Rocket Propulsion

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Lecture No. # 14
Unconventional Nozzles and Problems in Nozzles

Good morning. In the last class, we discussed about contour nozzles. The shape of the contour nozzle is in the form of a bell. Initially, you expand the flow by a large angle and you compress the flow later on, such that you get a very small value of divergence angle of let say 2 to 5 degrees at the nozzle divergent. Initially, you expand the flow at a larger angle say between 20 to 50 degrees and this shape of this contour is something like a parabola. You can fit with a second order parabolic equation for the shape or contour of the bell nozzle.

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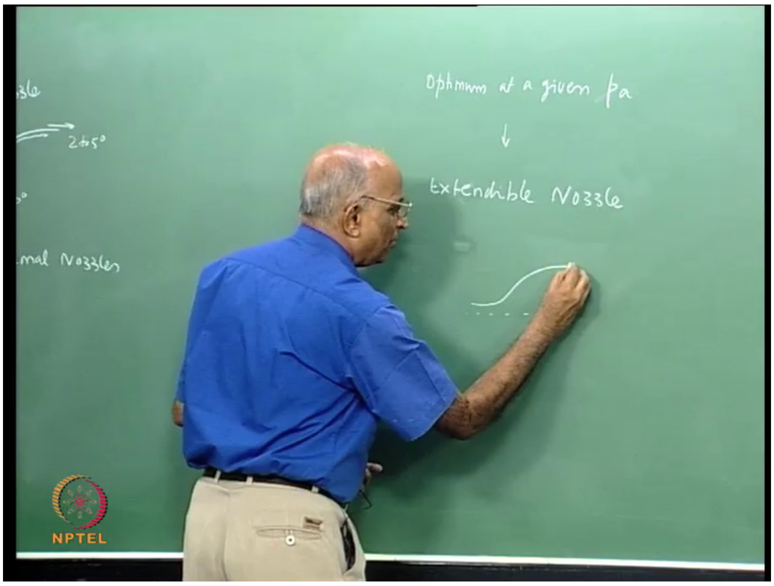

This is how a contour nozzle looks like. In today's class, let us look at some unconventional nozzles and examine whether there are nozzles other than conical nozzle and contour nozzle.

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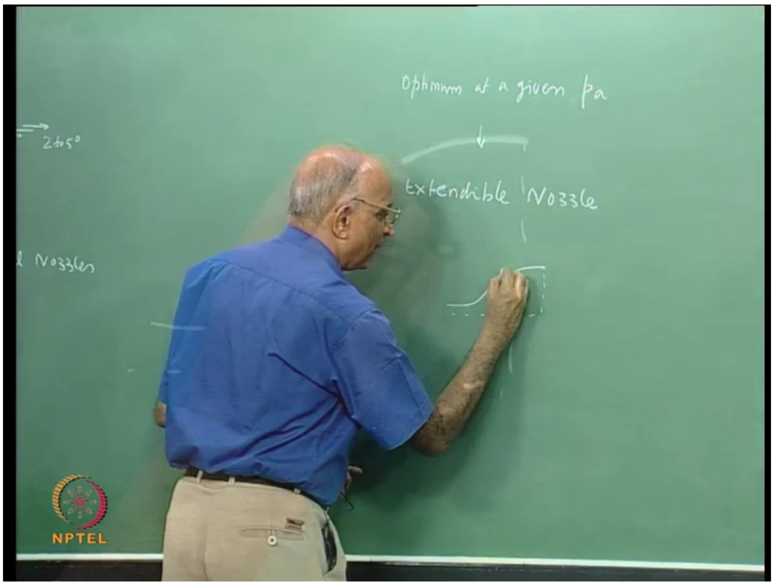

Also, we would like to note that the changes from conventional conical and contour nozzles must be such that they must give better performance. We keep this in our mind. Well, a nozzle operates at its optimum when the value of pressure at its exit is equal to the value of the ambient pressure. In other words, if we want to make a nozzle, we design it for operation, let say at 15 kilometres. But the rocket operates between 0 km and 30 km. We start operating the rocket on ground i.e., from 0 kilometres and the rocket goes up to 30 kilometres. During the initial stages of its operation, viz., between 0 km and 15 km, it is not optimum as the exit nozzle pressure would be less than the ambient value. At 15km, it is optimum and beyond 15 km again the nozzle exit pressure is greater than the ambient. It is under expanded and not optimum. We start off with the nozzle in an over-expanded mode which after the design point operates in an under-expanded mode. Therefore, can we have say a nozzle in which we can have an extendable nozzle with varying area ratios in which the exit pressure is matched to the ambient pressure as the rocket moves up?

Let me give you an example. Suppose, we have a nozzle and this nozzle is operating at lower altitudes or higher at values of ambient pressures. Now, we want to make it optimum and therefore we have to shorten it with a lower value of the exit pressure at the nozzle exit. May be we have to expand it out to larger values of area ratios when the rocket operates at higher altitudes. Therefore, we initially have a nozzle something like this. we extend the last part over here and then lock it. The area ratio has now increased and the nozzle has become longer. This initial lower area nozzle operates at low altitude while the larger area ratio nozzle operates at higher altitude.

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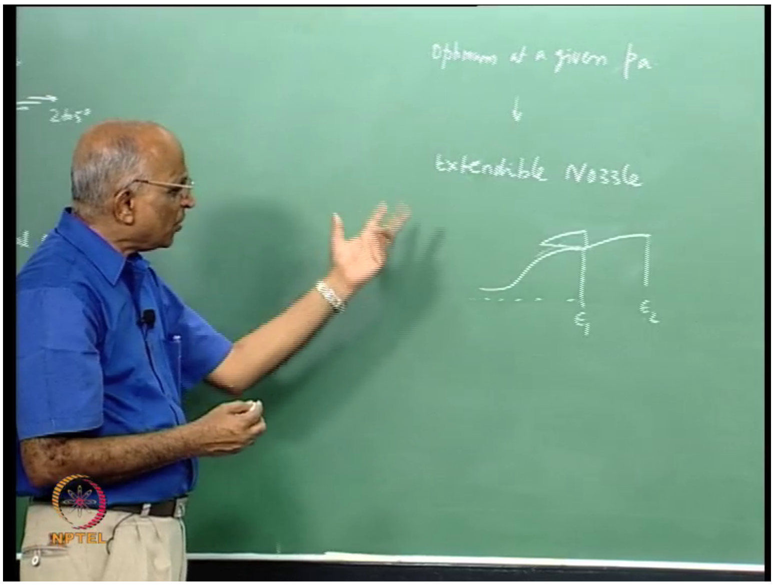

When the rocket reaches higher altitude, we shift the second half of the nozzle divergent through some mechanism. We shift this over here and thus increase the area ratio of the nozzle. That means, this becomes our initial low area ratio nozzle and correspondingly we have a higher area ration nozzle. That means, we extend the nozzle to provide higher area ratios during the flight. At the lower altitude, we use the smaller area ratio. During this time we keep the larger area ratio segment on top of the lower area ratio portion. When the rocket reaches a particular altitude, we push it out. The nozzle length increases and the area ratio increases and this is what we call as an extendible nozzle. This was tried in a flight. It is not used in practice even though it has been tried. We call it as extendible nozzle. We could have several segments coalescing one on top of the other at lower altitudes and being pushed up as required by a mechanism in an extendible nozzle.

If, instead of having an extendable nozzle, are there other alternatives? One such alternative is a double bell nozzle or something like a dual bell nozzle.

In this we have a nozzle like this and I want to increase the area ratio still further. What we do is that we put something like a step at the exit of the lower area ratio and then we continue the nozzle profile like this. Now, the centre line of the nozzle is the same. Both the portions are permanently in place. Now, what is going to happen? At lower altitudes, the flow expands to the ambient pressure and flow separates at the step and flows over. At higher altitude, because the pressure is very much higher than the ambient pressure, the flow reattaches at the junction or step and flows into the second part of the bell. Therefore, we can get area ratio ε_1 and area ratio ε_2 corresponding to the lower and higher altitude of operation respectively. This is known as a dual bell nozzle. In fact, this month's issue of AIAA journal has a paper on this dual bell nozzle looking at the optimum conditions. That means, work is still pursued with the dual bell nozzles and it may have some promise.

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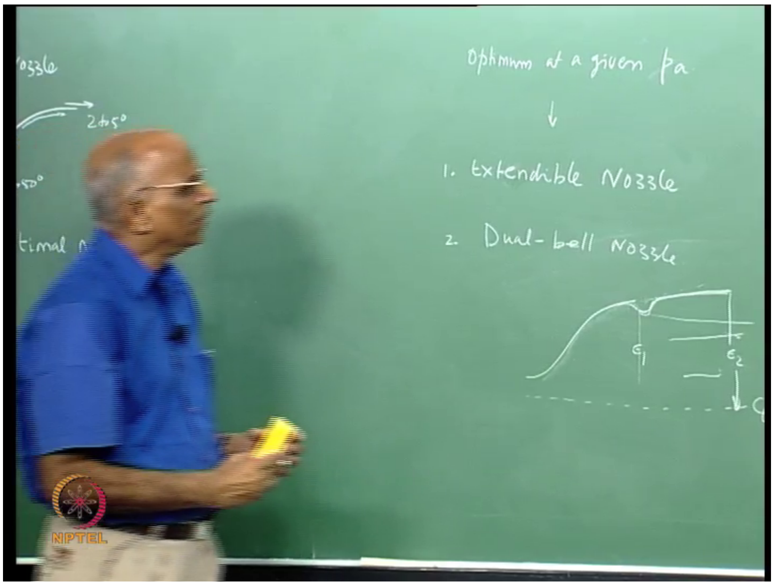

Therefore, this is the second unconventional nozzle. First is the extendable nozzle and second is a dual bell nozzle. The third is something like a radial flow nozzle. By the radial flow nozzle; what do we mean? Something an expansion deflection nozzle. Let us sketch a throat followed by a divergent. Let us say, this the convergent and we have a divergent following the throat. This is the centre line. At the throat, put a block centrally making the flow in the throat to be annular; something like a plug in the throat. We allow the flow to take place in the annular space between the blockage and the throat and what happens? The flow is guided by the contour wall in the divergent part. At the centre, it is not guided. Therefore, we have something like an expansion wave. By this centre expansion, which is avalable, the nozzle is able to adapt to different altitudes.

