Introduction to Computational Fluid Dynamics
Prof. M. Ramakrishna
Department of Aerospace Engineering
Indian Institute of Technology - Madras

Lecture – 09
Laplace equation - Jacobi iterations

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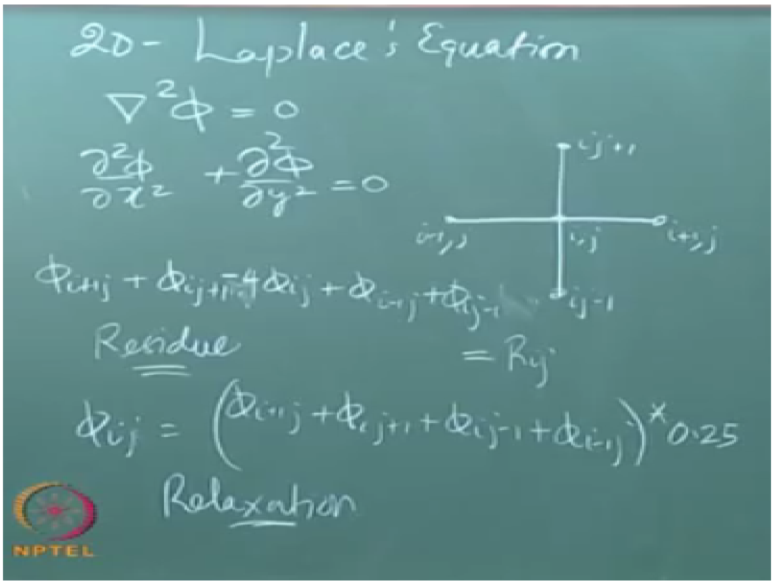

Fine, so we will take a look at the Laplace's equation last class we ended just short of my assigning a problem on Laplace’s equation, right, so we will get back to that maybe along the way, so we will take a look at Laplace equation and see what we are trying to do, okay Laplace’s equation, we will write it in terms of phi, nabla squared phi equals 0, right, so in 2D, maybe I will write 2D Laplace’s equation.

We will (()) (00:53) ourselves to 2D right now, 3D will follow in a similar fashion, so in 2 dimensions Cartesian coordinates, this turns out to be rho squared phi * x squared rho squared phi * y squared equals 0 and we wrote for an equally spaced mesh and I am writing only for one set right now, you wrote for an equally spaced mesh at the point i, j in terms of i + 1j, i - 1 j, ij – 1, ij + 1.

We wrote this differential equation representation let we could use on the computer as phi i + 1j + phi ij + 1 either way + phi ij + phi i 4 times phi ij, phi i – 1j + phi ij - 1 equals 0, so this equation, the representation at this point ij, right we have written in this fashion and we got a truncation error, there is a representation error, we got the truncation error for that representation; for this representation, okay.

Now, right in the beginning - 4 very important; -4, right in the beginning I would mentioned that this equation actually gives us allows us when you are given a differential equation in a sense, you are given a question, find phi and if you find the phi, if somebody gives you a candidate phi, somebody says I have a solution, you would verify it by substituting into this equation and checking whether they have the solution or not, right.

And if they give you something that is the solution, the right hand side will turn out to be 0, the left hand side will be 0, it will equal the right hand side. On the other hand, if they give you something that is not the solution, it will leave what is called a residue okay, so if I give you a phi, which is not a solution, if I give you a function which is not a solution, you substitute it into this equation, it would not give you a 0, it will leave a residue okay.

I am repeating again just something I said earlier in the semester, it leaves a residue, the residue is what you get if you substitute a candidate function, right into our differential operator here and it should be 0 but it is not 0 and what you end up with is the residue, fine, you write our equation in this fashion something equals 0, the left hand side should be 0, it is not 0, what is left is called the residue.

But clearly if on a mesh I give you discrete points on a mesh like I mentioned in the last class, you cannot substitute back into the original equation but you can substitute back into this, right, so if I were to give you various values of phi ij, at the various grid points, you could actually substitute that into this equation and find out whether this algebraic equation is satisfied. If the left hand side is not 0 that is it leaves the residue.

If it leaves the residue that means it does not satisfy, am I making sense, so if you end up, if you substitute you have a candidate solution you give me a candidate solution saying here is the solution, so the way I verify it is I substitute it and I find out, I try to find out what I get and if I get a residue, which I will call R ij because it is the residue at the point ij okay and that residue is nonzero, so now I change it a little.

Every time you give me a candidate solution, I will substitute it into this equation and see evaluate the residue and I will ask the question, is the residue 0, okay. If the residue is 0, you have given me a solution at that point, if the residue is not 0, you have not given me a solution at that okay, what you have given me is not a solution at that point. So, we have these 5 values okay, the algorithm that I proposed in the last class which was the phi at ij is phi at i + 1j + phi at ij + 1 + phi at ij – 1 + phi at i – 1j times, this divided by but times 0.25, one fourth.

So, what we could do is; if you give me something so that the residue is not 0, what I could do is; I could reset the value at ij, I can evaluate a value at ij as the average of these 4 values and obviously, if I substitute back in it will be 0, you understand. So, in a sense I have adjusted the value at ij, so that it satisfies Laplace's equation the discrete form at that point okay, this is called relaxing, it is called relaxing the process is called relaxation.

I will relax the value of ij, I have relaxed the value it is as though it is in tension and I relieve that tension, you understand, I will relax the value of ij, I will relax the value of phi ij by taking the average and substituting the average value there it is satisfied equation so it is obvious because I got it from the same equation, it should be satisfied okay, right. So, what is the algorithm that we are talking about now?

