Introduction to Computational Fluid Dynamics
Prof. M. Ramakrishna
Department of Aerospace Engineering
Indian Institute of Technology – Madras

Lecture – 38
Calculus of Variations – final & Random Walk

So we will I will just clear up 1 issue that we had left over from last class and there is 1 topic that I want to talk about because that brings me to a summary of some of the things that we have been doing in this course and then in tomorrow's class we will do a closure so you can tell your friends everybody to be ready to try to recollect what we did in the last 40 plus class is that fine. So we will try to make that collect affairs.

We will see if can together recollect what happened in the last bunch of class. So that is fine. Now before I go, so we had there are some issue.

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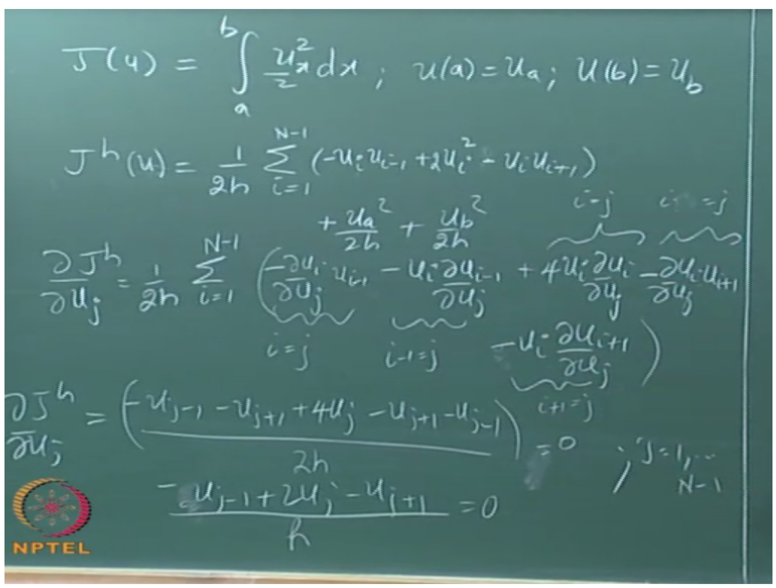

We had problem. We had J of u as integral a to b x square * dx given u(a) we called it u sub a and u(b) as u sub b. This is where you are and I had discretized using Hat functions and got something that looks like this. I will just sort of write this out. You can work out if there is some issue. The whole point is actually you have to just go through it blindly the minute you start counting you can start making mistakes. So this turned out to be a 1/2h if I remember.

I am doing this for memory, so just make sure that I do not make any mistakes. I = 1 through n - 1 - ui - ui - 1 + ui square * ui + 1. I should really write secular terms in front so that there is no confusion with and I made a mistake - I get a 2 ui square. And then of course we had these terms which is + ua square/2h + ub square/2h. So what we do is we want to find the maximum. We want to find that depend the maximum and minimum depends.

In fact, on whether you how you write, whether it is - ux x square, or -ux x or ux x. I differentiate this with respect to uj as I said because the second example I gave I did this funny counting and if I want to check this out you can actually count and see. So what does this give me? This gives me 1/2h summation I = 1 through N - 1 and we just go through this blindly so this gives me - dou ui/dou uj ui - 1 - ui dou ui - 1 dou uj + 4 ui * dou ui/dou uj - dou ui/dou uj * ui + 1.

I am short of running out of space so I will write it underneath so - ui * dou ui + 1/dou uj and this whole thing is under the summation. Constants of course disappear. So if you say j = 1 or j = some particular value then obviously depending on what is the dou u2 * u1 is 0 and so only if I = 1 it works. So this going to work so each of these derivatives is going to work so j = 1 only if I - 1 is 1 it works. So you find out when these are non zero.

So this is nonzero I = j. This is nonzero if I - 1 is j. This is nonzero if I is j. This is nonzero if I + 1 is j and this is nonzero if I + 1 is j. This should be I = j. So if you say I = j then that becomes j - 1 and that becomes j - 1 so this is - uj - 1 this is uj + 1 and this one is 4 uj - uj + 1 and this should be uj - 1. This is the derivative. So j = 1 to n - 1 now we are going to pick each of the j. So each one of the js j = 1 we differentiate it = 0 get an equation. j = we differentiate set = 0 get an equation.

So we get a bunch of these equations. We are going to set it = 0 this is for j = 1 through n - 1. This is 1/2h outside that I have left out. This is just by way of otherwise this is uj - 1 + 2uj - uj+ 1 = 0. You multiply and divide by each it obviously looks like the second derivative being = 0. So I just wanted to clear that up. The other possibility is actually if you enumerate since it is in 1 dimension you can actually enumerate you can count them.

