Rocket Propulsion

Prof. K. Ramamurthi

Department Of Mechanical Engineering

Indian Institute of Technology, Madras

Lecture No. # 10

Nozzle Shape

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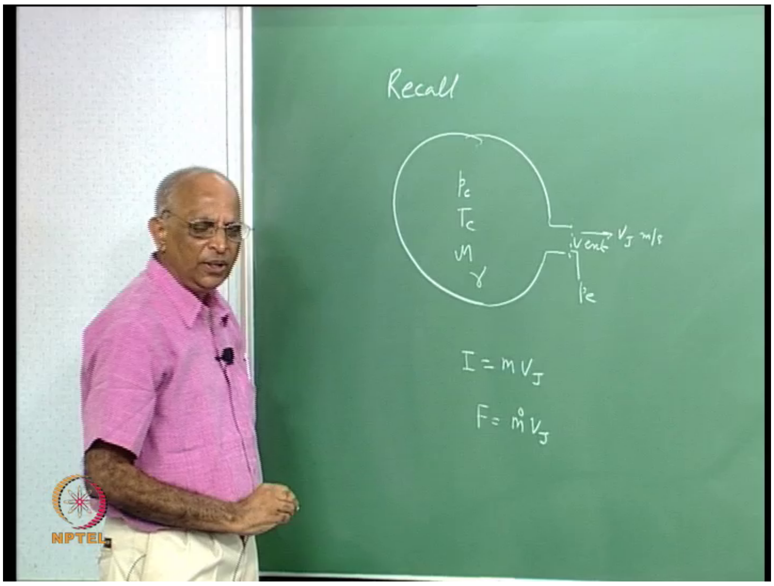

We will continue with the subject of nozzle today. If we recall what we did in the last class, we had a chamber and this chamber had a gas at a pressure pc, temperature Tc, the molecular mass of this gas was M gram per mole. And we expanded this gas out through a small opening, which we called as a vent; we were able to calculate the exhaust jet velocity in meters per second. We found out the condition for which V_J is quite large; we found that the temperature of the gas must be large, the value of the chamber pressure must be large, and the molecular mass of the gas must be small. We also examined the variation with respect to gamma, found that gamma is not very influential, especially when pc is small or the ratio of the exit pressure pe to the value of pc is somewhat high or equivalently the pc value is small compared to pe, then gamma is not influential; this was seen from the expressions which we derived.

Now, for a rocket, impulse(I) is equal to mass which is ejected out multiplied by the jet velocity. We also said that the force is equal to d/dt of I, which is equal to m° into V_J.

Therefore, we are interested in a high value of this velocity with which the mass efflux leaves the chamber.

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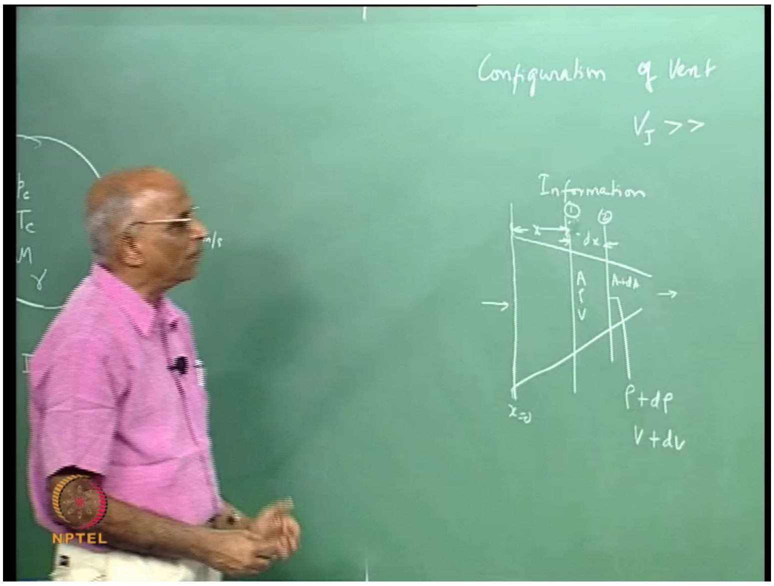

Therefore, the question is whether there anything like a configuration or shape of vent, which can give a high value of jet velocity. And this is where we get started with and afterwards we will try to find out the conditions for this high jet velocity to be realized. We will have to look at matching of the exit pressure with respect to the chamber pressure and that is what I propose to do in today's class. We will also physically try to understand, if there is anything like some information transfer between the outside and inside of a chamber. Let's get started with this background.

We have a nozzle and we are interested in the shape of this nozzle; therefore let us have shape like this. The shape is such that at a distance, let us say this is x and is equal to 0 at the beginning, at a distance x from the initial origin the area of the vent is A. Let the density of flow through this be ρ, let the velocity of flow be V, at this particular section x at a distance x which is the reference plane. Let us consider the variation in the properties at a distance x plus dx.

The area is different from A, and I want to find out the configuration of this particular vent which gives me the maximum V_J; therefore let the area at x + dx be A + dA, let the density at this section be ρ + dρ and the velocity at this section V + dV. My main aim is to find out what must be the shape, such that I get a high value of V_J. Therefore I just look at these two sections; let say section one at x, at which the area is A, density is ρ, and velocity is V. At section two, dx away from this section the area is A + dA, small change in area, I think according to this figure dA should be negative, but I just have a general notation A plus dA, let the density be $\rho + d\rho$, and the velocity V + dV. Now I say that the flow is steady or constant, in other words whatever comes in at x here flows out through this particular opening or vent. The mass flow rate m° in kg per second is equal to ρ A V

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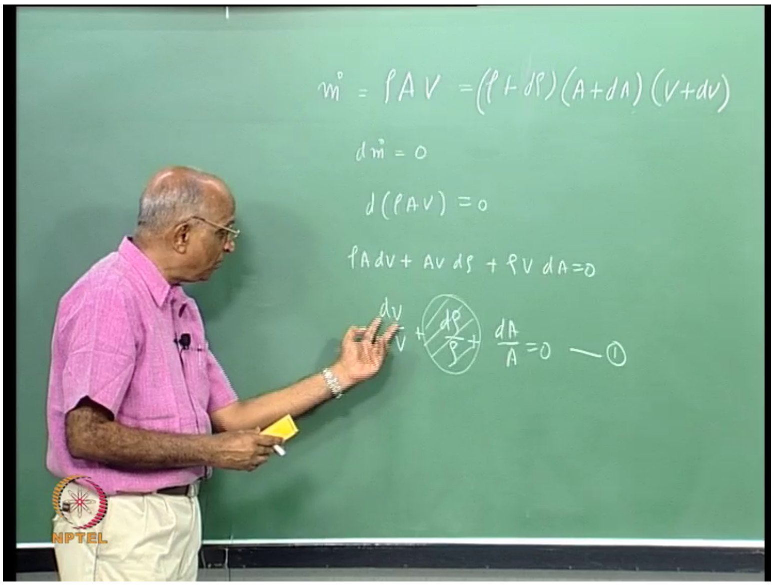

through the section one and the same thing flows through the section two, which is equal to $\rho + d\rho$, that is density at the section two × A + dA × V + dV; and this is mass balance equation. Since the flow is steady, the same mass flow rate flows through this section and this section. Let us solve this equation and what do we get? Mass flow rate is a constant or rather d (m°) must be 0, because mass flow m is constant. Therefore, I get the expression $d(\rho A V)$ is equal to 0. And therefore, now if we expand this expression we get $\rho A\, dV + AV\, d\rho + \rho V\, dA = 0$.

Or rather I divide this entire equation by ρAV, which is a constant and we get $dV/V + d\rho/\rho + dA/A = 0$ and this becomes my mass balance equation in the differential form. I could have derived this expression by saying ρAV is a constant, therefore logarithm of ρAV is constant and differentiating this would have given me $dV/V + d\rho/\rho + dA/A$ is equal to 0; which we call as the continuity or the mass balance equation. But, why did we have to derive this. We wanted to it find out what is the change in area, which will give us high value of velocity and therefore, our aim was to relate dV by V with dA by A. Unfortunately, we are left with dρ/ρ and we would like to get rid of this term dρ/ρ. Because the flow, we assume is compressible, the density is changing; and therefore, I am left with this one term here dρ/ρ, which we should express every in terms of dV/V or dA/A to be able to find the dependence of velocity on the change in area. Therefore to do that I again ask, can I write one more equation; let say the momentum equation.

