Introduction to Computational Fluid Dynamics
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Lecture - 31
Demo - One Dimensional flow

So we will look at a demo. Before I start the demo we will try to figure out what is the actual problem that we are going to solve right. So I think I maybe suggested some boundary conditions and so on to you but we will make it little more explicit right now. So I have a program that I have written which has 2 different flavors of solvers. Both of them require and I add second order and fourth order dissipation.

So I am not doing any of the more modern techniques that add dissipation in a regulated fashion you understand because we have not covered that material in this class and I am not proposing to cover that material in this course. I have just given you enough, I have brought you just up to that point where you should be able to comfortably read the material that is out there, the literature that is out there okay.

And maybe in some other advance class you are able to get that high resolution schemes and so on. So one of course is FTCS because we have been looking at FTCS right through okay. The other is four-step Runge-Kutta method as it turns out that the time step that I can take is much larger. I will start with that because the time step that I can take with it is much larger, things move a little faster right.

So the demo moves a little faster and just for completion maybe I will do FTCS. The things that you have to observe from this is we have looked at linear wave equation; we have seen certain behaviour especially in the previous demo's we have seen certain behaviour. The one dimensional equation we have already seen is a combination of right the combination of these kinds of wave equations.

So there are consequences the fact that the 3 propagations means there are consequences. So I want you to pay attention to this right, what are the propagation speeds? Can you figure out what is moving? How is it moving okay? So the actual problem so it is one-dimensional flow so it is going to be just a pipe right, it has a reservoir at one end and it is opened to possibly to atmosphere or some other reservoir in the other side and we will decide on boundary condition.

The problem that I am going to run is going to be a relatively easy problem. I want to run the demo for a Mach number of 0.5. So the steady state solution will be 0.5 and you will understand as we do the demo as to why I pick 0.5 right and I will let you play around, once you get a code working I will let you play around with various other possibilities that you can run. Is that fine? Okay.

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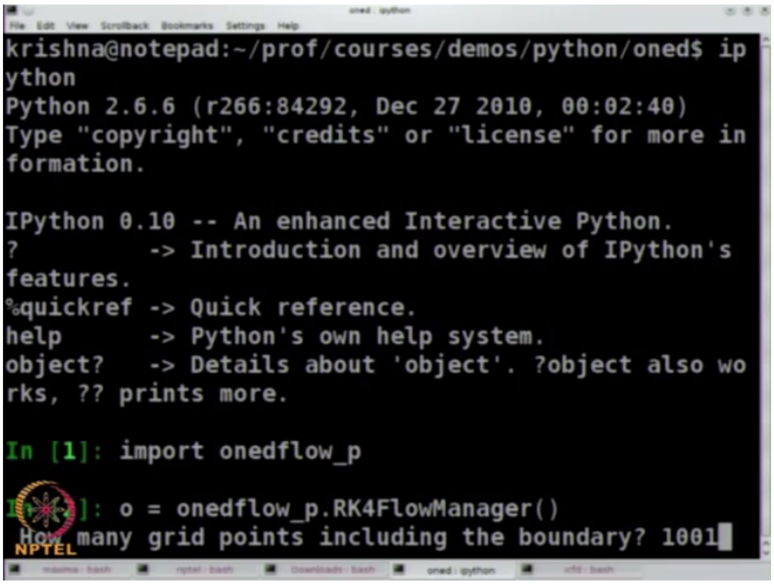

So let me start off and this code is written completely in python that is one of the reasons why I am running Runge Kutta because python there is a penalty that you pay but anyway and I am using grace plot like I did last time. The particular flavor that I am going to use for you today is called has a subscript p. It does not matter why right. So I will create a solver. So there should be an RK4.

So there is Runge-Kutta 4-step flow manager so I will create this flow manager then it is going to prompt me for various and sundry things right. So how many grid points do you want to run? I want to choose something reasonably large okay so I am going to pick like 1001 that is simply because most of the time I ask students to do some assignment and they run 5 grid points, 10 grid points, 12 grid points you understand right so 1001 grid points.