You can count them and you may get a recursive formula here or you may get a different expression. You could write it in a different fashion because they are sums. You can combine them in different fashions. So if you wanted to collapse. We wanted to combine this with that you could actually do it if you rewrite the sums a little.

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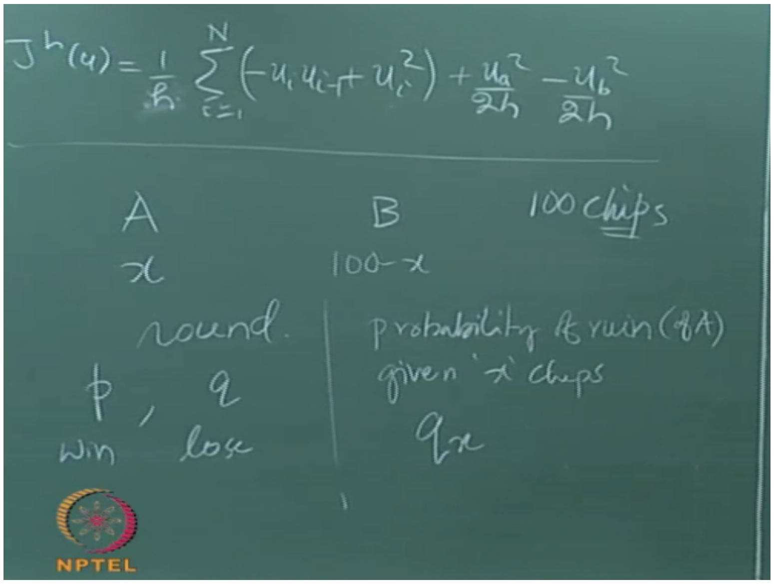

You could write this as Jh(u) = 1/h and I am just doing this from memory so you can check it out later summation - ui * ui - 1 + ui square + ua square/2h, I = 1 to n so the n is there - ub square / 2h. You can check this out. The Nth one term so you can rearrange these terms it is just a matter how you collect the terms. The different ways by which you can combine the terms so we could get something of this sort if you wanted to combine the I + 1 term and the I - 1 term.

But by suggestion is all of this. You can do this kind of stuff very easily when you are dealing with 1-D 1 dimensional and the problems are relatively separate. It gets a little messy when once you go to multiple dimensions. So I bring the calculus of variation it is not as I said it is not quite finite element method, but there is you have an idea that you can use these Hat functions and various basis functions that we develop to solve differential equations.

And there is a variational forms. You have the differential equation form and there is a variational form that is associated with the differential equation there is a relationship between them. So there is a completely different class of techniques that you may or may not have seen earlier. There are completely class of technique. Now what we will do is we will change the gears a little.

So it is an abrupt change from what you have been doing so far then it is okay it does not matter since we are nearly end of the semester it is natural progression. You know generally, I am going to put a topic which is normally taboo in most cultures which is gambling. So gambling is illegal supposedly, illegal in India. I say supposedly because if you go and do the right paper work I can actually come to you and make a bet.

I could bet for instance that I am going to I bet 1000 rupees that I am going to die tomorrow and you can say no no I bet 1 Lakh rupees that you will not. You would of course have been an insurance company and I would be a person who is coming to you paying 1000 rupees as a premier am I making sense. So there is business of gambling is very strange. It is very strange things. So there are lots of situations if you actually look at it you can finally decompose it to some form of a gamble.

So I give that example of insurance to rationalize in the fact that I am going to take gambling of motivation, game of chance. Let me call it instead of gambling let us call it a game of chance. So there are 2 players in this game of chance. The 2 players are for lack of anything better I call them A and B 2 players A and B in this game of chance and between the 2 players they have some tokens say 100 chips, tokens are what they use to figure out who is winning who is losing.

So between the 2 players they have 100 chips. So at any given time if A has x chips, B has 100 - x chips so what we call is 0 sum game. So if A wins B loses. So it is there is no win win situation here. So I say it is a 0 sum game. So we simply if B wins A loses. So we know that if we know the story of 1 of them you know what is the story of the other? It is a complementary story. We would like to know we will just look at A.

We will now ignore B. We will just look at the story of A. What is the story of A you can be wondering what this have to do with CMD. So we will see whether the story has anything to do with CMD or what you have been doing so far. So if A has x chips, now you have concern. Now you ask the question what is the probability that A will be ruined? By A will be ruined means it shows sequence of units that A has 0 chips. So what are the events?

The event is what is called a round. After each round whatever that round is there is a probability P that A will win the round and if A wins the round 1 chip is transferred from B to A. So with the probability P at any given round A will win that round. With the probability q see B is win with the probability q A will lose the round. Is that fine everybody? So yes this is all very nice. This is at single round, but I would like to know what A would like to know what I would like to know is if A has x chips what is the probability of win.