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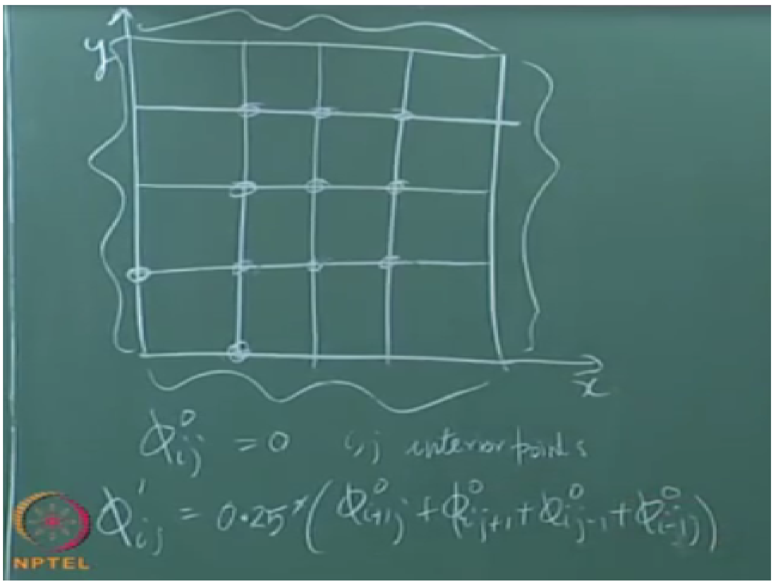

So, yesterday this is the problem that I started off with at the unit square, a unit square at the origin, so I have the unit square at the origin, we can divide it up into, if I remember right, I divided it up into 9; divided up so that there are 9 interior grid points, you could; we could of course choose more okay and what you do is sort any of the interior grid points, we do not have the values the problem is defined, so that boundary conditions are given on all 4 sides.

So, the boundary condition is prescribed on all 4 sides, okay, so in theory right now you do not have, I have not assigned the problem, so you could just assume any boundary condition, right you can just make assumptions on values of phi and I say any boundary condition, values of phi on the 4 sides of this unit square, okay, you can make an assumption on the values of phi at the 4 sides of this unit square, so that sets your problem okay.

So, you can take phi equals 100 here, maybe phi equals 100 here, phi equals 200 there, phi equals 200, you can pick values either constant or you can pick something varying as a quadratic curve right, you can pick values; you can pick values along on the boundary only on the boundary and the way Laplace's equation the problem is specified now that I give you the boundary conditions, the question is to determine the values at the interior okay.

And we propose to use the averaging process that we just talked about right now okay, so the first candidate solution that I proposed; the first candidate solution that I proposed, so I will give it a superscript 0 to indicate that right it is the first guess that we have got at ij and ij are interior points, I will say that it equals 0, i, j interior points, this is the first candidate solution that I propose.

So, if you were to substitute it into the equation you would be left with the residue at every point if you check the residue, you will find that in fact because these values are not necessarily 0, you may not have guessed them to be 0 then as these are 0, you will find that 0 here is not the average value okay, so this has to be relaxed fine, so you set this value to the average of the 4 neighbouring values, am I making sense.

So, we come up with the algorithm, I repeat that again but this time with the superscript, so you say phi ij 1 is 1/ 4 th of phi i + 1j, 0 + phi ij + 1, 0 + phi ij – 1, 0 + phi ij; i – 1j, 0, is that fine okay and of course wherever it is on the boundary, so wherever it is on the boundary you take the boundary values, wherever it is on the boundary you will take boundary values, so for the very first one j - 1 will be on the boundary, i – 1 will be on the boundary okay, for the very first one, everybody is with me.

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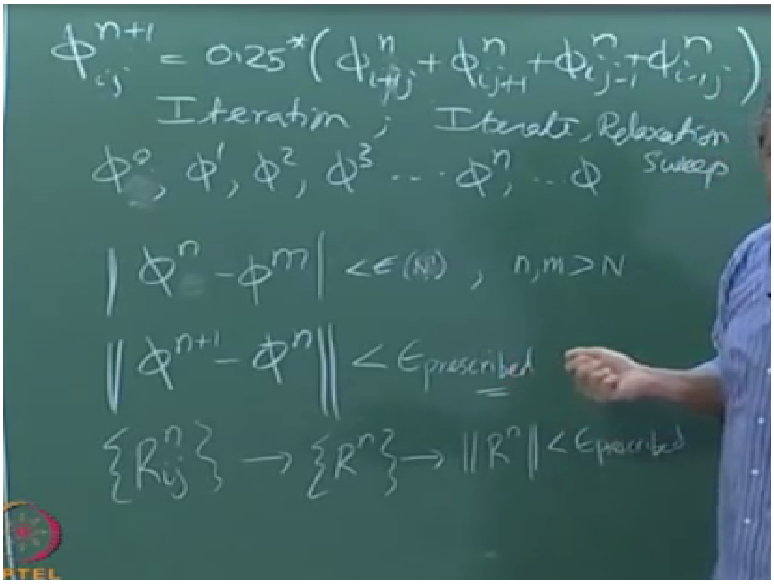

So, in general what we could write is; in general, we can write phi ij n or n + 1, yes this is the relaxation scheme that we are talking about n okay, at the end one, so we get n + 1 from the n1 + phi i + 1j, ij + 1, n + phi ij – 1, n + phi i - 1j, n, is that fine, so I have a mechanism, I have an automaton, I have a way by which I can now automate it, right I have something given the value I have you; I have an equation given the values at n, I can find the values at n + 1, okay.

So, what the mechanism that I have presumably which we hope, the mechanism that I have is a mechanism that if you give me a guess, I hope I can get a better guess, okay and given a guess if I can always get you a better guess eventually, I will converge, so solution that is the hope, so what am I generating, what am I proposing to generate here; I am saying you give me phi ij, so I will drop the i because it is for all the interior ij's phi 0.

And then you are going to generate phi 1, phi 2, phi 3, phi n, you understand what I am saying, you are going to generate this value; you are going to generate a sequence of solutions, how do you check whether the sequence converges or not? There is a various test that you are familiar with right you could do; we could use the equivalent of a Cauchy test; equivalent of a Cauchy test, right Cauchy test, basically says; you remember Cauchy test.

