Introduction to Computational Fluid Dynamics
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Lecture - 06
Demo - Hat functions, Aliasing

Hi, so what we will do today is we look at I show you a demo right, the use hat functions. And what we have seen so far is the intervals are important, number of intervals that we have, the size of the interval is important for orthogonality. And the fact that overlaps do not, the intervals do not overlap is important for orthogonality right, the what was it? The size of the interval so far did not matter right, we took them as equal sizes right, that is what we basically had.

The number of intervals increase the accuracy with which we are able to represent the straight line for box function, if you think back to the example of the box functions. How well we were able to represent the straight line, it got better the error reduces as the number of intervals that we use increases. So we will try to see if there is a relationship between the function that we are able to that we want to represent right, and the number of intervals, is that fine.

And I am going to use hat function to do the demo, so this is so there will be a combination of things that we are going to get out of this demonstration.

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I am going to use a scripting language called Python, a particular version of it called Ipython, just so that what comes on the screen, that is a sound strange to you. I am going to use 2 packages, I would suggest that you find for yourself some package with which you are able to plot right, plot graphs and check them out on screen. There are lots of packages out there, I am going to use a package called grace plot, which I will this is as I said, so that you understand as a full of those packages what it means, what I am doing.

And to represent arrays, I am going to use package called numpy I guess for numeric Python Numerical Python but anyway numpy. This is all that we basically need right. So the objective is to see that if I take a function say for example sin x, and I use 10 intervals on which to represent sin x. How well do I do it? And then we will change the function sin x, sin 2x, sin 3x, sin 4x to see whether it gets better gets worse, you expect that maybe it gets better gets worse anything before we start.

If I use 10 intervals and I tried to use I tried to represent sin x has suppose or sin 2x or sin 3x sin Nx, you expect that a representation to get better, the representation to get worse, you expect the representation to get worse. And the reason why I pick trig functions, they are made actually numerous reasons why I pick sin x, one of them of course is that it is possible for me to scale it. I can change there is an inherent length scale associated with sin x.

And by changing going to sin 2x, 3x, sin 4x, I am changing those length scales right okay. So let me start it off. The first time around I will set it up maybe subsequent classes, I will not go through as much of a setup and doing going through the full setup this time, so that you see what I am doing okay. So I am going to run Ipython, and I am going to setup my grace plot initially it is going to be very intense I change the background color.

I create the plot going to be an intense white color, so I will quickly resize it, and I have set up something that you give it the more not so bright color okay. Now what I will do is I will now get my numpy, from numpy import star you can go learn Python if you want to use this programming language, most of the stuff that I am doing I am not going to do anything unusual, I will create an array, so numpy has in it various things including definitions of pi sin and so on right.

So I will create a row vector, there is a peculiar way by which you do it, going from row vector whose values go from 2 0 to 2 pi, and I want 11 points that is 10 intervals, so if you say what I have managed to do, that is what I have managed to do. So I have x going from 0.628 so on to 6.28 which is 2 pi right close to 2 pi. Now it is very simple, I create y which is sin of x, and since my plotting utility is going to connect these points by piecewise straight lines.

Which means that inherently underneath I am doing hat functions, I just use it to plot okay, now if I were to use cubics, quadratics I have to be a little more careful, I cannot just use this straight lines, I have to be a little more careful. I have to actually plot the interpolating polynomial right, but in this case by coincidence it happens there is going to join them by straight line, so I am using hat functions okay.

I say g dot plot x, y and I get that, what do you say are you happy with the function? You have to a bit critical, is it any problem with this function, first of all it does not seem to reach the maximum right, it does not reach 1, there is a small gap here. Why does not it reach 1? Because I am not sampling the function at that point, so I am using the word sampling here, I am not sampling the function at that point I am not sampling the function at the peak okay.

Which is the reason why anything else, it looks reasonably like sin x, it has a reasonable semblance to sin x okay, maybe if we try higher number of grid points or whatever it is maybe it may be right. So what about sin 2x, shall we try sin 2x, sin of 2 star x, I do not have to keep typing this every time.

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What now so is it any better, it is worse, it really does not look like I do not know maybe the peaks also do not seem that great, but actually if you look at it, it will turn out that the point where it crosses the 0, the 0 crossing so to speak. These are the 0 crossings will turn out to be quite good okay, on this graph of course it may not be it is not that easy to find out, but you can actually evaluate it and check it out.

You will see that the 0 crossing are actually fine okay, the point at which the function crosses the 0 is fine, and the number of 0 crossing is fine. The number of peaks and valleys is fine, the number of peaks and troughs is fine right, the function value is seemed okay not great. And the 10 intervals this is not looking that good. So we will see what happens if I go to 3x okay, so tell me what we have I see a lot of smiles, tell me what we have right.

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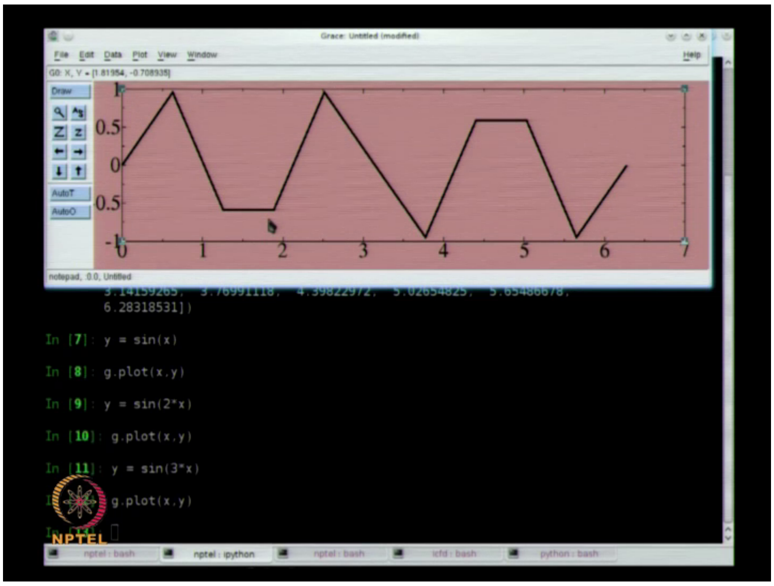

So here obviously that minima is gone minima and maxima here are gone okay, and as it got at least the frequency does it seems as though it got the frequency as number of 0 crossing that I am going to have right divided by 2 in this case, so it is like this so here to here. I can count it looks like the frequency information has been picked up properly, the 0 crossing seem to have been picked up properly, amplitudes are totally off okay.

So now I am doing sin 3x with 10 intervals okay, so there is clearly some relationship between the number of intervals that you are going to pick, and say the underlying frequency or length scale that you are trying to the represent okay. So that is one of the things that we want to out of this. Let us go ahead, I mean we will just try 4x, and see what it does, sin 4x. Now this is like nothing I do not know, you tell me what do we have? We have anything at all.