What is the probability that A is ruined given that A has x chips. You understand what I am saying. So we will call that probability, probability of ruined because of A. Given x chips it is just setting up the problem. We are doing the so that is q of x. Is that fine? Probability of ruin is qx. So, now we will start. We basically say that A starts with x chips. You start the process. You go round 1, round 2, round 3, and round 4 and you ask the question what happens.

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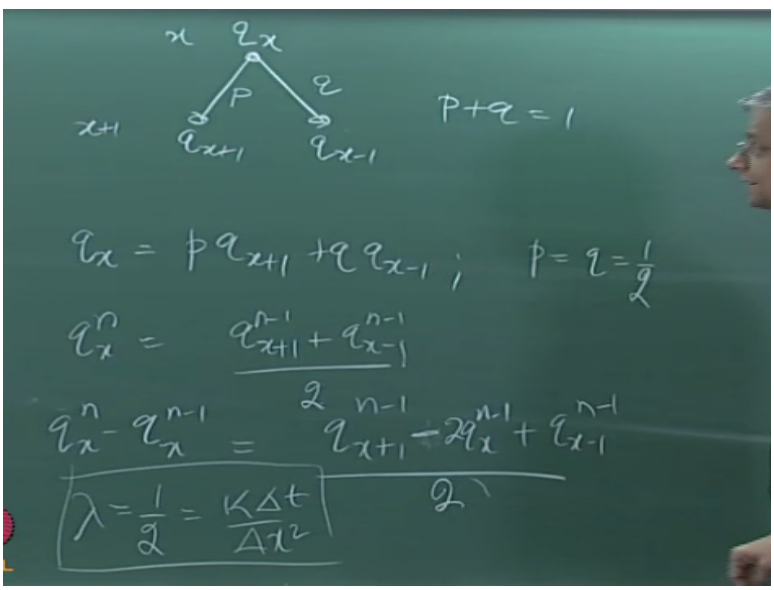

So if you are at round 1 and you have x chips. So q sub x is the probability that you get that A gets ruined. After 1 round with probability P it could be x + 1 chips and the probability that A gets ruined with x + 1 chips qx + 1 what is the other possibility that you lose. With the probability q and you have q of x - 1 chips. So the probability that you get ruined having x chips is in fact the probability that you go to x + 1.

And then get ruined p * q x + 1 plus the probability that you go to x - 1 and get ruined + q * qx - 1. So just say it can be if one of them is not cheating then P is likely equal Q these are not cheating, I say Pq whether one of them may be cheating, but if they are not cheating then p is likely = q, but we know that p + q is 1. There are only 2 possibilities I have enumerated the 2 events. There are only 2 possible events.

So you either go from here. You transition from x to x + 1 with the probability q you understand what I am saying you are transition from x to x - 1 with a probability p here, with the probability q there. So qx could be this. Now if you turn out that neither them is cheating, and it is an even game, then p = q = 1/2. So this tells me that if I substitute qx = qx+ 1 + qx - 1/2. This is round and this is round and this is round n - 1. I will count the other way around.

I will count the other way round so this is I will determine instead of incrementing and now this is starting to look familiar. So what I do is I subtract qx - qx n - 1 from both sides so I get qx and - qx n - 1 = qx + 1 n - 1 + 2 * qx n - 1 + qx - 1 n - 1 so I have - /2. So this 2 comes because in the denominator I have a 2. Just looks like heat equation. It looks like the discretization of heat equation. So we started off with 2 gamblers.

They are talking about the ruin and we end up with the heat equation what is the deal? How does this help us? What does this do for us? What does this do? It looks like the discretization of heat equation. This is 1/2 you remember the stability condition for heat equation that is we created we had a parameter this is lambda 1/2 is lambda. Lambda = 1/2 if you go back to the heat equation which was kappa delta t/delta x square that is exactly at that stability margin.

On the edge of the stability envelope so it is the heat equation it looks like the discretization of the heat equation. So let see. So we had the heat equation. We need boundary condition what where do the boundary conditions come from? These are all very nice. This looks like the discretization of the head equation where does the boundary condition come from. You go back here. If x has 0 chips then the probability that x looses is 1 because that is it we have nothing.

The game does not move on anyone. You understand what I am saying? So q sub 0 = 1. And x has 100 chips then the probability of loss is 0 because you have already won. So the boundary conditions are is q of the probability of ruin if you have 100 chips is 0. So here it looks like our usual heat equation discretization of the heat equation we have the boundary condition.