Cauchy test basically says phi n - phi m will be < than epsilon; epsilon will depend on some capital N for little n, little m > capital N, for any little m and > n, once you have passed this capital N, right there is an epsilon so that these only will be that close; will be close enough right, well we cannot actually do this, so what we will do is; we will do an engineering approximation, right.

So, we will basically ask ourselves the question what happens to phi n + 1 – phi n, right, I will write it as our norm because given these nodal points you can actually come up with functions okay and we want this to be less than some prescribed epsilon; prescribed, so you will prescribe this epsilon okay, so now it is clear, we are generating a sequence of phi's and all we have to do is check whether the phi is converge, is that fine.

The other way to do it; the other way to do it since we are on convergence and then I will get back to the earlier or the other way to do it is to look at R ij at each of the points, you will have similarly a sequence of R's, right you have a sequence of R ij's in fact, I will drop the idea just like I did here, so you have a sequence of R's which are the residues and what do you want to happen to the residue, it should go to 0.

So, we want the norm of the residual to go to 0, so the other possibility is from here, we can say we want the norm of this to be < epsilon prescribed, is that fine, so you can test whether this actually tells you whether it satisfies the equation or not okay, so you can either say; you can either say I am generating a sequence of phi's, do the phi's converge and generating a sequence of R to the R's converge okay.

This will do for a first order right, for now for you to start writing some code or whatever it is this is enough, we have to actually do make this a little more precise okay but right now, this test will work for you and test to see whether your code works or not fine. Are there any questions? Okay, so yeah, so if there are no questions, we will just get back here, so what we have is you will notice that the iteration okay, so each one of these; each pass through this is called the iteration, okay.

The phi that you get out of it just this is the job then you have to learn this, the phi that you get out of it is called an iterate, right in the process that you are going through is called an iteration okay, through each iteration through each relaxation it is called a relaxation also, so it is a relaxation sweep, right so you will hear people use the term relaxation sweep; relaxation sweep so it could be; you could call it a relaxation sweep, right.

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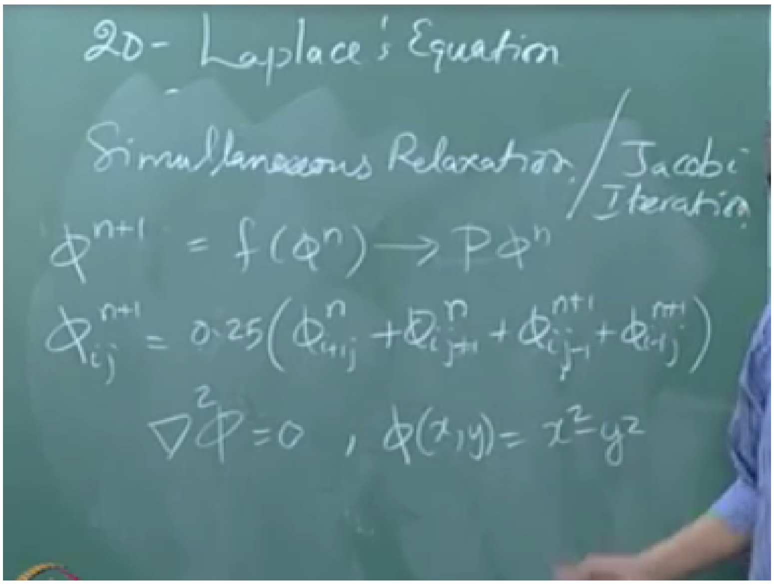

So, each relaxation sweep or each iteration will give you a new improved phi i at n + 1, right, n + 1 iterate from the nth iterate okay because this is the n plus first iterate from the nth iterate, it will go back there to the; if that because it is the n plus first iterate from the nth iterate, it is called simultaneous relaxation; simultaneous relaxation, you are basically saying that phi at n + 1 comes from phi at n, okay, is that okay.

So, in fact I think as someone had pointed out last time we look at this in greater detail it is actually a linear combination, so it is some P times some matrix times phi at n, in fact that is what it is, okay fine now, there is another possibility, so if I have phi ij n + 1, what I have done is; I have done if you look at if you come here and you look at this, when I do, when I relax this point, I take the average of these 4 points and I get a value here.

So, what I can do is; I can replace the value here; I can replace the value at that point by what I have just calculated because it satisfies right, it satisfies Laplace's equation at that point, my approximation to the Laplace equation. So, when I come here I can use the latest value that I have got, I do not have to use the old value, I can use the latest value as and when I get it, you understand what I am saying, okay.

So, then we are not relaxing, they are not at the same level, it is not simultaneous relaxation, it is called successive relaxation, so when I come to this point I can use the latest value there and consequently get the latest value at that point, right and repeat that get the latest value at that point, so as you progress, as I progress through taking the averages; as I progressed through taking the averages, you will see that I am using the latest value at any given point.

So, when I come here what happens? The i - 1 value is the latest value in general and the j - 1 value is the latest value in general, okay, so I can in fact write this as phi i + 1j, phi ij + 1 at n + phi ij - 1 n + 1 + phi i – 1j n + 1, is that fine and this is called successive relaxation; this is called successive relaxation, this is just a name right but when people talk about successive relaxation you should understand that they are talking about using the latest value okay, fine.

The first one simultaneous relaxation is also called Jacobi, see that the depending on which direction you are coming from they have different names and they are attributed to the people that have brought up the thought of the algorithm Jacobi iteration, simultaneous relaxation is also called Jacobi iteration, is that fine, okay. Now, how do we figure out whether the codes working or not, how do you find out whether the code works or not?

