Rocket Propulsion

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Lecture No. # 12

Characteristic Velocity and Thrust Coefficient

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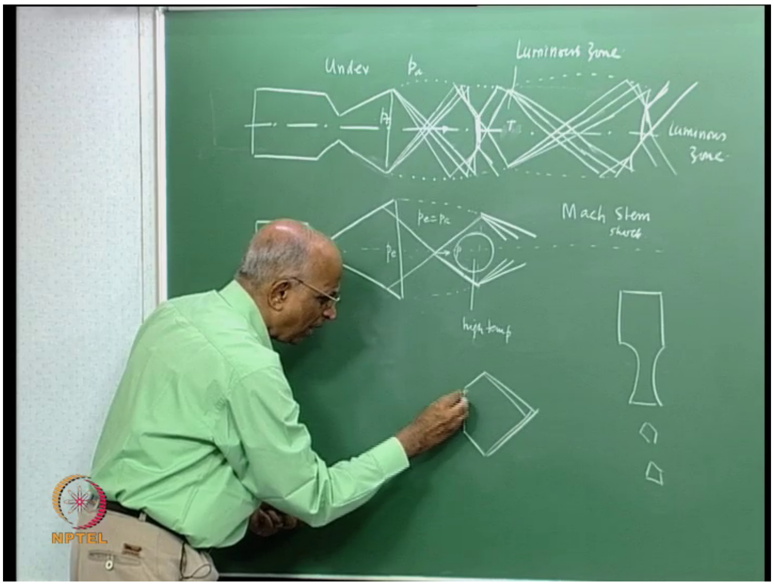

Good morning. We continue with what we were doing with nozzle area ratios. The plume from an under-expanded and over-expanded nozzle is shown here. When under expanded, the exit pressure was higher than the ambient pressure. So, the flow continues to expand; how does the flow expand when its exits a nozzle? You have series of rare fraction fans and when these rare fraction fans hits the boundary of the hot plume, they are reflected as compression waves which merge to form oblique shock waves. These oblique shock waves when they when they interact with boundary of the hot plume from the nozzle, re reflected back as rarefaction fans which again forms oblique shock waves and this process continues.

Therefore, when I look at an under expanded jet i.e., jet formed from the under-expanded nozzle as it were, I start a high value of Pe compared to Pa. I get first a set of expansion fans, which are followed by a oblique shock waves. Behind the oblique shock waves, we have compression. We therefore get a higher temperature, and this higher temperature region shows as a luminous zone. Following this, we have rarefaction fan, again compression waves and oblique shock waves. Therefore another luminous zone is obtained here.

Similarly, if we have an over expanded nozzle, the value of pressure at the exit is than the ambient pressure. Therefore, I form something like a shock wave which matches the pressure and therefore, to be able to get the value Pe equal to Pa. I have something like an oblique shock wave. The oblique shock waves continue to interact and we have a system of interacting oblique shocks. However, along the center line, after the interacting oblique shocks, I need to have the velocity at the center which is still going axially straight and the compression process causes the pressure to exceed the ambient pressure.

This is followed by an expansion following the high pressure region. Therefore, in this particular zone, I have something like a high pressure and a high temperature region. The high pressure and high temperature region promotes chemical reaction and emits light and shows up as a luminous zone.

Therefore, what is happening in an under expanded nozzle? We have a set of initial rare fraction fans, followed by oblique shock wave. In the case of over expanded nozzle, I have oblique shock waves, which thereafter result in the expansion fan. Therefore, there is a distinct change in the distribution of the luminous zones.

If the absence of expansion i.e., in the zones of compression, we should get some brighter spots light these bright zones are due to the shock wave heating and these are known as shock diamonds. We get this both for the under expanded flow as well as the over expanded flow. The only difference is that there is a phase difference in the location of the shock diamond. If we have an over expanded flow, the shock diamond comes much earlier as shown because we do not initially get the expansion fan. We start with the oblique shock wave. If we have under expanded nozzle, the luminous zone is after the first set of rarefaction fans where the pressure is higher than the ambient pressure. Therefore, a trained eye can determine if a nozzle is under-expanded or over-expanded by looking at the plume.

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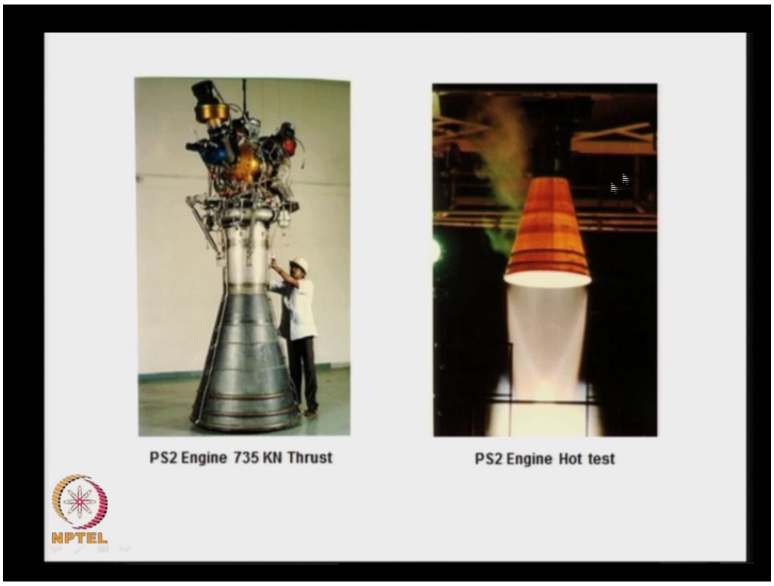

See in this case, we have oblique shock wave at the nozzle exit and the plume is seen to collapse inward. The shock diamond is formed a little later as is seen here. But in this particular test water is used for cooling the plume so that subsequent shock diamonds were not visible.

In the plume from the exhaust of the nozzle, we saw one shock diamond followed by many shock diamonds. With an inviscid flow, you could have infinite number of these diamonds. But with viscosity, there is some dissipation and that is where I showed this particular slide which shows the space plane SR 71 in which case may be you do see a shock diamond followed by second one, third one, fourth one and keep on going.

Therefore, the shock diamonds are the reflection of under expansion and over expansion in a nozzle. It is very fascinating to see some of these pictures and try to conjuncture what is really happening. But in practice there is another problem. With oblique shocks, when they interact at high incidence, Mach stem shocks are formed.

And we have instead of having a regular reflection like what we have discussed so far, we have Mach reflection and therefore another shock is formed. And this shock wave is known as a Mach shock wave. That means, we have an incident shock, we have a reflected shock and a Mach stem shock in between. I have the incident shock over here, reflective shock and the vertical Mach stem shock. Thus the shock diamond pattern is different with a Mach stem shock being formed.

And that is, how you see the diamonds in particular pictures? The diamond pattern is actually something like this wedged shape pattern. Normally, I would have expected may be incident waves like this, what we said was the diamond pattern is something like the interacting shock pattern but most often we have a Mach stem shock. We do not observe this regular reflection pattern.

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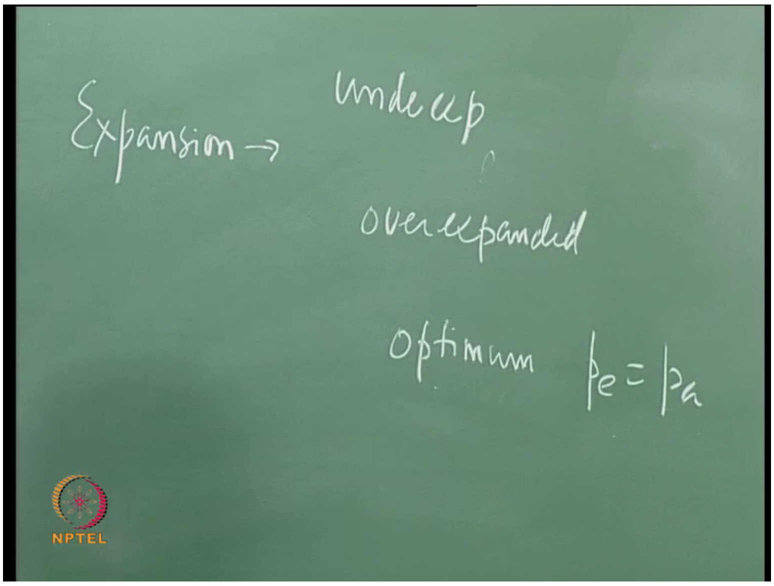

And when we get a diamond pattern it is of interest to see that whether it is due to under-expansion or over-expansion in the nozzle. The optimum is when the exit pressure of the nozzle is equal to ambient pressure in which case it goes straight without the formation of the shock diamonds.