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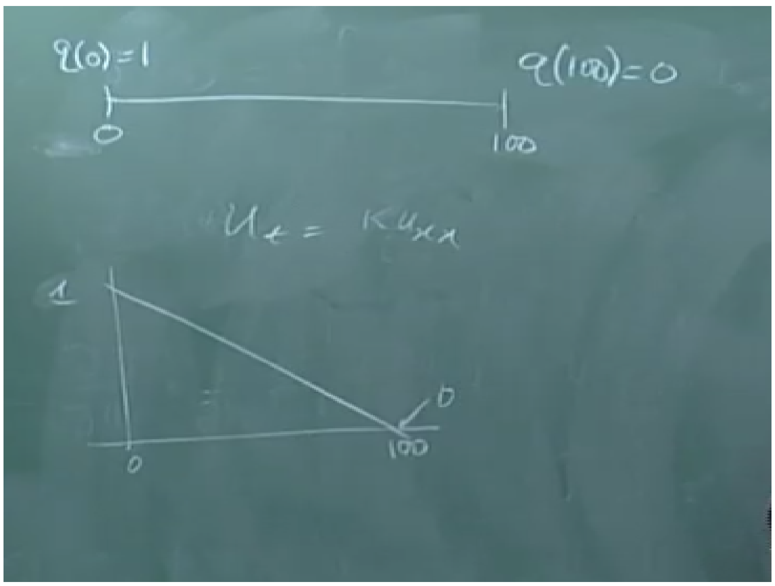

So that is o that is the grid point 0 and it is the grid point 100 and q of 0 is 1, q of 100 is 0 and what you do is you take in between you take values and you would take the averages to keep evolving and time and what is the solution going to be. What is the solution to this equation? ut = kappa ux * x it is going to be straight line going down from 100 to 0. So that gives you the probability distribution basically.

It is going to be a straight line going from well probability 1 to 0 in the y value there is 0. Of course, there is a difference there and this is the continuum. There are discrete chips. this is the continuum that means somehow I have to fragment the chips if I want to do this is the limiting process I have to keep breaking the chips and I have to keep breaking the chips now. What we do now is the limit of that equation as delta x goes to 0 as the size of the quantization of the chip goes to 0 it goes towards the heat equation.

So that equation is consistent to the heat equation. So before we set about asking ourselves a question instead of solving the heat equation so that is the finite difference representation of the heat equation. So before we go about wondering we talked about probabilities and all that and can we use those probabilities directly to solve this problem. Instead of working at the wave we have just worked it.

Right now this would be this is called the Fokker–Planck equation you have heard this term? Maybe not. So what we have basically doing is we are basically showing that this random process there are 2 people playing, they are tossing a coin or something of that sort is random process. There is a continuous equation associated with it where u is the probability and you can solve it using the finite difference techniques that we have got, so we have done so far.

The point that I want to make why do I bring this up at this point of the semester. The point that I want to make is it is consistent. The scheme is consistent whatever that process that I explained is consistent. It is consistent with the heat equation it converges to the heat equation. If you have molecules bouncing around you do some kind of limiting process or averaging or whatever it is consistent with the heat equation if you are looking at energy equation.

If there is no motion in the sense that no drift velocity, no mean velocity then the energy equation did degenerates to the heat equation. If I do a finite different scheme in this fashion if I do a finite different scheme in this fashion it is consistent with the heat equation. So if I start with the heat equation and I discretize it using a finite difference scheme or I discretize it using calculus variations and differences.

I discretize it using finite element method whatever it is I generate a automaton, I show it is consistent all of these there are a variety of models at the discrete level whether they are molecules, whether it is some A and B gambling. All of these in some limiting sense are govern by the heat equation. So in CFD what we are basically doing. We have the fluid flow which is a process we have done continuum and got a differential equation there is a limiting process.

Then we are saying there is numerical scheme there is consistency, convergence, and all of that stuff going on they both model the same equation. They both in the limit go to the same equation. So we are saying that I will use the discretization that I have done to model the reality of these molecules am I making sense they are basically saying that this is equivalent to that because both of them in the limit go towards this differential equation.

Am I making the sense is that okay. So what is the underlying automaton that you do when you discretize using finite difference method, you discretize using finite element method, you discretize using something like this we will figure out how to do this. We do something like this. So there are different ways by which there are different, it is not microscopic. So microscopic could be do it at the molecular level that is too much.

Because you already know that the number of molecules you are talking about is of the order of Avogadro number of molecules it is a huge number 10 power 23. So you turn, we back out from there and say well here are these 100 grid points that seem to go to the same problem that seem to go the same differential equation. Am I making sense? So the underlying process that you pick so what we are trying to do here is we are trying to come out with the discrete representation.

Some kind of a discrete representation which goes towards the same differential equation that are alternate discrete the one that we are trying to simulate goes to here though it is discrete is that fine. So that at 1 level that is the point that is what I wanted to show you something that was very different. Now in the point is I have shown you from here I have shown you that we get the same governing equations I did this simply to show you that you get the same equation.