How do you test to see whether the code works or not, you know how to check convergence, if it converges does that mean that if it converges does that mean we have a solution, see we have the following questions; how do we know our code is working, how do you know if it is converging that you are getting the right answer right, so we need to answer these questions, how do you know if 2 of you get 2 different answers, you right run the program and the 2 of you get 2 different answers, which answer is the right answer.

Is there a way for us to decide right, is it possible that 2 people get 2 different answers, am I making sense these are natural questions that you have to ask yourself because if you go out and you start solving this problem, right and 2 different people are solving the same problem, is it possible that somehow they end up with 2 different answers, right, why would you say it is not possible but how do we know that?

Is there a way we can show that so, you are saying Laplace; the solution to Laplace's equation is unique okay, solutions but how it about the discretization, for approximation is the solution to our approximation unique, you understand, so in a partial differential equations course, you may have learned that the solution to Laplace's equation is unique okay, that the solution exists when that it is unique.

But how about for our discretization right, you understand because we are doing; we are only approximating, so is it possible that we have already seen when we talked about machine epsilon that there are a bunch of numbers that are represented by one number, so we have to have a little anxiety saying that I may; are we going to get the same answer or all of us going to get the same answer, okay.

So, we have these set of questions that need to be answered, so first we look at; is there a way for us when you are given an equation, nabla squared phi equals 0, and we have a discrete representation for this equation, is there a way for us to set up the problem, so that we can check whether the program that we write is generating, so we need an answer, we need a solution to this.

If you have a solution to this, you are set okay, so Laplace equation fortunately is easy that is why I have pick Laplace equation is the first problem that we look at right, any analytic function is a solution to Laplace's equation right, so the simplest thing is to take z or z squared z is a; z being a complex number is not that interesting z squared s, so we can take either real or imaginary of z squared, let us start, I want to keep it simple.

So, if a solution is x squared - y squared and I invariably start with something like this because I want to keep it easy, so if I were to solve this problem I will pick a simple, so is this a solution to Laplace's equation, you can substitute and see and it is indeed a solution to Laplace's equation, so all the boundaries here; all the boundaries here, we can pick the boundary condition as it comes from x squared - y squared.

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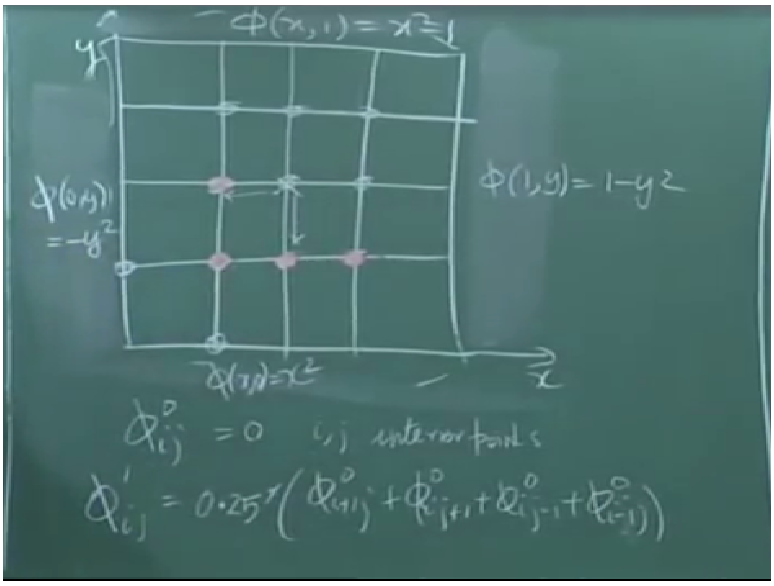

So, on the bottom boundary, you can set phi of x, 0 equals x squared on the right boundary here, you can set phi of 1, y as 1 - y squared on the top boundary, we can prescribe the boundary phi of x, 1 is x squared – 1; sorry, x squared - 1 and on the left; phi of 0, y is - y squared and then you would expect that these points would be samples from x squared - y squared they would actually be sampling the function x squared – y squared.

The solutions here should correspond to x squared - y squared, so this is something that you can try out okay and now this is something that you can try out, you have an answer to the problem and therefore, you can test to see whether your code actually converges to that also, is that fine okay, right and you can see how well it; how well does it if it converges, how well does it converge is it; does it go actually go to the answer what does it go to, right you can find that, okay.

Second question is everybody going to get x squared - y squared, is it possible that the numeric goes somewhere else, okay so we will do; we will we will do something that mirrors, we will do something that mirrors the typical; the continuous function the derivation that you have seen and continuous function I will just draw this here.

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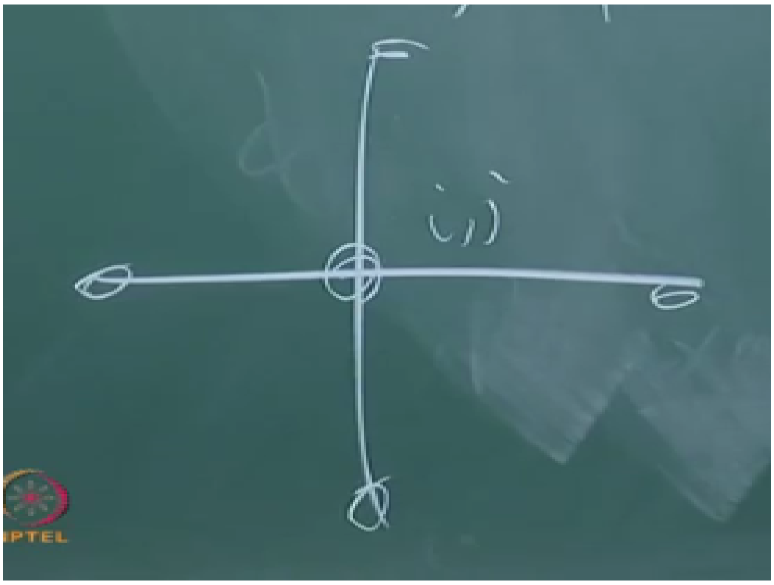

So, if you have these 4 points what are we doing here, what is at the point ij, at any given point ij at the interior, what is phi ij? Phi ij is the average of its neighbours, so what is an average, the average is not larger than the largest neighbour nor is it smaller than the smallest neighbour, so any given point here let we take the average for actually, I guess I could have done it just here any given point that you take the average, for any given point for which you take the average is not larger than its largest neighbour nor smaller than its smallest neighbours.