Therefore, by putting this centre blockage, I can make this particular nozzle operate at different altitudes. In other words, I have the outer wall, which guides the flow. Inner part is free. It can adapt different altitudes and therefore, this is known as an Expanded Deflection (ED) nozzle or expanded deflection nozzle. What we have to do is that we want the nozzle to operate at different altitudes.

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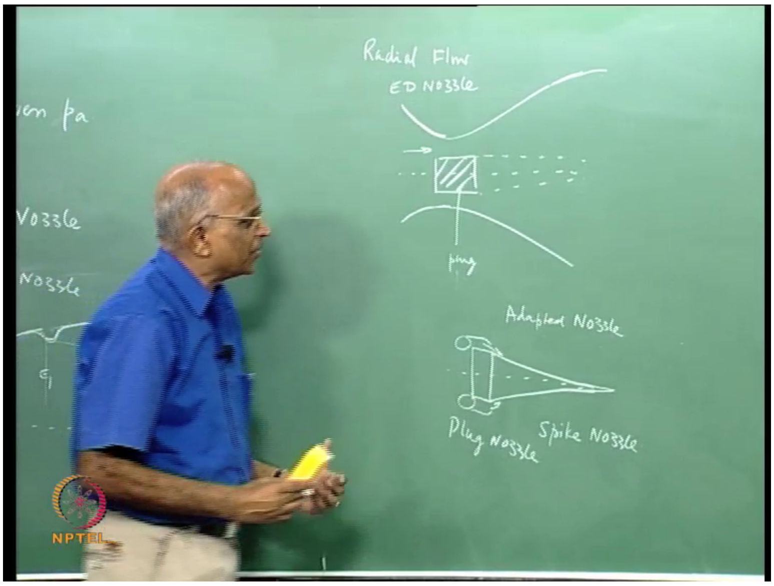

Therefore, what we have done is, we have introduced a plug in the throat region and we allow the contour to change the pressure of the outer flow. But, the inner region of the flow is keept free to expand. This is the principle of the expansion deflection nozzle.

As an extension to this type of the plug nozzle, we could also have a different type of plug nozzles. Instead of having plug here and allowing the outer divergent contour to guide the flow in a nozzle, we could have a nozzle and I can have this plug in the form of a contour over here. Centre contour along the plug as shown. What is it that we do? We guide the flow over here, from the chamber. In other words, I have a annular chamber which leads to the annular throat over here and we allow the shaped plug to guide the flow. We allow the flow to come along the plug. We allow the inner contour instead of the outer contour from the divergent to balance the flow. Keep the outer open such that the flow is free to expand. This is again a case of an adapted nozzle.

The shaped plug at the annular throat can adapt to any ambient pressure condition. I could have the contour of the plug in the form of a spike, in which case, we call it as a spike nozzle or simply we call it as a plug nozzle. I could still have some more variations.

This particular plug which is at the centre: we could terminate it a little earlier instead of ending up at a point. We do not allow the total spike. In other words, we have a primary flow along this and I have the shock waves here and the secondary flow here. I have recirculation and a base pressure here and this becomes what we call as the aero-spike nozzle. What we are saying is that the combustion is happening over here; in an annular combustion chamber. We push the flow along the spike or contour and we allow the spike to expand the flow along the contour. The free expansion in the outer portion adapts the flow to the ambient pressure.

Instead of having an outer boundary which regulates the pressure, we have an inner boundary which corresponds to a spike, which we call as a plug nozzle or a spike nozzle. Of course, this is a plug and the same plug which we used in the case of a contour nozzle; Here, the outer is free such that the nozzle can adapt to different ambient pressures of operation. This is therefore the case of an adapted nozzle.

Aero-spike again: we would have additional thrust coming from the base of the plug if truncated as shown and which we call as a Aero-spike. But, why should we always think in terms of a cylinder or a bell or something like that. Why not open out the bell. Make it something like linear or planar. If we do not have a cylinder, but have an opened out cylinder we call it as a linear nozzle. What do we mean by a linear nozzle? Well, we open out the nozzle something like a two dimensaional sheet and we have the surface such as a ramp in the shape of a contour. Now, we allow the flow over this contour surface, this ramp as it were, and we use the ramp to expand the flow to the ambient pressure.

In other words, I have a two dimensional surface as shown. Let say, over here, it could be a shape of something like an aircraft wing. Flow comes along this, guides along the surface and comes out. The shaped surface is not confined and it could as well be a part of an aeroplane, like let say a wing or a fuselage.

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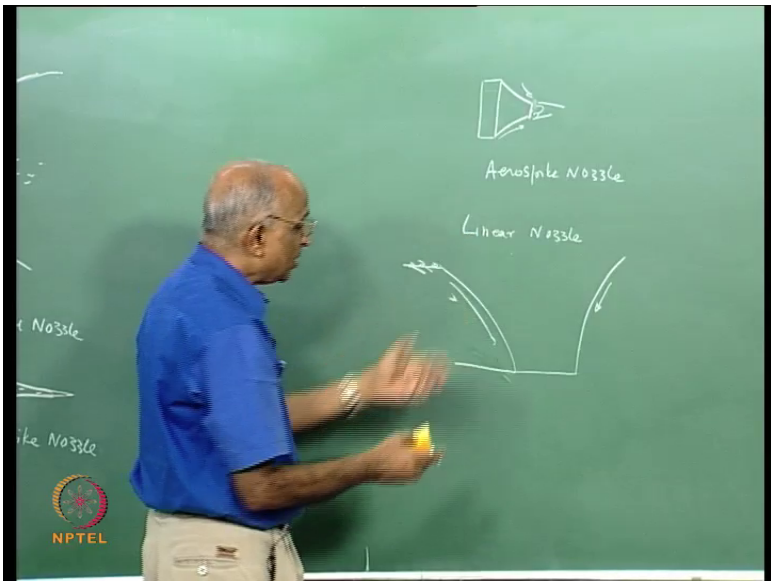

We allow the gas to come along it and flow along the surface. It is known as linear nozzle. This has been used for space plane. These follow the same principle what we have discussed on nozzles so far.

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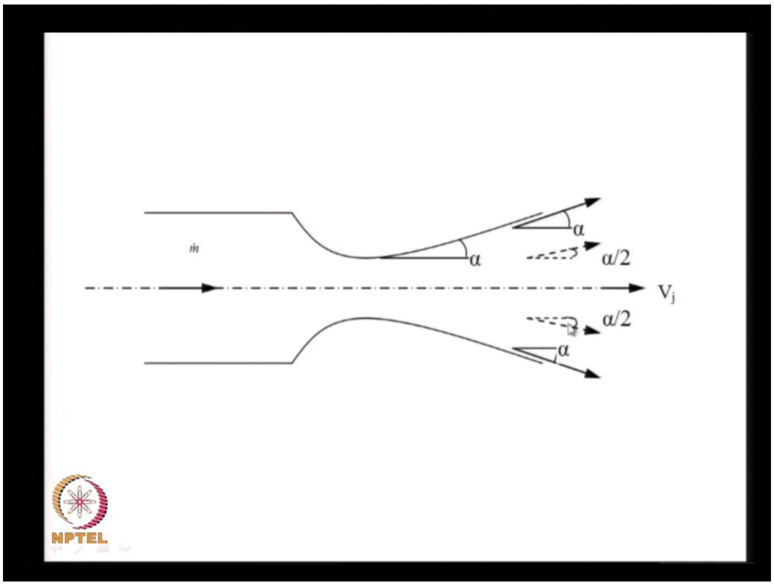

We now summarize what we have learnt in nozzles through a series of slides. In the first slide, we look at the divergence losses, may be, alpha and how we got the divergence losses coefficient.

This was the value of Δ, which we decided as percentage loss in thrust versus the angle α.