But this is not how I am going to solve it. If I solve this, I am solving finite difference method. If I solve this equation, then I am saying instead of gas flow or instead of a solid rod being heat conduction being governed in a solid rod by the heat equation I am saying the heat equation governs this and I will solve it using finite difference method that is not the objective here. You could do it, but that is not the objective here. What we will do instead is we will ask the question what we have stated here is the problem can be modeled as such can it actually modeled as such.

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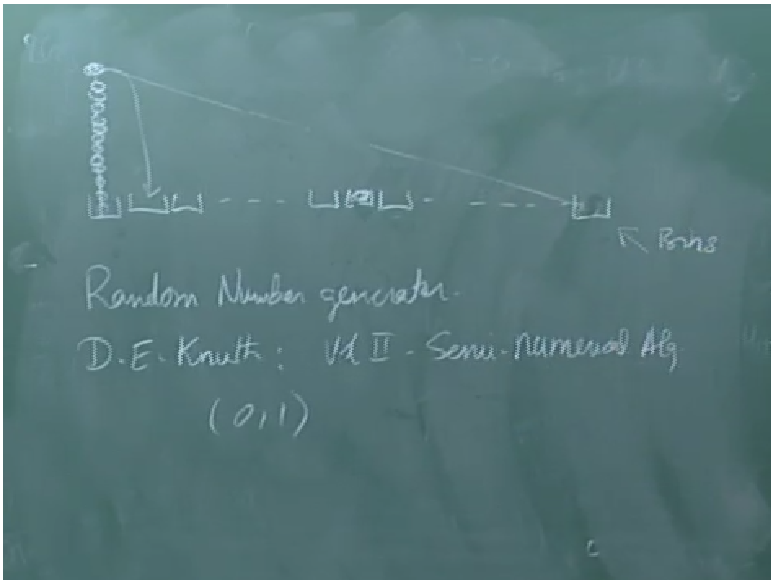

So in order to do that you need to create 101 bins containers, you need to create a 101 bins and if you have a chip or a coloured ball in that bin for that you can toss a coin and you can decide whether that the ball goes to the right or the ball goes to the left. You can decide whether the ball goes to the right or the ball goes to the left with the probability in this case because p and q are both equal to half with the probability half, the ball will either go the left or the ball will go to the right.

Am I making the sense? So that much is clear. So we have already done this it is important. I want you this picture so given that your x that either goes to the right or goes to the left. This picture is important. So we come back here so what do we have. So we have each one of these how many balls does the first bin have? Well it depends on what is the discretization of the probability that you want to do. So the probability in the first 1 is 1.

The probability in the first bin is the probability that the ball in the first bin the probability is 1. So what I am going to do is I am going to make sure that there are I quantize so the probability is 1 that 0 to 1 that magnitude 1 I quantize using 100 balls which means that each ball represents 100 to a probability. So whatever happens here so if you start off with 100 balls and you basically say I am going to toss a coin or whatever and decide what to happen.

What is the n result after that 1 round? It is possible that 1 ball ends up here. It is possible that 1 ball ends up here or the ball goes in oblivion which does not make sense but away so there are for each of these if you do that then if there are only 99 balls or 98 balls then I pop it off because it is supposed to be 100 at all times. In the last bin there is a ball or a chip that shows up there I remove it because I supposed to be num there is that fine and this is the game that you play.

So you just basically say that I have a chip here where I have a ball here I toss a coin, the ball either ends up here or here and what you will see is all of these balls jumping around normally unfortunately we do not have time for demo or this you could actually try this, this is very easy to program. The critical thing that you need here is a random number generator. Need a random number generator.

So to generate pseudo random numbers I would suggest that you read Knuth, the art of computer programming volume 2 semi-numerical algorithms. There is lots of material out there on generating random numbers. I think if I remember it this book has like 100 pages or 150 pages it is quite a lot of space dedicated to it and it is very important as I jokingly say I forget who said this, but random numbers are too important to leave to chance so you have to generate them properly.

So you cannot just generate you understand what I am saying. So the standard random number generator that you see out there may not be the best generator. There are lots of random number generators, but make sure of that you are using a good random generator. Your computer may have a random number generator which is a true random number generator giving that it is based on the thermal state of a diode somewhere in the CPU is actually using a A to D converter to read of the random voltage that is coming from it not a good idea.

We are writing a program, you want a random number generator, but you want to use what are called pseudorandom numbers which is what he is going to give you because you are not sure when you are developing a program you do not know whether your program is working or not and to throw into that some random effect is not what you want so you have random numbers that are actually not random numbers I mean there are sequence of numbers that look random.