So, this point, this is not larger than its larger neighbour, smaller than and this is true of all the interior points, it is true of all the interior points okay and therefore can I conclude that the maximum and minimum will actually occur on the boundary, the only values that I do not touch that I do not change are in the boundary, every other point I ensure, every other interior, every point on the interior I ensure is larger than the smallest neighbour and smaller than the largest neighbour, you understand.

And all of these satisfy that condition they are all larger than the smallest neighbour and smaller than the largest neighbour consequently, the maximum and minimum to the solution must occur on the boundary fine, okay so at maximum and minimum over the solution occurred on the boundary what can we say; the maximum and minimum occur on the solution on the boundary of the solution they occur on the boundary what can we say?

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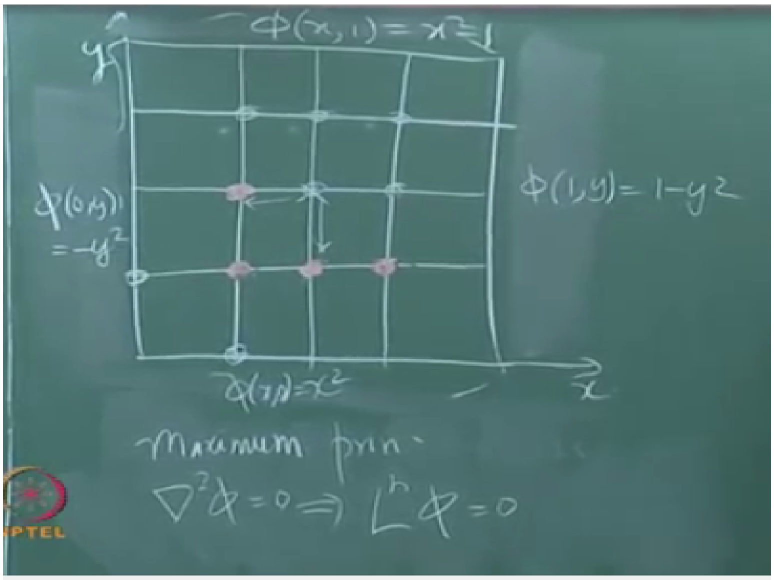

We will use this argument now, this is called the maximum principle, I just write, I am saying this just for okay and you just say maximum principle okay, fine, so what we are saying is if I instead of writing the system of equations, so if I have, I still write it as nabla squared phi equals 0, so I will replace this by some; what shall I call it; L of h phi equals 0, right, so this is some system of equations that is what we called it right, h is the grid size okay.

And the discussion that we are having now is; is it possible for 2 people; is it possible for 2 different students to get 2 different answers and it is important that L is a linear okay that we get a system of equations, is it possible now for 2 people to get 2 different answers. So, let us say 2 people; 2 different students get 2 different answers okay.

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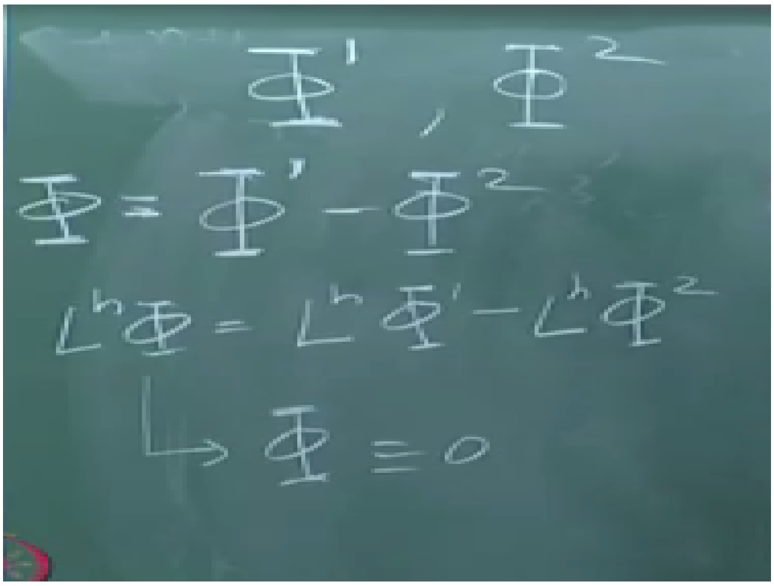

So, the 2 answers; 2 different students get 2 different answers; one gets the answer capital phi 1, so I am making it capital phi, so that we do not confuse it with our iterates okay and the other gets the answer capital phi 2, okay, so we do the usual mathematical trick because we know we have a linear equation, what are we going to do? We are going to look at what is the difference between the 2 answers; phi 1 – phi 2.

Because the equation is linear, this is also a solution to Laplace's equation, right because the equation is linear is also solutions, so if this is some phi, this is the difference L h phi is Lh phi 1 – Lh phi 2, you understand because it is linear, the L actually distribute across it because it is linear, it is just a system of equation linear system of equations in this case and therefore this satisfies Laplace's equation, right.

What are the boundary conditions that it satisfies? Remember both phi 1 and phi 2 satisfy my x squared - y squared boundary condition therefore, phi 1 – phi 2 the boundary condition is 0, now we use the maximum principle, the maximum and minimum occurred on the boundary and that is 0, right. So, the only function for the maximum and minimum occur on the boundary and that is 0, this phi is identically 0, is that clear, okay.

