Rocket Propulsion

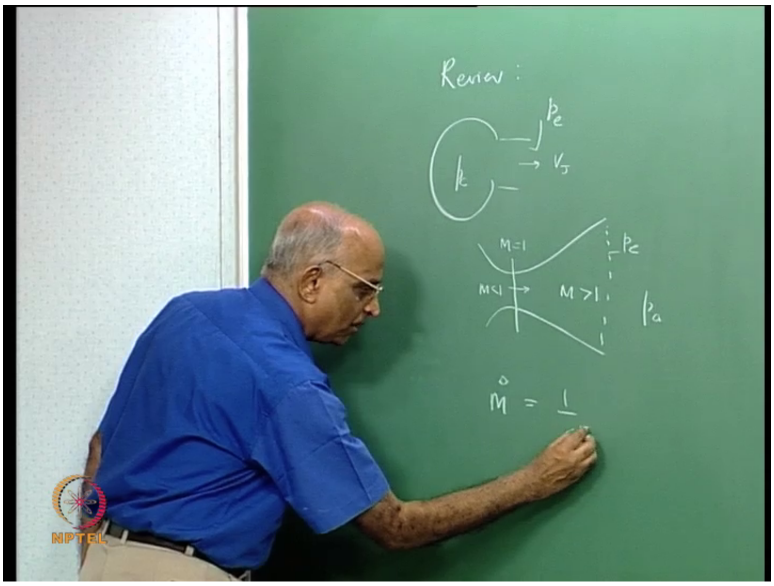
Prof. K . Ramamurthi
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture No. #13

Divergence Loss in Conical Nozzles and the Bell Nozzles

Good morning. In today's class, we continue with nozzles. First, I will review what we have done so far and then, we will see whether there are any gaps, anything which has happened in nozzle development, which we have not covered. We said that nozzle is something like a vent, and we first learnt how to calculate the jet velocity. How did we calculate? We said that we have a chamber, which is at a pressure pc. Suppose, at the exit I have pressure pe; I can calculate the jet velocity V_J. We wanted V_J to be as high as possible and to be able to get a high value we need this vent or opening to be in the form of a convergent divergent shape and we also put a condition that the minimum area which we called as a throat, we should have a Mach number equal to 1.

(Refer Slide Time: 00:28)

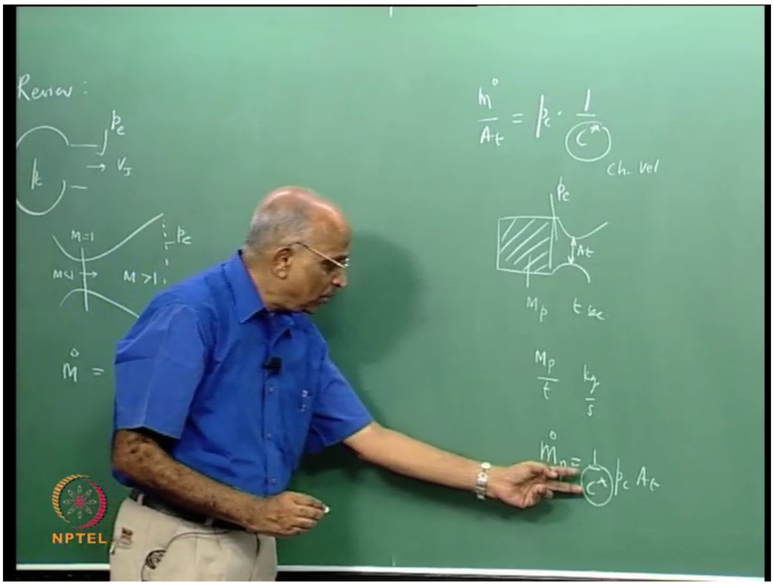


We called this nozzle as a de Laval nozzle or a convergent divergent nozzle. In the convergent part, the Mach number was less than 1; in the divergent the Mach number was greater than 1. After having done this, we said at the exit I have exit pressure which is pe; the ambient pressure is pa and if the exit pressure does not match to the ambient pressure i.e., is not equal to the ambient pressure, we could have some shortcomings in the nozzle performance either due to under expansion or due to over expansion.

After this, we took a look at what is the flow through the nozzle, we got the equation for $m^\circ = pc \times At/C^*$ as the mass flow rate. We got an expression say m°/At which is mass flux as equal to chamber pressure pc × 1 over C*. C* had units of meter per second and we were able to correlate it with the pressure built in the chamber.

The pressure built in the chamber is related to the mass generated or rather to the mass flow rate through the nozzle. We saw C* as a transfer function to develop pressure in a rocket chamber. Let us take one example just to clarify things. Suppose, I have a rocket in which the mass of propellant is let us say Mp kg.

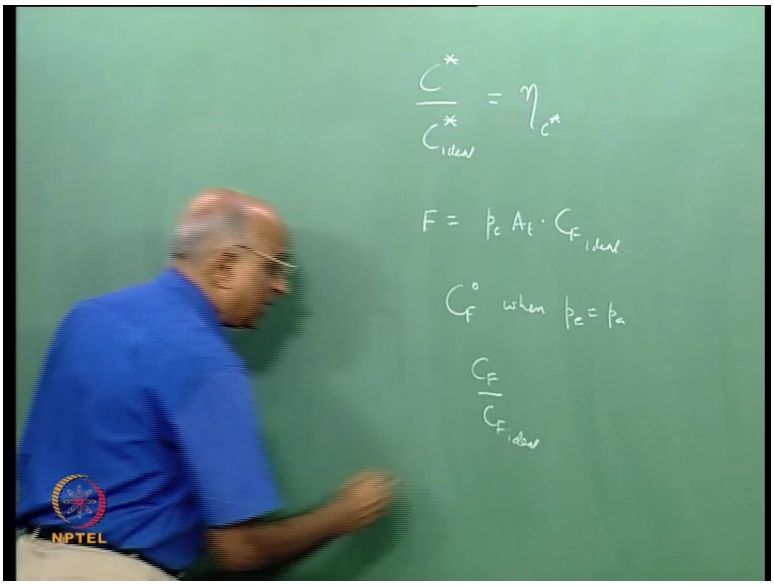
(Refer Slide Time: 02:17)



Let the rocket fire steadily for a period of t seconds. The mass flow rate through the nozzle is therefore equal to Mp/t kg/s since Mp is in kilograms and the flow rate is in kilogram per second. If the pressure built in the rocket chamber is pc and if the nozzle throat area is At, we can directly write that $Mp/t = m^\circ = (1/C^*) \times$ chamber pressure pc × At. We can therefore determine this value of C*. If we do an experiment and determine the value of C star based on the measured value of mass of propellant, time and pressure and then we compare the value of C* which I actually measure to the C* which we derived ideally, both would not be equal. We calculated C* is equal to $\sqrt{RTc}/\Gamma$; we would find that the ideal value may be a little more than the actual experimental value because, some flow losses are taking place. The measured value to the ideal theoretical value was called it as C star efficiency of a rocket or rocket chamber.