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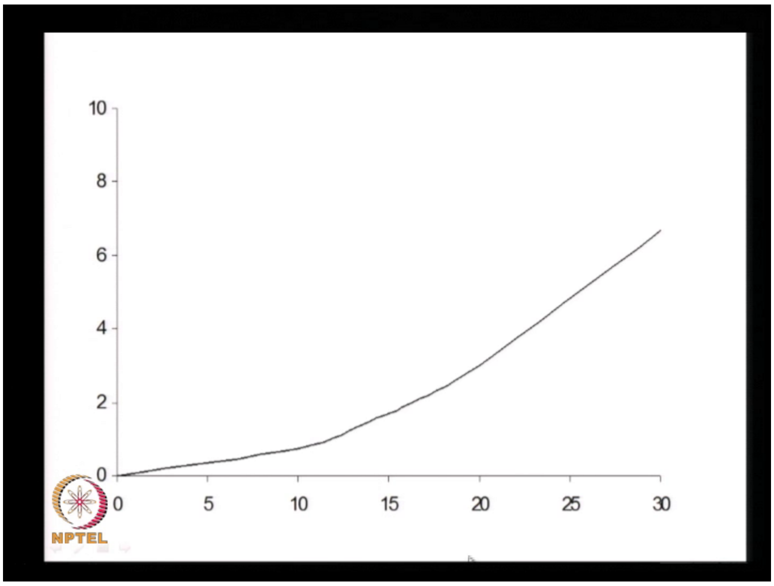

We derived this and we had said that the nozzle half angle is around 15 degrees or so for a conical nozzle. We talked in terms of a contour nozzle instead of having a conical nozzle wherein we initially expand the flow to α_i before bringing it back to α_e.

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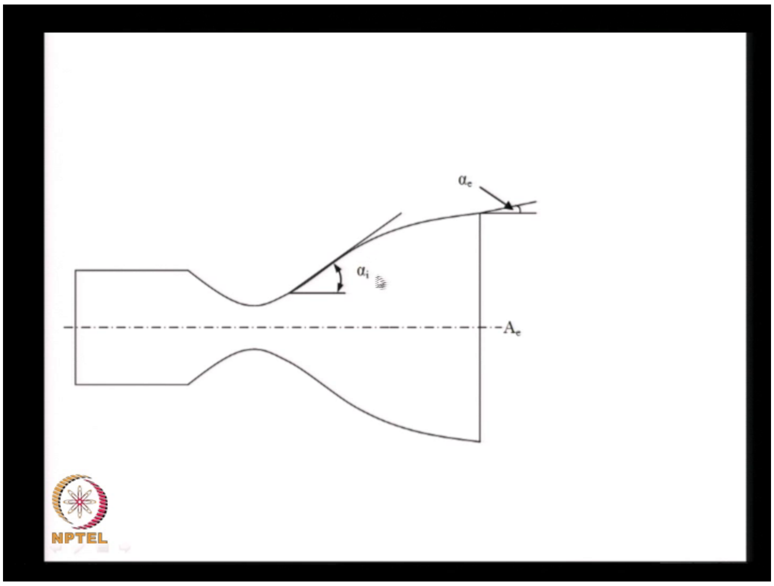

The value of α_i is between 20 to 50 degrees and then, bring it back. We have a low angle over here at the exit of something like 2 to 5 degrees or so. Smaller the angle at the exit, smaller is the loss.

This is the extendable segment.

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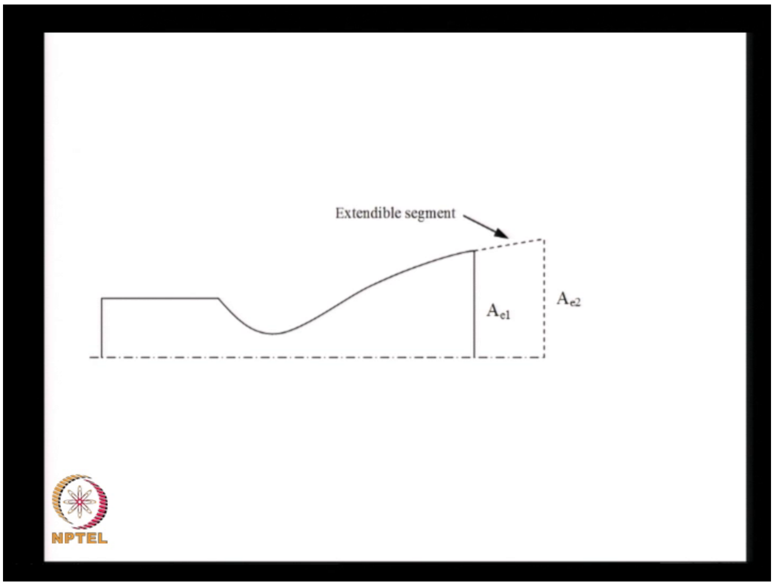

Initially, we have a small area ratio corresponding to exit area Ae_1.We store another segment on top of this a segment between area ratio Ae_2 and Ae_1 and at higher altitude deploy or push itto get the larger area ratio nozzle.

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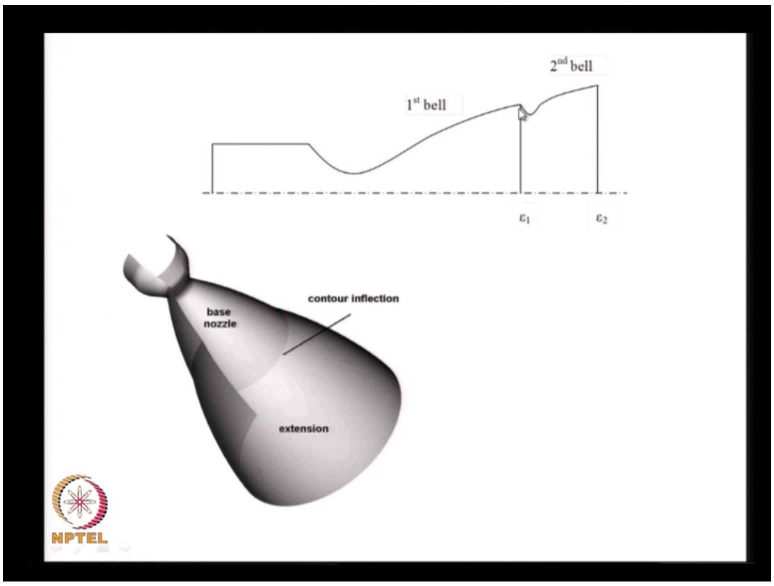

This is a dual bell nozzle. As you see along the divergent contour we have a step at which for higher ambient pressure the flow separates but at lower ambient pressures follows the contour of the second part of the nozzle. As I told you, there was a research paper in the AIAA Journal this month in which dealt with the dual bell nozzle.

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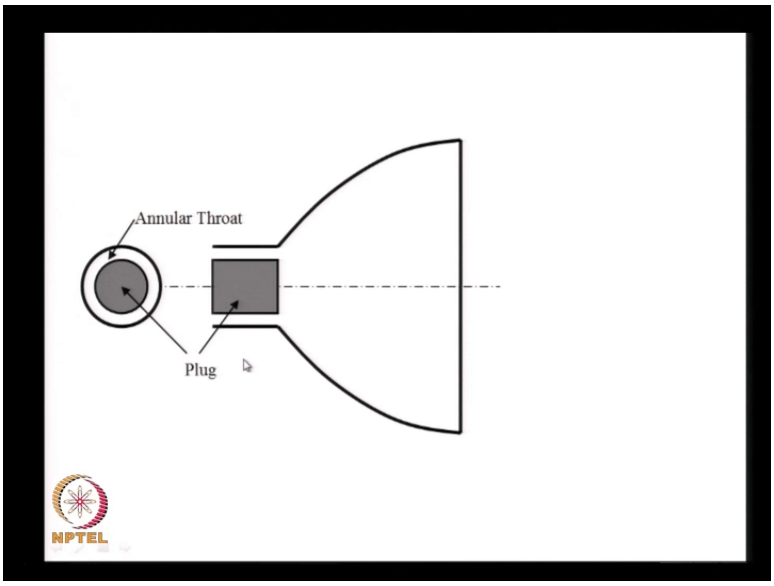

Those who are interested should go through it. This is a plug nozzle. I put a plug in the throat to make an annular throat. We have an outer surface, which guides the flow. The inner surface is free therefore, it can adapt to the ambient pressure. What you have is an annular throat, instead of having a cylindrical throat.

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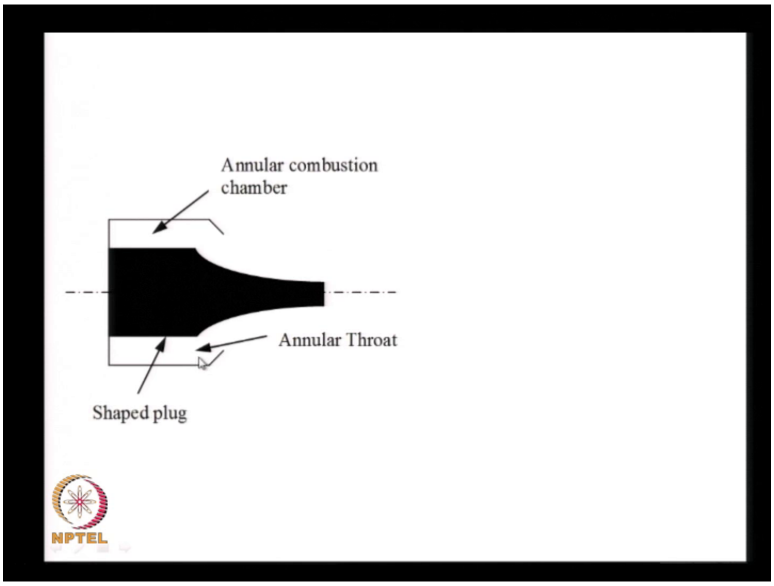

This is what we called as a plug nozzle or a spike nozzle. We have a spike following the annular throat. The flow comes from the annular combustion chamber. The hot gas is generated in an annular chamber instead of a cylindrical chamber. We push the flow onto this inner contour surface and this surface guides the flow. Outer surface is free; therefore, the expansion can adapt to the altitude. It is not used in practice. This summarizes what we learnt about nozzles.