So, as I said this sort of mirrors, the proof that you would have seen possibly and for the differential equation itself, so yes it will turn out that if you do; if you solve this problem; if 2 different students get 2 different answers, the possibilities are both of you have the wrong answer, right that is one obvious possibility or one of you has the right answer, right and one of you has the wrong answer, okay right.

After you have been programming enough, you start with the feeling that you do not start with the confidence, my answer is right, you are feeling always is that slightly that your answer is wrong, you have to start with the assumption, the presumption is my answer is wrong and you have to work to show to yourself, convince yourself that the answer is right, is that okay fine, great.

So, here we have; what should I say, we have shown that they have shown that the solution the answer is unique, we need to show now whether the scheme always converges we look at that right but before we go there, I have just quietly written this as Lh, let us figure out what is this L, what is the nature of this L, what is the nature of this operator L, okay fine.

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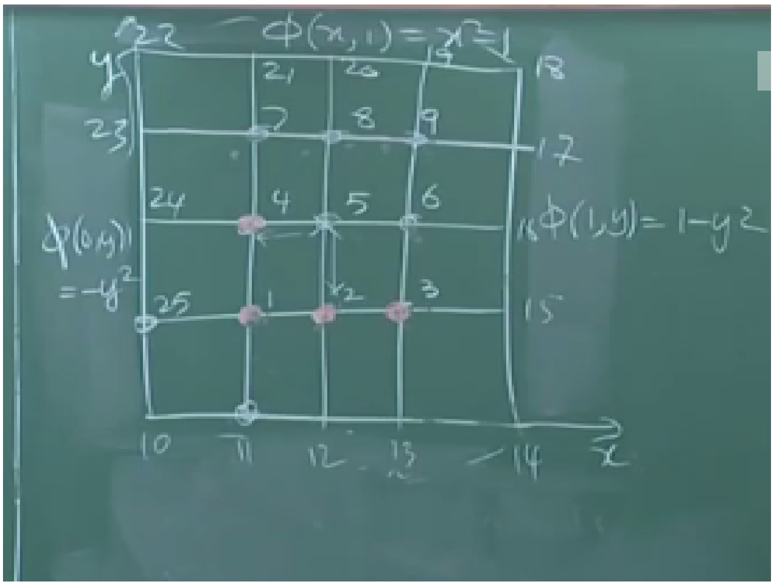

So, back here, so there are 2 possible numbering schemes that you can; the different ways by which you can number these, okay intuitively if you look at the way I have written this, if you look at the way I have made these pink coloured dots, the assumption is that I have numbered this, this possibly is 1, 1 or 0, 0 or whatever it is and you know I have numbered them in an increasing fashion of y and an increasing fraction of j, right.

In fact, I do not even have to tell you instinctively a lot of you will assume that is what I am doing, okay so we number this in this sequential fashion, right that is what we have done but is it possible for me having gone through this process, I now understand that it is only the interior points that are unknown, they need to be determined, so possibly I can number the interior points in a sequential fashion and then number the boundary points okay, fine.

So, one way to do it is you say, you use 2 subscripts, the other way to do it is that you basically number the boundary points after this, so that is 9, so and this becomes 10, 11, 12, 13, 14 and so on okay, you could also number them in a random fashion, would that make a difference do you think? So, this is something for you to think about, if I what should I do this sequentially would it matter if I started my iterations from the top.

Would it matter, if I iterated, I relaxed this point, this point, this point and did it in the reverse fashion, okay or can I just do it in random; can I just do it at random, you should always get the answer. The question that still there we should always we would expect that we will get the answer, we will look at; we will see whether we can throw some light on that idea, right now intuitively, I am getting a lot of head shake saying that you do not expect.

I do not expect it either, you do not expect from what we have seen, you do not expect that the answer will change, we are taking averages, you do not expect that the answer will change, so there are different ways by which you can sweep through the domain not necessarily from left to right, bottom to top not necessarily in that direction that you can sweep in any direction and instinctively, the way the number it is the way we tend to sweep.

And therefore the way we number it becomes important okay, so we will definitely get to the answer, then the second question is how fast we get to the answer, right okay, so we look at those issues right now, let us stick to this, while numbered it in this fashion; 1, 2, 3, 4, 5, 6 and we can write for each one of these, we can write an equation and as a consequence, we will get a linear system of equations.

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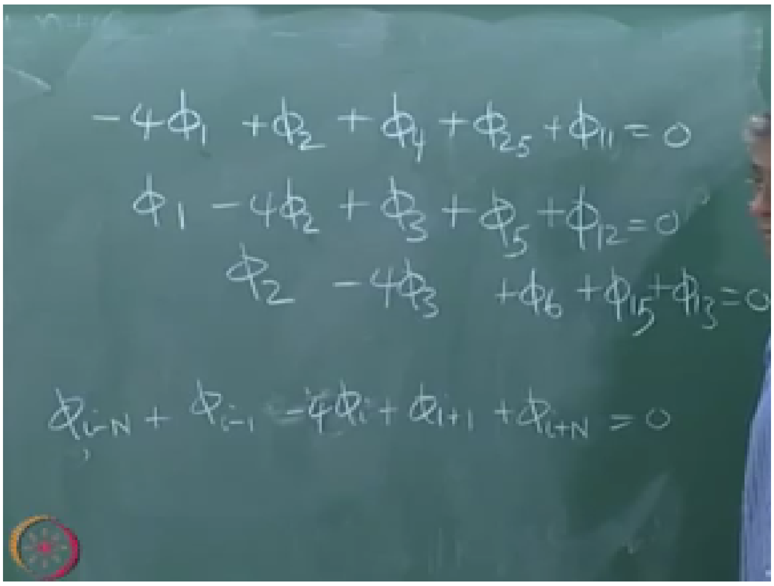

So, for each one of those, we write an equation for the first one, I get phi at 2 + phi at 4 or phi at 4 + phi at 2, it does not matter - phi at 1, if you want – 4, I keep forgetting, let me start the other way around. I will do it in a sequential fashion; - 4 phi 1 + phi 2 + phi 4 + phi 25 + phi 11 equals 0, is that fine, what is the next one turned out to be; phi 1 – 4phi 2 + phi 3 + phi 6 or phi 5 + phi 12 equals 0, am I making sense.