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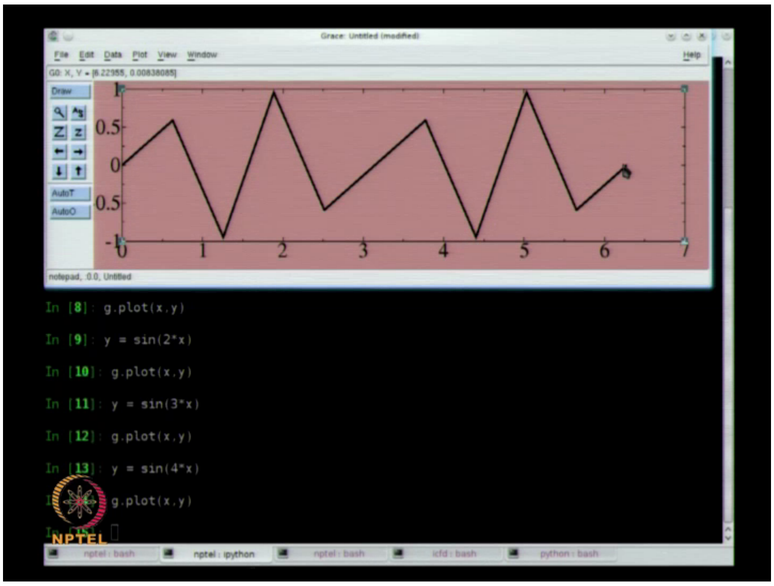

Do we have any is the frequency information correct, is the number of 0 crossings is correct? How many times does it crosses 0? There is 1 here, 2 here, 3 here, 4 here, 5 here, 6 here, 7 here, 8 here, 9. 4 x that looks about right okay, so in some sense the frequency content is there right, and the function representation that is if I were to show you this, you would be hard pressed to say that it is actually sin of any anything sin 4x or something of that sort. So it is difficult to say that sin 4x okay.

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Let us try the next one, what do you think will happen here? It has been getting bad, how bad can it get, this is I should actually this is we have to look at the scale before you look at this right that is 10 power-15 okay, 10 power-15 just for the fun of it just so that you can say that kind of stuff that you may have to do when you are running or exploring these programs. I will reset the scale here to -1 to +1 and that is what you get.

So what happened? We sample the functions exactly at the 0 crossing, now this is a bizarre situation we picked up the 0 crossing exactly, but we have nothing right we do not the functions is just totally disappeared we are not picked up the peaks anywhere, so we really have nothing is that fine. So let us go on we go on and see what we get, after all we have 10 intervals 11 grid points 10 intervals.

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So what do you expect for 6? Going to be the same as 4 is the prediction that I get, this it is the same as 4 or similar to 4? Is it the same as 4? there is a sign change there is a sign change. So you would expect and I think if all of you are familiar with this, I do not have to go on.

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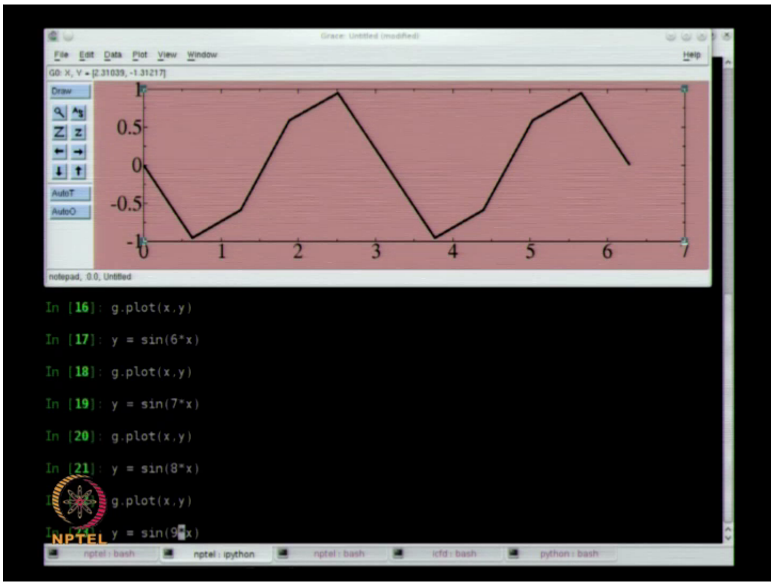

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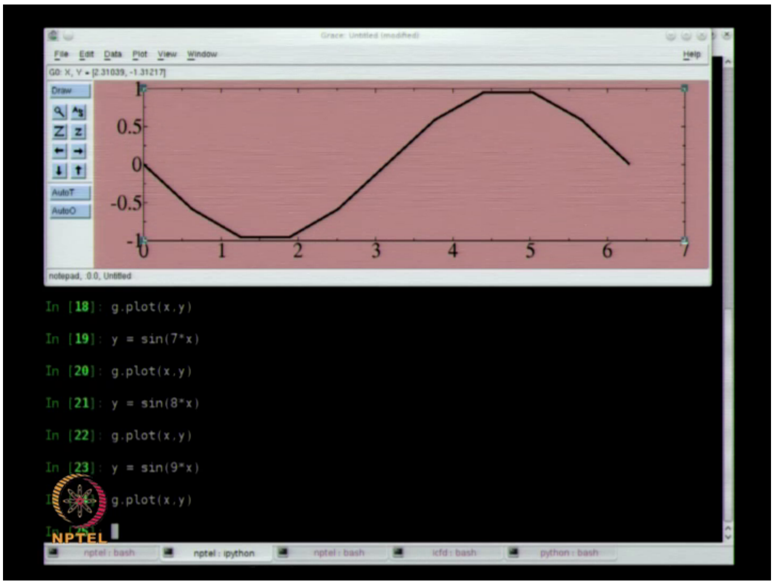

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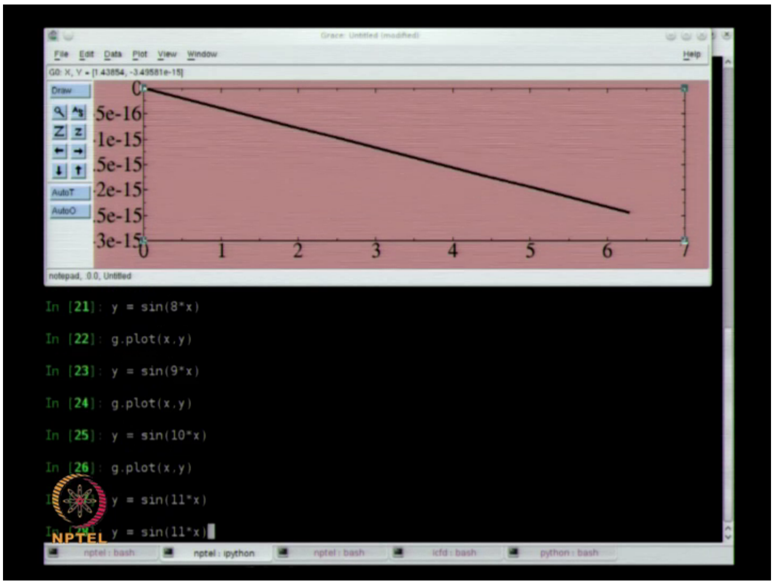

So if I go to 7x okay and we see that the pattern is repeated, the pattern is going to repeat, this is what I expect. I go quickly to 8x, and I plot that right and 8x is getting better but it is not 8x. 9x what we expect for 9x yes a bit strange it is clearly the sin is slipped. And 10 will be 10 should be a straight line of course I mean I know I am not going to rescale it again, but 10 should be a straight line.

But we will see gets the same actually we get the same value it should be a straight line, but remember we are dealing with numerics, and it is not quite the same but yes the scale is still 10 power-15, so if I make it to -1 to+1 it would just be a straight line, the value is essential is 0 it is within machine epsilon values essentially 0 okay. So and if we go on what do you expect to happen, if I were to repeat this process what do you expect to happen is going to keep repeating.