But you know what they are before hand and then you can debug your program, because every time you run it is not going to do something different and you are saying is it because there is a bug in the program or because my random numbers have changed. So you generate the random numbers, debug your program, and then you can do whatever you want, but I would still suggest that you stick with pseudo random numbers.

So if you have a pseudo random number generator that generates uniform distribution on 0, 1 then it is < 0.5 it is head. It is > 0.5 it is tail it is very easy. Basically what you are trying to do is you are trying to generate random numbers in fact what you want is just 2 states - 1 and + 1. The way I have implemented it I just generate the - 1 and + 1 and I add it right I shifted to the appropriate, I shift to the p0 bin and I shift to the next bin.

So and what would you expect is that you would sort of see movement and then there will be a point when you are finally get through a solution that you are not going to get that is only remember that you are dealing with random numbers I do not know whether you have dealt enough with probability theory and so on, but you are dealing with random numbers. You are not going to get the straight line.

It not just going to go the straight line and sit there it is going to be jumping about that straight line. So what you are going to do, you are going to get variations about the straight line. This straight line is the expectation of your random process the straight line is the expected line. So you are going to see the function grow and sort of it is going to be jumping about that is going to be pouring about that.

As I said there is an another way that we can do this, here what we have done is we have taken bins and we have kept track of what is coming and going out of the bin that is like an Eulerian method in the fluid mechanics lingo just like an Eulerian method, the other possibility that you do a Lagrangian method. Lagrangian method is you follow a particle. So you follow a particle and get back here as I promised.

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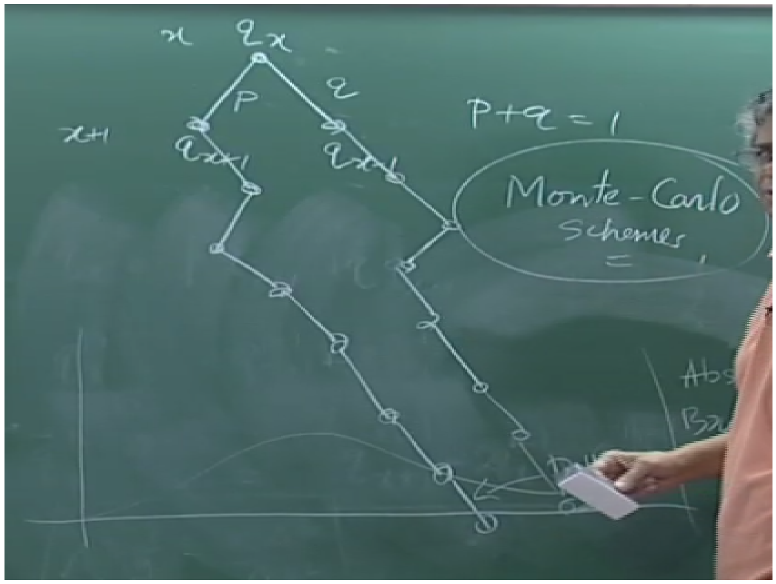

If you follow a particle so you are here you toss a coin, you go from this point to that point with probability half and then you toss a coin again so one at each pause these are actually bins you understand what I am saying these are actually bins is moving from 1 bin to another bin, but right now I am following that solitary particle and in a similar fashion you could have so this is called a path, it is a particle ray so it is called a path.

So you could start off each one of these in a similar fashion. So it is possible that some of them are biased in some fashion so you can trace and I have seen to have biased at this fashion that need not be biased. So you start of in a path. So you can make, you can track, you can create, generate multiple paths, so from this points you could generate 200 odd paths and at any given time you can find how those 200 odd paths would end up at that time level and you will get a distribution.

You may get a distribution of some kind and I make you sense you get a distribution of some kind and so it gives you an idea of that if something starts here what is the probability that it lands up in any given spot. Is that okay. Of course in this case if it goes to the zero side, it is just going to get eaten up so 0 is what we call the absorbing boundary condition it is called an absorbing condition.

It just eats up anything that comes to it is gone anything that comes to the 0 side has gone this is called the absorbing boundary condition and since I am talking about random numbers and distribution of some kind and since I am talking about random numbers under distribution of something and the toss of a coin and so on, this is class of schemes are called Monte-Carlo schemes.

You could actually this would be x you know you could actually start off at different points and go through the same process and you could and so this is used in a variety of ways now that I mention Monte-Carlo I will just say a little something about Monte-Carlo schemes. The most important part of Monte-Carlo scheme of course is a random number generator. Since I am not sure as to what you are background of probability theory.

And I do not think have you guys looked at seen any have you done probability theory. So maybe I do not say anything about stochastic process. So there is a second step that you will go to called stochastic process. I will just write out the equation because I intended at least that you will be familiar with it.