What does the momentum equation tell? The momentum equation specifies that the rate of change of momentum must be equal to the impressed pressure or rather impressed force.

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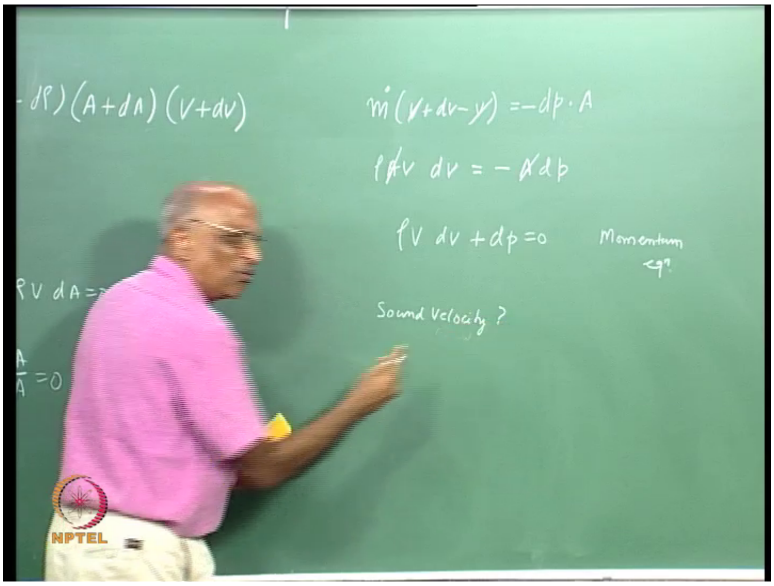

Therefore, let us write the momentum equation; the mass flow rate is m°, the change in velocity is from V to V plus dV, that means V plus dV minus V. The rate of change of momentum is therefore m° × dV.

And this is balanced by the force and what is the force that we get? we have the change in pressure across is dp and the area is A, we find dp is higher therefore, the force acts in the direction of change of momentum and therefore, we have change of momentum is equal to the force. And what is m°? m° = ρAV. A and A and also V and V get canceled. ρV dV is equal to minus A of dp, and therefore we get ρVdV plus dp is equal to 0, which becomes the momentum equation or pressure balance equation.

We must be able to generally derive the momentum equation by just applying the Newton's second law. Seeing that, I have a mass which flows across, the change in velocity is dV, the rate of change of momentum is m° into dV and that you have the pressure force, which acts on the mean area into dp is the change in the force and therefore, this is the force balance equation. Therefore, now we find that though we wanted to get rid of dρ/ρ, we have got an expression in terms of dp. So, how do I still get rid of this term in some way or the other. To able to do so, we look at the sound speed. What is this sound speed or velocity? You know it is very central to gas dynamics and compressible fluid mechanics. I talk to you and when I talk to you the sound waves travel, we say at the speed of sound; and this we say is the speed of sound denoted by a m/s.

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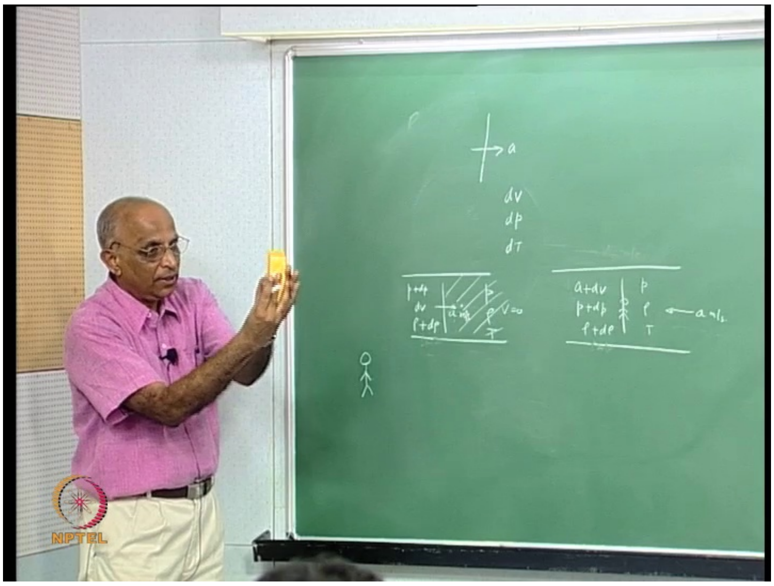

I have a sound wave, which is travelling at a speed of a; and when you hear the sound, you get a little bit of pressure on the ear drum which you perceive depending on the response of the ear drum. How you get some velocity from the pressure? May be, I get some velocity change from the pressure change. I could even get a very small temperature change from the pressure change. If the sound intensity is large, may be the temperature change could be high; therefore, let us take look at what this sound wave means and how do we introduce sound wave into that equation. And towards the end of this lecture, we will try to see the importance of sound when we talk of the flow.

Let us imagine that we have a pipe and a planar sound wave propagates in this pipe. I take the same medium of a gas, let the pressure of this medium be p, let the density of this medium be ρ and the temperature of this medium be T. The medium, let it be stagnant like the room in which we are seated. This means that the velocity V is equal to 0. In other words, I am just looking at the sound wave propagating in a stationary medium (at rest-the velocity is zero), of pressure p, density ρ and temperature T. And what happens when the sound wave propagates through the medium? Let say this sound wave, it increases the pressure little bit by p plus dp. Initially, velocity is 0, the sound induces a small velocity dv. we have the density now to be ρ + dρ and temperature could also change.

Now, I want to write an equation for this particular change. What is happening? I am standing over here, this is my frame of reference. I am watching the sound wave go by, the sound wave processes this medium, which is initially stationary, increases the pressure by dp, increases the velocity from 0 to dv of the particles, may be the density changes from ρ to ρ + dρ and that is what I am watching. And for me, to write the equations standing here to see the wave traveling at the velocity of sound a meters per second is difficult because the sound wave is also moving as also the particles in the flow are moving. And therefore, we transform the frame of reference; instead of me standing here and watching the wave go by, I position myself on the wave.

And if I stand on the wave and I am moving along with the wave, I see the gas coming towards me with a speed a meters per second, because I have told that the sound wave moves with a velocity a. And now, the conditions ahead of me here are p, rho and perhaps T, and velocity is a. And we had when the velocity was 0, we had dv therefore, the velocity here is a plus dv, the pressure is still the same p plus dp, the density is ρ + dρ. I think such transformations are important in the sense that I initially watched the sound wave go by but if I stand on the wave, this is that I see? Since I am moving, I put myself on the wave therefore, the medium is coming over here and I have the changes happening behind. Now, I write the equations for this frame of reference. What will be the equations that we will get?

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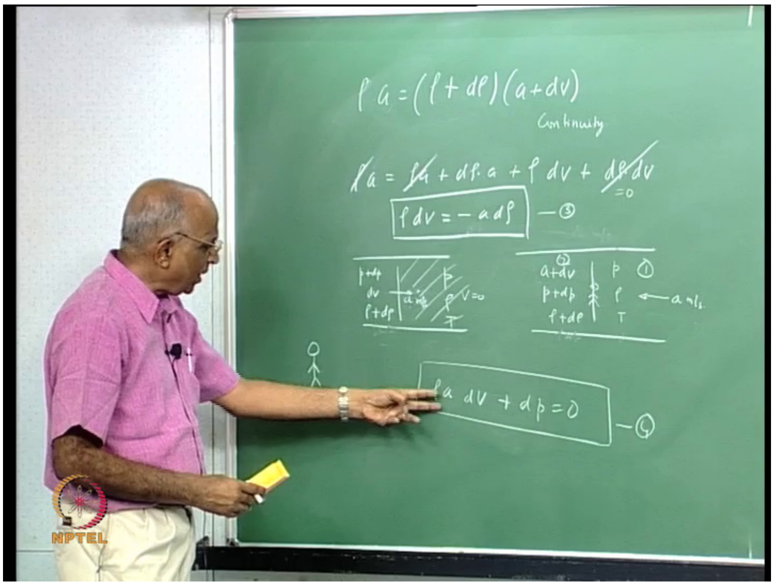

I have over here; on the right hand side the undisturbed medium, this is the disturbed medium and I stand on the wave between the two. The equation that I get for mass balance is rho into a into cross sectional area. I take the same cross sectional area, and we get $\rho \times a = (\rho + d\rho) \times (a + dV)$. This is the mass balance or continuity equation. You see the similarity of this equation to the earlier equation, wherein you have ρ A V being conserved with area change, I just wanted find out how the sound wave travels and therefore wrote the equation for constant cross sectional area. Let us simplify it. I have ρa is equal to ρ + dρ into a + dv. But mind you we are talking of sound waves; sound wave is travelling at a speed a meters per second.