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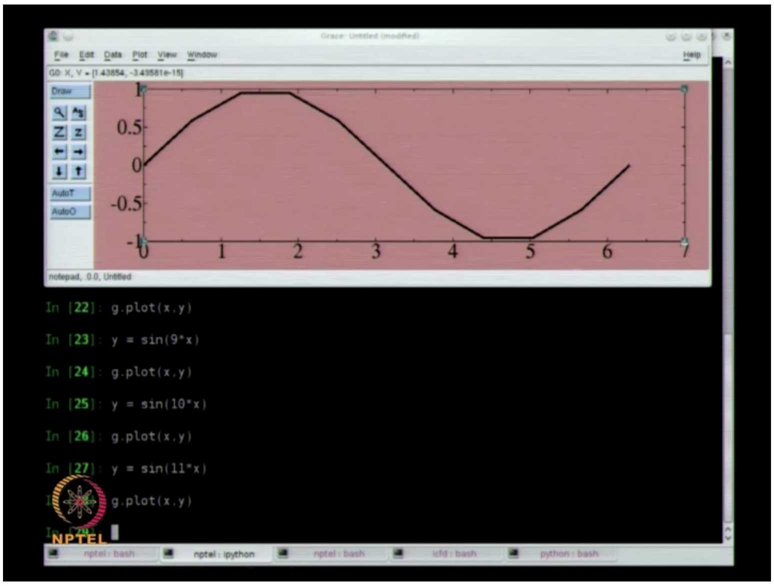

So the first lesson that we get out of here as if I have 10 intervals, if I have 11 grid points, there is the largest frequency that I will be able to represent with those grid points right. If I have 11 grid points, and I am using hat function there is a largest frequency that I am going to be able to represent using those grid points, is that fine, is that make sense okay fine. Now we can try it, we can try a different set of grid points just to see if we can infer anything else.

There is the largest frequency we had 10 intervals and the largest frequency was what? It was 4, so it looked like maybe right now you would say then N/2-1 or something of that sort, so let us pick some other number. Let us say we take x to have going to 2 pi, how many shall we take 21, do you want to take 21 take 21, 21 grid points is about 20 intervals fine. So we go through this business again y=sin of x, check this out.

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21 grid points is definitely better than right, 21 grid points is definitely an improvement on 11 grid points, 20 intervals is definitely an improvement on right 11 it looks like though it is pi/2, you know you are not going to actually sample it exactly at pi/2, but we are quite close. We cannot make out that, so it looks like it picked the peaks well, the function representation is quite clean okay. So this is one thing that we should bear in mind now.

The lesson that we take from here, that if you are a representing a function it is a good idea as sometimes either an engineering way to look at it would be to double the number of grid points to see whether there is an any improvement, there is something to bear in mind right. I will you will come back and visit this that suggestion at the end of the semester okay. So the function definitely looks a lot better.

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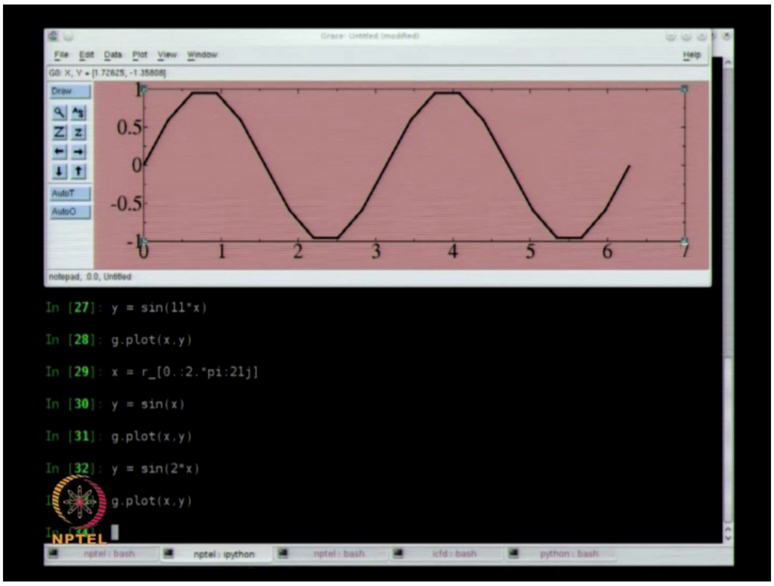

What do you expect will happen to 2 star 2 time 2x, this I would expect this would be the same as sin x on the 10 right, so now it is very clear it should be the same as sin x on the 10 right. So if I now go ahead, we will do a few of these and then I will sort of jump ahead to that.

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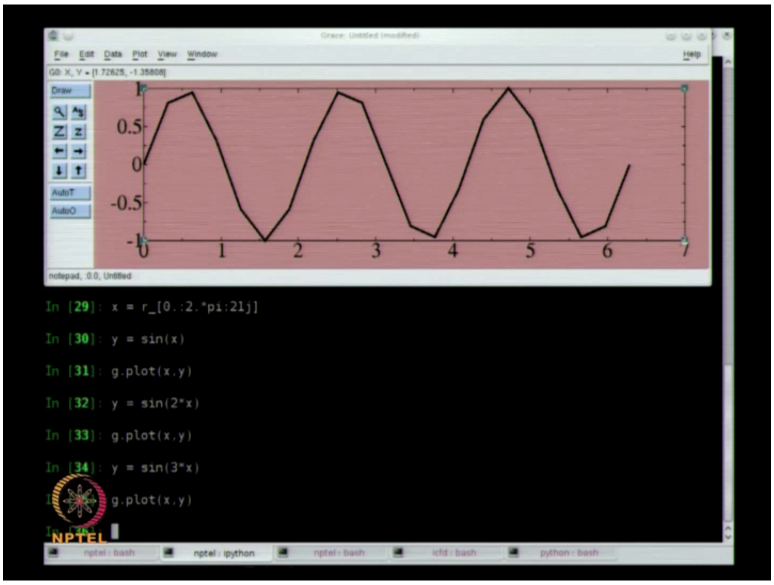

So if I do 3x, 3x of course has no equivalent there right, so it seems to pick the peak in some places, it picks the peak in some places, it lost the peak in some places, the 0 crossing are not there or I mean how good. Otherwise, you could say it is sin, I mean we can live with it but I want you to bear this in mind every time, so if you are going to if you are actually trying to represent a trigonometric function right.

It looks like if you are going to the represent sin x, it looks like if you want to capture the function well right. It looks like you need at least 20 grid points, 21 grid points, 20 intervals right, employing hat functions you need at least 20 intervals for a given wavelength that is one lesson you get out of this, you need at least 20 intervals for wavelength. So if I am talking and you say I want to model this in some fashion right.

And you want to model it so that you can pick up that signal, you actually pick up the pressure variations. Then you need at least 20, 10 that you could live with 10, but 10 does not that good okay.

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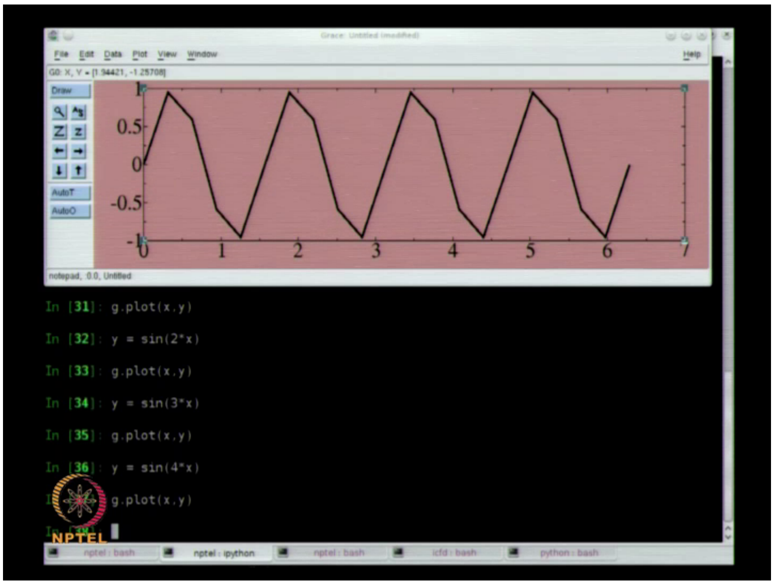

So let us try 4x should again look familiar, 4x is again familiar. But okay, let us skip ahead which one should we do? Let us try 7x, so you can get it is very clear that beyond the certain point, the function start looking very strange, and they do not really look like sin 7x that you want okay, that is the only reason why I am sort of bothering you with this.