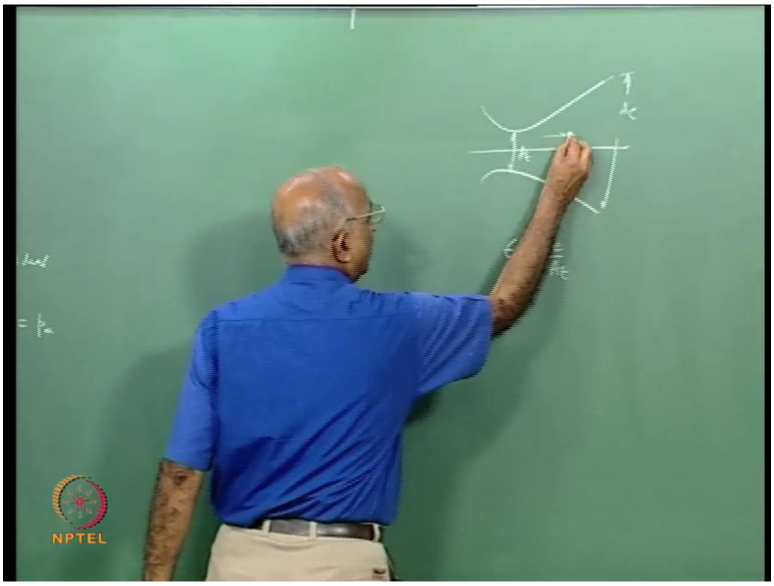
We also said that the nozzle has a divergent portion and we can also write the thrust of a nozzle as equal to the chamber pressure × At × a coefficient called thrust coefficent. We derived the equation for this coefficient. We called this as C_F^0 ideal when pe =pa. Further, we found that since the thrust also depends on the exit pressure and the ambient pressure, and I show exit pressure pe and the ambient pressure pa here, pe may not be equal to pa, but the thrust is a maximum in pe is equal to pa; we called this ideal thrust coefficient as C_F^0 when the exit pressure was equal to pa.

(Refer Slide Time: 03:56)



In a similar manner, when we have a rocket, which is firing or operating, we measure the pressure using transducers. We measure what is the thrust that is generated by rocket and determine the measured value of C_F. We also calculate the value of C_F, which we did using the ideal theory. The ratio of the experimental to the theoretical value was called as the thrust correction factor ζ_F. This is a summary of what we did. We also did something which was important. We said that instead of specifying the exit pressure and the chamber pressure, we can also specify a rocket nozzle in terms of a nozzle exit area Ae divided by the throat area At which we said was ε.

(Refer Slide Time: 05:54)



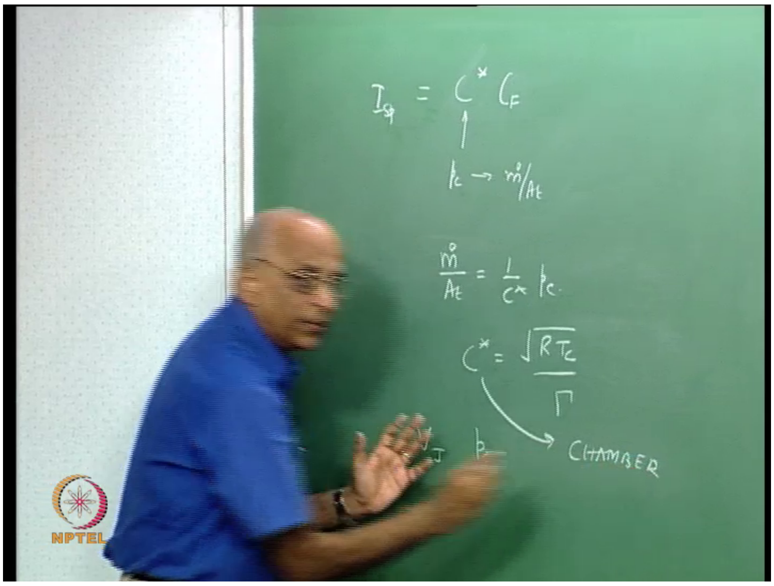
Are there any questions on what we have done so far? Mind you all that we have done is for an ideal case, an adiabatic nozzle. And one-dimensional flow you always said that flow is going straight like this to the right. The mass flow rate is contributing to the thrust and expansion. Are there any questions so far?

Your question is why we need to couple the C* with C_F to determine the value of specific impulse when the specific impulse can be readily evaluated based on nozzle flow.

Let us first clarify that C* is something which tells how much chamber pressure is developed when we provide a certain mass flow rate through the nozzle. The nozzle is identified by the throat area. In other words, the transfer function between mass per unit flow rate through the nozzle or mass flux through the nozzle at the throat to the chamber pressure gives me the value of the C*. We could get the chamber pressure for a given mass flux at the throat and this is the transfer function. What does it tell us? When we looked at the expression for C*, it was $\sqrt{RTc}/\Gamma$ where Γ is a function of γ viz., $\sqrt{\gamma} \times [2(\gamma+1)]^{2(\gamma+1)/(\gamma-1)}$. What does it really tell?

It tells, supposing I have a mass flow rate through the nozzle - mass flux through the nozzle throat,

(Refer Slide Time: 06:54)

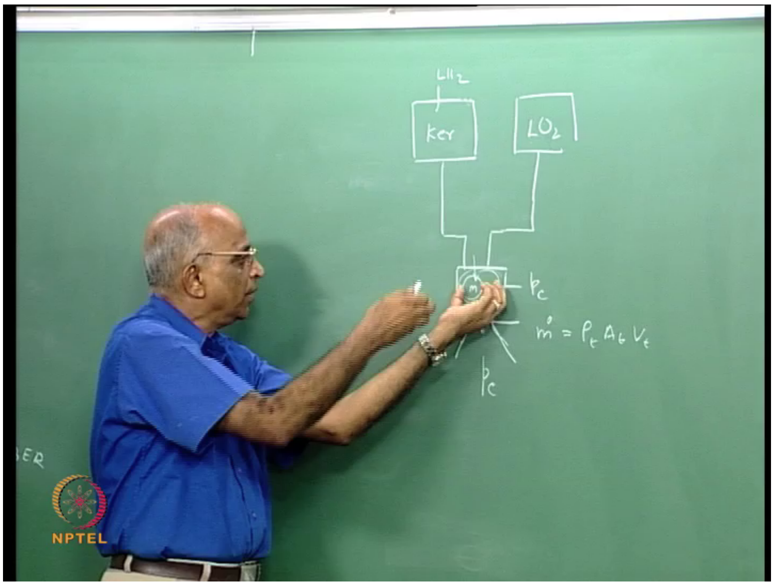


what is the value of chamber pressure that we get? To get a high value of V_J, I need a high value of chamber pressure. Therefore, this C* tells you the capacity of whatever propellant you have in the chamber to generate a high pressure. Therefore, C star is not a function of nozzle performance, but more like what how a chamber can build up high pressure. All what it tells you is let us take one or two small examples.

Let us take an example of a rocket, which burns a liquid fuel. We will study about propellants in the next series of classes.

Suppose, I have a tank containing let us say liquid kerosene. I have another tank containing oxygen. I introduce them or rather force or push them in a chamber and allow it to burn to generate a high value of chamber pressure pc. The value of C* for the kerosene and oxygen, introduced in the chamber, will tell the capacity of propellants kerosene and oxygen to generate a pressure pc in the chamber.