Are there are some problems we have with a under expanded and over expanded nozzles?

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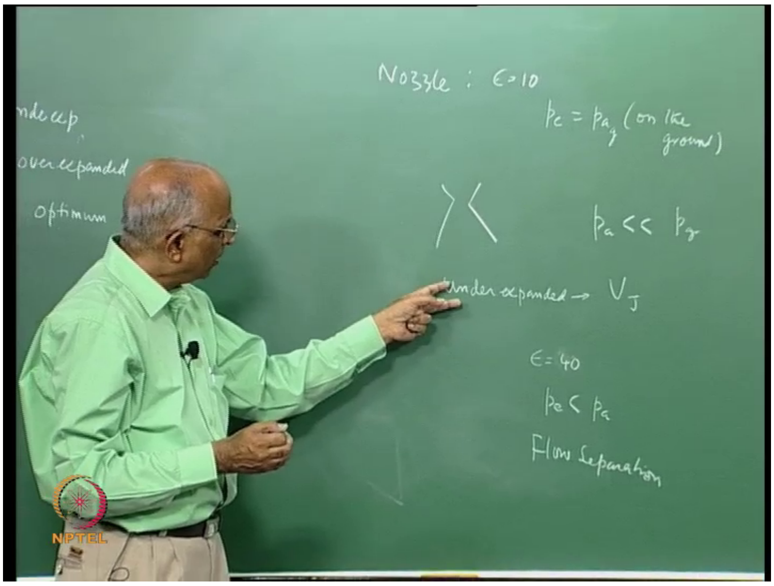

Supposing, we have a rocket nozzle with say an area ratio of let us say ten or so, which gives an exit pressure equal to the ambient pressure on the ground. Thereafter, this particular nozzle operates at altitude where the ambient pressure is lower than on ground. i.e., Pa is lower at a higher altitude than the on the ground. The exit pressure of the nozzle remains the same at the altitude as on the ground. However, the ambient pressure has reduced. The nozzle becomes under expanded at the altitude. And therefore, we will not be able to get a high value of V_J which would have been possible, had we used a higher value of expansion ratio or equivalently a higher area ratio nozzle.

Whereas, if on the other hand, we have a nozzle of area ratio of let us say forty which gives me an exit pressure which is much lower than the ambient pressure on the ground, the nozzle is over expanded. We will have flow separation which is again not desirable. I have shocks and I have as asymmetric flow separation and in essence I have side forces on the nozzle.

Therefore, we find that in general, we cannot always have optimum expanded nozzles as we use fixed area ratio nozzles. We have to live with under expansion, and because of this we loose out on jet velocity. But at the same time I cannot have over expansion because I cannot deal with flow separation as it introduces side forces. These are some problems in rocket nozzles.

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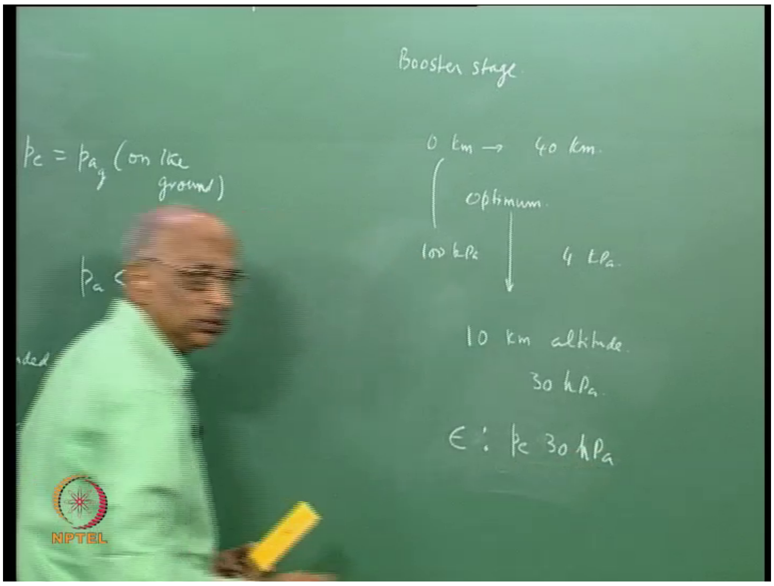

And therefore, supposing I have say a booster stage of a rocket and we define booster stage as a ground stage which has to fly let us say between zero kilometer to something say like up to forty kilometers or so. Then I cannot have a nozzle, which will be perfect or optimum for all the entire range of altitude. In other words, at zero kilometers I have ambient pressure of hundred kilo Pascal whereas at forty to fifty kilometer, maybe the ambient pressure may be something like four or five kilopascals or might be even lower. Therefore, my ambient pressure is decreasing and therefore, it is not possible for me to make a nozzle for ground condition with a single value of epsilon (ε) compatible over the range of the altitudes. We will lose too much on the jet velocity V_J.

Therefore, I make the nozzle optimum for an intermediate altitude; let us say between these two extremes - maybe the rocket has to operate from zero to forty kilometers. I design my nozzle for ten kilometer altitude in which case, the pressure may be somewhat different from hundred kilo Pascal and may be its something like nearer to let us say 30 kilo Pascal; that means, the exit pressure of the nozzle for the particular area ratio is such that the exit pressure of the nozzle Pe has a value of 30 kilo Pascal.

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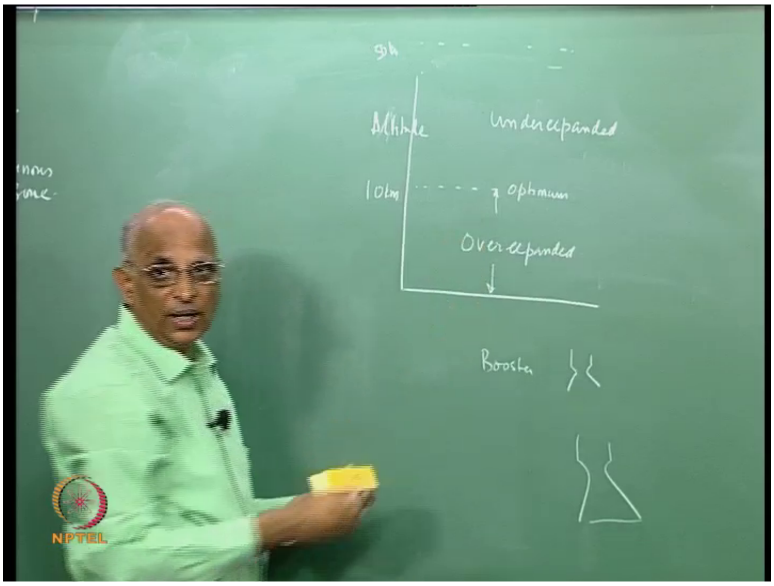

Now, this particular nozzle when the rocket takes off, what will happen? Is it over expanded or under expanded? Now let us show it the figure over here. The Y axis shows the altitude and we say this design for an altitude of ten kilometers. The rocket has to operate upto an altitude let us say of forty kilometers. This is where viz., at 10 km altitude I have the optimum nozzle therefore, initially the nozzle performs in an over expanded mode and thereafter, it performs in under expanded mode.

We are not getting the high value of jet velocity which we could have got instead of choosing a value of area ratio ε which gives me the exit pressure corresponding to ten kilometer beyond 10 kilometers. Had I chosen a value of epsilon which was corresponding to forty or fifty kilometers, I would get a much higher value of V_J. But then we cannot also afford to have an over expand the nozzle for the lower altitude of operation and get into side thrust problems. It is always that a given nozzle operates either in an under expanded mode or an over expanded mode, but we try to decrease the extent of these modes. You have seen in the earlier figures where we have shown a booster stage has a small area ratio nozzle. Whereas, if the rocket is going to operate at higher altitude, we will design a nozzle with larger value of area ratio such that it is more in the area of near to optimum throughout its flight altitudes. The optimum shifts to higher altitude for the upper stages and this something which we need to keep in mind.