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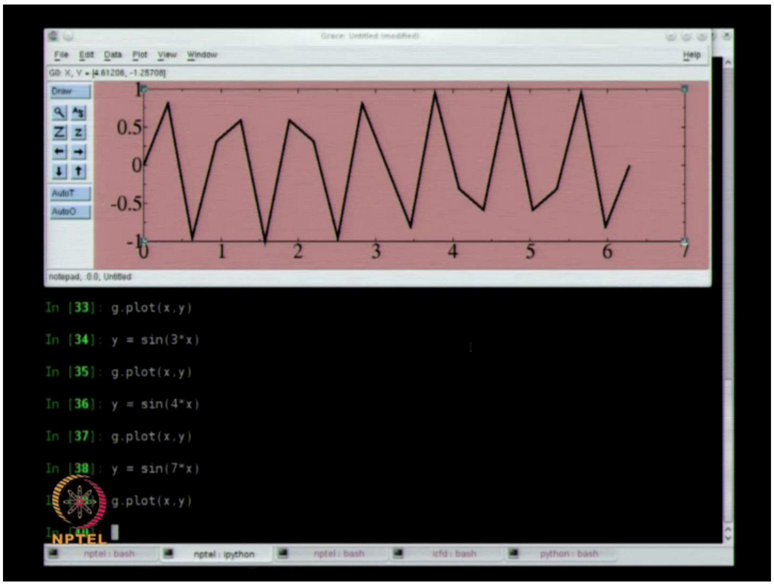

It also looks like if you look carefully now it looks like on top of this if I look at the envelope over this, there is a sin kind of something like a sin that riding on top okay, so I am sorry about that, there is something like the sin riding on top of that.

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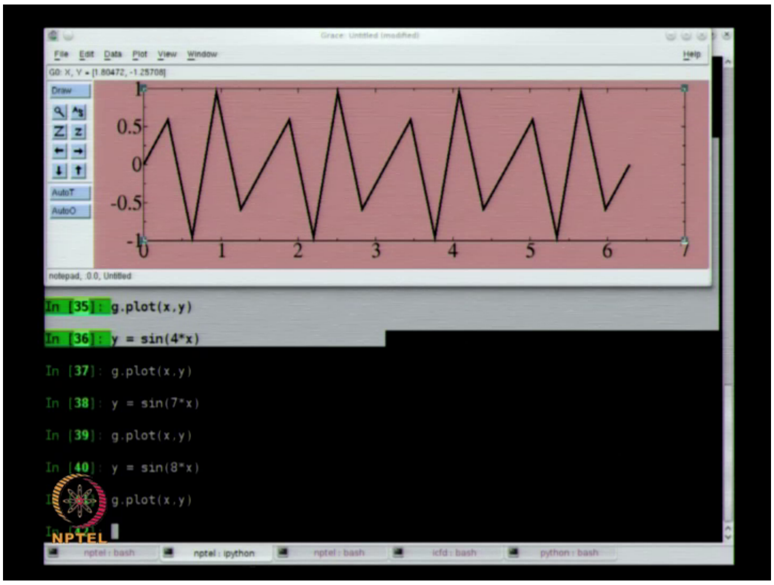

Okay, let us try 8x, 8x something that we have picked up earlier, 8x corresponding to 4x that we had, however, of course the wave number is different the frequency is different. Normally when you do it in space, when I am doing this when I am talking about some of these kinds of problems you will see me use wave number and frequency, and they are sort of an interchangeable fashion okay.

But normally when it is in space we say wave number, when it is in time we say frequency okay. So the wave number is a of course different but it looks very similar to 4x, if you look at it on the 0 to pi interval.

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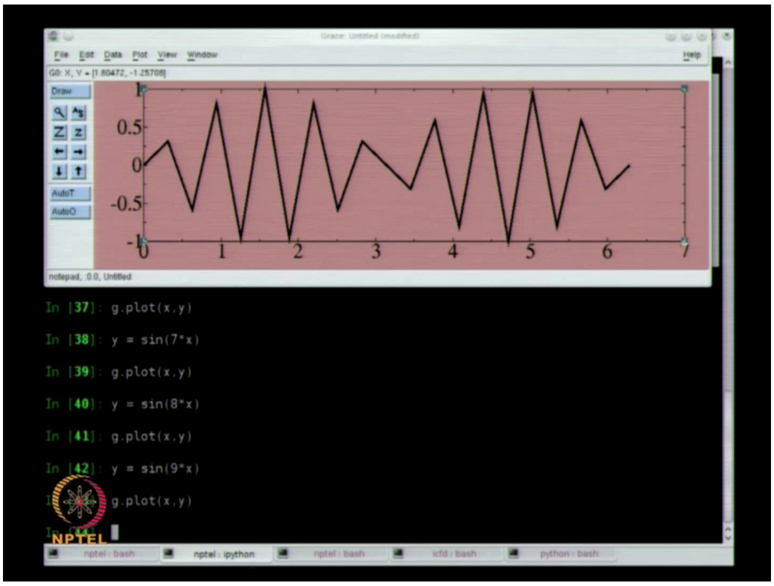

And what happens to 9? Why do I want to try 9? 9 should be the bad one, 9 and it has that the envelope again has that variation, so there is something riding on it sin x riding on it literally okay.

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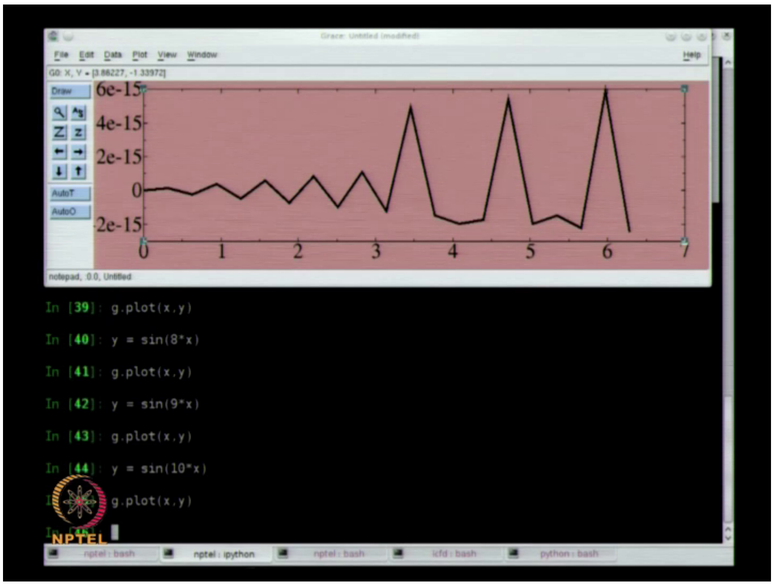

And what do you expect will happened with 10? 10 should be 0, each one has its own, it is 0 but each one because of the nature of the representation of the number itself on the computer right. It should be 0 but each one is the little different, and the magnitudes you can see that the maximum and minimum magnitudes do not necessarily seem to be the same, and we get the impression that has the value as you go further and further down, it is getting noisier and noisier.

That it is closer its smaller value here closer to the 0 value here, and much worse at higher at the higher angles okay, these all observations that we can make just purely from looking at what we have got so far right.