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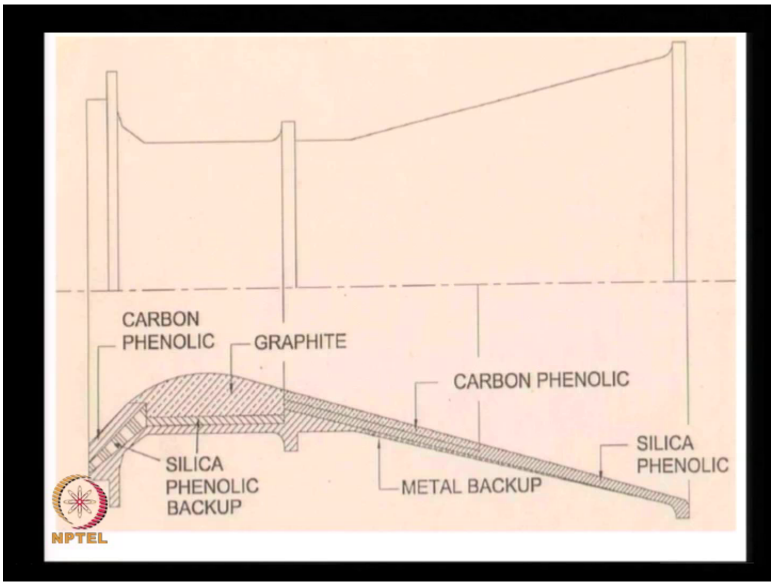

A nozzle runs hot as the hot gases in it are at high temperatures. Therefore, to protect the nozzle, we provide insulation on the inner surface. This is the conical nozzle. I give something like a carbon phenolic composite material as an insulation, which can withstand a high temperature The composite materials such as carbon phenolic are known as ablative materials. I will come back to it, when we deal with cooling of rockets. I will get back to this slide a little later in the course. But, this is how the construction of a nozzle looks like. This is the outer surface and this is the inner wall of the nozzle.

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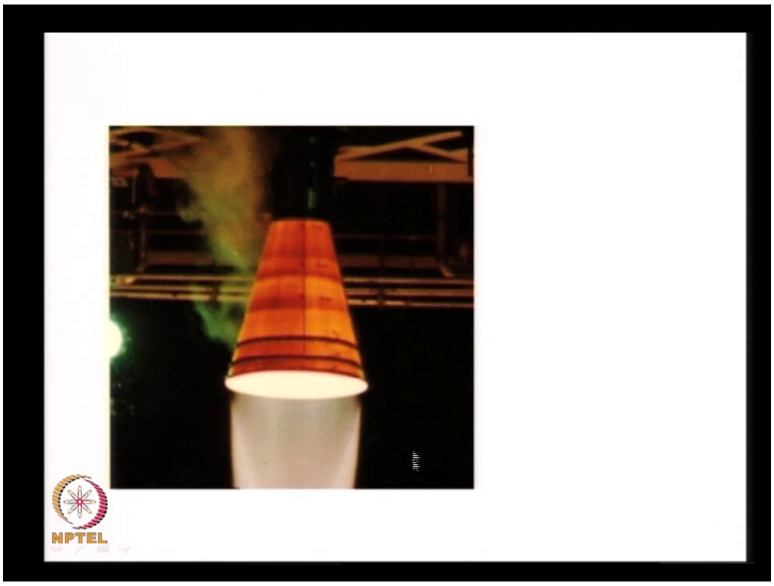

I repeat the case of a hot nozzle in this slide. A nozzle is firing for a certain amount of time. This is the conical nozzle. You see, that the nozzle runs red hot.

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Well, with that we close the portion on nozzles. But, it will be useful to do a couple of problems on nozzles. One of the problems which we do is related to this particular rocket shown in this slide. This rocket is known as Saturn 5. Saturn 5 rocket was used to put first men on the moon. We call it as Saturn 5 launch vehicle, which puts the Apollo capsule carrying three men on the moon. This is, by far the biggest rocket ever made in the history of rockets. It is the most powerful rocket and what does it consist of?

The first stage of the rocket, in the lower portion, consists of five rockets clustered together. Each one of these rockets is known as F1 rocket. It consists of five F 1 rockets clustered together. These rockets use liquid kerosene and liquid oxygen as propellants. Kerosene as fuel and liquid oxygen as oxidizer. The second stage consists of five rockets again. It is known as J 2 rocket. we will get back into the details of this later on while studying liquid propellant rockets. 5 J 2 rockets clustered together for the second stage. They use liquid hydrogen and liquid oxygen. The third stage consists of one single J 2 rocket. Therefore, what is it we are talking of? The Saturn 5 rocket consists of the first stage, which consists of five rockets and these are 5 F 1 rockets clustered together. The second stage similarly, consists of a cluster of 5 J 2 rockets. J 2 rocket uses liquid oxygen and liquid hydrogen as fuel. This first stage uses kerosene and liquid oxygen On the third stage, you have a single J 2 rocket and on top of this sit the particular capsule, which is the Apollo capsule, where the three astronauts who travel to the moon are housed.

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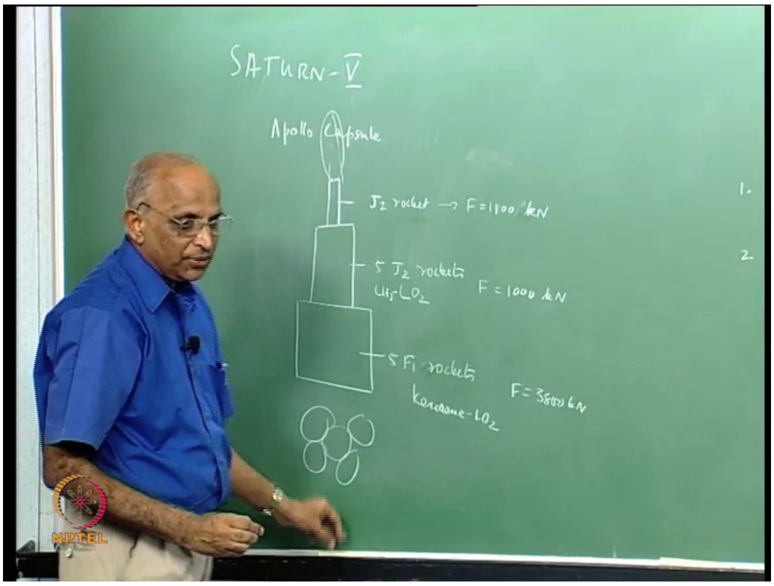

We would like to do an sample problem. Let us take an example of F 1 rocket. However, before that, let us put some numbers. J 2 rocket in the third stage has a thrust of something about 1100 kilo Newton. Each of the J 2 rockets here, mind you, the same J 2 rockets when used for the second stage, has a thrust of 1000 kilo Newton. Each of the F 1 rockets has a thrust of something like 3800 kilo Newton. Let me make sure about the numbers. This has something like a thrust of something like 110 ton thrust. Because kilo Newton, therefore, we are talking of 10 Newton is equal to 1 kilogram. Therefore, we are talking of a huge force here. Therefore, why is it that the same engine when used in second stage produces less thrust than when used for the third stage? Altitude. That means, higher the altitude, I get more specific impulse and therefore we get more thrust.

Let us work out a problem concerning the F 1 rocket. Out of all these five, let us do the nozzle problem related to one F 1 rocket. The thrust of this rocket is equal to 3800 kilo Newton; is that what was said? No my numbers are not correct. The thrust is very much higher. 6800 kilo Newton, I am sorry for the wrong numbers.

Each F 1 rocket has a thrust of 6800 kilo Newton. The mass flow rate through the nozzle is equal to 2600 kilogram per second. These are typical numbers. We should keep in mind that we are not talking of 1 kilogram per second. We are talking of something like, almost 3 tons of propellant gases going through the nozzle per second. The area of the nozzle is equal to 0.65 meter square. That means, if we consider the diameters, a man can easily stand at the throat or walk through through the throat of this nozzle.

The molecular mass of gases which are passing through the nozzle is equal to 22 grams per mole. The temperature of the combustion products in the chamber is equal to 3300 Kelvin and the chamber pressure is equal to 6.65 Mega Pascals. That means, something like 66 bar. This is little below the standard pressure of 7 MPa, which we are talking of as a standard value of pressure for specific impulse.

Mind you, this rocket was developed in the period of 1960s and we had the moon mission by 1969. Therefore, we are talking of an old rocket. But, mind you, it is still the most powerful rocket ever developed in the history of rockets and that is where I thought, maybe we should do a problem on this rocket.