We also know that the pressure change from the sound wave dp would be small, and similarly dV is small, the density change dρ is small, therefore the product of these small quantities for all practical purposes can be neglected. And therefore, what is it that we get from this equation? We find that ρ a and ρ a get cancelled and we get the value of $\rho\, dv = -a\, d\rho$, this is the continuity or mass balance equation across a sound wave. This equation is derived as I sit on this sound wave and I see the medium being processed by it.

I now want to write the momentum equation. What should it be? Well, I am standing here on the wave, I see the change of momentum: that means, I see ρa is mass flux, which is coming over here and what is happening? I have the velocity changing from a to a plus dv giving ρ a dv as the value of the rate of change of momentum; and that is balanced by the change in pressure.

I do not need to repeat this again viz., as rate of change of momentum per unit area is the change of pressure and this becomes the momentum equation for the sound wave. ($\rho a\, dV + dp = 0$)

Now, we look at this equation, which we call as equation three, because we have already derived the continuity equation which was Eq. 1 and we have the momentum equation that we call it as Eq. 2 for the flow through the section. Looking at the mass and momentum equation for sound wave, which is Eq. 3 for mass balance and Eq. 4 is the momentum equation. Therefore, if I have solve these two equations (3 and 4) for the sound wave together, I have the following:

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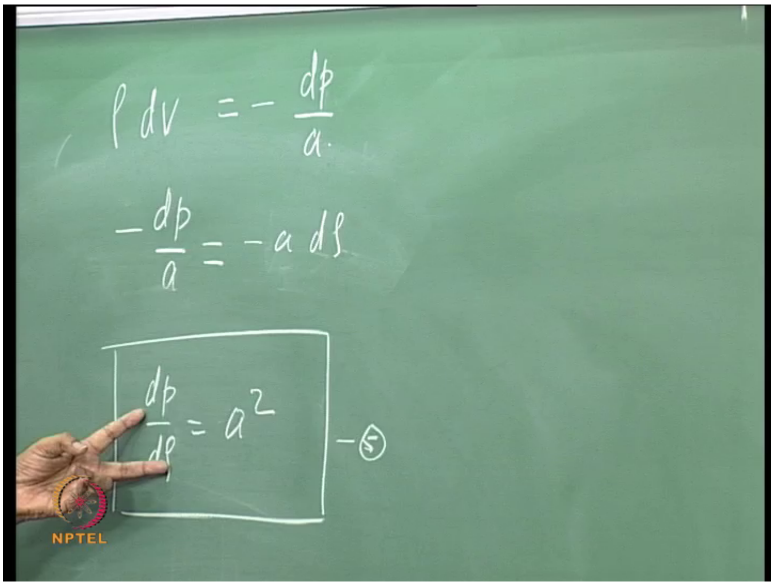

$\rho dv = -\, dp/a$; therefore, if we substitute ρ dv over here from the mass balance equation 3 as $-a\, d\rho$ what is it I get? Minus dp/a is equal to minus a dρ. It gives us the dp by dρ is equal to a^2. That means, the velocity of sound square is equal to the ratio of the differential of pressure to the differential of density. That is when I am talking to you; the perturbations in pressure across the sound wave to the perturbations in density across it equals to the sound velocity squared. I call this relation as Eq. 5.

Let us come back to what we set out to do i.e., get rid of term dρ by ρ. We have dρ is equal to dp / a^2.

Getting back to the equation dv/V + dρ/ρ + dA/A = 0 and substituting for dρ by ρ, we get for dρ is equal to dp / a^2, we get dp/ρa^2 + dV/V + plus dA by A = 0.

But then from the momentum equation 2, we have dp = − ρV dV and therefore, if we substitute the value of dp as − ρ V dV what is it we get? We get the equation as dV/V – ρ V dV and ρ and ρ gets cancelled therefore we get V and on top that is dp is equal to V by a^2 into the value of dV plus the last term that is equal to dA /A = 0.

What is it that we have done? We substituted the value of dρ by ρ in terms of dp by a^2 into 1 / ρ, because we found the dp by dρ is equal to a^2 . And then we wanted to write get rid of dp in this expression and therefore, we use the momentum equation, which we wrote as dp = − ρv dv. So, dρ/ρ became v into dv by a^2 with a minus sign.

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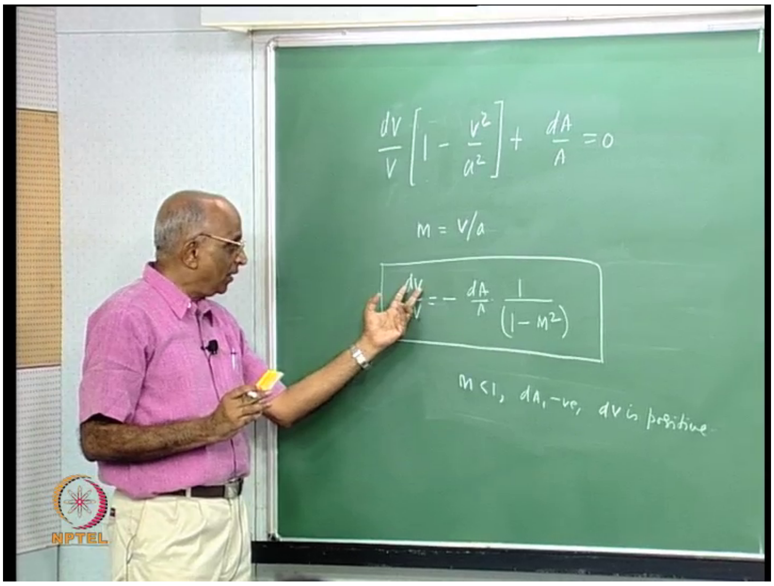

Let us simplify and write it down over here. We get dV/V × (1+ plus an expression involving V and a. Since we took V outside, in the denominator we get V^2 and it is divided by a^2. We still have the term plus dA/A, the sum of which with the previous term is equal to 0.

Let us call as Mach number M the ratio of velocity of the medium divided by the sound velocity. I will come back to the physical significance of this later. Therefore, I can write $dV/V = - dA/A \times 1/\{1-M^2\}$.

Therefore this is the final expression that we get. Now, whenever we derive an expression, we must analyze what the expression means or signifies. What do we find? If dA is negative like what we have drawn earlier and what did we draw? We say area is decreasing as x is progressing. If dA is negative, the term becomes positive. And if we say M is less than one, when dA is negative, then dV is positive. What does this mean? If we want the velocity to increase as the flow progresses, if the Mach number M is less than one, we get this to be a positive number. Unless we have area which is decreasing as x proceeds, I cannot have an increasing value of V.

