain range of pressures are known as plateau burning propellants.

Well, some propellants have even a slightly different behavior and some may have their burn rates going down. We have r versus p. In fact, we should have written ln r versus ln p over here and that is when I would have a straight line. So, this should have been ln r and ln p. Sometimes the burn rate begins to fall at higher pressures. It all depends on the chemical constituent in the propellant and such propellants which exhibit a drop in burning rate with increase in pressure are known as mesa burning propellants.

Therefore, what is it we did in this class today? We looked at the rate at which mass gets generated from a surface. We found out that the surface area is important - Sb is important. Then we found for the double base propellant we have two or three zones during their burning: like a foam zone followed by a fizz zone in which temperature increases to some intermediate T1, then a dark zone in which the temperature is more or less constant followed by a zone in which temperature further increases.

We looked at the events happening at the at the fizz zone and not in the foam zone. We wrote an equation and we were able to say that the velocity of the gases in this zone are dependent on pressure and the initial temperature may be the $T_1$, may be the surface temperature Ts, activation energy E. And since the velocity of gases is directly proportional to the burn rate r of the propellant, we said that the law is $r = a\ p^n$ which is known as Saint Robert's law or Vielle law.

Thereafter we found that since we have the reactions happening directly at high pressures; that means, a dark zone is not present at high pressures and therefore at high pressure the temperature increases from surface temperature directly to the flame temperature and more heat gets conducted. Therefore, n is higher at the higher pressures. Therefore you have small n followed by a large n for the double base propellants. If we add some salts and alter the mode of burning we could have something like a plateau burning propellant or a mesa burning propellant.

In the next class, we will take a look at composite propellants, and then put the whole thing of burn rate together and then address the design of a solid propellant rocket.