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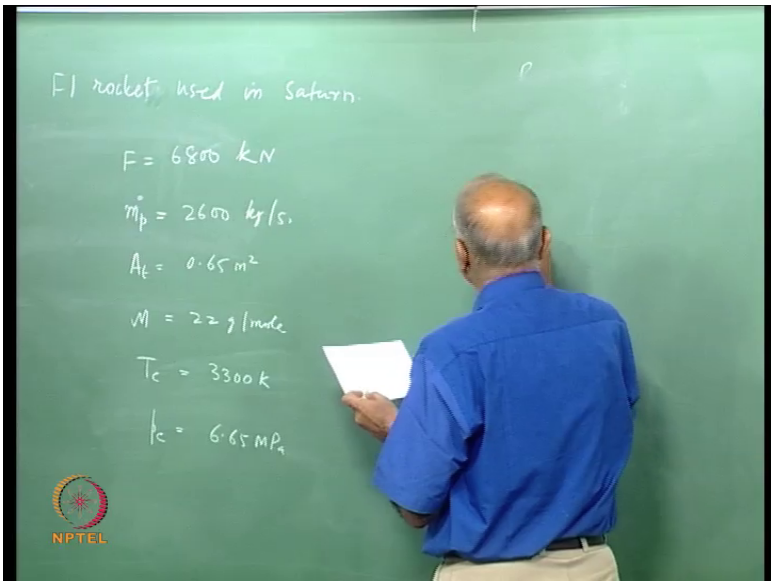

Now, I want to find out for this F 1 rocket, the value of η_{C*}, the value of Isp and the value of thrust correction factor ζ_F. Let us do it. We should be able to do it since the area is available and you know how much propellans are burnt per second.

To be able to get the value of η_{C*}, I must get the value of C*, which is actual and I must also get the value of C star, which is calculated under ideal conditions. The ratio of these two (measured/calculated ideal value) is the c star efficiency η_{C*}. How do I get the ideal value? Well, we already know it is equal to $\sqrt{}RTc/ \Gamma$. The value of capital gamma Γcan be determined. We get the value of the specific gas constant R as equal to the universal gas constant divided by the molecular mass of the gas and multiply this with the value of Tc to find RTc. Now capital gamma Γ is equal to $\sqrt{\gamma} \times (2/\gamma+1)^{(\gamma+1)/2(\gamma-1)}$. The value of gamma for the gases is equal to 1.22. Therefore, we substitute the value of gamma is equal to 1.22 and the value of capital gamma works out to be equal to 0.652; $\sqrt{1.22} \times 2/(1.22 + 1)^{2.22/(2\times0.2)}$.

To get C* ideal. For C* let us substitute the value, R_0 the universal gas constant 8.314 joule per mole Kelvin, and molecular mass as 22 g/mole..Please write the units whenever we do a problem. The value of Tc for this particular propellant combination is given as 3300 K. The value of the molecular mass is equal to 22 grams per mole but, I am talking in terms of joule which is related to kilogram. Therefore, we take 0.022 kilogram per mole.

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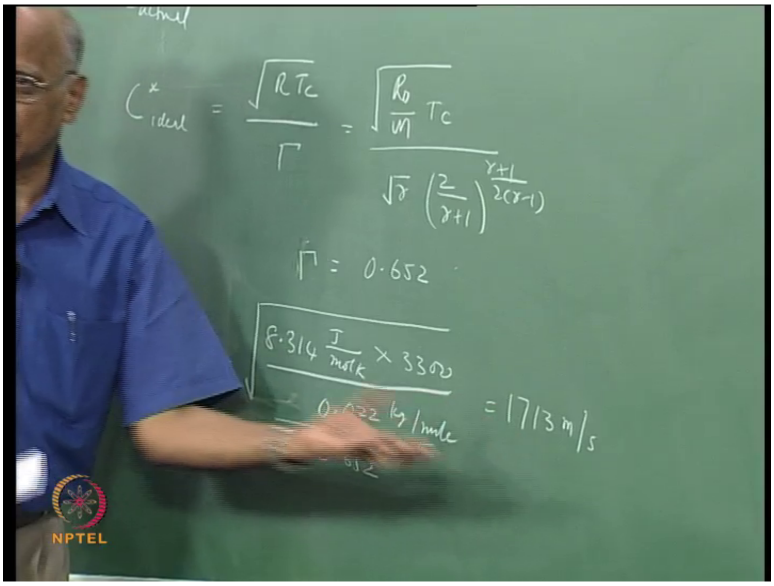

This is important. Many of us, mainly I find students just putting 22 here, which is not right because, when I say mole, I want the value of the soecific gas constant R in joule per kilogram Kelvin. Therefore, it must be kilogram per mole. The value of capital gamma, we already said is equal to 0.652 and this must be the numerator for this one. The value of C* ideal becomes 1713 m/s.

This is how we calculate the ideal C*. That indicates the capacity of the propellants in the chamber to generate hot gases at high pressure in the chamber. Let us repeat this. The capacity of kerosene and liquid oxygen to generate chamber pressure is given by C* ideal and this capacity is 1713 meters per second. Now, we want to get the measured value of C*. We have to calculate the actual expermiental value. How would I do it? I go back to look at the problem. The mass flow rate is given to us. The mass flow rate is given as 2600 kg/s equal to 1/C* × pressure × throat area. Pressure is given as 6.65×10^6 Pascal. The throat area At is given as 0.65 square meters. Therefore, the value of actual C* can be calculate from these values. This is 2600 kilogram per second. C* will come out to be equal to $6.65 \times 10^6 \times 0.65/ 2600$. This is equal to 1663 meters per second. The value of η_{C*} is therefore $1663 \div 1713 = 0.97$.

In fact, you find that the c star efficiency is quite high even for a rocket made in the 1960s. The present rockets like the space shuttle main engine, has a C* efficiency of the order of 0.99. This is the way they are and are very efficient. There is hardly any room for improving the combustion any further. We have to understand that, when we do liquid propellant rockets, we will try to understand how come we get such values and what are the factors which govern it.

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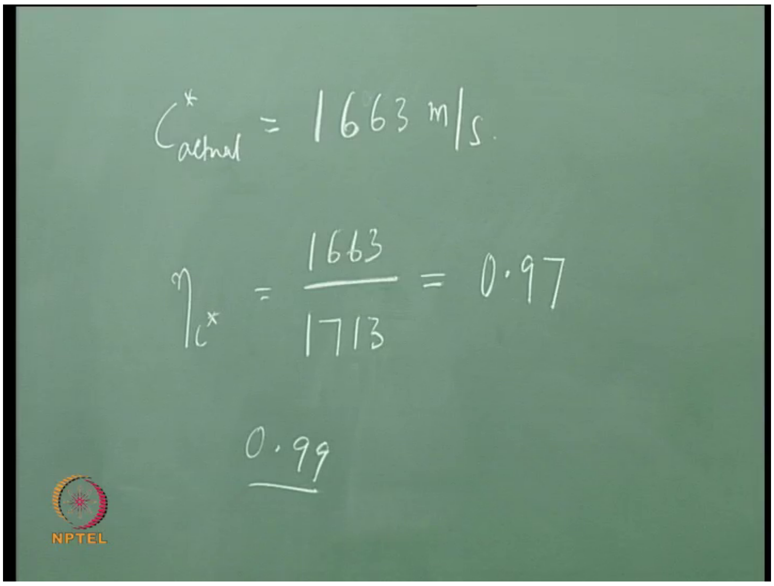

May be, we should use some of these tactics in the other propulsive devices also. Therefore, we have done one part of it, namely, what is the c star efficiency of this particular F 1 engine. The next one, I would like to find out what is the value of Isp. How do I do it? What is the specific impulse of this engine? Yes, I know the thrust is 6800 kilo Newton and I know the mass flow rate is 2600 kg per second. Well, it is simple; is it not? Specific impulse is equal to Impulse I over mass of propellant Mp which is equal to I over t divided by Mp dot. The specific impulse Isp is therefore equal to force or thrust divided by mass flow rat eof propellants. This is already avalable. Th evalu eo fspecific impulse is therefore 6800 into 1000 Newton divided by 2600. The value of specific impulse comes out to be equal to what 2710 Newton secod per kilogram. This is the value of Isp.

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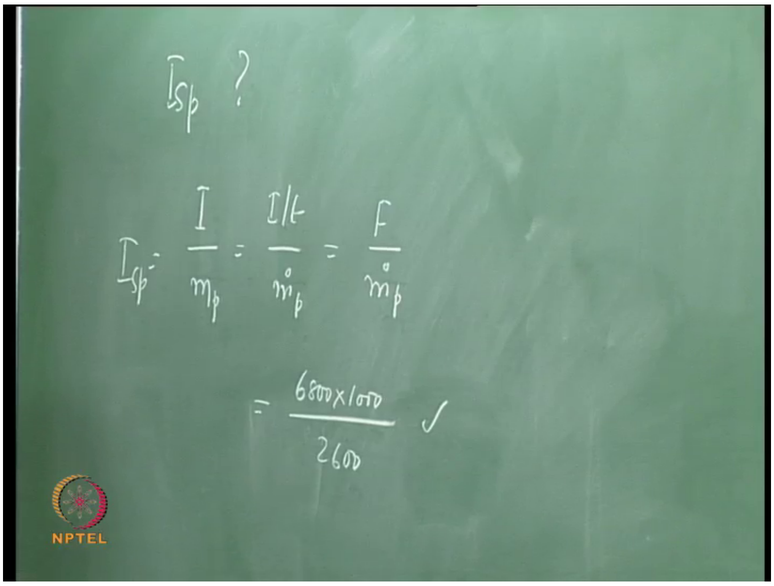

Now, we want to get the value of the thrust correction coefficient ζ_F. For this, we need to do little more calculation. Let me erase this part of the board. We get ζ_F is equal to the ratio of actual thrust and ideal thrust. How do I get the ideal thrust?

This particular rocket develops a thrust of 6600 kilo Newton, when the exit pressure is also sea level or rather, when the value of pe of this rocket is equal to pa which is equal to 0.1 MPa because, it is tested under sea level conditions. The test has been done at sea level, for which the exit pressure is equal to pa. We are assuming here that the nozzle exit pressure is equal to the ambient pressure.