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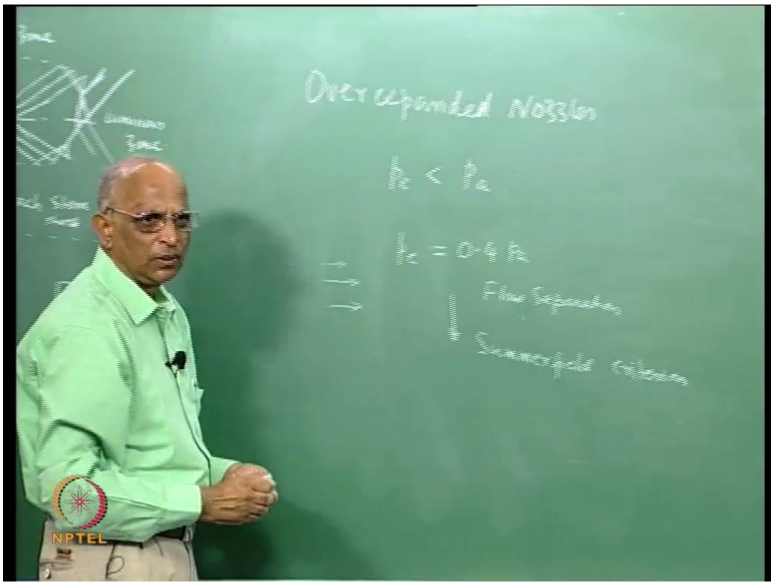

We had said that the nozzle, if over expanded, implies that Pe is less than the value of Pa. But in general, gases flow at high velocity in a nozzle divergent and therefore, we have the inertial force available as well. And in practice, when we have the exit pressure less than something like 0.4 times the ambient pressure, then only we have this problem of flow separation and shock formation even though, ideally we said that Pe just less than Pa (the ambient pressure) causes flow separation. Experiments have shown because of the inertial forces the rocket nozzle can operate without flow separation at a much lower value of exit pressure Pe than the ambient pressure. The value for flow separation is $Pe < 0.4 \times Pa$. This condition was devised by Summerfield and it is known as Summerfield criterion. What is the implication of this?

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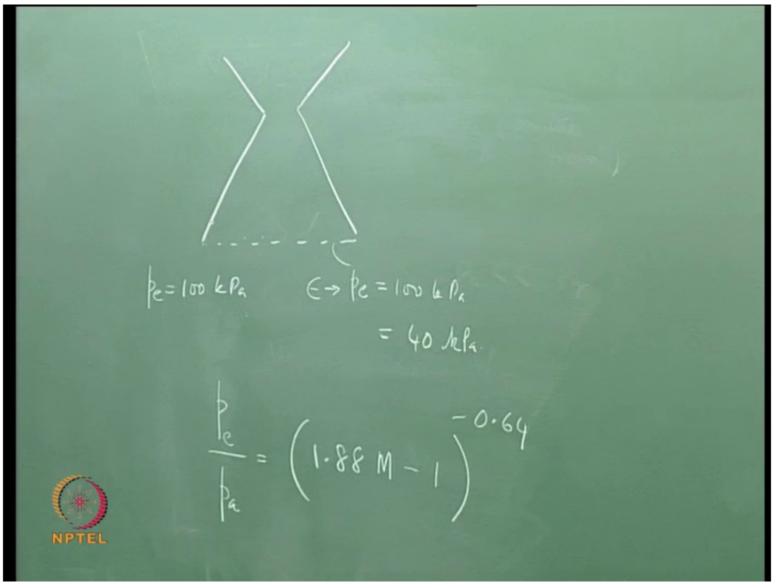

Supposing I have to make a nozzle perform on the ground as well as at altitude. The choice of the area ratio is such that the exit pressure would be about 40 kPa instead of 100 kPa. It is not necessary to have the nozzle designed for an area ratio ε for an exit pressure of hundred kilo Pascal. But it is okay for me to have a much larger area ratio such that the exit pressure is equal to much lower value of forty kilo Pascal. The inertial forces help in delaying the onset of separation. Having said this, it is also necessary to note that, this is a very accurate condition but only a sort of a thumb rule. In fact the local Mach number at the zone of flow separation affects the flow separation criterion. And the value is given by the value of Pe by Pa at the zone of separation is equal to $Pe/Pa = (1.88M - 1)^{-0.64}$. In fact based on experiments, it is not only the inertial force which delays the flow separation as a constant value of 0.4 times the ambient pressure, but it depends on the local Mach number at which the flow separation takes place. This criterion based on Mach number at the zone of separation is again based on experiments.

Therefore, I hope by now we get a feel for nozzles and the problems involved with it. We find that the nozzle area ratio ε depends on the ambient pressure. Ambient pressure keeps varying in the flight and therefore, we need variable area ratio nozzles. The moment we talk of fixed area ratio nozzles, it is also necessary for us to consider the problem of under expansion and over expansion. We lose performance by under expansion. We get side loads from over-expansion, which is harmful. Having seen these aspects, let us go to the next phase wherein, we also would like to know a little bit more about, what is the thrust? Or what is the thrust generated by the expansion in the nozzle?

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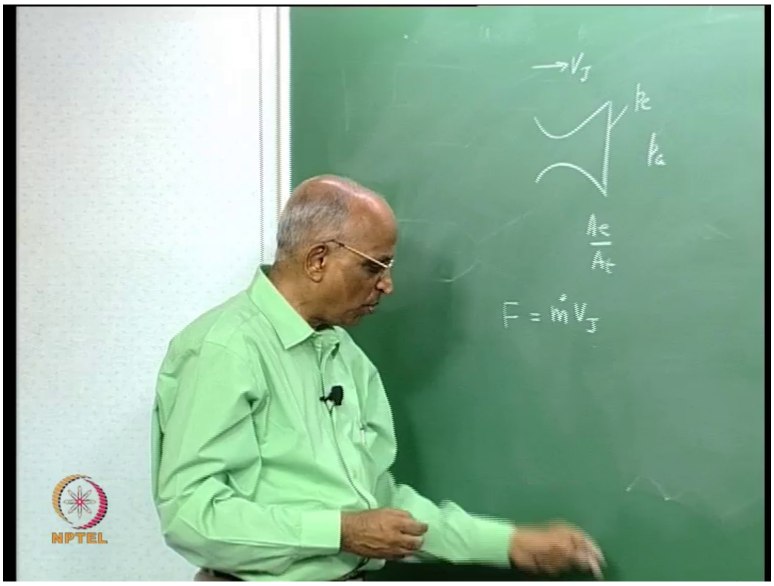

So, far we have considered the jet velocity as a function of the chamber properties; we said a convergent divergent nozzle is required; we look at the area ratio Ae/At of the nozzle. We talked in terms of the under expanded, over expanded and optimum nozzles. We are clear about this, but now the question is, what is the thrust generated by nozzle? How is the thrust generated? Thrust from change of momentum or momentum thrust $m° \times V_J$. But we have been telling that depending on the nozzle area ratio ε, the exit pressure Pe could be different from ambient pressure Pa.

Thrust could come from pressure also, because I am not able to fully expand the gas. Therefore, I have some pressure thrust just as momentum thrust. Let us write an expression for the force developed by a rocket, which we have said so far is equal to rate of change of momentum.