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So what you expect will happen to 11? 11 will repeat so it looks over so this is for this reason 10 would be called a folding frequency, it looks as though the figure fold over right, fold over in the sense that 9 is same as the 11 except for a sin, right 8 is the same as 12 except for the sin. So it says that we have folded over right the folded it over that frequency, and anything else that we can say?

So if I give you N intervals or N+ 1 grid points right, if I give you N+ 1 grid points then N/2 seems to be the largest N/2 in fact seems to be the troublesome frequency, it is 0 there. So let me try a different set of grid points again, let us try something we have both the ones that we have tried odd 21 grid points 10 interval, the number of intervals is even. Why do not we see what happens when we change? Okay.

So the first question is, do you expect any change? We expect change x= we expect change okay, how many shall I do? I do 20, or let us keep it at 10 in that way we do not have to okay, of course the trouble with 10 that does will not look that good but it does not matter.

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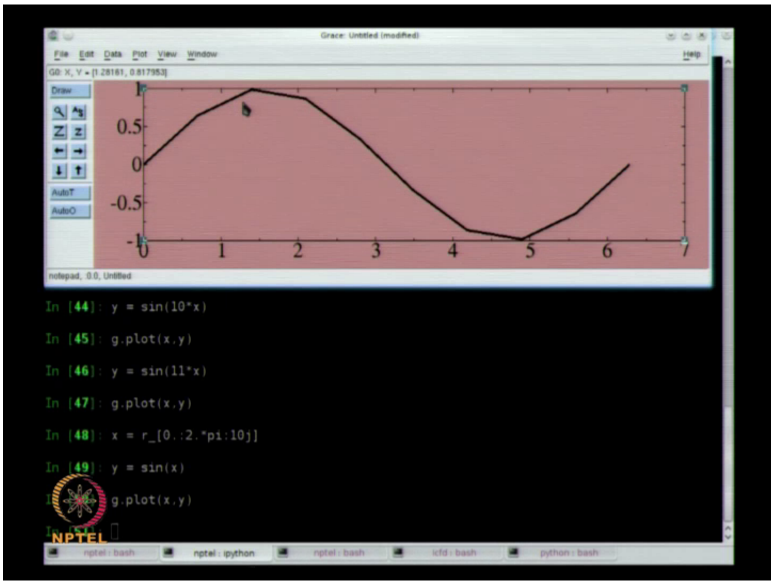

It is not bad one improvement of 9 intervals over 10 intervals is that we pick the peak better okay, let us not draw major conclusion from that right, I mean it is possible just happenstance that may just be chance that that is happened. The function does not look bad right; the function does not look bad.

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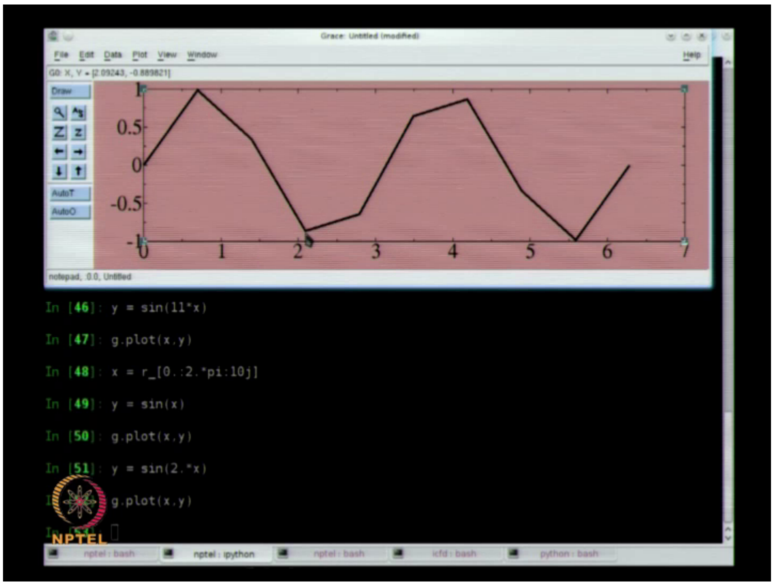

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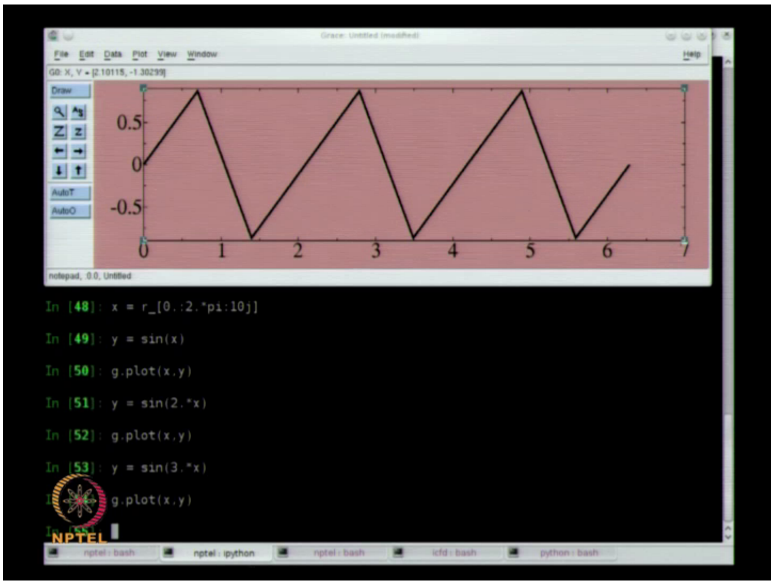

What about 2x again like last time we pick the peaks better, this peak is off that peak is off. 3x okay this is a nice saw tooth, we have picked the amplitude right, we have pick the frequency right, the 0 crossing are right, it just is in sign that is the only problem right, it is got all the information. Of course if you talk to your electrical engineering friends, they will tell you to convolve it sinc function or something of that sort extract for the signal.

But it does not matter right for us as far as we are concerned, we are not going to do complicated things like that I just want the sample points and I just want to use linear interpolants to represent a function, and it bothers me that this is what I get okay, it bothers me that this is what I get okay. 4x give me a guess, what do you expect? You expect it to be 0 okay.

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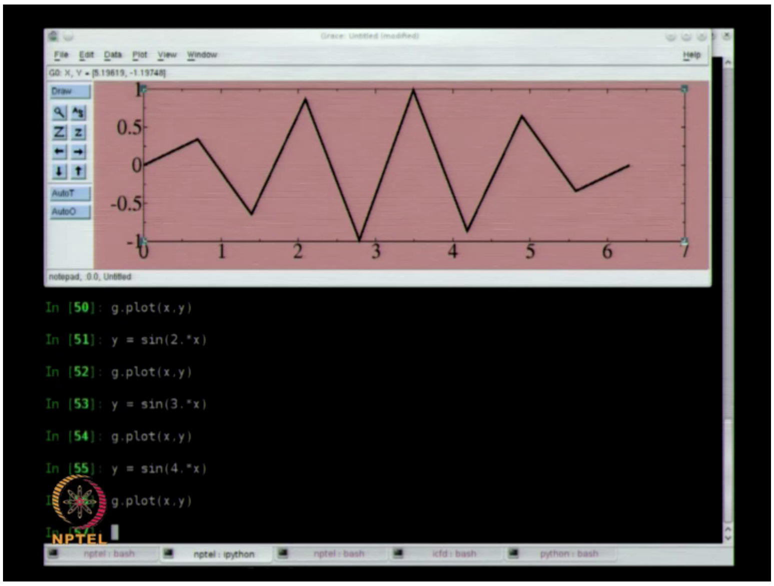

So it does make a difference whether it is even or odd, so the N/2 where N is the number of intervals, the N/2 seems to be significant right. So N/2 so if you have N is even N/2 is going to give me pick up that 0, or put the other way if you know you start off by saying that this is my frequency of wave number or whatever. You may make your choice as to what is the number of sample points that you are going to take right.