(Refer Slide Time: 09:07)



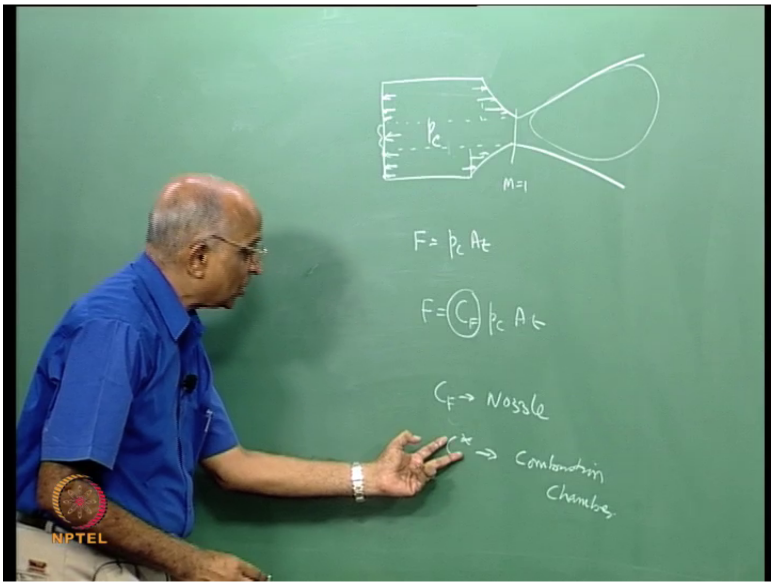
A higher value of chamber pressure pc will be obtained with a higher value of C star. But then it is for the chamber. However, we did specify the mass flow rate and the throat diameter. We did not look at the divergent part of the nozzle even though we did solve for the nozzle flow. We had written that the mass flow rate m° is equal to $\rho_t \times$ At × Vt. Therefore, we did not really look at the divergent part of the nozzle; we looked only up to the throat. Therefore, C* is representative of the chamber to be able to generate high pressure gases when some mass is flowing. If instead of having kerosene and oxygen suppose, I have say liquid hydrogen and liquid oxygen may be the C* could be higher. Therefore, we would prefer this propellant combination to kerosene oxygen. C* therefore becomes a capacity of the chamber for a given propellant to generate high pressure gases.

Normally, the value of C star is around 2000 to 3500 m/s; with a lower performing rocket or propellant that is not that good, will give a lower value of C*. Propellants, which are extremely energetic, will give higher values of C*.

Now, what is C_F? We defined C_F as equal to thrust divided by pc × At. In other words, if we had terminated the rocket at the nozzle throat itself we would perhaps have got a lower thrust than in a convergent divergent nozzle. Let us qualify this further. If I have a rocket nozzle and I terminate it at the throat, where the Mach number is equal to 1, what would be the thrust? We have chamber pressure pc and now for all practical purposes pc is acting on all this area. The pressure is also acting normally over the head end over here and over the convergent part of the nozzle. The force generated from the pressure gets cancelled as shown, except over the throat area.

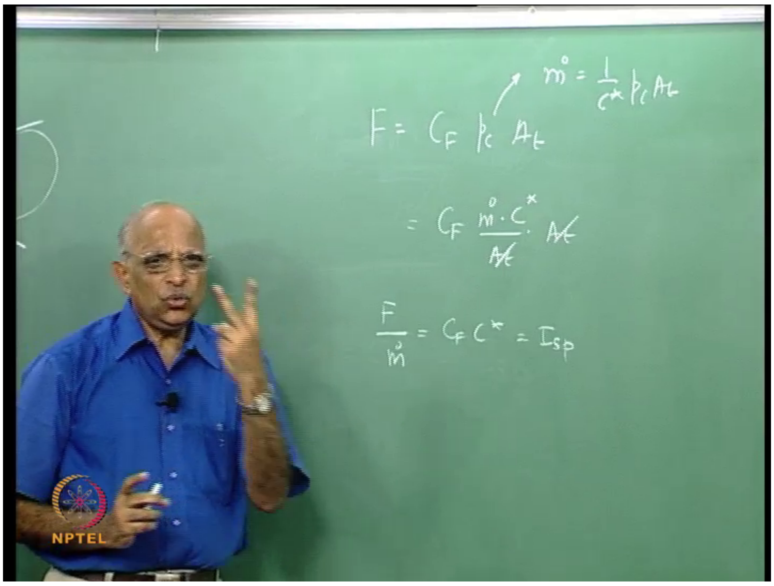
The thrust from the unbalanced pressure is over the throat area and is equal to pc × At. This gives the order of magnitude since we did not consider the variations in pressure along the length of the nozzle. But we have the divergent part like this because of which the thrust would have increased. The increase comes from the pressure acting on the walls of the divergent.

(Refer Slide Time: 11:38)



We can write F is equal to C_F × pc × At. The value of pc × At was the thrust if the nozzle was truncated at the throat. Therefore, C_F is something like thrust magnification due to the divergent part of the nozzle and therefore C_F is a quality factor for nozzle. In fact the values of C_F for most nozzles are between 1.2 to something like 3 or 4. We will work through some examples in the later part of this class. Therefore, we conclude that C_F is a quality factor for a nozzle while C* is a quality factor on the capacity to generate the pressure in the combustion chamber of a rocket. The product of C_F and C* is the net specific impulse Isp. What is Isp? It is the total thrust divided by the mass flow rate m°.

(Refer Slide Time: 13:34)



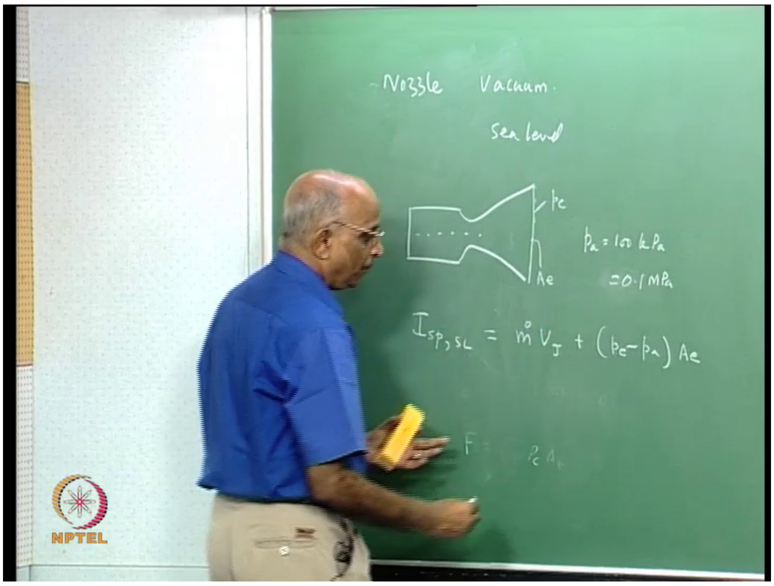
Let us put the expressions down again. Force or thrust = thrust coefficient C_F × pc × At. Well, pc can be written as in terms of mass flow rate m° = (1/C*) × pc × At. Therefore pc can be written as equal to m° × C* ÷ At. Substituting this value of pressure in the expression for thrust F, we observe that At and At get cancelled and we have thrust or force divided by m° is equal to C_F × C*. The value of mass of propellant is m° into time while force is impulse per unit time. Either way the specific impulse is impulse per unit mass of propellant or thrust per unit mass flow rate and works out to be the product of C_F × C*.

Therefore, the specific impulse of a rocket has two attributes or properties in it; 1. the capacity of chamber to generate high pressure and high temperature gases and 2. how you expand the gases to get high velocity. Therefore, let us keep this terminology very clear. I have nozzle factor and a chamber factor, which gives us the net Isp. I will dwell on this further after a couple of minutes. But does this answer your specific question, why C*? Why C_F? And what is the relation? How I got the specific impulse to depend on C* and C_F?

Let us take one example: let me take the example of a particular nozzle which operates let us say in vacuum, and let me take another nozzle which operates on the ground, let us say at Chennai which is at sea level. I have a chamber, which generates high pressure and high temperature gases. The exit pressure is equal to say pe. At sea level the ambient pressure is equal to pa which is equal to 100 kPa or 0.1 MPa. Now, I want you to tell me what is the relation between let us say Isp when the rocket operates at sea level and in vacuum. I want Isp at sea level condition and at very high altitude conditions where the pressure is almost zero.