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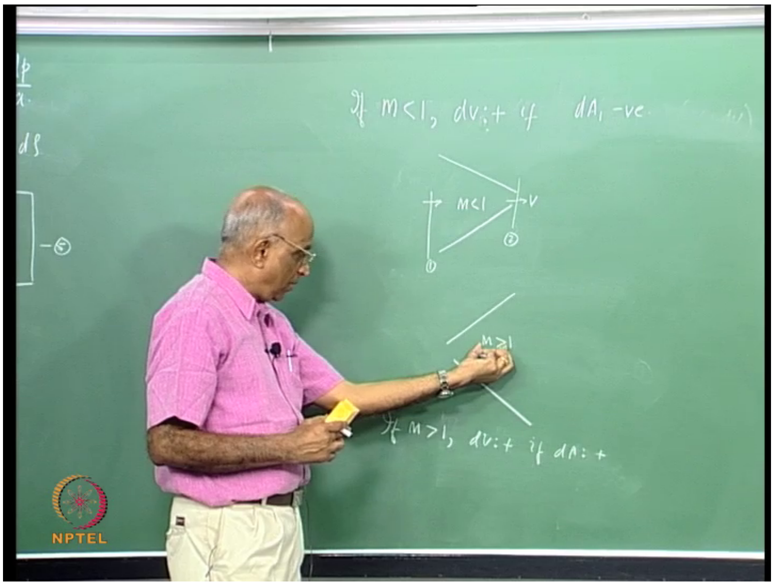

In other words, we tell that flow will accelerate or flow will increase in velocity, if the Mach number is less than one; dV will be positive, if we have dA to be negative. In other words, all what we are saying is the cross sectional area, if we have a subsonic flow with Mach number less than one, then we should have something like this converging section for velocity here at section two to be greater than at section one. That means, here it enters at lower velocity V, thereafter dV gets enhanced and we have higher velocity over here. That means, for the case of a subsonic flow or a flow for which Mach number is less than one, flow will accelerate in converging passage.

On the contrary, using the same set of arguments, if we have a case wherein the flow takes place in diverging configuration, and if we have Mach number greater than one, then what happens? Mach number is greater than one; this becomes negative, negative and negative gets cancelled, therefore, dV by V goes as positive of dA by A. In other words, the flow accelerates, if Mach number is greater than one, dV is positive since the change in velocity dV is positive. See through the simple argument of looking at the mass balance and the momentum equation, we are able to come out with a conjecture that in a converging section velocity will increase only if Mach number is less than one. On the other hand, if we have a diverging section and if Mach number is greater than one, then only the flow velocity will increase.

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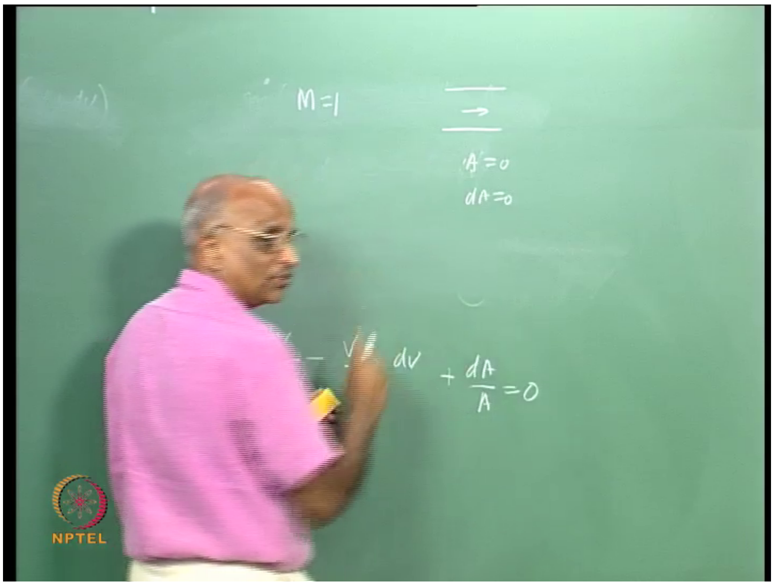

What will happen if the Mach number is one? If Mach number is one, the equations sort of break down. In fact, we find that if Mach number is one, then we have one over 0 and unless I have dA by A equal to 0, this equation cannot predict anything at all. Therefore if Mach number is one, maybe we must have a constant area; that means, A is constant or rather dA must be equal to 0 for flow to take place.

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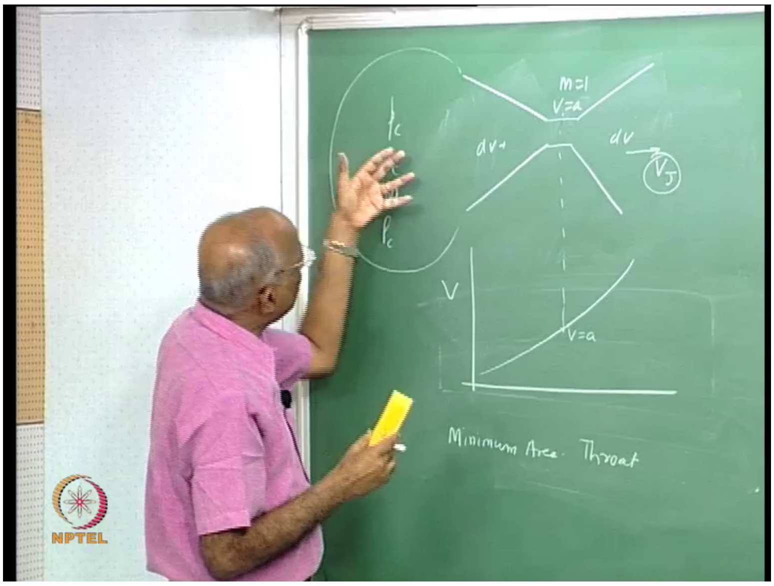

So far we looked at the continuity and momentum equations, and found that in a vent or in a small opening, we should have initially the flow should come like this, it should come to a value of M is equal to one. And then if you pass through the divergent, such that dA is positive here, we have dV is positive and therefore, I can have acceleration of flow. And therefore, along this particular length if I plot the velocity V will keep increasing, and at dA = 0 the velocity must be equal to the sound velocity. Therefore, in order to get a high value of jet velocity, what we require is we must have a minimum area and this minimum area like a constriction; we call it as throat. Therefore, we start with a large area, we converge it, we increase the velocity to a value is equal to the sound speed at this section (the Mach number is equal to one) and thereafter when the Mach number is greater than one and the flow velocity increases and therefore, I can have high value of jet velocity.

Therefore the configuration of the vent should be a convergent followed by a constant section which we call as throat followed by divergent, if we have to get a high value of jet velocity. Supposing by chance, the mass flow rate is such that (like when we considered the balloon) which had a pressure of Pc, a temperature Tc and a molecular mass of gas with density ρ_c; if the Mach number at the smallest section throat is less than one, then what is going to happen?

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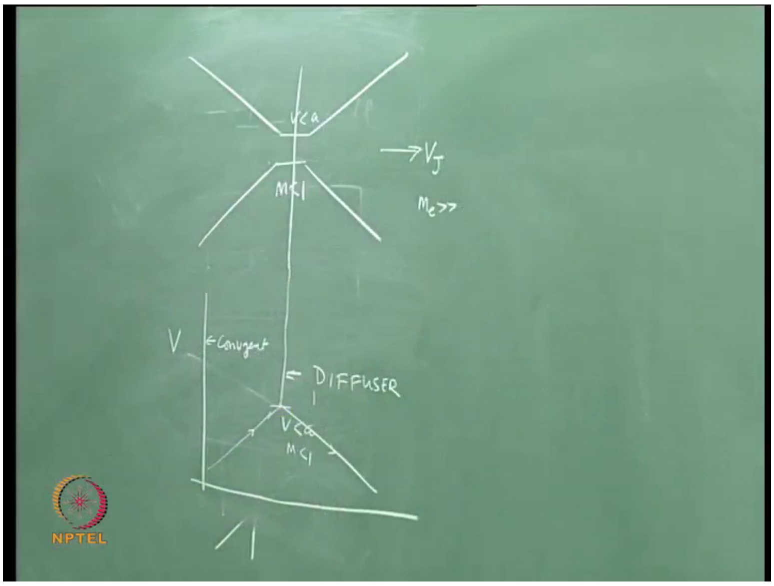

Let us say that we have the same configuration of the vent or nozzle; in other words we have the convergent followed by the throat, followed by a divergent. If, the velocity of flow is less at the throat giving the Mach number to be less than one at the throat section, then what is going to happen? Well, we now plot velocity as a function of distance, the velocity keeps increasing up to the throat, but I find that V is still less than the sound speed i.e. Mach number is less than one. And therefore, the velocity begins to drop as flow progresses further into the divergent. That means, in the convergent the velocity increases in the divergent velocity decreases; and this portion divergent is what we call as diffuser.