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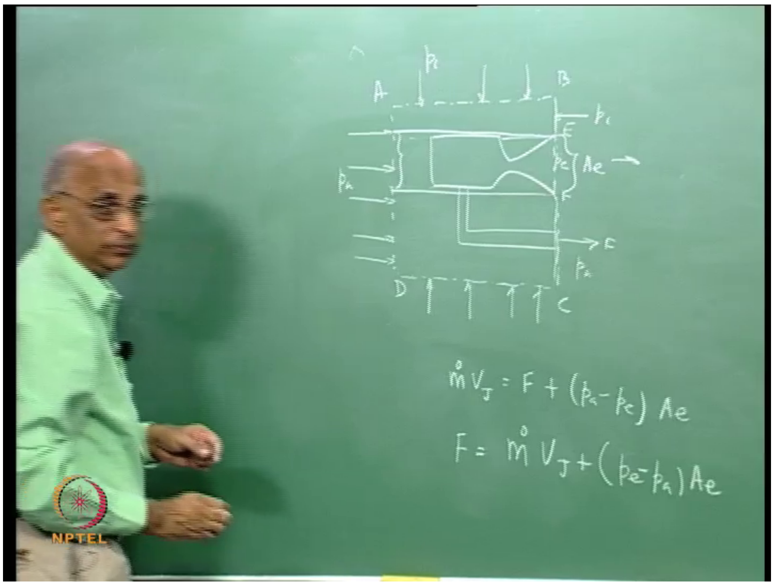

Let us put the diagram in the form of a control volume. We again start from basics. Let us imagine, we have a rocket as shown here; let the pressure at the nozzle exit be at a pressure Pe. Now what I do is, I want to find out what is the force is generated? Therefore, we clamp the rocket on the ground and we attach it to a fixture over here. We hold it on firmly with a force F such that the rocket is stationary while it is clamped but in operation. Now, we want to find out this force F which rocket develops? The exit pressure is not balanced by the ambient pressure all along the rocket and we make a control volume about the rocket as shown by the straight dotted lines.

Now we want to find out the force F. We find that everywhere on the on the walls of the control diagram, that is this imaginary dotted lines, where the pressure is the ambient pressure? Let us call these lines as AB , B C, CD and DA. The nozzle exit is EF along line BC. We find all along the lines the pressure is the same and the only place where we have a difference in pressure is in this region EF. That means, over this particular area corresponding to nozzle exit area EF, we have ambient pressure Pa is acting upstream of it and Pe acting downstream. Along all other lines, the pressure Pa is balanced acting in the opposite directions such as on AB and CD and similarly on left and right sides of the lines outside the boundary of the rocket.

Now let us write the force equation for this control volume. The momentum thrust is balanced by the pressure forces and the restraining force F the momentum component which is issuing out of the nozzle = F + force from the pressure unbalance; that is: $m^\circ V_J$ = F + (Pa –Pe) ×exit area of the nozzle Ae. Rather the thrust F = $m^\circ V_J$ – (Pa − Pe)Ae. The momentum thrust is partially offset by the unbalanced value of pressure force at line EF giving a value of force (Pa – Pe) × exit area of the nozzle in the opposite direction.

Therefore, the force or rather the net thrust is equal to $m^\circ \times V_J + (Pe-Pa)A_e$. We have now modified the thrust equation in which we originally considered only the momentum thrust and we now incorporate the pressure thrust in it.

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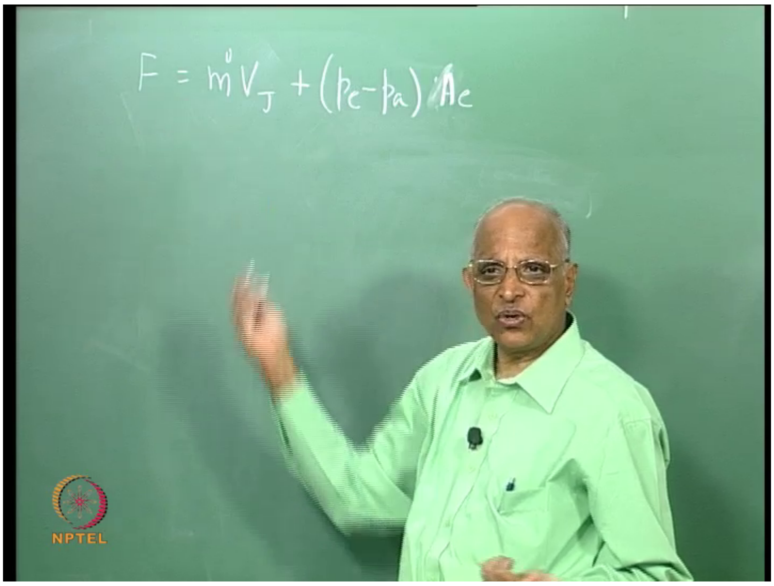

To repeat; we had got an expression as F is equal to $m^\circ \times V_J$. Now we add the pressure term (Pe –Pa) × the value of the exit area. Well, if we want to get the thrust developed by the rocket; we need the expression for $m^\circ V_J$. We have already derived the value of V_J. This was $\sqrt{2\gamma RTc/(\gamma-1)[1-(pe/pc)^{(\gamma-1)/\gamma}]}$. Therefore, if we can derive an expression for m°, we can find out the momentum thrust to which we can add the pressure thrust. Let us now calculate the value of m° that is required.

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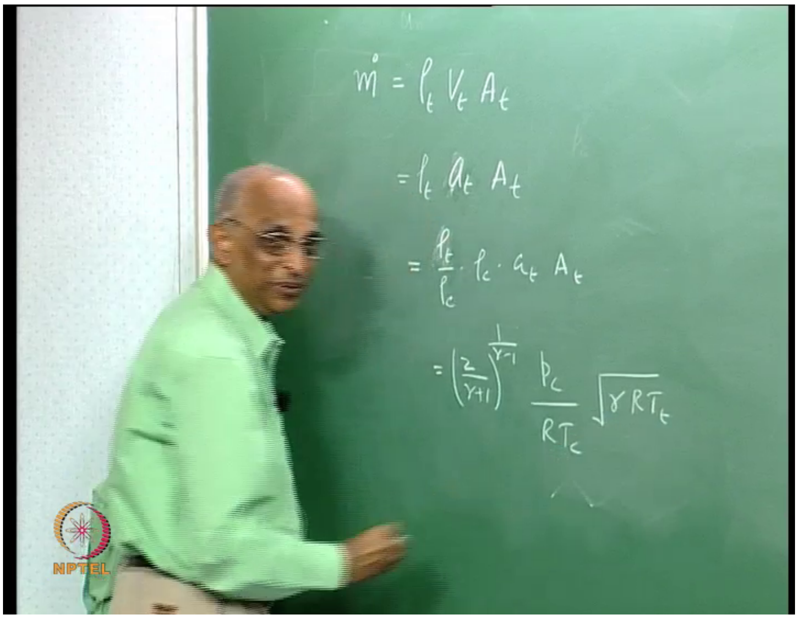

$m°$ = ρ_t V_t A_t and is better that we use the reference as At, because at the throat the flow Mach number is unity. Therefore, we substitute V_t = a_t viz., sound speed at the throat. Now can we write it in a form which is easy to determine. Let us do it. We have ρ_t/ρ_C = $[2/(\gamma+1)]^{1/(\gamma-1)}$. We derived this expression for ρ_t/ρ_C in the last class from Pt/Pc and Tt /Tc .

What is ρ_C? Pc/RTc from the ideal gas equation. The sound speed $a_t = \sqrt{\gamma R T_t}$ and we need to consider the local condition here at the throat. A_t is known.

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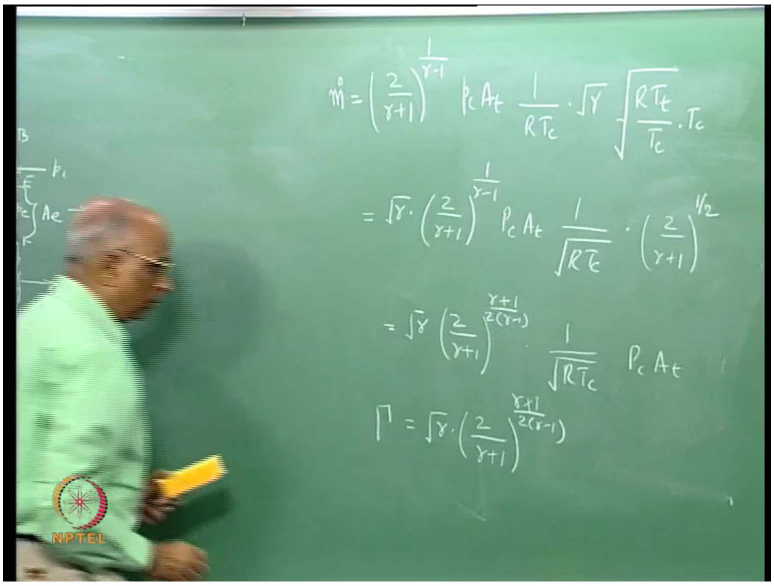

Therefore, what is the expression for the mass flow rate m°? It is equal to $\sqrt{}(2/\gamma+1)^{1/(\gamma-1)} \times p_c /(RT_C) \times A_t \times \sqrt{\gamma RT_t}$. T_t can be written $T_t/T_C \times T_C$ and $T_t/T_C = 2/(\gamma+1)$. $\sqrt{RTc}$ in the numerator with RTc in the denominator gives $\sqrt{RT_C}$ in the denominator. We therefore get the value of $m° = \sqrt{\gamma} \times [2/(\gamma+1)]^{\gamma/(\gamma-1)} \times Pc \times A_t \times 1/\sqrt{RTc} \times \sqrt{T_t/Tc}$.