We can the thrust developed as equal to m dot V_J plus I have pe minus pa into what? Ae. The thrust comes from the momentum thrust plus the exit pressure minus pa into the exit area where Ae is the exit area of the nozzle. Mind you we derived this, and we said we had control volume and therefore, we found a pressure thrust in addition to momentum thrust.

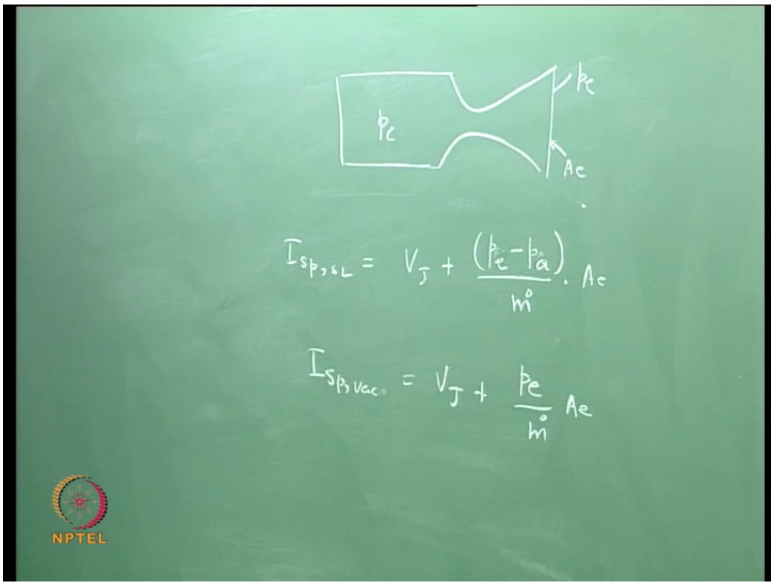
(Refer Slide Time: 15:33)



Now, we would likw to write the equation for thrust of the rocket using the same nozzle, when it operates in vacuum instead of operating on the ground at sea level conditions. Let us say the nozzle now operates in the vacuum, the same nozzle, the same area ratio this is the value of the exit pressure is the same and it is pe. The chamber pressure remains the same at pc. The thrust now becomes m dot into V_J plus the pressure thrust pe into Ae.

Now, I want to find out what is the specific impulse at sea level. Specific impulse at sea level is therefore equal to V_J + [(pe – pa) /m°] Ae .

(Refer Slide Time: 17:24)



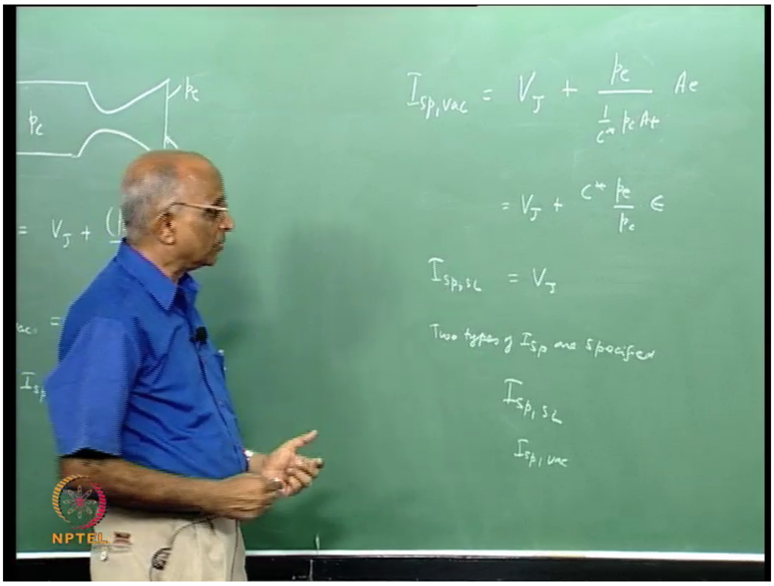
Now, what would be the value of specific impulse for the same nozzle functions in vacuum; I call it as vacuum specific impulse. Vacuum is equal to what would be the value of pa; it is 0. Therefore, we have V_J + (pe/m°) × Ae. In other words, the same nozzle when it is fired in vacuum gives me a higher thrust because, minus pa is missing in the above expression. Can we relate these two?

Let us say Isp at vacuum with the Isp at sea level. I find therefore, the specific impulse of a given rocket operating in vacuum is greater than when it operates on the ground. In other words, the specific impulse corresponding to operation in vacuum is greater than when the same rocket operates at sea level conditions. Now, we want to derive a slightly modified relationship relating the two.

Therefore, we write Isp at vacuum $Isp_{,vac}$ = V_J + pe / m° × Ae and what is the value of m°? m° = 1/C* × pc × At. If in terms of the C star, the Isp becomes the following: I have the value of Ae; Ae by At is the nozzle area ratio ε. The vacuum specific impulse becomes V_J + C* × pe/pc × ε . This is the nozzle area ratio ε in this expression. In other words, compared to a nozzle which gave me V_J plus this value of (pe – pa)/pc×ε at sea level, we get a much higher value. If this particular nozzle at sea level was such that I have optimum expansion namely pe was equal to pa, the Isp at sea level would have been just V_J alone.

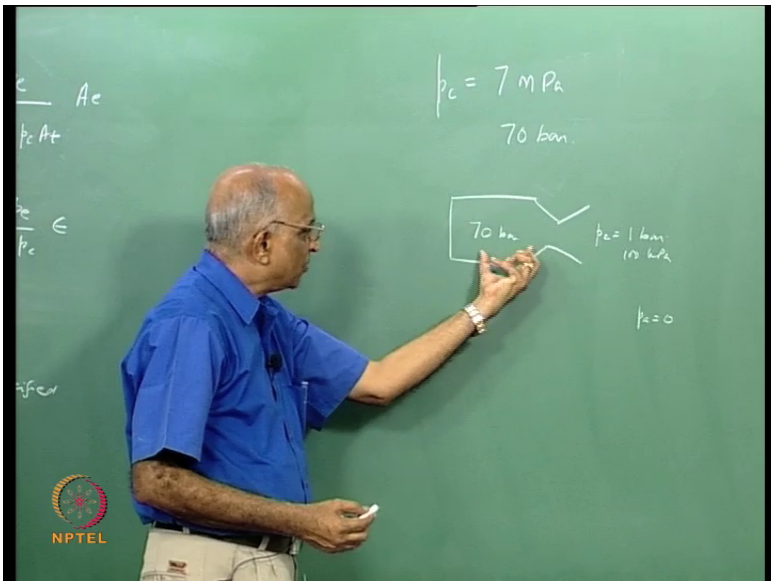
All what I am telling is, if the nozzle was such that that the exit pressure was same as the ambient pressure for which we told ourselves the C_F is a maximum, we would have got the value Isp = V_J. But the same nozzle when operating in vacuum, we get an additional contribution C*×pe/pc×ε. Therefore, now the question comes how do I specify the specific impulse? If we tell that the rocket is operating in vacuum, we get a higher value of specific impulse. If it is tested on ground, we get a different value. Therefore, we must be clear in our terminology and therefore, two types of specific impulses are given; one is Isp corresponding to sea level operation and the second is Isp corresponding to vacuum. Therefore, whenever the performance of a rocket is specified we must be careful to know whether sea level or vacuum operation is being specified.