So you may take at least 2N sample points 2N+1 sample points right, sample the function at 2N+1 points at least okay is that fine okay. So what do we have what is the so we have what are the other conclusions that we can draw from this? **"Professor - Student conversation starts"** (()) (26:09) So frequency information is it lost here? The frequency information is still there; the peaks are gone right.

So until up till that points, so up till the point so one of the big conclusions that we can draw is the there is largest frequency that can be represented okay. **"Professor - Student conversation ends."** So if you give me I will call it a mesh or a grid right, if you take an interval a, b you break it up into equal number and for now because that is all we have been doing we cannot draw any other conclusions equal number of intervals right N equal number of intervals.

Then N/2 this is sort of the highest frequency that can be represented okay, turn it the other way around for a given mesh whether something is high frequency or low frequency depends on the mesh, am I making a sense.

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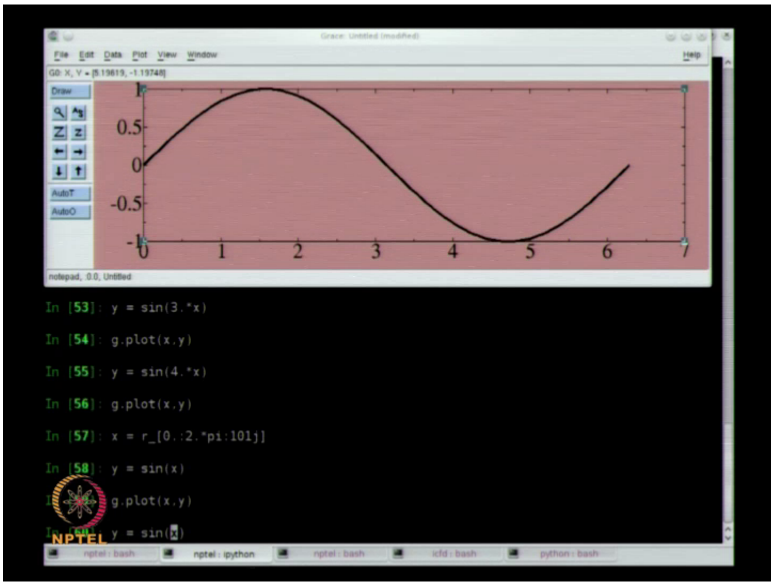

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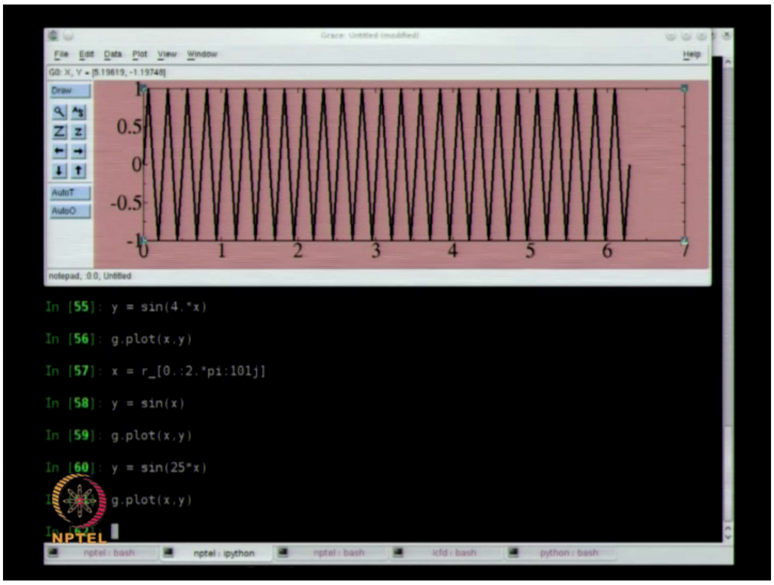

So if I were to pick very large number say 101 intervals, if I were to pick very large number right, so it comes out sin of x comes out extremely smooth, but like I mean it is pretty obvious that I can actually represents say 250, I am sorry say 25x sin of 25x, am I making sense. It is not a great representation right; it is not a great representation but on 10 intervals I cannot pick it up at all. It is obvious right now what we have seen it is always 10 intervals like.

But that is not, I want you to pick I want you to, so when we talk we talk about high frequencies and low frequencies. As we go along you will see that we keep talking about high wave numbers low wave number, high frequencies low frequencies, so the point that I want you to get out of this is when you say a frequency is high right, in our world then you are performing some computation and you are trying to determine a function, when you say a frequency is high.

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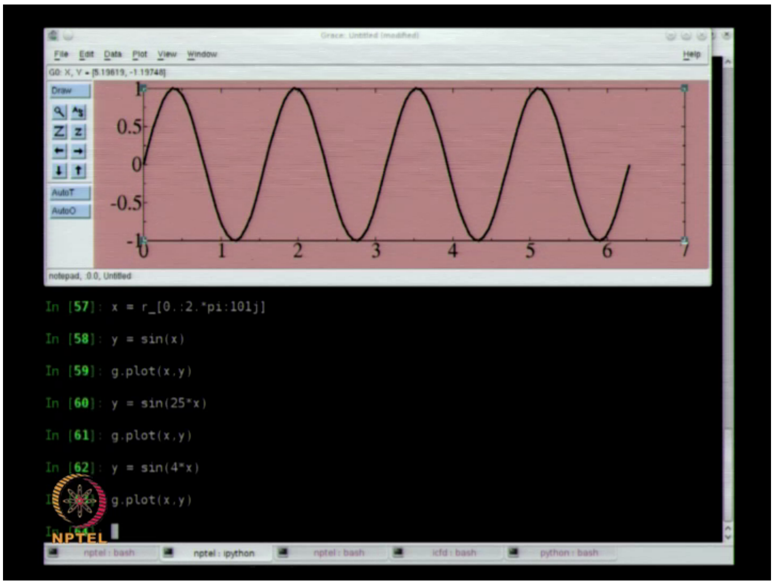

The frequency is high with reference to the underlying grid or the underlying mesh that you have used, you understand. So what does high frequency that is wave number 4 right on a grid of intervals 10 is a relatively low frequency, on a grid of 100 intervals in fact we pick it extremely well, you understand. So the mesh will pick up frequencies wave functions which are low with reference to the mesh it will pick up well.

And those that are high with reference to the mesh it is not going to pick up as well okay, is that fine. So when we say high wave number low wave number, when you are talking with respect to the physical problem you may have actually an understanding as to what is the high wave number? What is the low wave number? And when we are talking with respect to computation when we say high wave number low wave number.

You have to ask for clarification say what is the grid size? Otherwise, there is no sense unless they give you the other piece of information, what is high and low does not make sense right, high and low are compared to terms high with respect to what? Low with respect to what? Right, that information has to be given, is that fine okay. Are there any questions? okay, so I think I would suggest that you possibly can go onto trying out different functions.

My suggestion is try out different functions right with this hat functions as such try out representing different functions with hat function. I would also suggest that maybe you can try the quadratics and cubics okay. And you may be surprised, so I will let you try quadratics and cubics yourself, then I will come back and we will do a little demo again with that okay, is that fine, are there any questions okay.