A contraption, which decreases the velocity and enhances the pressure, is what we call as a diffuser and a contraption, which increases the velocity is what we call as a nozzle. And this total is what we call as a nozzle. If I can have a Mach number one at the throat and then I have a convergent followed by divergent, I can get a high value of jet velocity since the velocity increases in both the convergent and the divergent; that means, Mach number at the exit will be a large value much greater than one.

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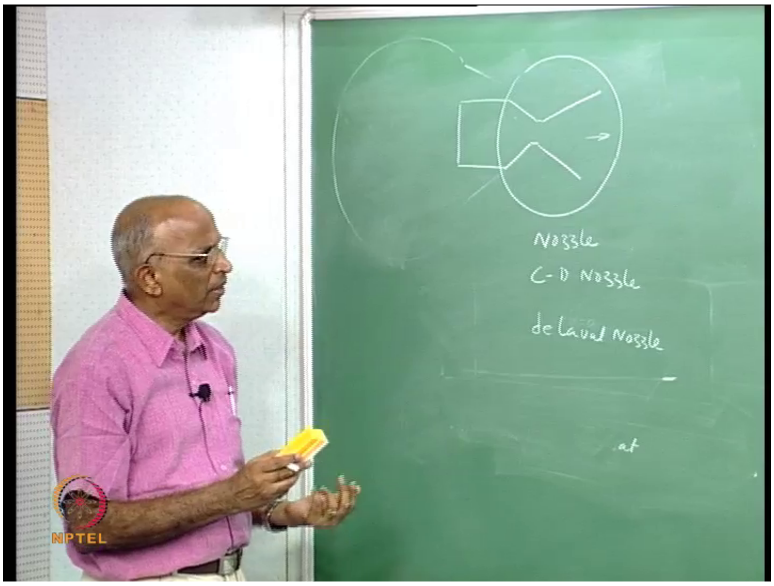

Therefore, from the continuity and momentum equations, we find it necessary to have a convergent section, followed by a throat section, followed by a divergent section and this is what gives me a high velocity and this is what I call as a nozzle. For getting the high velocity, the Mach number at the throat should be unity. This is the convergent divergent nozzle, which in some text books is also referred to as de Laval Nozzle.

But let us be very clear, if by chance I do not get the Mach number at the throat as equal to one, well the buildup of velocity in the convergent is a lost in the divergent and the velocity drops. And this is the configuration of a nozzle - a convergent divergent nozzle. Let us go through an exercise involving variation of parameters, because the convergent divergent nozzle is central to having a high jet velocity. Let us find out the variations of parameters across a convergent divergent nozzle.

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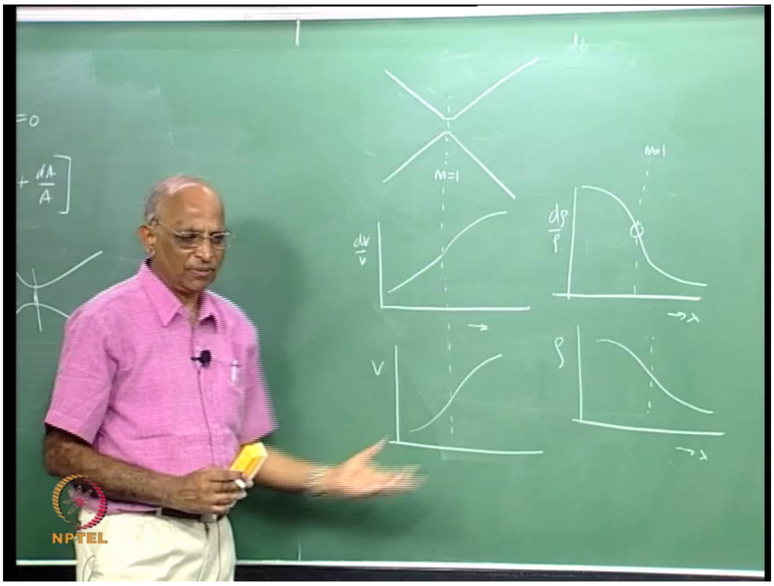

Let me first sketch a nozzle, converging till the throat, followed by a divergent and let us assume that the flow rate through the nozzle is such that the Mach number is equal to one at the throat. Now, let us first find out what is a change in dV by V along the length. We just now saw that dV by V increases up to the throat since the Mach number is less than one and the variation in dA is negative as x increases.

And then it further increases in the divergent, because the Mach number is one at the throat and characteristic of the equation changes as $1 - M^2$ becomes negative and therefore, dV by V increases. If dV by V is given by this trend, well V the value of velocity would also keep changing and what is it that we get? I get the velocity to continually increase.

If the velocity increases, what is going to the happen to the pressure and the density. Let us now plot a few more parameters, instead of dV by V, we are going plot the variation of pressure that is let us say dp by p. How should it look like? To determine this let us first determine the variation of $d\rho/\rho$, because we already have an expression, which we derived for it.

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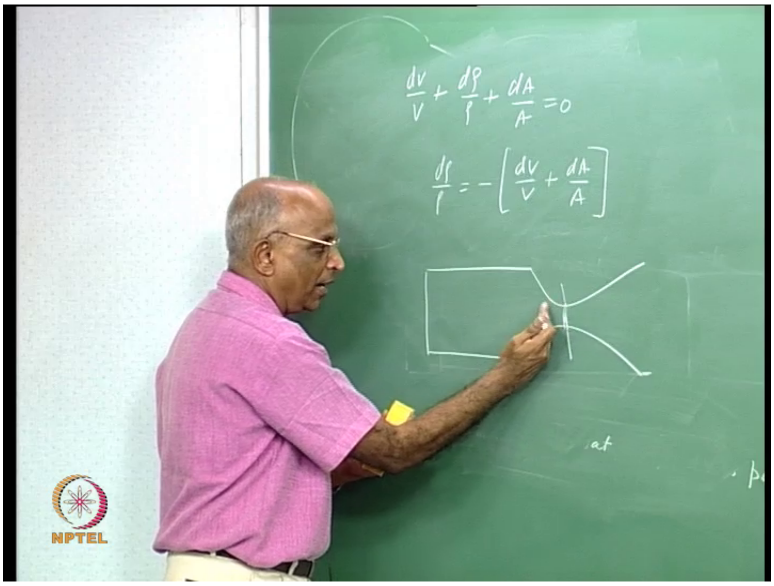

We had the expression dV/V +dρ/ρ + dA/A = 0 or rather we have dρ by ρ is equal to minus of dV/V + dA/A. I know the value of dV/V increases based on the earlier discussions and therefore, dρ by ρ will be negative. Though dA by A in the convergent is negative, dV by V has a higher value since it is divided by a fraction given by $1-M^2$. In other words, if we say this is my throat region and we have Mach number equal to one at the throat here, this is my length x along the nozzle which I am considering, we find therefore, the density will keep falling. And what happens at the throat region? At the throat region, we had one minus Mach number squared and therefore, I have something like a step gradient in velocity; we have very step region dρ by ρ also. This is important.

In other words, the density keeps decreasing more rapidly at the throat region where the Mach number is around unity. The region at the throat is a region of decreasing density as the flow progresses. In fact, we will find and when we get into this problem of combustion instability, we will find that when we have a rocket nozzle, let say have a rocket with a convergent divergent section like this. The rapid decrease in density or the change in density over here acts as a sort of reflecting surface with the disturbances in the chamber being reflected back into the chamber. We will come back to this point later on. That means, at the throat portion, you have region of decreasing densities and therefore, if we now plot the density as a function of x here, we have the throat here well the density keeps decreasing. Therefore, for a convergent divergent nozzle, for which the

Mach number is equal to one at the throat velocity change increases subsequently while the density change decreases.