Therefore, for this condition, the value of F ideal is equal to $m^\circ \times V_J$ because, there is no pressure thrust coming; pe minus pa is 0 because pe is equal to pa and how do we get the value of V_J? We have derived the expression V_J^2 = square is equal to 2 of the enthalpy difference which came out to be equal to $\sqrt{2\gamma R_0 Tc/(\gamma-1)M\{1-(pe/pc)^{(\gamma-1/\gamma)}\}}$. Here pe is equal to the ambient sea level pressure. Put in the numbers, R_0 is 8.314 joule per mole kelvin, Molecular mass is equal to 0.022 kg/mole and temperature is given 3300 and gamma is given 1.22.

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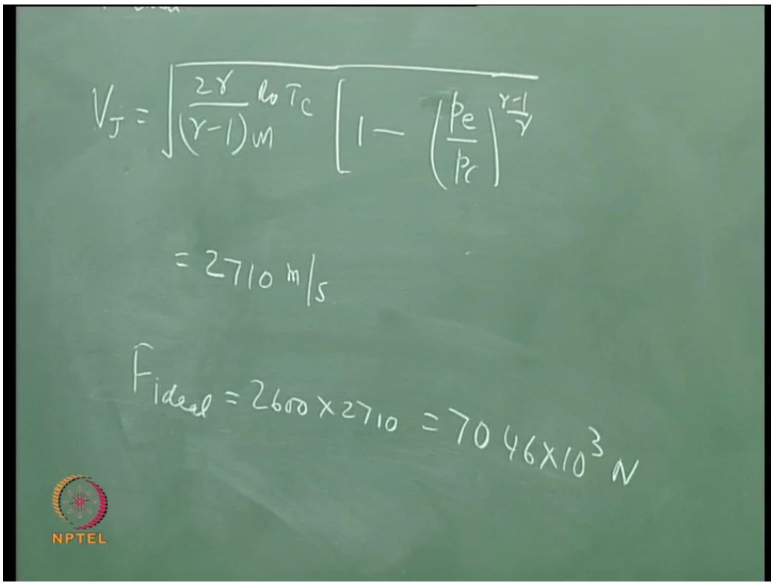

The value of pe is equal to 0.1 MPa, pc is equal to the value what is given here that is 6.65 MPa. You substitute it and you get the V_J is equal to 2710 meter per second. Therefore, what is the value of ideal thrust? Thrust is equal to m° × 2600 kilogram per second multiplied by this value of V_J. Therefore, the value of F ideal is equal to V_J multiply it m° which gives you the 2600 into 2710, which is equal to 7046×10^3 Newton.

What is the actual value of thrust? 6800 kilo Newton. But, how do I get the value of zeta F? Therefore, our immediate reaction or anybody's immediate reaction would be to take the value of ζ_F, i.e., the thrust correction factor is equal to F actual divided by F ideal. F actual is equal to 6800 kilo Newtons.

The ideal value is some what larger, that is 7406 kilo Newton and therefore, you will tell me, that this value is equal to 0.965. This is what one expects. But actually, you know, we have is an actual rocket, in which we must also consider the effect of C* efficiency. ζ_F is actually the thrust correction factor. In other words, Isp at the actual thrust goes as η_{C*} × the thrust correction factor ζ_F into the value of Isp.

Therefore, if I were to correct for theefficiency of C*, I should have $\zeta_F = 0.965/\ \eta_{C^*}$, which we got as 0.97. Rather, this works out to be 0.995, because this is only for the nozzle. We looked at that total problem and the total problem gave us this value and we have to isolate the correction to apply for the nozzle. Therefore, the correction factor for the nozzle is 0.995. Whereas, the contribution from the combustion or from the value of pressurization or c star is 0.97.

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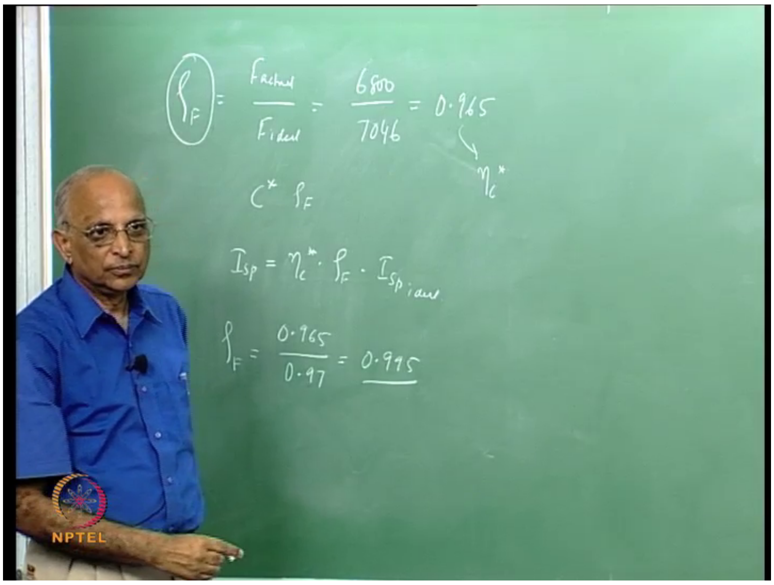

I think this is how we get the efficiencies. Well, I would be happy even if we put a number 0.965. But, let us keep in mind that 0.965 also includes the value of η_{C^*}. That is why, I had to remove it and that is where I got this particular number 0.995.

Let us take one more problem that is problem of rocket being propelled at different altitudes. Let me pose this problem to you first. Yes, let us say a booster rocket operates between sea level (0 kilometres) to 30 kilometres altitude and the chamber pressure of this rocket pc is given to be 7 MPa.

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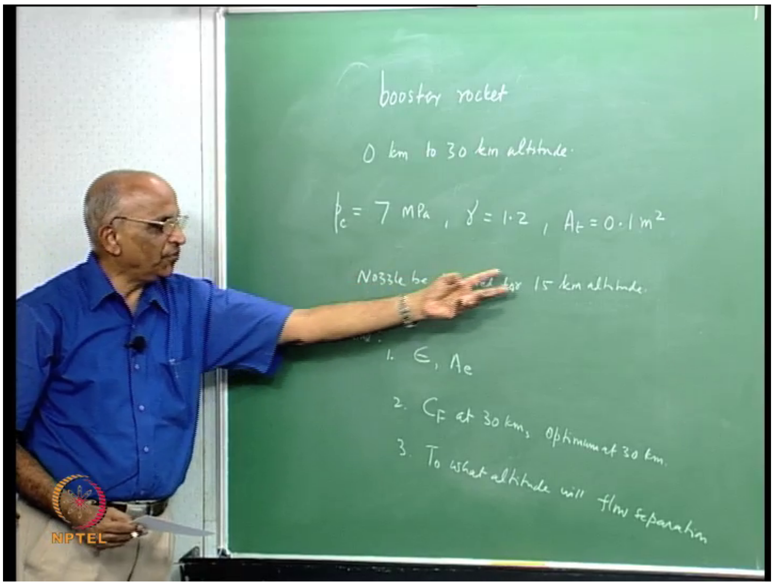

The specific heat ratio of the combustion products is given as 1.2 and the throat area At is equal to 0.1 meter square. Since, this rocket performs between 0 to 30 kilometre altitude, the designers felt that the nozzle could be designed for mean altitude of operation, say 15 kilometre altitude. Now, I want to determine the following: First, the nozzle expansion ratio, ε, and the value of the exit area Ae. Second, the value of the thrust coefficient C_F at 30 kilometre altitude and what is the optimum value of C_F at 30 kilometre altitude and third, we also want to know, till what height or till what altitude will flow separation occur. In other words, we assume that we have a conical nozzle which operates between 0 and 30 kilometres. The nozzle is designed for an altitude of 15 kilometres. We want to know till what height flow separation takes place in this conical nozzle. We also want to find out the area ratio, the area at the exit and the thrust coefficient at 30 kilometre, optimum value of thrust coefficient at 30 kilometres and to what altitude will flow separation persist. Let us do this problem.

We need the data and the data on ambient pressure which are normally avalable as ICAO tables. In these Tables, the height in altitude versus the ambient pressure is given. ICAO stands for international civil aviation organisation. This gives some standards and they will list the altitude versus the pressure, ambient pressure in Newton per meter square, temperature , density. If at sea level, the pressure in Newton per meter square is 101325 Newton per meter square. If the altitude is 4 meters height, the value is 61660 Newton per meter square. If the altitude is 8 kilometres, the value is 36651. You see, the value keeps decreasing. Let us put two or three more values of ambient pressure at the different altitudes as shown in the following slide:

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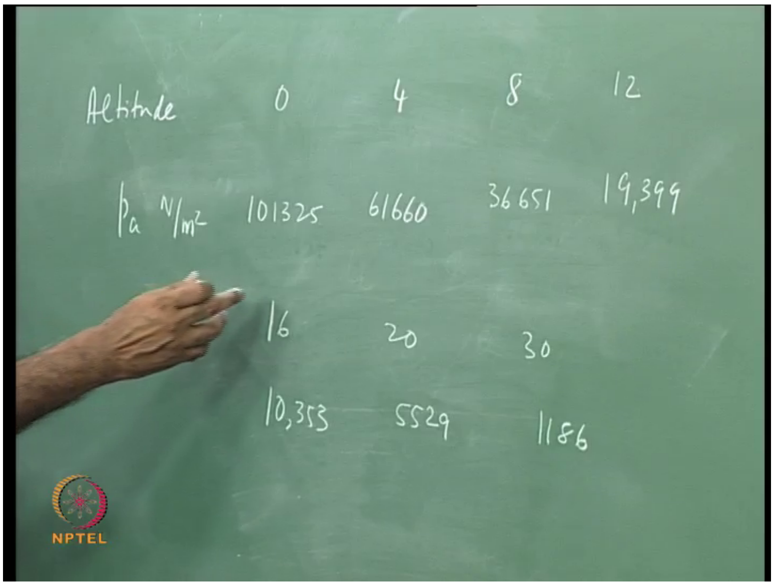

12 kilometres, the value is 19399 and if the altitude is 16 kilometres, it is 10353 N/m^2. If it is 20 kilometres, it is 10353 N/m^2. If the alttude is 20 km, the ambient pressure is 5529 N/m^2 and if it is 30 kilometres, the value of pressure is equal to 1186 N/m^2. Since, I do not give the value of ambient pressure at 15 km height, let us assume that the nozzle is designed for 16 kilometre altitude instead of 15 km. So that, the ambient pressure table is available to us.