We have taken $\sqrt{RTc}$ outside and are therefore left with $\sqrt{Tt/Tc}$.

Now instead of $\sqrt{Tt/Tc}$, we can write it as $\sqrt{2/(\gamma+1)}$ and we now we get the final expression. It equals $\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times 1/\sqrt{RTc} \times Pc \times A_t$. The terms containing γ are function of γ alone. Let us therefore define $\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} = \Gamma$.

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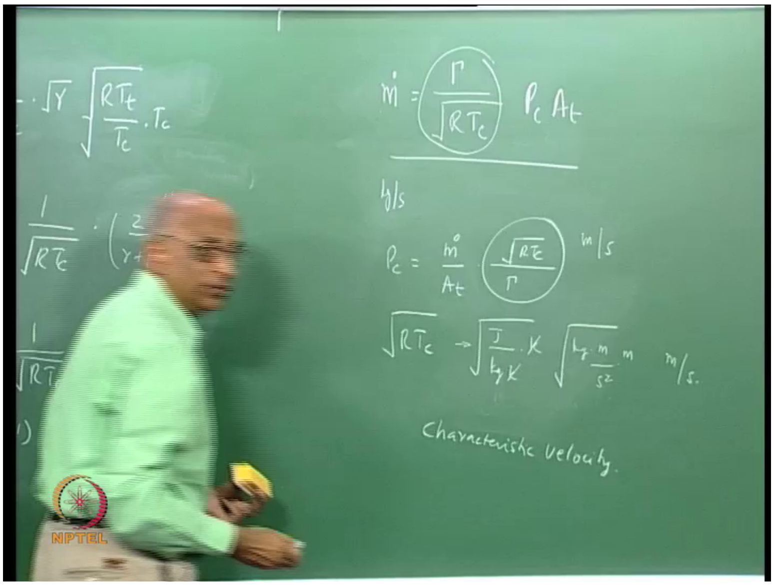

In which case the value of $m^\circ = \Gamma/\sqrt{RTc} \times Pc \times A_t$. This gives us the net flow rate m° and is the mass flow rate through the nozzle.

You find that for a given mass flow rate in kilograms per second, if we were to use this expression, the term capital gamma divided by under root $\sqrt{RTc}$ transfers the mass flow rate into pressure Pc. We have something like a transfer function. What do you mean by this transfer function? If I were to rearrange this equation, we get Pc as equal to $m^\circ \div A_t$, that is mass flux through the throat, $\times$ $(1/\Gamma)/\sqrt{RTc}$ or rather $\sqrt{RTc}/\Gamma$. We can interpret this by the following: for a given mass flux through the nozzle this represents something like as a transfer function which will give me the chamber pressure; that means, $\sqrt{RTc}/\Gamma$ is a function which converts the mass flux into pressure.

Let us examine the unit of $\sqrt{RTc}$. R has units of Joule per kilogram Kelvin, Tc is in K. the unit is therefore Joule which is Newton meter. Newton meter is equal to kilogram meter per second squared into meter and therefore the unit is meter per second. The unit is of velocity. Γ does not have any units. Therefore this transfer function given by $\sqrt{RTc}/\Gamma$ by capital gamma has unit sof velocity and is called as characteristic velocity C*.

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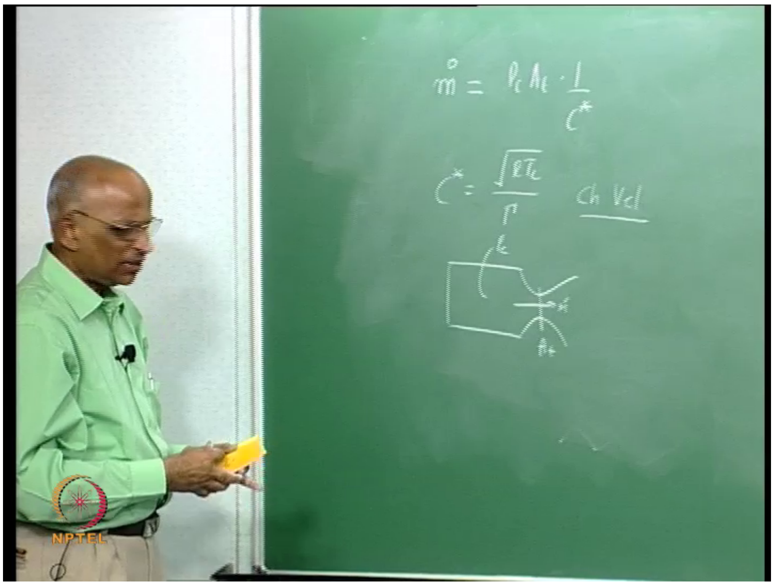

Therefore we can now write, the value of $m° = Pc \times At \div C^*$, where C star ($C^*$) is defined as a characteristic velocity which is equal to $\sqrt{RTc}/\Gamma$.

We have just defined the transfer function term C^* and because we find that the unit is velocity and we call it as characteristic velocity. And therefore, given a rocket; assume have a rocket chamber with the throat of area A_t m^2, if we know that the mass flow through the nozzle is so many kilograms per second, we can go back using this transfer function, find out what is the value of chamber pressure by using the transfer function which we call as characteristic velocity C^*. It is an extremely important parameter, to characterize the mass generation rate of a rocket. We will come back to this later on.

Therefore, what is it that we ended up doing? We wanted to find out the value of m° and the value of m° was $\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times 1/\sqrt{RTc} \times Pc \times A_{t.}$ This multiplied by V_J + (Pe –Pa) ×Ae is the force or thrust generated by the rocket.

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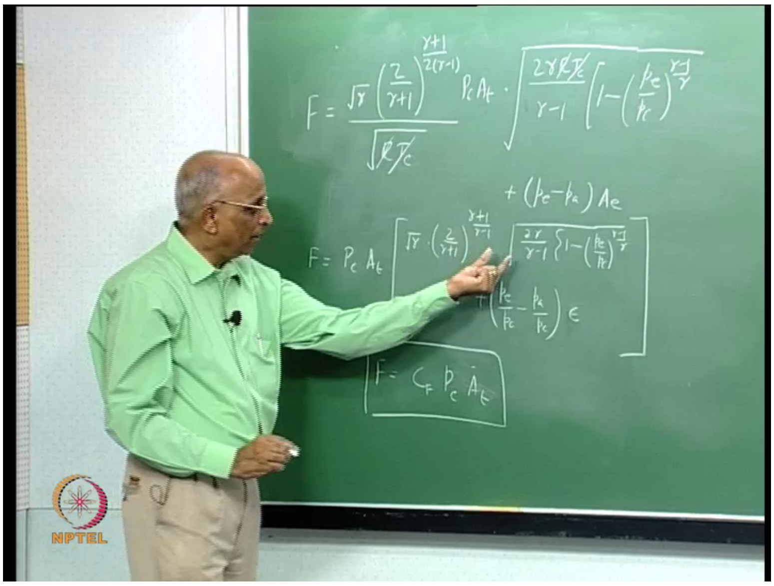

Let us simplify this expression and get the value for the thrust generated by a rocket. We find that F = $\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times 1/\sqrt{RTc} \times Pc \times A_t \times V_J$ given by $\sqrt{2\gamma RTc/(\gamma-1)[1-(Pe/Pc)^{(\gamma-1)/\gamma}]} + (Pe - Pa) \times A_e$.