Now, there are two other parameters which are left. What is going to happen to the temperature? What is happen going to happen to the pressure? We have made an assumption that the flow through the nozzle or vent is adiabatic and is reversible.

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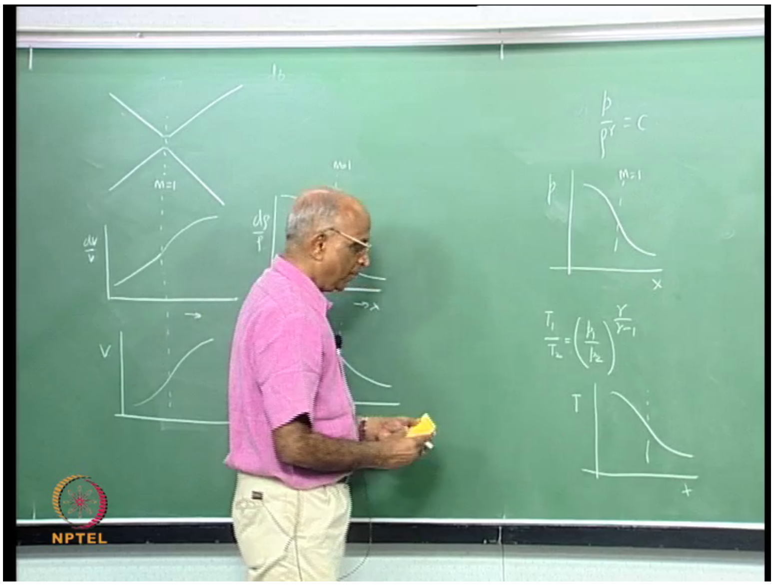

Therefore, p/ρ^{γ} is a constant. I know density changes and therefore, the pressure if I have the Mach number at the throat equal to one, well the pressure should also decrease this will be my variation of pressure with respect to x.

And we also have derived an expression in which we found $T_1/T_2 = (p_1/p_2)^{(\gamma-1)/\gamma}$. Therefore, the temperature, if I have the throat here should also decrease and this will be by temperature variations with respect to the distance x.

The net result is that the pressure decreases in a nozzle monotonically, the density decreases in a nozzle with rapid changes in density taking place at the throat. Well the velocity increases; this is all what we have deduced thus far.

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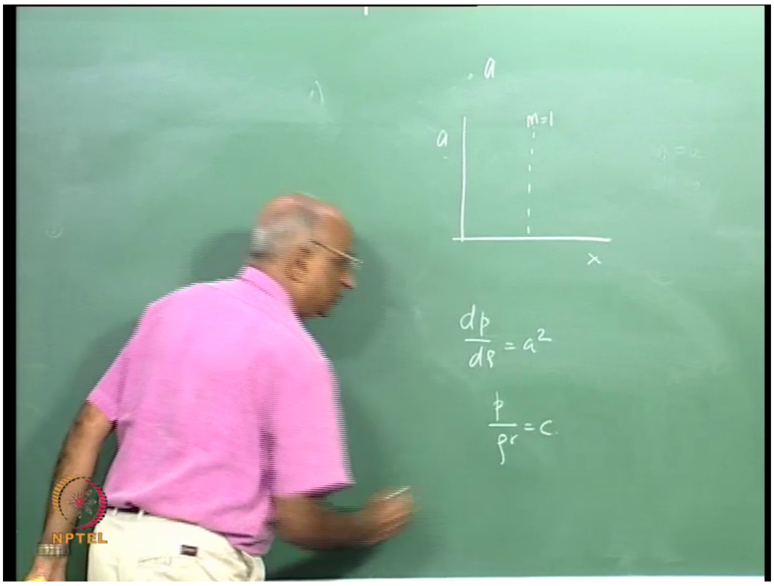

Now, let us include some other parameters. If we want to plot the sound velocity variations what will it look like? In other words, we have the expression for sound velocity a. We want to plot how the sound velocity will vary along the length and at the throat the Mach number is one. We again go through the expression what we derived today. We had dp/dρ is equal to a^2. Now we told that the nozzle flow is isentropic that is adiabatic and reversible therefore, we have p/ρ^γ is a constant. Therefore, what is a square?

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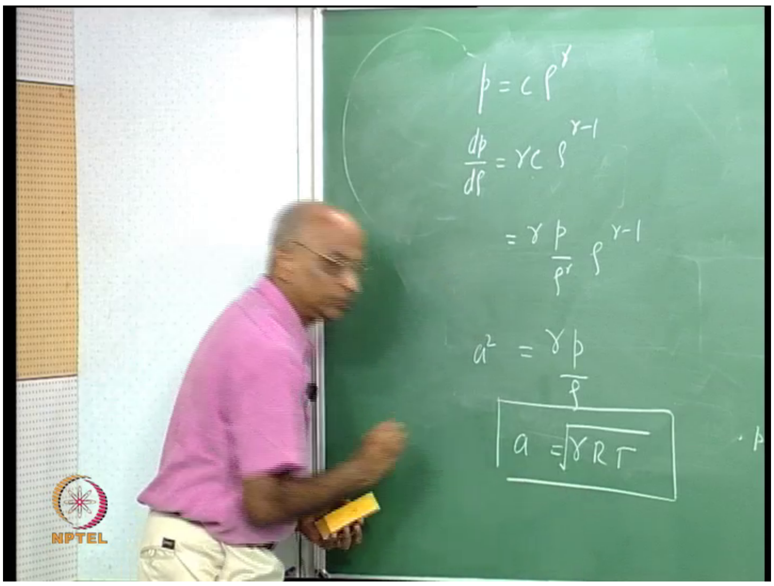

Let us write the term p = constant c × ρ^γ. Therefore, $dp/d\rho = \gamma \rho^{\gamma-1}$ × by constant C. And therefore, now if I want write the value of the constant C, it is p/ρ^γ. Hence, the expression for $dp/d\rho$ becomes $\gamma\, p\, \rho^{-\gamma}\, \rho^{\gamma-1}$ and this is equal to $\gamma p/\rho$. Therefore, we find that the sound speed $a^2 = \gamma p/\rho$ and for perfect gas or rather for an ideal gas p is equal ρRT. The sound speed is therefore equal to $\sqrt{\gamma RT}$.

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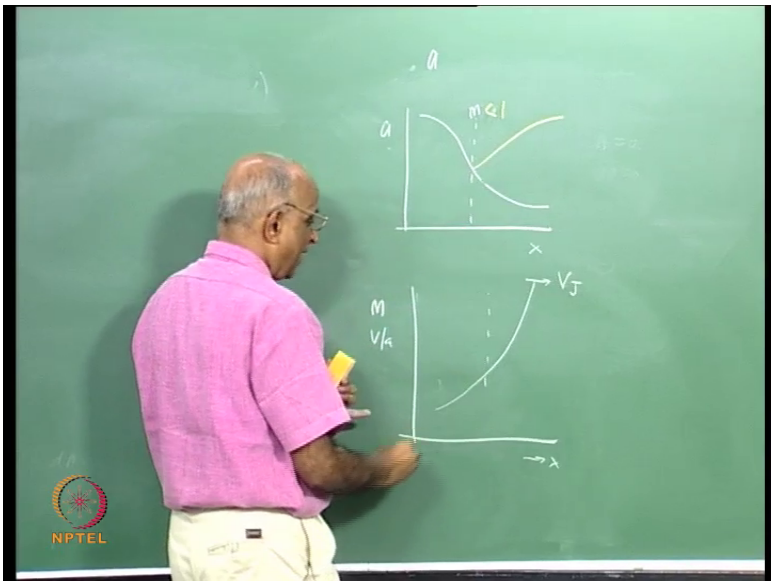

Therefore how would the sound speed vary occur along the length of the nozzle? Well the sound speed starts with a high value of sound speed corresponding to the high chamber temperature; it keeps on coming down and it like this along the length of the nozzle as the temperature drops.