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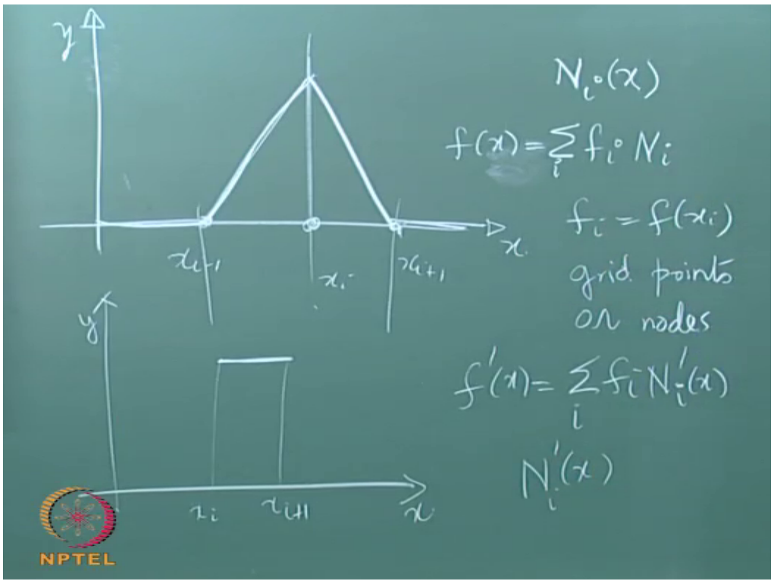

So there what do is I will stop by and I will go back to my regular session of what we were doing earlier, what we had was we in the last class right is that fine, in the last class we saw that if we had or okay, why do not I continue first with the I just say something a little something about this hat functions and so on, and then maybe we will connect up to the last class. I just want to make a point here.

So if I have a hat function that is xi-1, xi, xi+1 this is x co-ordinate, I take one hat function is that fine, I take one hat function. And the question that I have is we saw that we are able to represent the function reasonably well when we have sufficient number of intervals, in the last class we also saw that we may want to use differential we may want to use Taylor series or something of that sort to represent derivatives.

So then the question crops of how well can I represent derivatives here using hat functions okay. So we have looked at 3 classes of functions so far, we have looked at box functions right, we have looked at hat functions okay, so representing derivatives of box functions. If you represent a function itself with the box function, then the box function is a constant right it is a constant on the interval, the derivative is always 0 in the interval does not work okay.

What about hat functions? Hat functions there is some hope right there is some hope, but the only point is that the slope is a constant in the interval xi, xi-1, xi okay, so the derivative you can represent the derivative. And in fact if I were to take the derivative or the hat function that is centered at x, then what do I have? I would take the derivative of this hat function, let us restrict our interest only xi-1 to xi right elsewhere only in the support, elsewhere the function value is 0.

So the derivative N prime, so if I have let me let us say if you have a f(x) is fi Ni summation over i that is a representation, f sub i is the function f at xi at the grid points or the nodes these are terms that you will see use for this points. So the derivative if I represent the derivative as f prime x is in fact summation over i fi Ni of x derivative okay, and as a consequence I am asking the question what is the derivative N prime of some hat function N prime of i? okay.

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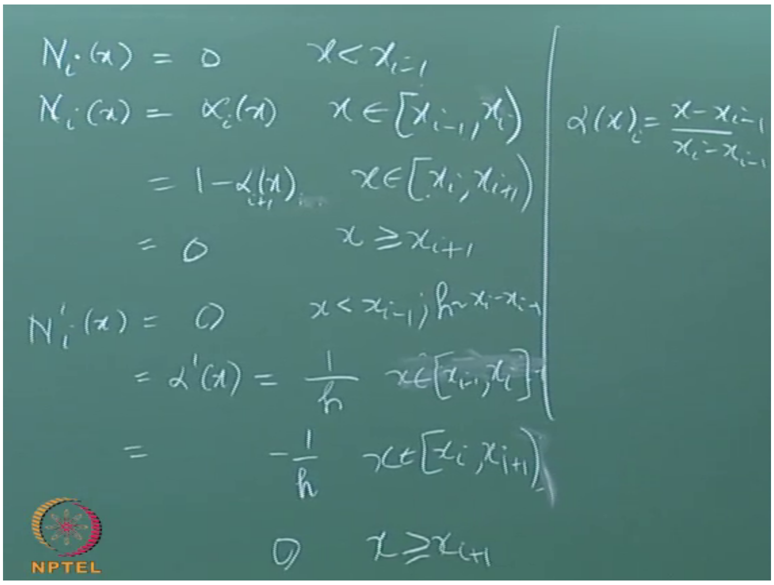

Remember the function Ni of x is 0 for x <xi-1 Ni of x is alpha of x for x in xi-1 to xi, and 1-alpha of x for x in xi to xi+1, and 0 for x>=xi+1, is that okay. So the derivative of course as I said I am not really interested in this because these are 0 anyway, but it is important they are 0, so N prime of i is 0 for x <xi-1=alpha prime of x, you remember what alpha was maybe I should remind you what alpha is, alpha x=x-xi-1/xi-xi-1 okay.

And I really need a subscript here, so if I make this i either i and i+1 or whatever right, I really need a subscript there okay. So what is this going to turn out to be? 1/xi-xi-1 and we call that h right h is like xi-xi-1, and otherwise it is -1/h, sorry h is like xi-xi-1 this is for x in xi-1, xi and this is for x in xi, xi+1, and it is 0 otherwise. So what does this turn out to be? What does the graph of this function turn out to be?

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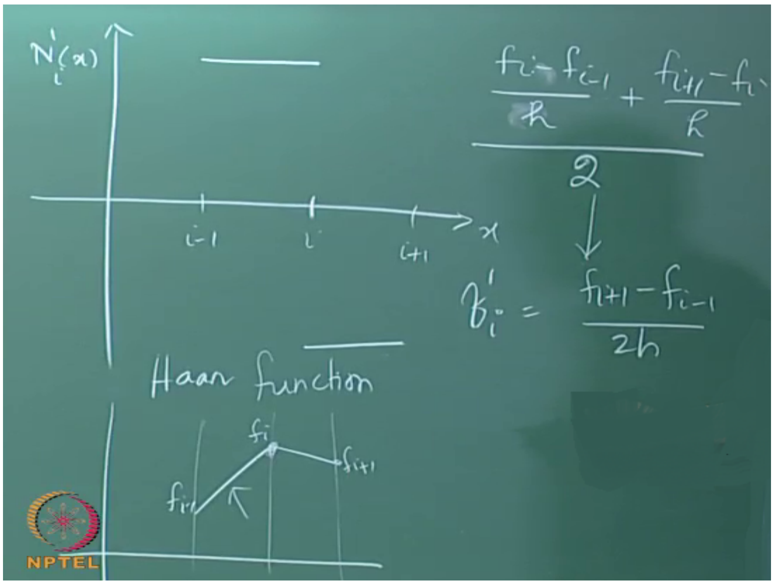

The graph of the function, what does the graph of the function? Is 1/h on i-1 and i, and -1/h on i i+1, okay. So given the relationship between this and hat function it looks like a box function, but it is not quite the box function right. So we use the box function for convenience of this class for what we are doing and so on right, we use the box functions for that reason, so if you look here we could take this interval.

We could take another interval they are orthogonal because the intervals were not overlapping that is all very nice right for that purpose it was nice, but it is very likely that often we will use the hat function right to do a representation because we are going to be solving differential equations. And if you are going to be solving differential equations, it is nice to note that the derivative follows some function that we are using to represent other functions right.

So this may be preferable to the box function, it belongs to it is named after Haar called Haar function, so it is clear that the derivative is the constant but it can be represented. What do we do at the node i? What is the derivative at the node i? There is a jump is there a way I can estimate it, is there a way I can estimate the derivative at the node i, any suggestions? Why do not I take this?

So if I have a function that is varying in this fashion, it has the constant slope in this interval, constant slope in that interval, I know the slope on either side of this value, why not I just take the average? Has the value here maybe it works, let us see what we get if we do that just for the fun of right, let us see what we get if you do. So if this is fi-1, fi, fi+1 right, what is the slope of the first line segment here? What is the slope of the line segment there? fi-fi-1/h.