We would like to first calculate the area ratio of the nozzle and the exit area. What do we tell? We say, well, the nozzle is designed for 16 kilometre altitude and therefore, for 16 kilometre altitude, we have pe is equal to pa. Therefore, what should be the value of pe? For the nozzle? 10353 N/m^2 or Pa, because the nozzle is designed for this particular altitude.

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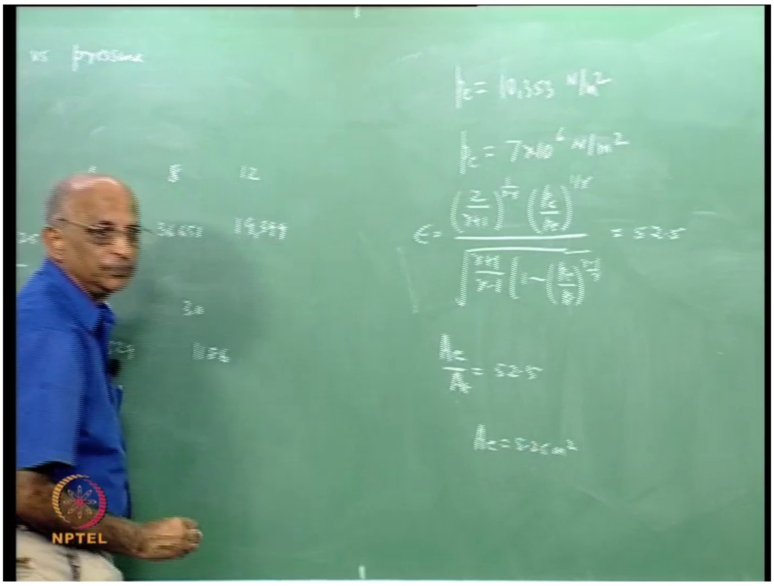

That means, we have pe as 10353 Pa. Other data viz., the value of pc is equal to 7 MPa. We say 7×10^6 Newton per meter square. The value of γ is given as 1.2. Therefore, we immediately write out the expression for the expansion ratio ε. Let us go back to your notes.

Epsilon is equal to $[2/(\gamma-1)]^{1/(\gamma-1)} \times (pc/pe)^{1/\gamma} \div \sqrt{[(\gamma+1)/(\gamma-1)]\{1-(pe/pc)^{(\gamma-1)/\gamma}\}}$. You can easily derive it out. It is not difficult. I do not want us to memorize anything. You substitute the values and you get the value as equal to 52.5. Area ratio of the nozzle is therefore 52.5. The value of the exit area Ae/At is equal to 52.5. Rather since At is given to you as 0.1 meter square, the value of Ae is 5.25 meter square. Is it alright? It is simple. You know, the calculations for rockets tend to be extremely simple.

In fact, rockets are very simple. In India, we still have not made good diesel engine or internal combustion engine or gas turbine engine. We have been taking time to do it and we have still to do it on our own. Whereas, rockets being easier to do, we see spectacular progress in making of rockets.

Therefore, you have Ae is equal to 5.25 meter square.

Let us go to the next part of the problem. C_F at 30 kilometres. How do we evaluate it? What will be involved in this? We had derived the expression for thrust coefficient C_F. Let us go back and take a look at it. $C_F^0 = \sqrt{2\gamma^2/(\gamma-1)[2/(\gamma+1)^{(\gamma+1)/(\gamma-1)}\{1-(pe/pc)^{(\gamma-1)/\gamma}}$.

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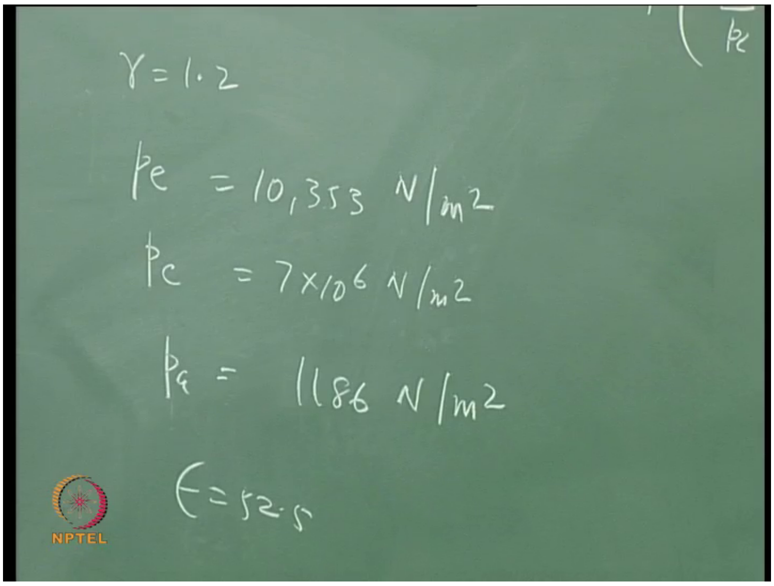

You will recall, we did this earlier. pa by pc and ε. Please check as to what are the values. I am interested at thrust coefficient at 30 kilometres. What are the values that we substitute? Well, $\gamma = 1.2$. What is the value of pe and what is the value of pc and what is the value of pa? Epsilon ε? We have already determined it as equal to 52.5. What is the value of pe? Which value to take? Yes, the nozzle has been designed for 16 kilometre altitude and that is what the exit pressure should be. Because, it is now opearting at a higher altitude but, the nozzle exit pressure will not change. Therefore, pe is equal to 10353 Pa. Your answer is correct. pc, we know is 7 into 10 to the power 6 Newton per meter square or Pa. pa at the current altitude of 30 kilometres, 11806 Pa. We substitute these values in the expression for C_F and we get the value of C_F as equal to, I use the other side of the board, 1.828 for C_F^0 plus 0.0687 for the pressure contribution $\{(pe/pc)-(pa/pc)\} \times \varepsilon$, the total being equal to 1.896.

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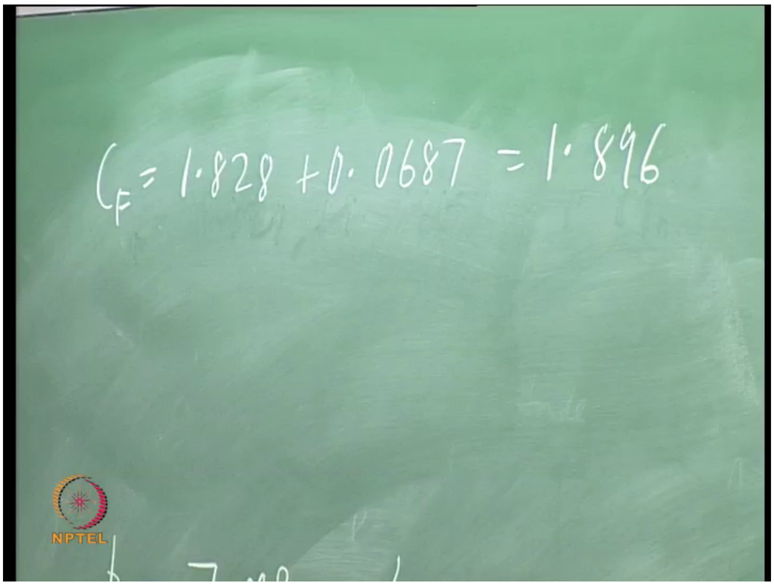

Please check these numbers. Now, what is the optimum value at 30 kilometres? Let us go back to the value of pe at optimum. From this expression itself you can tell me that gamma is still the same and what will be the optimum value at 30 kilometre altitude.