One last parameter, I can still think of is the Mach number variations along the length of the nozzle. I know the velocity variations which we had previously determined along the length x over here in this particular form and the value increases. We also found that the sound speed keeps coming down along x; and therefore, if I find out the Mach number variations as a function of x, what is it that we get? The numerator is increasing, denominator is decreasing as the Mach number is equal to V by a, and therefore, there is a much greater variation of Mach number along the length of the nozzle as is shown like this.

The above parameters are central to nozzle flows. Let us repeat it again. The change dV by V progressively increases along the nozzle length when the Mach number is one at the throat. The velocity increases while the density decreases and pressure also decreases. The temperature decreases and the sound speed decreases along the length of the nozzle. The Mach number increases along the length. And we are interested in a high value of Mach number or jet velocity at the exit.

And if by chance the Mach number at the throat is not one, that means, we have insufficient pressure over here to give at the throat high flow velocity equal to the sound speed; in other words, I now get the Mach number to be less than one, what is going to happen? dV by V is going to increase over here in the convergent, it is still less than the sound speed that is V is less than a and therefore, it falls down in the divergent portion; Velocity increases up to the throat, but it is still less than the sound speed and therefore, it begins to droop in the divergent portion. What happens to the density? Density falls up to the throat and thereafter increases; therefore the divergent acts as a diffuser.

When the Mach number is less than one at the throat, the pressure recovers in the divergent part here i.e., instead of falling it increases. Similarly, the temperature recovers in the divergent. The sound speed falls up to the throat and recovers in the divergent. In essence, we have the Mach number going up in the convergent and coming down in the divergent, if the value of Mach number at the throat is less than one.

I would request you to go back and study these figures again, because this is relevant in the study of nozzles. What we find is we must have a convergent section followed by a divergent section and in between I must have a constant area section, which we call as a throat. And the Mach number at the throat must be one for continued expansion to get high jet velocities at the exit of the nozzle.

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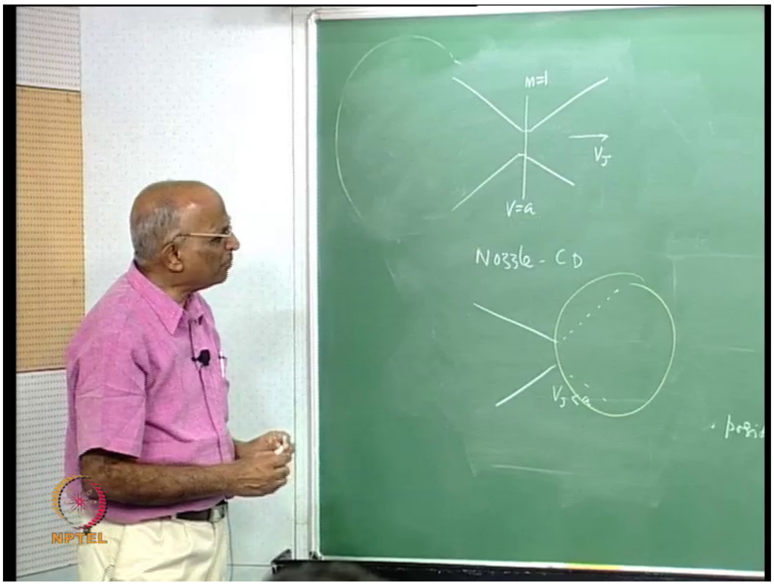

If we want to have a high value of jet velocity, it is essential for us to have a convergent followed by a throat section, followed by a divergent section, with the condition that the Mach number must be one viz., velocity must be equal to a sound speed at the throat. And with this condition the velocity keeps increasing. We can get a high jet velocity at the exit and this is what the convergent divergent nozzle does.

If we find that we have inadequate mass flow rate like for instance, I have a cold gas I have inadequate pressure and I cannot have a Mach number equal to one at the throat or velocity is equal to sound speed at the throat, then its better my nozzle is like this – consisting of convergent portion alone, such that my jet velocity is still less than the sound speed. That means, if I have inadequate pressure or inadequate conditions here, such that I cannot effectively use the divergent get then I must do away with the divergent section. Because if I have this divergent section over here, I am really loosing the velocity and I am really not gaining anything. Therefore, whenever rocket nozzle has to be designed, we have to ensure that the flow velocity at the throat must be equal to the sound speed at the throat.

Now, we ask one last question: What is the significance of sound speed? Is there something very significant about it and the velocity of flow being equal to it at the throat?

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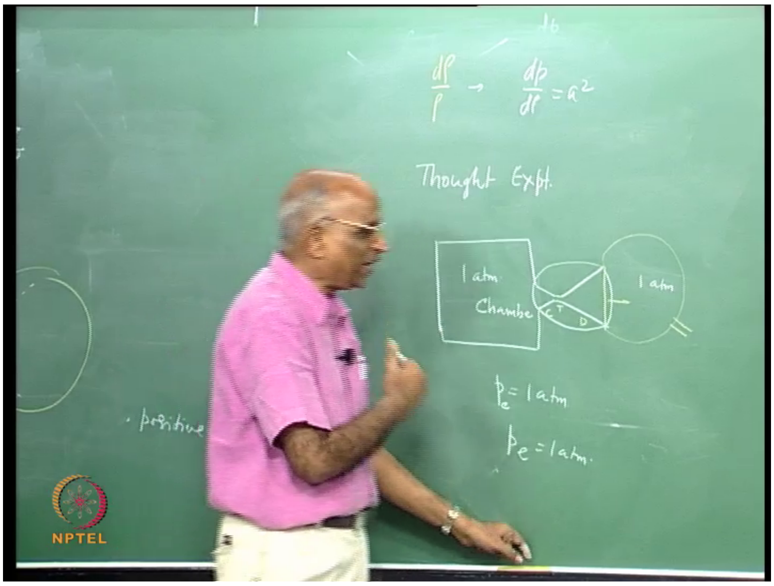

What we did while understanding the convergent divergent shape was that we introduced the sound speed through the equation for conservation of mass in order to eliminate dρ/ρ in that expression. We substituted dρ by dp by noting that $dp/d\rho = a^2$ i.e., sound speed square and then got al the parametric variations in the nozzle. But does the sound speed have some implication. To be able to answer the question, let's do a simple experiment or let us say a thought experiment and what is this thought experiment? Let us say we have a tank or a chamber something like this; it is at ambient pressure - one atmosphere pressure, I am really not bothered about temperature at this point in time. Maybe we attach a convergent divergent nozzle to it this is my thought experiment; let us draw it properly.

Now, what we do with in this experiment? We attach a vacuum pump here at the exit of the divergent and we suck the air out of the chamber through this particular nozzle. And what is the construction of this nozzle? It has this convergent, divergent portion attached to the tank, which initially is at atmospheric pressure. And then we start pumping out or start sucking the air out of this tank through the nozzle.

Now, what is going to happen? Now the pressure in the tank is initially one atmosphere, and this is the same as the pressure outside Pe or we say that the chamber pressure denoted by say Pc is one atmosphere, let us say Pe is also one atmosphere to begin with. When the pressure Pc and pressure here Pe are same, obviously, there is no mass flow rate. When we attach a pump downstream of the nozzle and start sucking out the air from this chamber, this may provide us with some clues on what really the flow velocity equal to the sound speed means.

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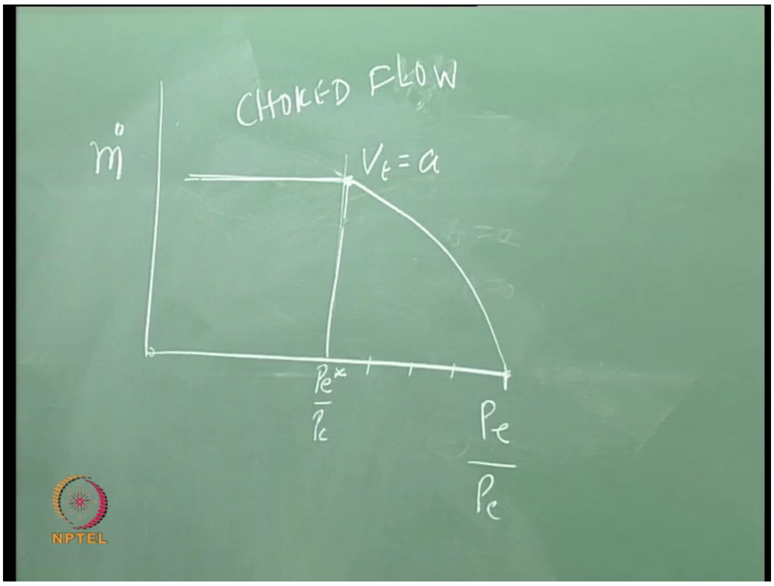

In this experiment, we plot the mass flow rate through the nozzle, that means, the rate at which mass is getting sucked out as a function of let us say the value of Pe that is the exit pressure to the chamber pressure as it is progressively decreased by the running of the vacuum pump.