(Refer Slide Time: 20:00)



Therefore, there are two ways of specifying the specific impulse whether vacuum specific impulse or sea level specific impulse; but then there is another problem. If we have a higher value of chamber pressure, we get a higher value of expansion ratio and I can get a higher value of the specific impulse. Therefore, we also need some terminology which says a standard chamber pressure and the standard chosen is we specify specific impulse for pc equal to 7 MPa or 70 bar pressure; that means, specific impulse is normally specified when the chamber pressure is equal to 70 bar. The choice of 70 bar comes as it is about 1000 psi in the FPS system of units.

(Refer Slide Time: 22:40)



When the ambient pressure is equal to 1 bar or 100 kPa pressure, we imply sea level conditions whereas, when we talk in terms of vacuum, we imply very rarified atmosphere. However, for each of these two conditions, we specify chamber pressure as 7 MPa. A rocket can fire at different pressures, but if we are to compare something, we need some standards and the standard is a chamber pressure of 70 bar and an ambient pressure of 1 bar for sea level Isp and 0 bar for vacuum specific impulse. The vacuum specific impulse is higher than the sea level specific impulse, which is V_J when the exit pressure is equal to ambient pressure. Are there any other questions on what we have done?

See so far, we have been talking of only one - dimensional flow in the nozzle. We discussed the divergent part and sketched it as a diverging cone.

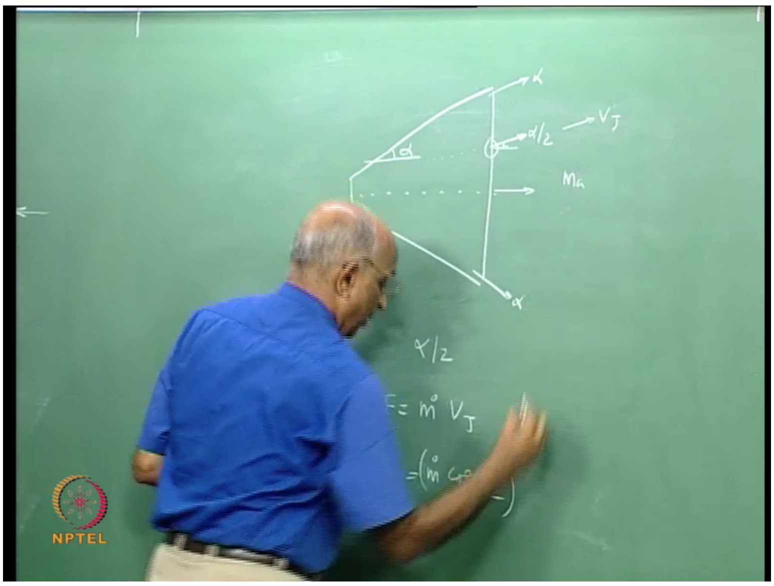
We have the convergent part, but I am really not bothered about convergent because, anyway at throat I had a Mach number of one and the flow lines stream out almost axially. We had a chamber of pc. Now, in the divergent part flow is diverging out, and if we look at the flow which is taking place, the gas which is flowing near to the wall will have a direction along the wall while for the gas along the center line the flow would be along the axis as per symmetry. Therefore, we may not be justified in assuming one - dimensional flow at the exit of the nozzle. It is really not correct and we have to make some corrections for may be the radial flow or for the divergence in the flow.

There is a simple way of doing this. The thrust is not going to be in the axial direction according to the figure. A component of thrust is going in this direction normal to the center line; it gets balanced out and only the effective thrust is in the axial direction. How do I get that value?

Well, there is an actual flow taking place along the nozzle; let us assume that the half divergent angle of the nozzle divergent is alpha (α). Now, the flow near the wall will be alpha. On an average, the mean flow direction could be alpha by 2 because here, it is alpha along the wall and zero along the center line of symmetry. On an average the mean flow we can assume makes an angle of alpha by 2. I can also define a small element and do the problem by integrating it out, but the above approximation is sufficient for me to give an answer. In other words, on an average the flow leaves at at an angle equal to $\alpha/2$. Is it ok?

We would like to determine the thrust. Let us assume that the nozzle is adapted; that means, ambient pressure is equal to pe here; F is equal to m dot into V_J over here. What is the mass which flows along the axis? It is equal to m° × cos ($\alpha/2$), that is the actual mass flow rate along the axis because on an average some mass flows at alpha, some flows at 0 with the mean direction being $\alpha/2$.

(Refer Slide Time: 24:00)

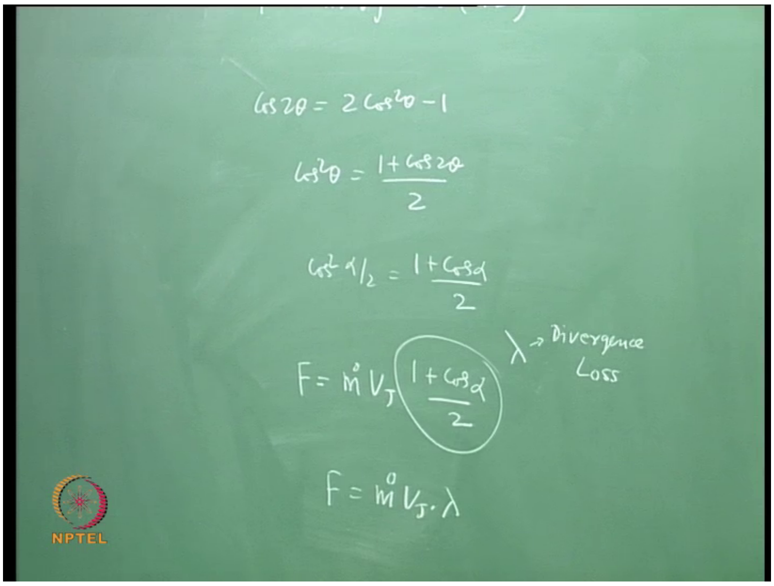


The average mass flowing along the axis is m°cos$\alpha/2$; V_J is again corresponding to this $\alpha/2$ as the mean direction of the velocity giving the axial component of velocity as V_J cos

$\alpha/2$. Therefore, the thrust due to the divergence being at an angle α will therefore be $F = m^\circ \cos \alpha/2 \times V_J \cos \alpha/2$ i.e., $m^\circ V_J \cos^2\alpha/2$. Is it all right? All what we told was flow is not all along the axis. Flow along the wall is at an angle α; on an average the flow is at $\alpha/2$ and therefore, the mass component along the axis is equal to $m^\circ \cos\alpha/2$. The axial velocity on an average is equal to $V_J \cos\alpha/2$. Therefore, the product of $m^\circ \cos\alpha/2$ into $V_J \cos \alpha/2$ make this $\cos^2\alpha/2$.

We would like to simplify this expression. We use the trigonometric expression $\text{Cos}2\theta = 2 \cos^2\theta - 1$. This gives $\cos^2\theta = (\cos2\theta+1)/2$. And therefore, we can write $\cos^2\alpha/2 = (1+\cos\alpha)/2$. This trigonometric manipulation is done because we can express it in terms of the half divergence angle of the nozzle. Mind you the total divergence is 2α; we said that α is equal to half divergence angle of the divergent.