We will write one more, so there is no phi 1; phi 2 – 4phi 3 + phi 6 and then you get 2 boundary points; + phi 15 + phi 13 equals 0, of course the boundary points we are going to take over to the right hand side, okay. In general, if I write for a general point in between as I go down to an interior point which is only surrounded by interior points, I will get phi, if I am at the point i, remember I have only one subscript now, okay.

Phi i - 1 + phi I – 4 phi i + phi + 1 that is the easy one, so you can see the second derivative; i + 1, i – 1, then what do you get; + phi i + n, where n is the number of points in that row in a given row, so phi 1, there were 3 in the row; 1 + 3 gives me 4; 2 + 3 gives me 5, 3 + 3 gives me 6, okay there are 3 in that row and subsequently I will get a phi i - n and this should equals 0, this is a general situation, okay.

Coming back here, if you look at it, if I have a general point if there are n of them in this row, this is -1, this is +1, this is – n, there will be; you will have to go back n, this is - n that is + n, okay, so with a single subscript, we can actually go i – 1, i + 1, i – n, i + n, is that fine, so we can really write this as a system of equations and really write this as a system of equations maybe I will write it in a bigger fashion in a system of equations.

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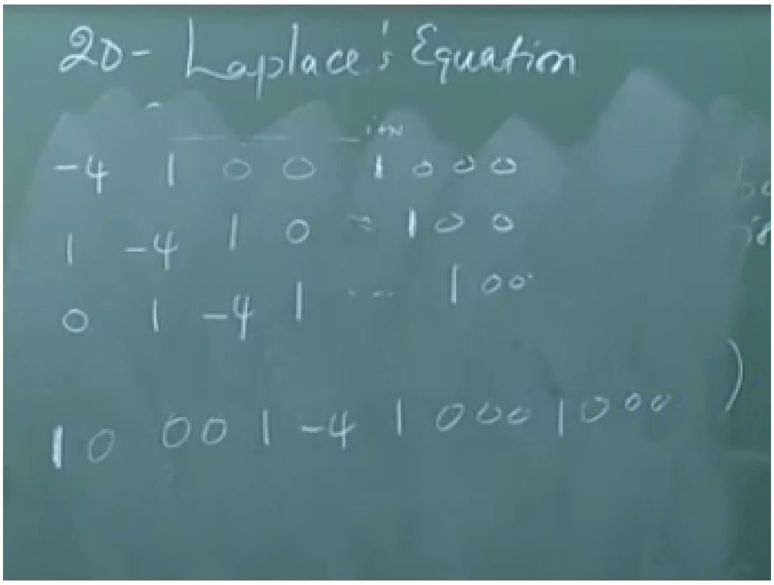

I will write it there in that first, can really write it as a system of equations, which will be -4, 1 and what; whole bunch of 0's, so this is the first one then you have to go to i + 1, i + n, so when you get to i + n, there will be a 1 there, lots of 0's, this is the structure of the matrix that you get, what is the next one? 1 – 4, 1, lots of 0's till you get to 1 on the diagonal there, lots of 0's, is that fine and you get 0, 1, -4, 1, lots of 0's, 1 on the diagonal.

So, the equation keeps on shifting to the right okay, lots of 0's after that and this will keep on shifting to the right till you get to a point where a 1 will appear, right till you get to a point where ever you gone to the second row, now from the first row something is contributing, while you are in the first row, the one below goes to the right hand side okay, so will you get lots of 0's, 1, -4, 1, lots of 0's, 1, lots of 0's.

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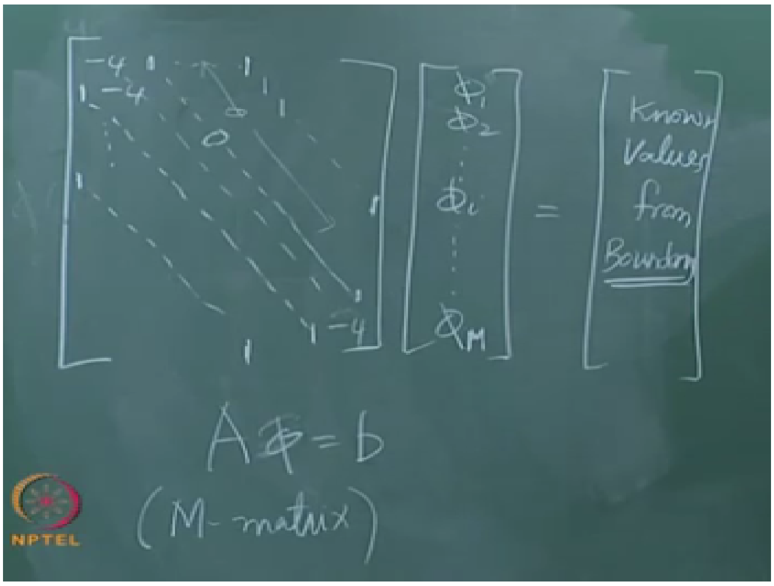

In case, this does not make sense, I am going to erase this, I will write it as a; I will just draw lines, so you get a - 4 on the diagonal and we are all agreed that -4 is there for all the points right, so you get a - 4 on the diagonal, there is nothing to the left of this but you will get a +1 on the super diagonal; on the first diagonal above the diagonal, fine and you will get a +1, so there is a -4 here, you will get a 1 here on the sub diagonal, fine.

So, that takes care of the second derivative in the x direction in a primary coordinate direction then what happens, then you have n away from this, you have a 1 all of these in between of 0's and all 0's, okay and you are going to have a bunch of 1's till you come to the right boundary, you get to the right boundary or the top boundary, right when you get to the top boundary or the right boundary at each of the boundaries you have to be a bit careful.