Initially the pressures Pc and Pe are the same and therefore the mass flow rate is zero. We thereafter start sucking of the air by decrease the pressure Pe; if we start decreasing the value of Pe, the mass flow rate will increase. That means, as we decrease the value of Pe, the mass flow rate will increase and therefore, do you think that as I keep on increasing the vacuum level i.e., decrease the value of Pe, the flow rate should keep on increasing or what should happen?

Let start our thinking process. Initially when the pressure outside is one atmosphere, pressure inside is one atmosphere; the velocity at throat is equal to 0. Then we start sucking air out, as we keep decreasing the pressure Pe. The flow velocity at the throat of the nozzle Vt will keep on increasing till it reaches a value Vt is equal to the sound speed at the throat a_t . In other words, let us just plot the mass flow rate versus the pressure till that time. We keep on decreasing the value of Pe, that means we keep moving on this sloping curve here till the time a stage is reached at which given the value of velocity at the throat Vt is equal to the sound speed a_t. Therefore, at that point in time what is happening?

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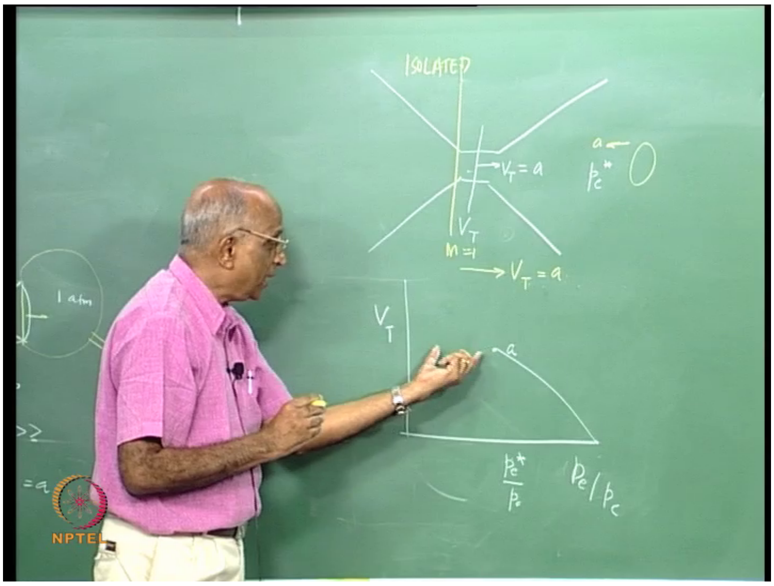

Let us plot the value of the velocity at the throat I call it as V_T. What happens to it? Now in the same figure on the X axis, we show the value of Pe by Pc, as a function of V_T. What we find is initially the velocity is zero for Pe/Pc equal to one. As Pe by Pc decreases, the velocity V_T keeps increasing till the time for a given value of Pe reaches a critical value of Pe*. At this ratio of Pe* by Pc, the value flow velocity at the throat is equal to the sound speed at the throat.

What is the implication of this? Now, what I find is the flow velocity at this particular point for this condition, when I have reduce the pressure to Pe*, the velocity at the throat is equal to the sound speed. Now, let us ask ourselves some foolish questions. What is that drives the mass flow rate from this chamber to the outside? Because I suck something, when I suck something what happens? I reduce the pressure here and this information of a reduced pressure is transmitted through the divergent portion, the throat of the nozzle and the convergent part to the chamber, which causes the mass flow. That means, I suck the air out, something the information is supposed to reach the chamber and that is why some mass is flowing out. As I keep on decreasing the pressure, this tank senses viz., it gets some information of the lowering of this downstream pressure being lower and therefore, the mass flows out. This is how information is fed to the tank for drawing an increased mass from the tank.

Now, as Pe is reduced progressively, a stage comes, when the flow velocity at the throat V_T is equal to the sound velocity at the throat. The information or disturbance travels at the speed of sound. Since the flow at this critical state Pe* by Pc is same as the speed of sound, any information here is travelling at the speed of sound. But if the gas is flowing at the speed of sound then no information on need for flow can reach it from the nozzle. But if the gas is not flowing at a velocity V_T equal to sound speed at the throat and is much lower, it can access the communication that gas is required to satisfy the pressure conditions. The information that additional flow rate is required can be reached against the flow till the sound speed is less than the sound speed at the throat. When the flow speed and sound speed are the same speed, no information can be passed on and therefore the chamber becomes isolated. The chamber over here cannot get any information of the reduce pressure here, when the flow velocity at the throat is same as the sound speed a_t, because information or disturbances travel at the speed of sound.

Therefore when the Mach number is equal to one at the throat, what is going to happen? You know I keep on decreasing the pressure Pe, but the flow velocity here is same as the sound speed and it effectively isolates and therefore, the tank is unable to know that a low pressure exists downstream calling for additional flow. And therefore, what is going to happen? The mass flow rate cannot increase any further as the reduced pressure is not able to communicate with the chamber. Therefore I have something like this. If the downstream pressure is such that I get the flow velocity at the throat to be less than the sound speed, the mass flow rate increase. Thereafter the mass flow rate is a constant since for any further reduction in Pe as a value of flow velocity reaches the sound peed. . And that means even though I am trying to suck or pull the gas, the information is not given to my tank. It supplies at the same constant rate. We call this condition as choking and say that the throat is choked and this is known as choked flow. This reasoning of choked flow comes from the flow velocity being same as sound speed. Therefore, when we have a convergent divergent nozzle and at the throat the value of the Mach number is equal to one, it corresponds to a choked flow through the nozzle.

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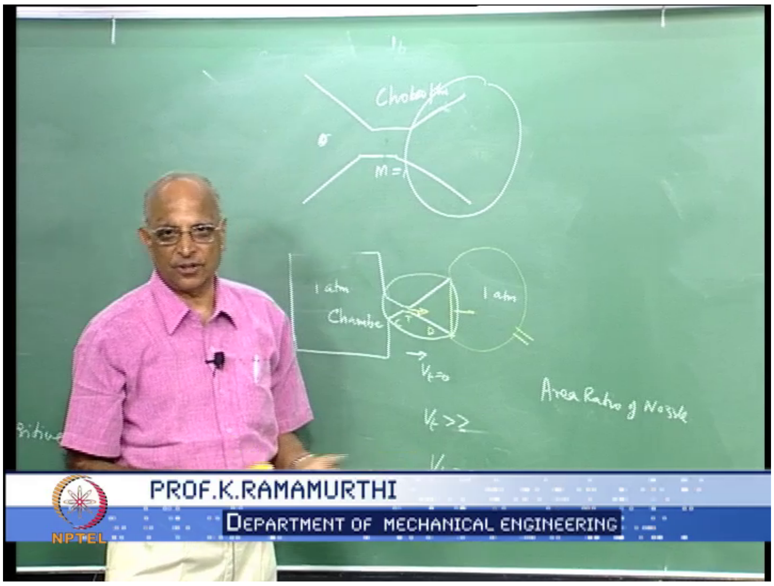

In other words, any downstream disturbances cannot really affect the flow for the choked condition; in other words, the concept of the flow through the nozzle throat at Mach number of unity viz., choked flow is central to the rocket nozzles. I will continue with this in the next class. In the next class, what we do is we will find out the value of the density corresponding to the choked flow, pressure corresponding to the choked flow and there after we will relate it to the area ratio of the nozzle.