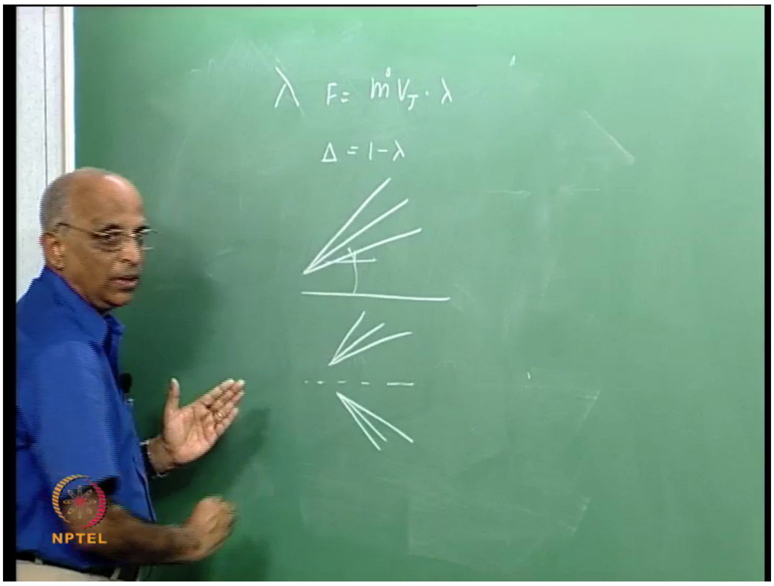
(Refer Slide Time: 27:35)



Therefore we get the thrust along the axis of a rocket F is equal to $m^\circ V_J (1 + \cos \alpha) / 2$. The term $(1+\cos\alpha)/ 2$, shown within the circle, is the loss factor due to the divergence. It is denoted by the Greek symbol lambda λ; and we say λ corresponds to divergence loss or $F = m^\circ \times V_J \times \lambda$. Well, λ is something we say loss due to divergence, but I am not really looking at a loss; see actually, we are just multiplying it by a factor and it is the diverging loss factor. Therefore, if we have to have a loss, the loss should be something different.

We say λ denotes the availability of the axial thrust out of the total due flow vectoring. What is the non-available part? The thrust not available is equal to Δ =1 −λ. In other words, λ tells us the fraction of the available thrust, which we call as divergence loss factor. The loss of thrust expressed explicitly is equal to Δ =1 − λ. Therefore, we have defined two terms and what are the two terms for the actual divergence effects? We defined λ as the divergence loss factor or rather we have to multiply the value of m° V_J by λ to determine be the thrust in the convergent divergent nozzle.

(Refer Slide Time: 30:40)



What is not available? The thrust, if the entire flow was axial everything would have been m°V_J therefore, 1−λ is not available. Therefore, we define capital delta as 1 −λ for losses. Now, why are we doing all this? We would like to have a nozzle in which we do not have too much of loss due to the divergent. Therefore, let us put some numbers for the losses.

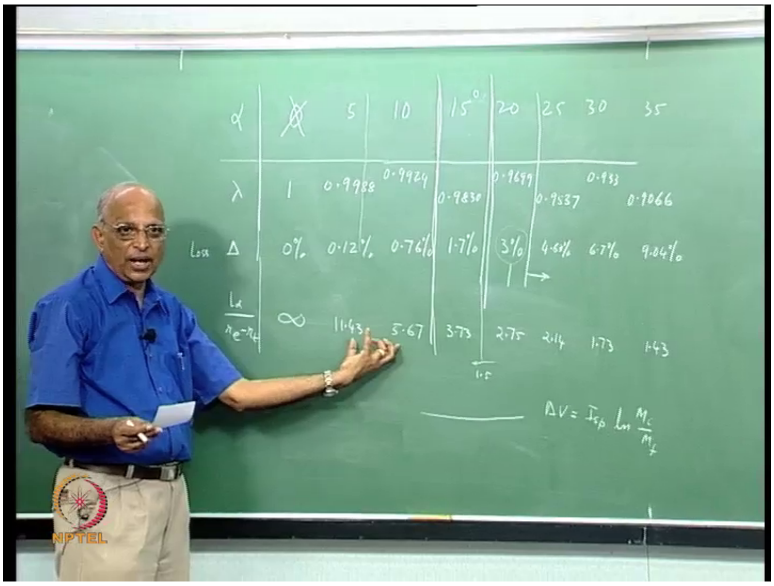
If the half divergence angle alpha of a nozzle is 0 and the loss would be zero. This is not possible because, I have a parallel walls for the nozzle. Alpha could be 5 degrees; it could be 10; it could be 15; it could be 20; it could be 25 or let us say 30. In other words, we are looking at different nozzles for which let us say half divergence angle varies from 0 degree in which case I have no divergence to other angles.

When $\alpha = 0$, cos 0 is 1; therefore, the value of 1 plus cos $\alpha/2$ is 1; that means, the entire thrust is available and the loss coefficient is 0 or in terms of percentage loss it is 0 percent.

When I have the value of alpha equal to 5^o, the value of lambda is 1 plus cos 5^o / 2; Cos 5^o is around 0.99 and the value of λ comes out to be 0.9988, and if I have $\Delta = 1$ minus lambda, it is equal to 0.12% ; that means, 0.0012. If the angle α is 10 degrees, λ is equal to 0.9924 and the loss Δ comes out to be 0.76 percent. Let us put few more values: for 15 degrees, the value λ is 0.9830 and the loss is 1.7 percent. If α is 20 degrees λ is 0.9699; $\Delta = 1 - \lambda$ gives me value of around 3 percent. If it is 25 degrees, λ is 0.9537; I will qualify these numbers and the loss is 4.63 percent or 0.0463. Well, the last value of α of 30 degrees for which lambda is 0.933 and the loss $1 - \lambda$ comes out to be 6.7 percent. If we were to put one more angle let us say 35 degrees, the value of λ is 0.9066 and the value of loss Δ is 9.04 percent. What is it that we are tabulating here? We are considering the divergent angle of the nozzles to vary from 5 degrees to 35 degrees and for each of the values, we get the divergence coefficient and also the loss in thrust in percentage. You find when I go from a semi divergent angle of 5^o, I am losing just 0.12 percent thrust; for 10^o I am losing 0.76 percent thrust; when I come to 15, I have lost already 1.7 percent thrust; when I go to 20^o the loss is quite high. We have lost 3 percent of the thrust; when α goes to 25^o it becomes 4, 6 and so on.

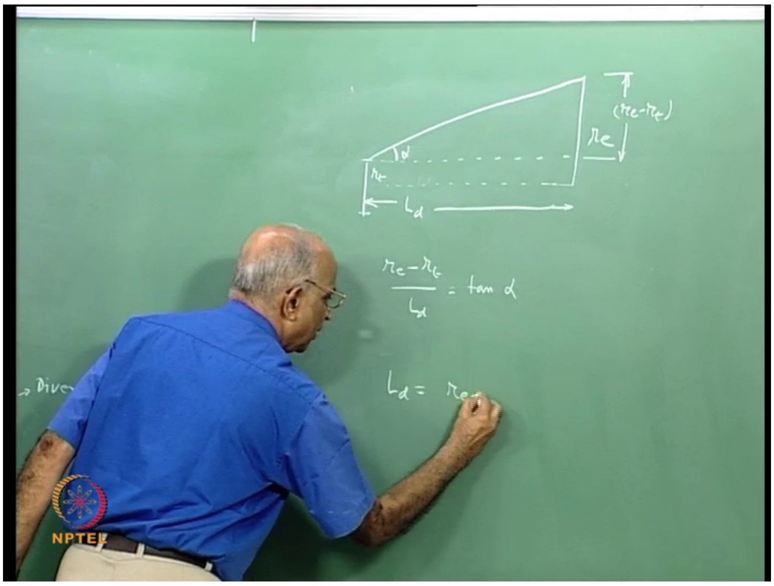
In other words, it does not appear meaningful to have any semi-divergence angle greater than 20 since the losses become substantial. In fact, we were to compare 10 and 15^o nozzle divergents, the loss for the 15^o nozzle is 1.75 times more.