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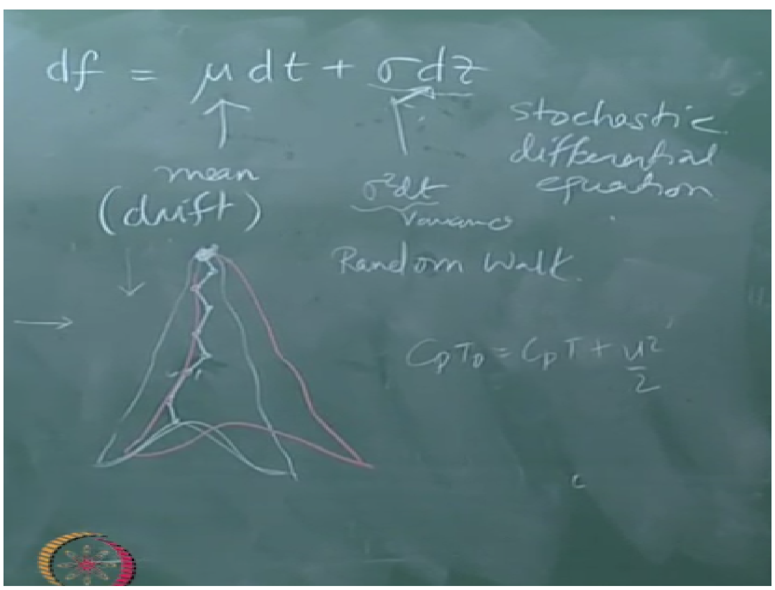

So, typical stochastic differential equation. So if I am talking about the probability distribution as a normal distribution Gaussian normal, this is the mean, also called the drift. This sigma square dt is the variance rather so these are the z or random dz are random numbers which are who have variance which are variance that looks like then the combination looks like sigma square dt. So if you think about it you may say what is this equation? What is this strange equation?

This equation basically describes that process there what I have shown. What I have shown there is it could be nu0 or if you look it if what are actually drawn is drifting to 1 side. So it is possible that you can imagine if you just had simple diffusion if adjust had simple diffusion, so simple diffusion is the best way for me to describe it is. You have some fluid flow in this direction.

I have I set up the fluid flow so that I have a time like direction in the downward direction as time progresses something that was here will end up is travelling at a constant speed we already know that. Propagation speed is A. it will travel downstream. so if I drop some dye there, some ink there it is the example that I gave in the beginning when I talked about linear wave equation I said I will put some chalk dust and the chalk dust is carried along advection and I said there is a diffusion, but we are ignoring diffusion.

Now I am going to talk about the diffuse. So I put dye there what happens to the dye as it goes along if I drop dye, so the dye is carried down and it also starts to diffuse. It also starts to diffuse. So the distribution of dye that you expect say something like this the distribution of dye is something like that which we have got by random walk that process that I wrote there is also called a random walk.

The process of generating parts is also by a random walk that is I take a particle and I go through I take so many steps randomly it is a random walk and this envelop basically shows may be 10,000 such parts that I will draw and I get a distribution that look when I look at the end this is the distribution that I get it is the random walk. That is because the flow is in this direction. What if I added small velocity so very tiny velocity component that way left to right?

So you expect that there will be the distribution will be slightly distorted. So what will happen is, you will get something and the distribution may be something like that. Am I making sense it will be slightly distorted. So the drift velocity nu that is drift and this is the pure Gaussian process and you have seen this before. So normally when we did as we said earlier when we are talking about for molecules or it is the same differential equation that finally comes up.

So when you are talking some element are statistical mechanics earlier in you physics or chemistry or whatever you would have seen that you break up the motion of the individual molecules which you have considered to be hard spheres you break it up into a drift component and a variance about that drift a change about the drift. The drift component gives you what you call the speed of the fluid at that point in the continuum and the variance gives you the measure of the temperature.

The variance becomes the internal variable. The temperature becomes an internal variable so therefore we tie it to internal energy so it becomes an internal way. So it is beneath what we see in the continuum that we can capture. The continuum we can only capture this, the variance becomes an internal variable. As I said the objective of doing this whole business was for me to make tie up a few of these. I wanted to connect.

I wanted to connect a few of these things together. So if you have a stochastic differential equation at the sort what did basically says is yes you have the drift component, you would normally call what would you have seen in your physics and chemistry as drift velocity and then you have added to it the random component about it which you capture in the form of some kind of a variance which is actually a measure of the temperature.

So in that sense the equation itself is very different from something like CPT0 is CPT + u square/2. One has a drift component of course this is the energy equation not the same be careful I am just writing this out I just stuck me randomly and I wrote it right now, but the relationship you understand what I am saying this is in terms of energy so it is not in terms of speed itself, but that is basically.