And what is the slope of the second one? fi+1-fi/h, I add the 2 and take the average, and these seems to tell me that an estimate of the derivative f prime at i is fi+1-fi-1/2h it is good okay. So from this function it is possible almost everywhere that we will be able to get it, whether this is an acceptable value or not something that we will have to figure out okay, fine right. So before we leave this Haar functions, let us get back here.

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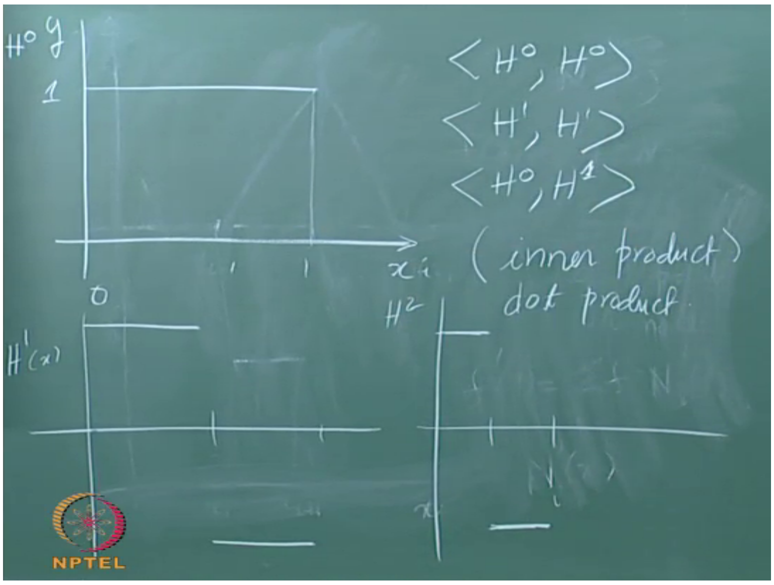

Before we leave this Haar functions I just want to point out something, I want you to try out something, before I get back to Taylor series and so on I want you to try out something. On the interval 0, 1, you consider a function it is a constant just drawn this vertical line it should be light vertical line, I will consider a function that is constant. And then consider so I will call this H0 right and I will call something H1 of x.

So up to 0.5 it has value 1, sum 0.5 to 1 it has value -1, so you try to find H0, H0; H1, H1; H0, H1 is that fine okay. So far just to keep it familiar I have been calling these dot products, maybe we should really call them inner the products okay they are called inner products, you can continue to use the dot product if you want, inner product. Because we use terms inner product, scalar product, dot product right, dot product.

So you can and I call this H1 I call that H0, because if you think about it if you look at what is happening on the interval 0 to 0.5 it looks like H0 that is just been squeezed down to 0 to

0.5, so I can again define a small function there which is positive negative right. I mean you can so you can see that there is hierarchy of functions that we can build on if I define an H2 I would actually normally put subscript also but it does not matter.

So I can have something that is 1 up to 0.25 right, and -1 up to 0.25. And actually I can define 2 of these functions I can define H2 subscript 1 that is here, and I can actually translate that function to the right, so this function just like so it looks like we are getting low frequency high frequency higher frequency right just like sin x, sin 2x, and sin 3x it looks like that something very similar to that, is that fine is that okay right.

So I just wanted you to be familiar, I just wanted you to get that idea right, now what we want is we will just tie up with the derivatives representation of functions representation of derivatives that is basically what we want, because we want to solve differential equations right.

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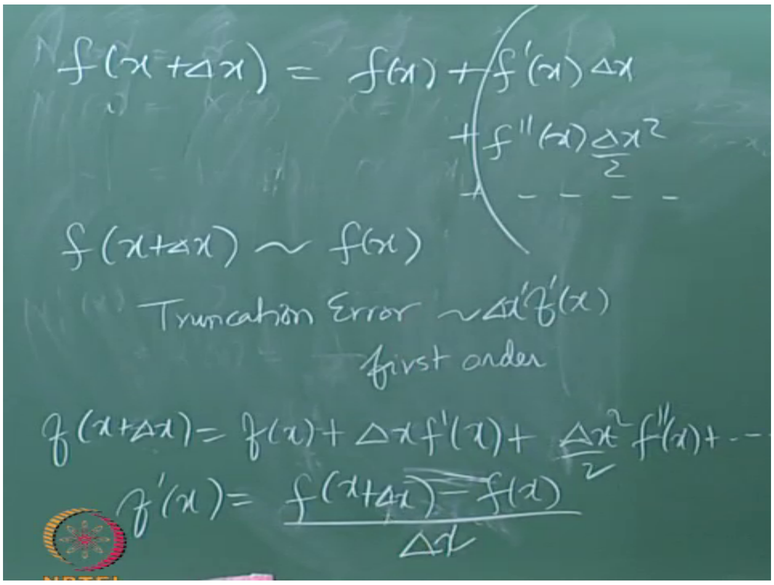

So in the last class we saw that if I have if I give you f at x, then we could use Taylor series prime that indicates differentiation and delta x+ f double prime. That we could use Taylor series to figure out what was the nature of the error that we are making, if we were to approximate right f at x+delta x as f(x) right. So in the last class basically said well if you have a function value at some point the easiest thing for us to do is assumed the same function value at a neighbouring point right at an adjacent to this point or whatever.

So that would be an approximation f(x)+delta x approximately like f(x), and what would be the error that we would make in that approximation, well we have an infinite series here and in this infinite series are basically truncated the infinite series in making this approximation. We created what is called the truncation error, this is a repeat of what I did last time, and the truncation error is typically indicated by the leading term.

So the order of the truncation error is f prime of x times delta x and because the delta x has an exponent 1 right, this error is said to be first order. So basically as delta goes to 0, this error goes to 0 in a linear fashion okay, is that fine. What about higher order representation? How can I get a higher order representation? I say f(x)+delta x=f(x)+delta x times f prime of x+ higher order terms delta x squared/2 f double prime of x I keep that +higher order terms.

It is clear that if I have the derivative which we saw when we did when we use cubics, so if you have the derivative information it is possible for us to get higher order a representation right. So there is a connection between what we are doing there, and what we are doing here, but the trouble is what if we do not have the derivative then we are in trouble right. So if you have the derivative information it is actually possible for you to look at what is.

So if you say that this is my function value right now, this was my function value yesterday right. So my bank account has so much money yesterday, my bank account has so much money today, then I can maybe predict what my bank account is going to get, if I know the rate at which that value is changing right that is basically what it is, is that fine okay. This also gives me the different clue.

If I look at this equation, this equation tells me right now we have been looking at saying that what happens in an x+delta x this equation tells me so if I have prime and if I have f(x) this equation basically tells me that I can predict f(x)+delta x right, in a little more earlier I was saying approximate, but now I am saying I changing it around, I can actually predict. So if from a differential equation you are able to get the derivative value you could actually predict what is going to happen at x+delta x given the value at x and the derivative at x fine okay.

That is one way to look at it, this is the argument we have been using so far. The other things that you can get from this equation once you have written this equation as you can write it as a way by which you can approximate f prime of x, what does f prime of x? f(x)+delta x-f(x)/delta x that is the other possibility that we can actually evaluate the derivative based on the values at x and x+delta x given that now you know f(x)+delta x, is that fine okay.

So we will see in the next class, how to go about doing this right. And we will see what is the error, so we will now go about representing derivatives, how do we represent derivatives? How do you estimate derivatives? How do you represent derivatives approximate derivatives? And how do you find the error N right, that we are making in that representation fine okay. I will see you in the next class.