(Refer Slide Time: 32:16)



If the half divergent value was 20^o, the loss is quite heavy. Therefore, the general practice therefore is to adopt some value around 15 degrees such that the loss is somewhat small. What loss? The divergence loss, but that is not the only reason. Let us try to put one more reason on to it. Let us consider the divergent part that I show here.

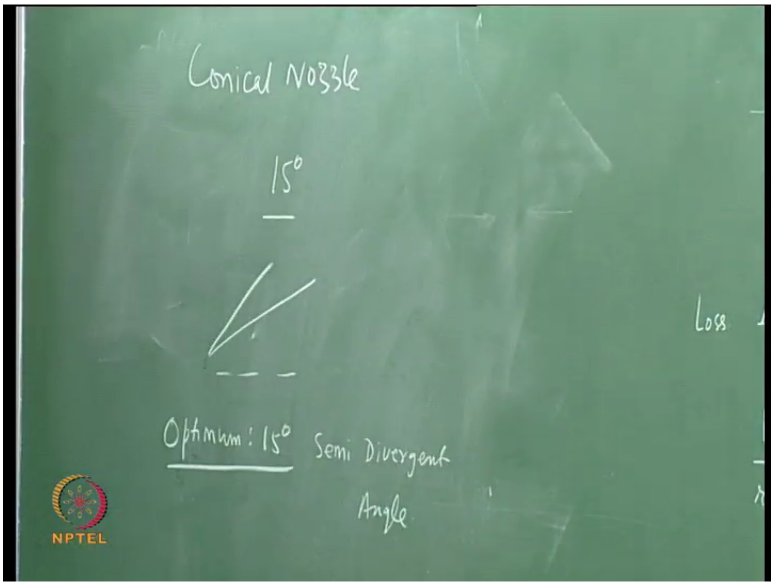
(Refer Slide Time: 37:27)



We have seen that if the divergence angle is very small say of 5 degrees or even 1 degree, the loss factor is almost going to be negligibly small. Therefore, why not have such a small angle. There is another implication as shown in the figure. We show the center line alone along the axis of the nozzle; we have the throat; the throat radius is r_t; the exit value of radius is r_e and what is α? tanα is a value of this angle: $(r_e - r_t) \div$ the length of the divergent Ld. The divergent length Ld is equal to $(r_e - r_t)/\tan\alpha$. Is it all right? All what we are saying is, if I have a small angle for alpha, a nozzle for the same exit diameter and throat diameter will be much longer. If the angle is 0^o, the length will be infinity. Therefore, let us put the length here into this Table. What do we find? Well, we just put the length of the divergent for a particular case of throat and exit diameter of the nozzle i.e., Ld divided by r_e at exit − radius r_t at the throat. This for the zero degree nozzle 0 is infinity.

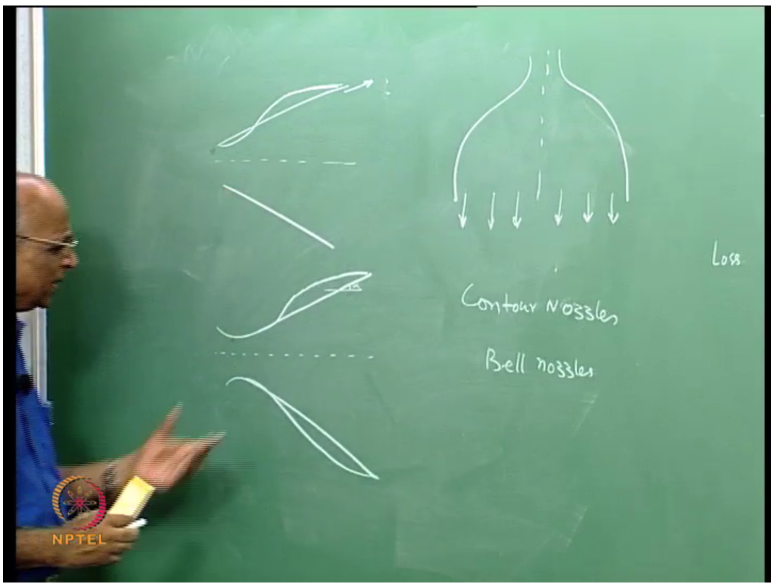
If we have a semi divergent angle of 5^o, the value becomes 11.43. If it 10^o, it is 5.67. If it is 15^o, it is 3.73. If it is 20^o, it is 2.75; 25^o it is 2.14; 30^o it is 1.73 and if it is 35^o the value is 1.43. What does this mean? The length of the nozzle is very large if the angle is very small and the nozzle length reduces as α increases., If we compare the ratio for a 15^o and 20^o degree nozzle, we have values of 3.73 with 2.75; the change is not as rapid as it is between 11.53 and 5.67 for smaller values of α. As the nozzle length becomes longer and longer, the mass of the nozzle becomes larger. We had also found earlier that ΔV, the ideal velocity provided by a rocket, is equal to you Isp or V_J into natural logarithm of initial mass to final mass of the rocket. The mass of the rocket will go up as the length of the nozzle increases. The mass of a nozzle and therefore of a rocket of small angle of nozzle divergence will be more. And as the inert mass of the rocket increases, the ideal velocity provided by the rocket will decrease. The general practice is therefore to choose a divergence angle around 15^o for a conical divergent. Mind you it is just based on the premise that we do not lose any further. We do not lose too much of thrust because of enhanced angle, but at the same time, we do not enhance too much the mass of the nozzle. We have lost only 1.7 percent of thrust and and the nozzle weight does not go up drastically as it is if we go for smaller angles. Therefore, based on this divergence analysis, we can summarize that a conical nozzle will normally have a semi divergence angle of about 15^o. We will not go for smaller angles because in that case, the nozzle becomes long and mass would go up; we will not go for larger values of angle because, if we go for larger angles, we will lose more of the thrust. Therefore, the optimum for a conical nozzle is generally kept at 15^o semi divergent angle.

(Refer Slide Time: 41:20)



Does it make sense? Now, if this part is clear I just have a few more things to tell in a nozzle. The question is, why did we address this divergence problem in such a major way? We said that the divergent part of nozzle is so chosen for α such that do not loose thrust and therefore require we smaller value of alpha. What prevents us from having a nozzle in which we can bring it back to give a very small divergence angle α at the exit? We can initially expand it out with larger angles and reduce the angle at the exit. We have the throat here; I have a conical nozzle. If we could have a small value of divergence angle at the exit, we would not loose out by the divergence loss.

(Refer Slide Time: 42:28)

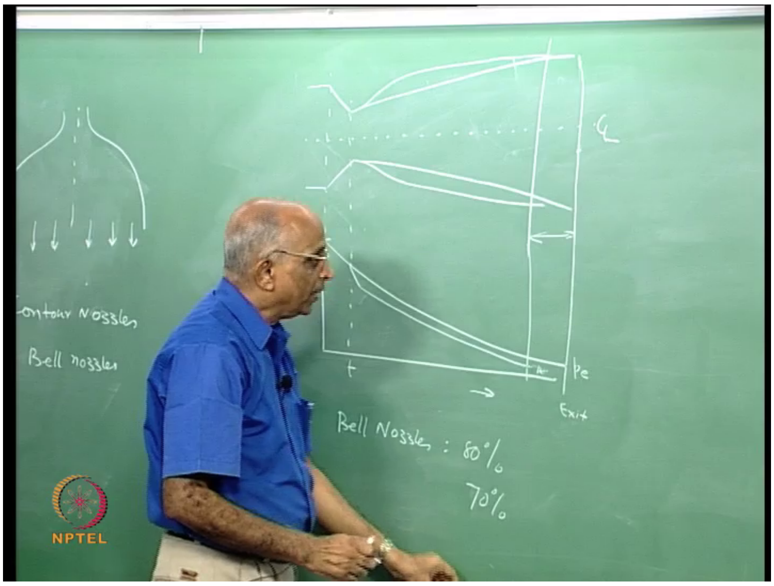


What is being said is that in the initial stages of the divergent portion, we provide a larger divergence angle and reduce it later on; that means, we have initially a higher rate of expansion followed by a smaller rate of expansion. The shape of the divergent then looks something like a bell: the shape of the nozzle looks like a bell here. This is the center line. Initially, we have larger expansion angle. We decrease the divergence angle such that the flow goes out more axially.