So here what you could do so he had a stochastic differential equation like this you can actually generate random numbers using normal distribution. Given sigma square or mu and 1 depending on how you want to do it. Mu and 1, Mu is the mean and 1 is the variance or this z would be 0 mean and unit variance and mu would have the would take care of the drift change in the mean and this would take care of the random steps.

Is that fine is that okay. Have any questions? So these Monte-Carlo schemes they can be used for various things. I will give you a last typical application because it is about distribution and I will give you the final simple application not to differential equations this is most probably the kind of thing that you may have seen in school I am not sure if you seen this in school.

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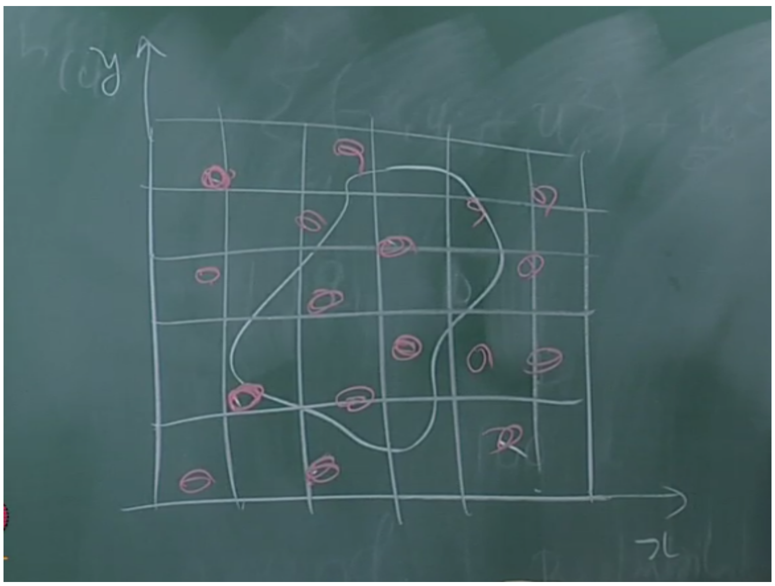

So if you have some random shapes for which you want to find the area. So if you are saying why are doing all those random numbers what is happening so let me give you a simple straight forward application. So if you want to find the area what you want to do is a process of integration actually. You can find the bounding square which you can scale and the easiest thing I am going to break it up into blocks because that is my nature.

But easiest thing to do is generate random numbers on the square, generate so this is x and that is y generate random positions so just say you generate 100 of them so these random positions maybe I should make a little, make them a little bigger has to be seen so you generate a bunch of these random x and y so this is the reason why I wanted to give this 2 dimensional one as an example I will tell you why because I have a warning at the end.

So you generate a uniform distribution in both of them in both directions so bi-variant uniform and count how many you have inside divided by the total gives you the fraction of the total area. It is a relatively simple minded way to do it and it really works quite well so you can generate instead of 100 random numbers you can generate 10000 random numbers locate them all over, find out how many are them are inside the area that you want find out how many of them are.

Or what is the total number you know so this is of course chosen to be a square so the area of the square is very easy to find. You can find out what is the area of that an estimate get an estimate of that area. The reason why I pick this is you want to make sure that the random numbers that you generate in the x coordinate direction and the random numbers that you generate in the y coordinate direction are not correlated.

So you really should be careful as I said it is not that simple of just generating the random numbers and using them. They are not so you have ensured, that the numbers are not correlated. The other thing that you do is you are talking about pseudo random numbers we go back right now to beginning of the course. We are talking about a finite representation of a number mantissa is fixed 24 bits, remember if I am generating random numbers between 0.

And 1 all that matters is a mantissa, 24 bits is 16 million numbers. That means that the best that you created every 16 million numbers you will have a cycle. Every 16 million numbers so if you have a good random number generate the site will be 16 million pathetic ration number, if you have bad random number generator every 100 it will keep repeating. Does that make sense? So you go to double precision so that will give you a larger number.

Or you use integers you do not need the experiments you are talking about numbers between 0 and 1 so you use integers and you are already at 4 billion random numbers if you have a good algorithm remember you can still have a bad algorithm. So you have good algorithm that loop that it gets so if you go to 2 dimensions I you generate 1 million random numbers, 1 million x coordinate, 1 million like we already done 2, if you understand what I am saying.

If you go to 3 dimension, 4 dimension you can imagine now that your random numbers may start getting correlated there may be other issues so there are I do not want to give you this impression that these are actually relatively easy to implement they are not that deficient they are relatively easy to implement the convergence properties are not that good most of the times so you will use them when you are in a difficult situation.

You typically use them when you are in a difficult situation, but otherwise what happens is you can go through this process, but please bear in mind that underlying all the Monte-Carlo scheme deep down inside the core of your scheme is random number generator. So you have to make sure that you understand that your random number generator is doing a right thing.