Therefore, now if we take PcAt outside and we simplify this whole expression again, can we remove some terms? We get RTc over here and this was equal to R into chamber temperature Tc. Therefore, RTc gets cancelled and therefore the expression for the thrust is $F = Pc \times At \times \{\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times \sqrt{2\gamma/(\gamma-1)[1-(Pe/Pc)^{(\gamma-1)/\gamma}]} + (Pe/Pc - Pa/Pc) \times \varepsilon \}$, where $\varepsilon = Ae/At$.

Let us understand each term properly. This whole term within curly bracket $\times\{\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times \sqrt{2\gamma/(\gamma-1)[1-(Pe/Pc)^{(\gamma-1)/\gamma}]} + (Pe/Pc - Pa/Pc) \times \varepsilon \}$ if denoted by a coefficient C_F; we can say the force generated by a rocket or thrust generated is equal to C_F into chamber pressure into At, where $C_F = \times\{\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times \sqrt{2\gamma/(\gamma-1)[1-(Pe/Pc)^{(\gamma-1)/\gamma}]} + (Pe/Pc - Pa/Pc) \times \varepsilon \}$.

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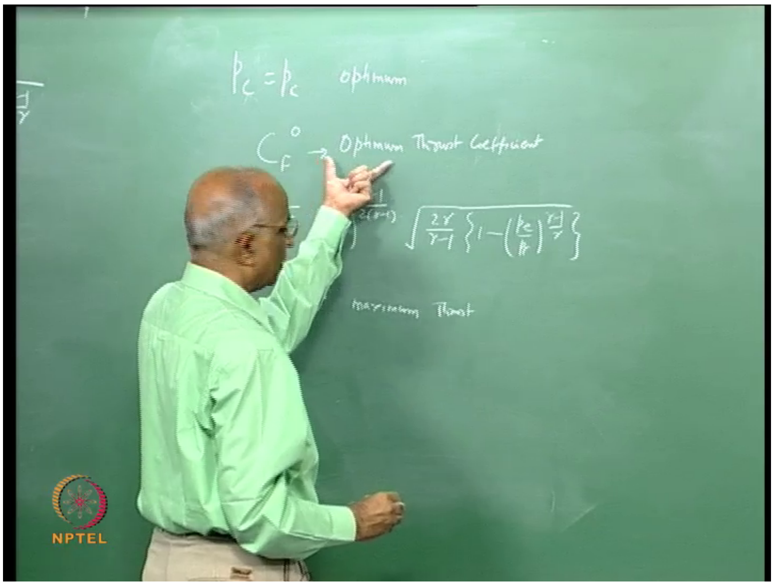

Let us first find out the value if we have the value of Pe by Pc as an optimum i.e., when the expansion in the nozzle is to the ambient pressure or Pe = Pa. The value of C_F when Pe is equal to Pa i.e., for an optimally expanded nozzle. There in this case this pressure term becomes zero. We denote C_F under optimum expansion as C_{F0}.

This pressure term is out, but then you are increasing the value of Pa. The value of C_{F0} corresponding to the value of Pe is equal to Pa will be a maximum and that is denoted by C_{F0} which corresponds to the optimum nozzle. This can be shown by differentiating with respect to Pe/Pa and equating to zero to find the maximum C_F. We get $C_{F0} = \{\sqrt{\gamma} \times [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} \times \sqrt{2\gamma/(\gamma-1)[1-(Pe/Pc)^{(\gamma-1)/\gamma}]}\}$.

If we have the exit pressure not being equal to the ambient pressure i.e., we have let us say an over expanded nozzle. We get a negative pressure thrust term. If we have an optimum expansion in which Pe is equal to Pa, this becomes zero. If we have an under expanded nozzle, we have more thrust compared to cases when Pe is equal to Pa. Hence we get the maximum thrust F when Pe is same as Pa.

This is because of the contribution of momentum thrust and the pressure thrust. The condition C_F is equal to C_{F0} implies the maximum thrust when the nozzle is optimally expanded. This means that an adopted nozzle always gives maximum thrust. Otherwise because of under expansion, we lose thrust. We are not able to get sufficient momentum thrust and the pressure thrust is not adequate to give a thrust greater than the deficit.

Let us take a look at some other results from this expression. You know, all we did was to derive this value of C_{F0}, which we got for the optimum value.

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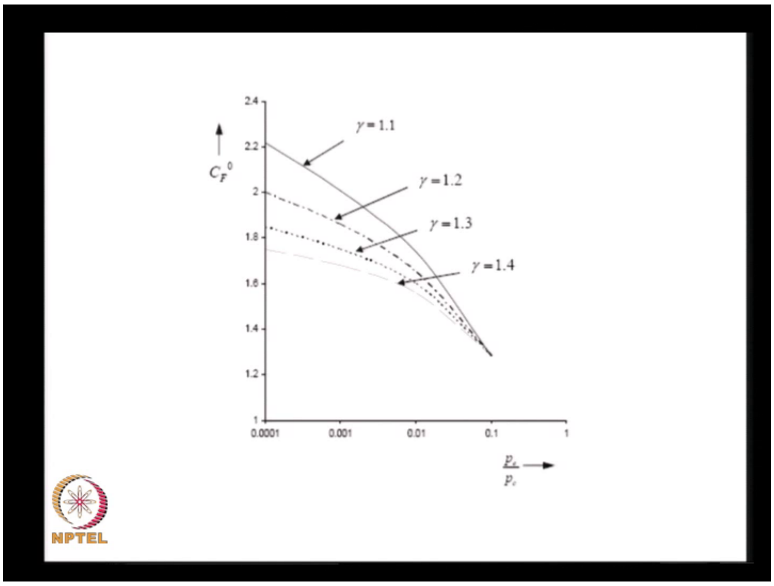

That is C_{F0}, the thrust coefficient also represented as C_F^0 for the optimum thrust conditions. What is it we find? As we increase the value of Pe/Pc, that means in this slide it is the inverse of Pc/Pe, we have higher expansion ratios, higher values of Pc or lower values of Pe, we get a higher and higher value of C_F^0. The values, which are realizable for values of $\gamma = 1.4$ are shown. It is seen that at the lower value of γ, we have a higher values of C_F^0 especially at smaller values of Pc/Pe. The value of the thrust coefficient is sensitive to the value of gamma, in addition to being a function of Pc/Pe.

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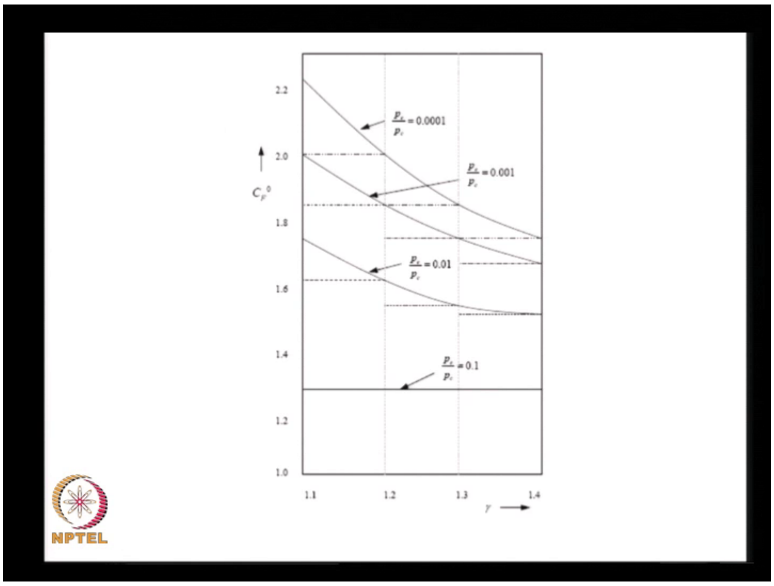

The same results are plotted on this particular slide, wherein the optimum thrust coefficient is shown as a function of γ. We find that at large values of Pe by Pc, γ does not influence the thrust coefficient C_{F0} coefficient at all. If we have a small values Pe/Pc, variations in γ cause a considerable change in the thrust coefficient. That means, a decreased value of gamma will give us a higher value of C_F^0.