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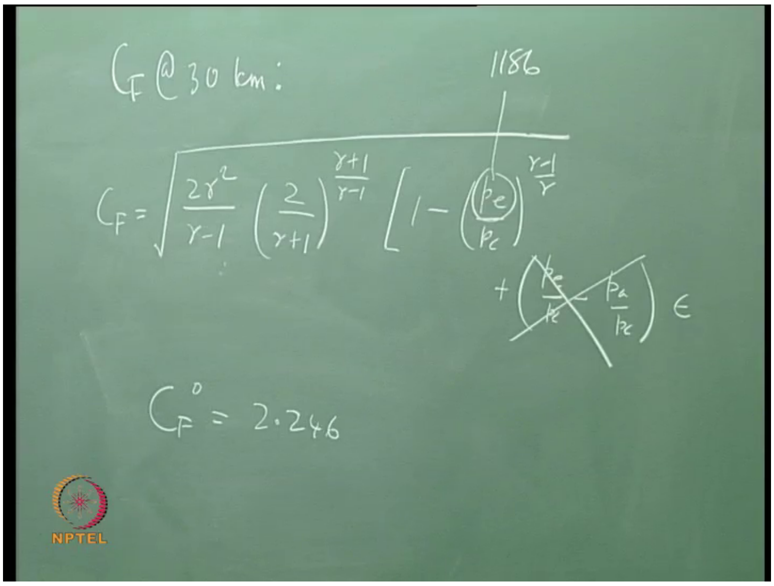

Well, pe must equal to pa. Therefore, for optimum, this term will get knocked out and what will be the value of pe, if it is optimum. Take a look at this table. Optimum at 30 kilometres, that means, I will have a nozzle which gives me this value, 1186 Pa. Therefore, I now change the value of pe as equal to 1186 and the pressure term is no longer there. The value of C_F which now represented as becomes C_F^0, will now become something like 2.246. You see, the thrust coefficient is typically around 2 to 3.

What is the percentage reduction from optimum? You had a nozzle. I think I will erase this out now. You had a nozzle which was designed for 16 kilometres. You are operating it at 30 kilometres. If it was designed for 30 kilometres, it would have been optimum at 30 kilometres. We would have the optimum C_F^0 as 2.246. But, the nozzle is designed for a lower value of area ratio and correspondingly a lower altitude, we get the lower value of the pressure thrust as 0.0687 and we do not get the net momentum thrust possible. We get the value of C_F as equal to 1.896.

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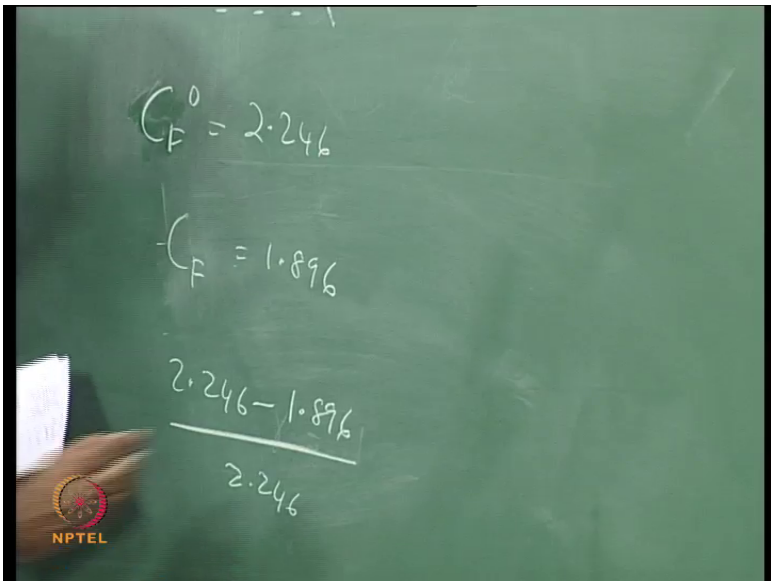

Therefore, we can now determine the percentage reduction from optimum is equal to 2.246 minus 1.896 divided by 2.246. In other words, I have something like 0.156 or something like 15.6 percent reduction from the optimum. Therefore, you see the importance. You know that we are not able to get the nozzle to expand to the ambient pressure at the given altitude and in fact we are having an under expanded nozzle. That is why, I am losing 15.6 percent thrust. Had the nozzle been designed for 30 kilometre altitude, we would have got a higher thrust. But then, we would have got a problem of over expansion and flow separation at the lower altitudes.

We now go to the next part of the problem and determine the altitude till which the nozzle is over expanded or the altitude till which the flow separation takes place. In other words, we want to determine the altitude till which the nozzle is over expanded.

For this, we apply Summerfield criterion, which states that, when the exit pressure of the nozzle is less than or equal to 0.4 times the ambient pressure, then the flow is over expanded. We have looked at a other criterion namely, involving Mach number also.

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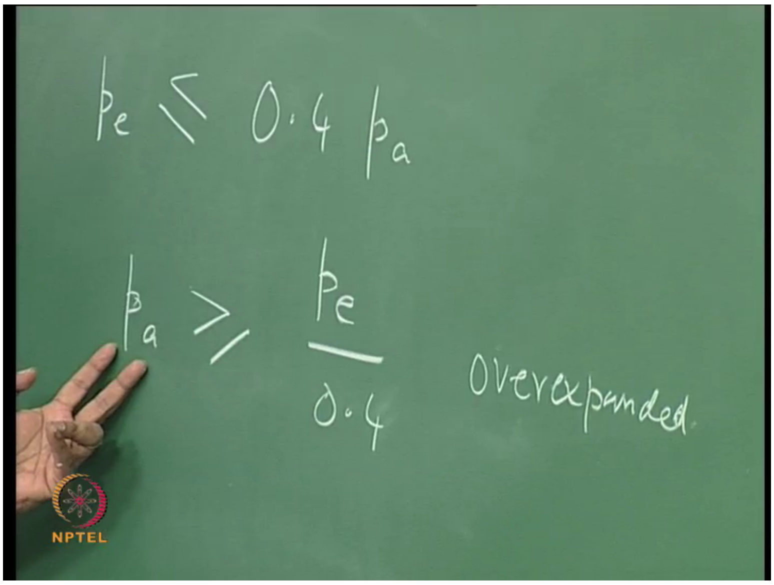

Let us use the Summerfield criterion. When the ambient pressure pa is greater than or equal to pe divided by 0.4, then we can say that flow separates in the conical divergent. Therefore, let us determine the altitude below which flow sepration is possible. Let us examine the changes in the ambient pressure with respect to altitude. Let me address the Table again to determine this specific altitude of interest. Let us make a table of altitude in kilometre and the ambient pressure pa in Pascal. Let us plot it for something like two or three altitudes for which we are interested.

At the altitude of 8 kilometres, the ambient pressure is 36651 Pascal. At the altitude of 12 kilometres, the value of the ambient pressure is now 19399 Pa. It has reduced, because the altitude has gone up. At 16 kilometres, for which this particular nozzle is designed, the ambient pressure is 10353 Pascal. The question is, the nozzle is designed for 16 kilometres and therefore, the exit pressure of the nozzle is 10353 Pascal, we want to find out the altitude at which the flow begins to separate or the nozzle gets to be over expanded. Therefore, we have to state here that flow separation, we have just written, pa must be greater than or equal to pe divided by 0.4 and pe for the nozzle is defined or defined as the ambient pressure at 16 km altitude viz., a pressure of 10353 Pascal. Therefore, pa must be greater than or equal to 10353 divided by 0.4 and this is equal to 0.259 into 10 to the power of 5 Pascal. Now, the question is what is the altitude when the ambient pressure is equal to or just greater than this value?.

Now, when pa is equal to 25900 Pa, it is somewhere between 8 and 12 kilometres. Therefore, we say at 8 kilometres, the ambient pressure is 36651 minus the value at 16 kilometres is 10353. But, we are interested in the altitude at 0.259 into the 10 to the power 5. Therefore, we have the value of 25900 minus the value at 16 kilometres which is 10353 Pa and the change in kilometre is from 16 to 8 corresponding it to a value of something like 8 kilometres. Therefore, we have 8 kilometres plus their change is 25 to 10353 and therefore, the value is 8 plus this so much kilometres.

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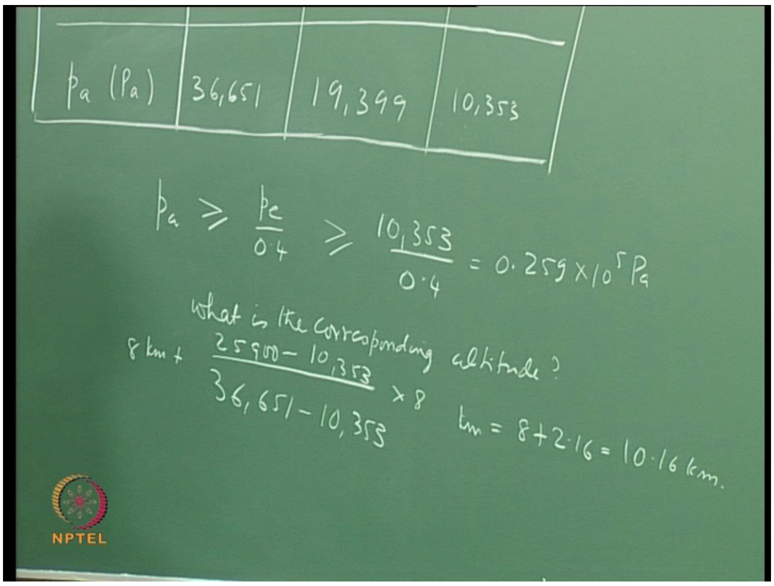

This works out to be something like 8 plus 2.16 which is equal to 10.16 kilometres. Therefore, this is the altitude at which flow separation ceases or thereafter the nozzle is either runs full or is under expanded. This is all about nozzles.

In the next class, we will start with chemical propellants. We will again keep it very very simple in the sense, we will look at what are the requirements of chemical propellants and then see, what are the propellants that, we must use.