So, here n away, you will get a 1 there, am I making sense, okay this 1 at the bottom and you will get 1's along this, 1's along this and 1's along this, this corresponds to the top boundary, this corresponds to the bottom boundary, there are 0's here which correspond to the bottom boundary, there are 0's here that correspond to the top boundary, here as you go along just like you did not have any entry here, as you go along, as you come to a left entry or a right entry, you will get a 0, as you come to the left boundary or the right boundary you will get a 0.

Does it make sense, I still see a few faces that are confused, when you have, when you read the boundary, then you go when your traverse along this when you come to this, there is no, the entry here is known, it is not an unknown, so that goes to the right hand side, so you do not get a +1 that will be 0 just like that with the very first one, there will be no i - 1 entry that will also be right, there will be no i - 1 entry that will also be 0, okay.

You go to the top, there is no i + n entry, when you go to the bottom there is no i – n entry, okay, the points right next to the right boundary has no i + n entry, the points right next to the left boundary have no i - n entry, the ones at the bottom will have no i - n entry, no i + n entry, okay, so that the no i + n entry, no i - n entry and then embedded in here, there will be 0's with a frequency of n, every time you go through, you come to the end there will be a 0.

Because there is no right hand, there is no point to the right, it is a known and all of those points will come to the right hand side, so this equation this matrix A, what does it multiply? It multiplies phi1, phi2, phi3; phi subscript 1, phi subscript 2, phi i, phi, I make it capital M because I n the number of values in the row, phi M; where M is interior grid points and this equals; these will be the values from the boundary conditions, known values from boundary okay, these are prescribed, fine.

So, we actually get something that looks like A phi or Ax equals b, there are system of equations okay, some observations, the equations are symmetric about the diagonal, equation that we get; the matrix that we get A is symmetric about the diagonal, the diagonal entries are negative, the off diagonal entries are positive, so it has some nice structure to it okay, you can just go look it up, I am not going to do this.

It falls into the category of something that is known as an M matrix, so just check that out, so the diagonal entries are negative, off diagonal entries are positive, it is symmetric right, so there must be maybe there is something need that we can do with this, we can check to see whether what kind of; right, what it is that we are but we have a system of equations, you have a system of equations.

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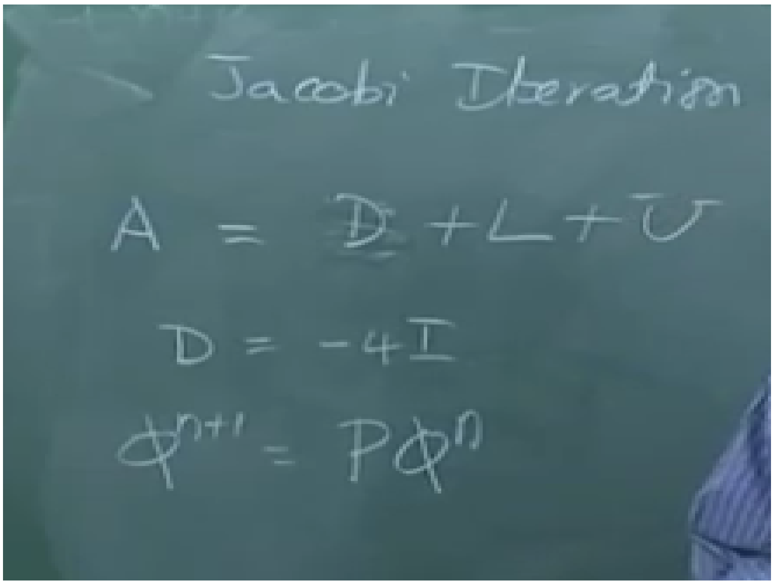

Let us now see whether, what it is that we are actually doing, when we solve the system of equations okay; what is it that we are actually doing when we solve the system of equations. So, when we do Jacobi iteration, okay what are we doing? The matrix A can be partitioned as the diagonal, a diagonal matrix D + the lower triangular matrix + an upper triangular matrix, just to give you an example.

The diagonal matrix D is - 4 times I, right the diagonal matrix D is - 4 times I, fine, the remaining part lower and upper triangular you can extract, I think that you can do, so when we do Jacobi iteration, what we are actually doing this; we are saying phi n + 1 equals; you can check this out D inverse phi n - ; well maybe we will work this out next time, I think they are coming close to.

So, what you can do is; you can just take the equation; phi n + 1 equals; we are trying to find just give you an idea as to where we are going, we are trying to find out what is the P, okay, we are trying to find out what is this P, so that we can say something about how this equation converges, so far what we are shown is that if you get a solution, it will be the solution and it is unique, you understand what I am saying.

Now, what we want to do is; we want to see whether does it actually converge, how fast does it converge, how fast does it get there, right, now we are talking about the operational part, it is nice to know that if 2 people get 2 different solutions right, I mean we know that you everything works 2 people should not get 2 different solutions, the solution is unique. Now, we want to know that we want to answer the question how fast do I get it.

I would like to get it today, I would like to get it in a few minutes, I do not want to take for hour to get the answers, how fast, what can I; the issue is if I can figure out, if I can answer the question how fast will it, how fast does it; how long does it take to get it then I can if there is a systematic way by which I can do it then I can ask the question, why is it taking for so much time, is there something that I can do to make it faster, okay.

Remember, always ask how good is it and if you are able to determine to answer that question, right the objective of asking how good does it is not just find out how good is it but if we understand the question in the way we answered it, it may help us improve right, whatever it is that we are asking, how well does it converge and we can figure out right, a way to improve the convergence if you are unhappy with the convergence, is that fine okay.

So, I will see you in tomorrow's class, we will look at this Jacobi iteration and see what is happening in tomorrow's class. Thank you.