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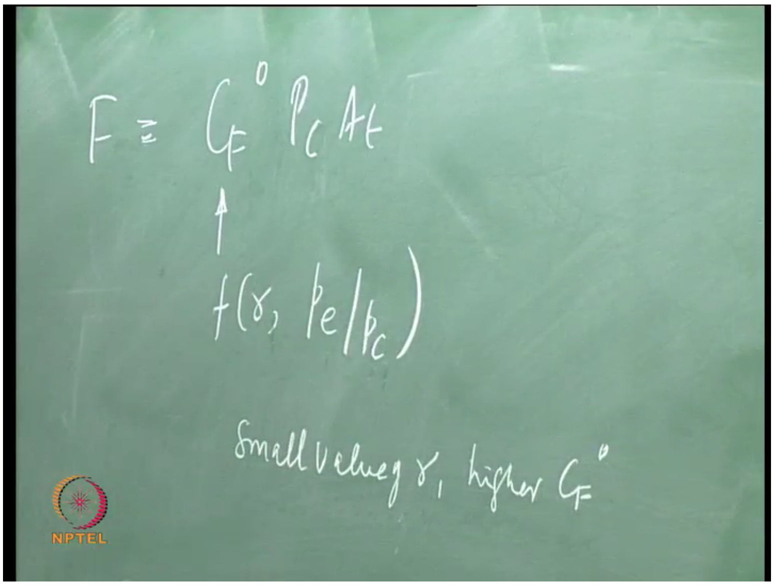

Therefore, we have looked at the thrust coefficient for optimum thrust viz., when you have a case of Pe = Pa, for which the thrust F = C_F^0 × Pc × At, we find that the value of

C_F is also, function of gamma in addition to being a function of Pe by Pc. A small value of Pe/Pc or a larger value of chamber pressure gives us a higher thrust coefficient. The value of gamma also influences the C_F. If the value of Pc/Pe is not large, for instance we have a low chamber pressure, then gamma is not very influential. A small value of gamma will give a higher value of C_F^0. This is all about thrust generated in a nozzle.

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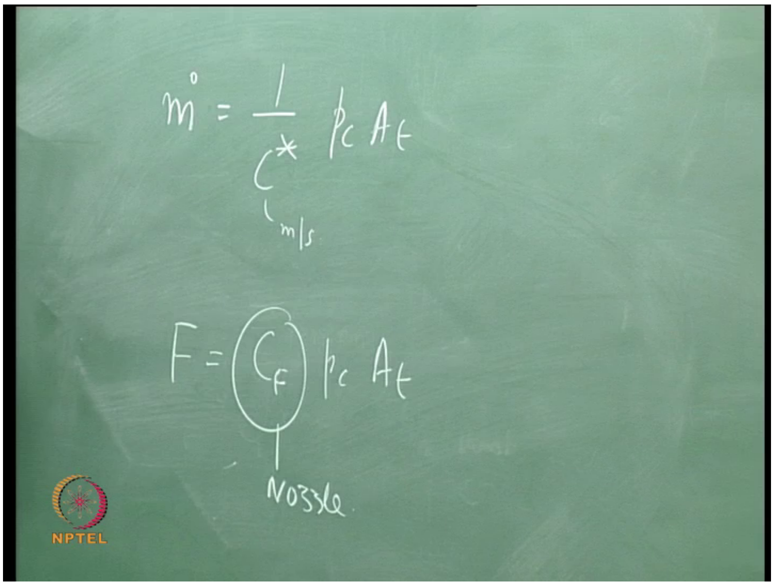

We told that m° viz., the mass flow through a nozzle is m° = (1/C*) × chamber pressure Pc ×throat area At, where C* is equal to √RTc/Γ, the unit being meter per second. And we said, force in the nozzle is equal to C_F (whether under expanded, optimum or over expanded conditions) into Pc into At. Therefore, whenever we make a rocket, we evaluate the it for C_F and C*. But C* does not come from the nozzle, since we are talking of the transfer function between mass flow rate and the chamber pressure. That means C* comes from the chamber. Whereas, the thrust coefficient C_F tells me what a nozzle is doing; it takes the chamber pressure multiplied by a coefficient, it gives me the force.

Can we put the performance of a nozzle together? I think we need to go a little deeper into these two particular expressions. Therefore, let us write out these two expressions in a slightly different form and then infer. So far we assumed the flow in a nozzle is adiabatic and reversible i.e., isentropic flow and also one dimensional flow.

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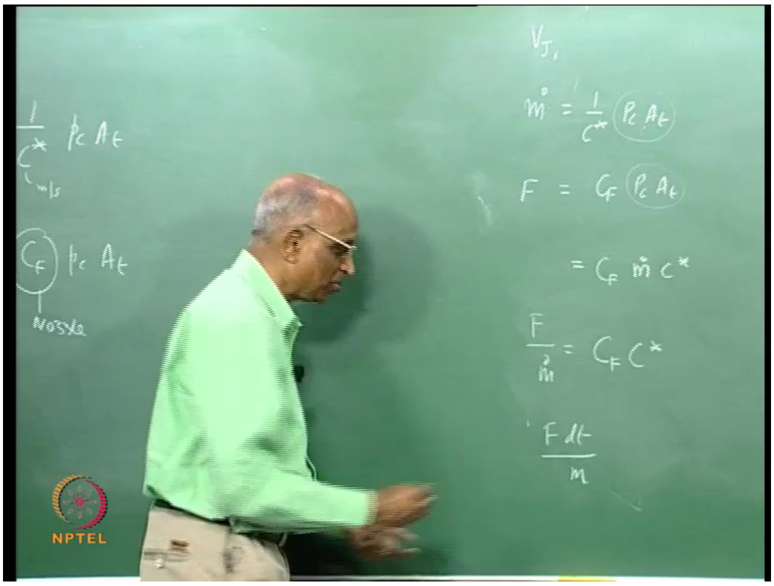

And for one-dimensional flow, we drive all the expression for V_J, we derive the expression for under expanded, over expanded, may be flow in the plume outside the nozzle and all that. And we find that we can write m° is equal to 1 over C* into Pc into At. We also derived the expression for the thrust and we find it is equal to the thrust coefficient C_F into Pc into At. Now if we play with these two equations, I find over here Pc into At is common for both these equations. Therefore, can we somehow put this together? Can we substitute Pc into At from mass flow equation into the force equation?

If we were to do it, we get Pc ×At equal to m° into C* and substituting in the expression for thrust F, we have F = C_F × m° ×C* .

And what is F divided by m°? What is the thrust per unit mass flow rate? That is we are talking of F d t impulse divided by mass of the propellant m°dt, or impulse per unit mass is a specific impulse Isp. We therefore have Isp = C_F × C*. This is an expression for specific impulse.

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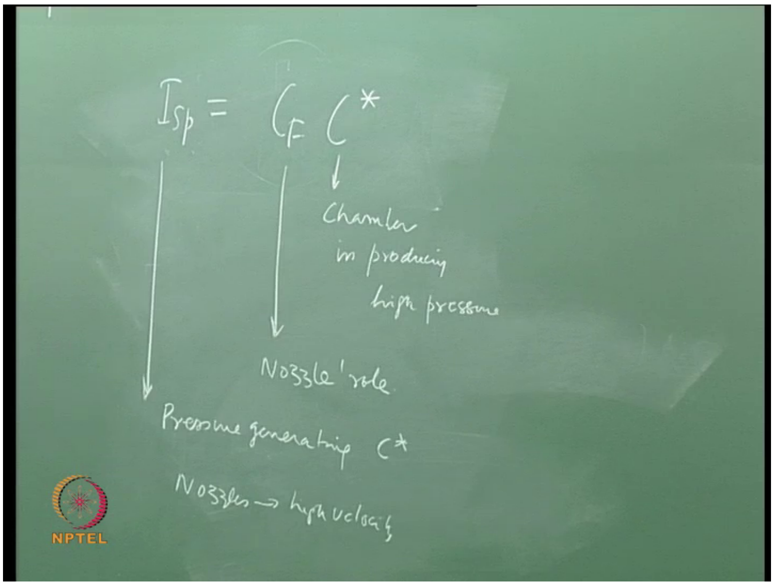

The expression for Isp was V_J. What was V_J? V_J was, when the exit pressure was matching the ambient pressure that is when we derived the expression for the jet velocity. Now you have the contribution coming from the exit pressure also.