In other words, we have a contour for the shape of the nozzle and such nozzles are known as contour nozzles or simply as bell nozzles. I think we could discuss further for a couple of minutes on the contour nozzles. We initially expand out the gases using larger values of divergence angles. Let us plot the pressure distribution along the length of the nozzle. We know how to do it. The pressure at the throat is equal to $[2/(\gamma+1)]^{\gamma/(\gamma-1)}$. Let us first consider a conical nozzle divergent with a divergence angle of 15 degrees. We are only following up with the one - dimensional analysis. We plot the value of pressure in the nozzle as a function of distance over here. This is from the chamber; this is at the throat "t" followed by the divergent portion.

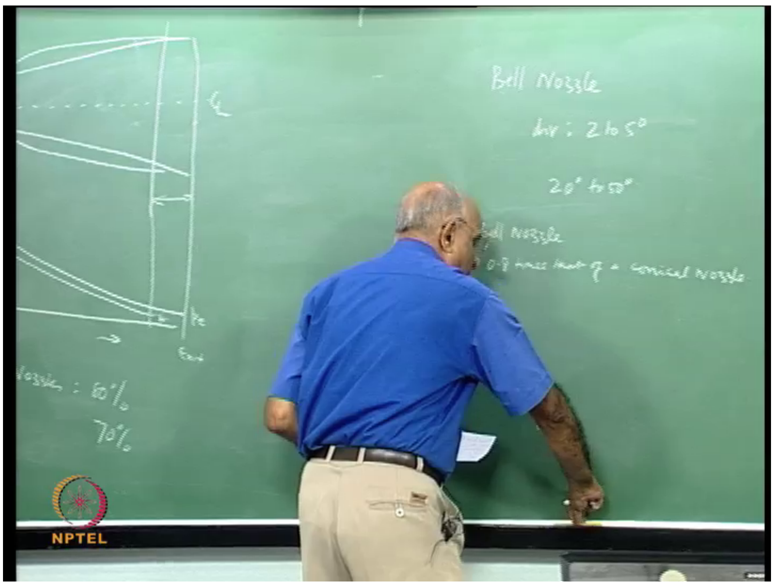
Now, the pressure keeps falling as we progress towards the nozzle exit. Let us say this is the chamber pressure value at the throat; we know how to calculate it. The pressure keeps falling further and this is the exit value of the pressure. This is for a conical nozzle. At the entrance to the divergent, wherein the pressure is still quite high, we have a more rapid expansion in the case of a contour nozzle. Since the pressure is high the flow cannot readily separate from the walls of the divergent. The divergent walls of the nozzle will continues to guide the flow. Afterwards, the angle is reduced and the flow is guided to be more axial.

(Refer Slide Time: 44:34)



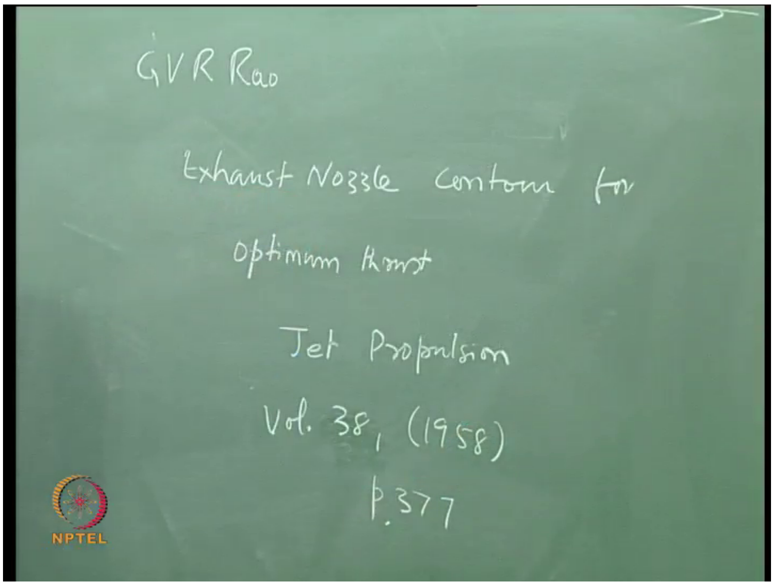
In this case it is seen possible for us to even reduce the length of the nozzle further and therefore, these bell nozzles are specified in terms of let say 80 percent bell or they say 70 percent bell. What is meant is the following. A bell nozzle whose divergent length is 80 percent or 70 percent of the divergent length of a conical nozzle is all what is required. We can terminate the length here itself because we are able to get the exit pressure or equivalently the exit area ratio. We can more effectively employ a bell nozzle than a conical nozzle.

(Refer Slide Time: 46:51)



The exit divergence angle of a bell nozzle could be between 2 degrees to something like 5 degrees, thus giving a very low divergence loss. Initially, we expand it rapidly; here we provide a large value may be 20 degrees to 50 degrees. We can make the nozzle a little more stubby or shorter with than the length of the bell nozzle being a fraction of the conical nozzle. To repeat, 80 percent bell nozzle means, the length of the bell is 0.8 times that of an equivalent conical nozzle. What do we do in a bell nozzle? We immediately expand downstream of the throat where the pressure is higher and allow the divergence to be smaller in the region of the exit such that we have less divergence loss. In fact, one paper, which we could read on this subject is by G V R Rao. He worked on it at Rocketdyne a long time ago. The paper is on exhaust nozzle contour for optimum thrust and is refereed to as Rao nozzle.

(Refer Slide Time: 48:16)



This was published in the journal 'Jet Propulsion' which preceded the AIAA journal. The volume number is 38 and the year of publication is 1958. The page number is 377 to 381.

In a bell nozzle we initially expand the gas rapidly; we have something like rapid expansion and then we have something slow expansion which does not lead to adverse pressure gradients near the walls. We can afford to have a shorter nozzles compared to a conical nozzles. Most rockets make use of the bell nozzles. If we have a bell nozzle, none of the earlier criterion like flow separation are relevant because we have higher pressure gradient along the wall. Therefore, whenever we talk of Sumerfield criterion saying exit pressure is 0.4 times the ambient; it is more applicable for conical nozzle and not exactly for a bell nozzle. I think this is all about nozzles; conical nozzle and contour nozzle. I want to spend another few minutes on different types of nozzles.

(Refer Slide Time: 50:13)

The ambient pressure decreases as the rocket moves up to higher altitudes. We had said that for maximum thrust coefficient, the pressure at the nozzle exit should be the same as the ambient pressure. Is it possible to have a different type of nozzles which can adapt to the altitude of operation. Can we make a nozzle to adapt to different altitudes starting from 0 kilometers and keep going up to 10 kilometers or 100 kilometers height? We shall deal with this in the next class and work out one or two small problems.