Therefore, what is it the net inference? The specific impulse Isp depends on the performance of the chamber, and what does a chamber do? Chamber is generating or producing high pressure gases. It gives C*. And what does a nozzle do? It expands the gases and provides a value of C_F? C_F is representative of the nozzle converting this high pressure into high velocity. Something like the nozzle effectiveness or the nozzles role and therefore C_F is the figure merit of the nozzle. It is an index of how effectively, it converts the high pressure gas into velocity

And C* is a figure of merit of the chamber, in that it tells us how high pressure is made available in nozzle? Therefore, you have a composite index of C* and C_F. Therefore the specific impulse or Isp is a product of pressure generating capacity in the rocket and the effectiveness in the gases being expanded.

And how is pressure generated? It is through C*. Whereas, once the pressure is generated, the nozzle helps in generating the high jet velocity. Therefore, you have both the chamber and the nozzle contributing to the jet velocity or specific impulse. There is a propellant to generate hot gas and C_F which means the effectiveness of the nozzle. Therefore, Isp $= C_F \times C^*$.

But so far, we considered only ideal one dimensional isentropic flow and it is necessary for me to go back and apply corrections since everything cannot be ideal. Therefore, there has to be some something like an efficiency.

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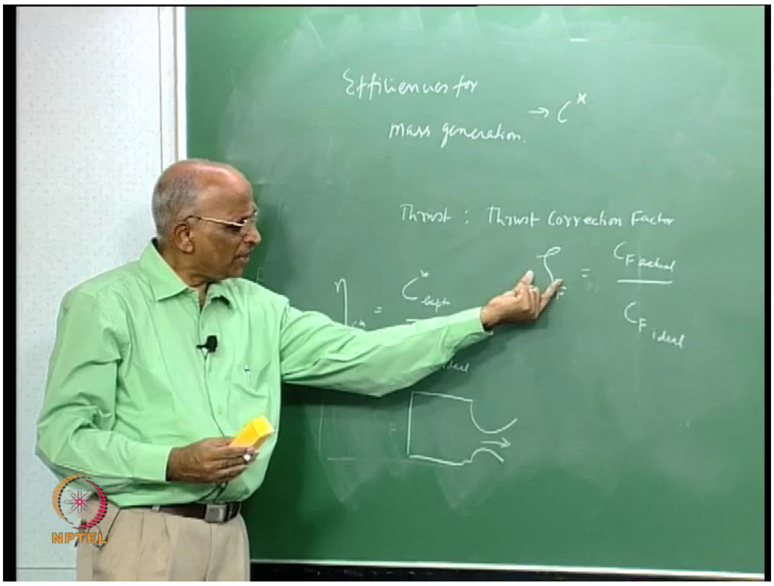

Therefore, can we talk in terms of efficiencies of mass generation and efficiencies for thrust production? The mass generation, we decided in terms of C*; therefore, if we have to have an efficiency for mass generation, it is equal to C star which is experimentally observed divided by the ideal value which we derived in this class to be ideal value and equal to $\sqrt{RTc}/ \Gamma$.

Therefore, all what we do is an experiment; we have a rocket and find out the rate mass at which the mass is delivered out. Find out the ideal using the calculation, and you say this is eta C* (η_{C*}). We will find that the values are quite high of the order of 98 to 99 percent. We will do some problems on this later on.

How do you get the thrust efficiency? We call it as thrust correction factor ζ. And you denote it by ζ_F, a correction factor which is equal to C_F actually measured in a rocket chamber divided by C_F which we calculated under ideal conditions. And we use the ideal value, to find out the efficiency or the correction coefficient. And again these are quite large for nozzles of the order of again about 97 to 98 percent. Therefore, what we have done in today class is, we looked at the high temperature and high pressure gases generated by the propellant and being expanded in a nozzle.

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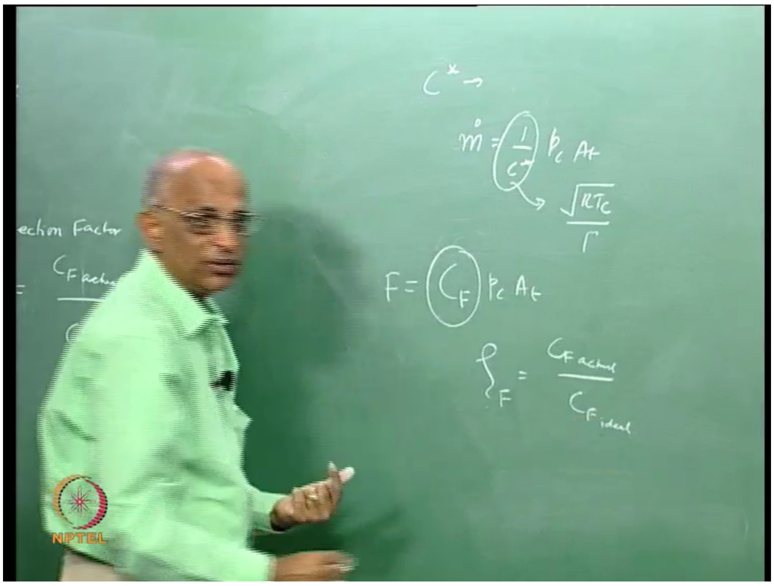

We expressed the mass generation rate in terms of C^*; that means, C star tells us what is the rate at which hot gases get generated from the propellant? And therefore, we wrote it as $m^\circ = (1/C^*) \times Pc \times At$ and we called C^* as a transfer function. And the expression for this is extremely simple, is equal to $\sqrt{RTc}/\Gamma$.

We also talked in terms of the thrust coefficient C_F, which described the thrust developed by a rocket; $F = C_F \times Pc \times At$. We determined the expression for C_F, and also how C_F varies? And we talked in terms of a thrust correction factor ζ_F, which is equal to C_F actual divided by C_F which is calculated based on ideal, one dimensional isentropic flow in a nozzle.

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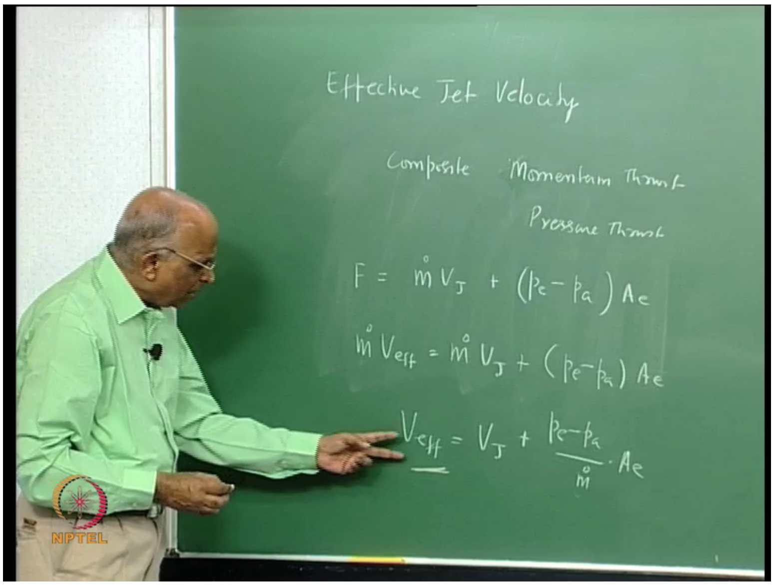

Sometimes an effective jet velocity is defined in the literature, to determine the composite of V_J or the velocity thrust or the momentum thrust and the pressure thrust. Let us examine what this effective jet velocity is? Let say that the total thrust is given by the momentum thrust which is equal to m° into V_J and the pressure thrust is pressure (Pe –Pa) × Ae. This becomes the pressure thrust. Now if we have to put both the momentum and pressure thrust in terms of an effective jet velocity, we could say F is equal to m° × V effective and this is equal to m° × V_J + (Pe – Pa) × Ae. And therefore, the effective jet velocity is equal to the jet velocity plus, you have (Pe –Pa) ÷ m° × Ae. This is defined as effective jet velocity and when a nozzle is not adapted to the ambient pressure.

This means that when the exit pressure is different from its ambient pressure at that particular altitude, the effective jet velocity is different from the jet velocity at the exit of the nozzle; this is to take care of the contribution of pressure thrust in addition to the momentum thrust. We will continue with nozzles in the next class, but in the next class we shall try to take a look on how to shape a nozzle? And about the deviations from the ideal cases that we have studied and the approximations.