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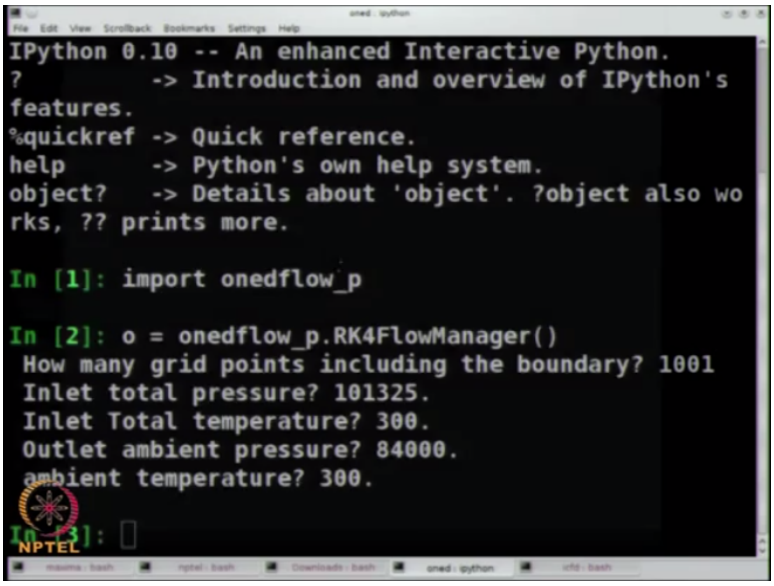

You could make it larger but then it will take too much time 1001 grid points. So if I take the inlet total pressure to be atmospheric pressure for no particular reason and the inlet total temperature to be 300 kelvin simply in my mind I am thinking reservoir has been sitting there it is an equilibrium with the right atmosphere outside though 300 is really not the temperature in Chennai but anyway we will live with that we will go with that.

Outlet ambient pressure, now we have to be careful. Now we are defining the problem here, so outlet ambient pressure I want it to be what? I want it to be so that I know the solution right just like we did for Laplace equation. It is nice to know the solution. So I have already pre-calculated it. If you sit down and think about it, it is around 84,000 or some 85,000 so I am going to run for 84,000.

Is that fine? Right and this should give me a Mach number around 0.5 okay. I am not actually calculating the exact Mach number that the steady state solution should have and we are looking for a steady state solution that is what we decided. We are looking for a steady state solution. I have told you before that ambient temperature is not relevant parameter. Why am I prompting for ambient temperature?

Changing the ambient temperature does not change the solution right but this is for the initial condition, this is not for a boundary condition. I am asking for the ambient temperature here so that I can set the initial condition because as we have decided my valve is at the left reservoir the P0 end okay. So the inside of the pipe is completely at 84,000 pascals and 300 kelvin okay.

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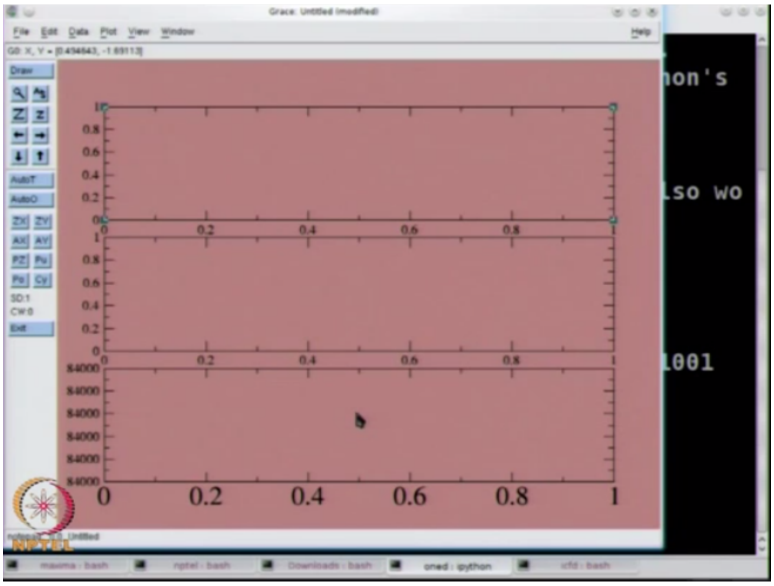

So that is done, so this should start something off, yes it does, I will quickly so that your eyes do not get zap by that intense white. I will change it to a more pleasant color okay. So the other scales do not matter. It is a little difficult actually. This demo took me a little time to get right simply because of the fonts and so on. So for the people in the video world do not worry about this now I will read out the numbers okay.

You are in the class can most probably see them. I will read out the numbers as they are relevant okay fine. It is just not worth trying to make the letters so large that they are visible out there okay. So what do we have?

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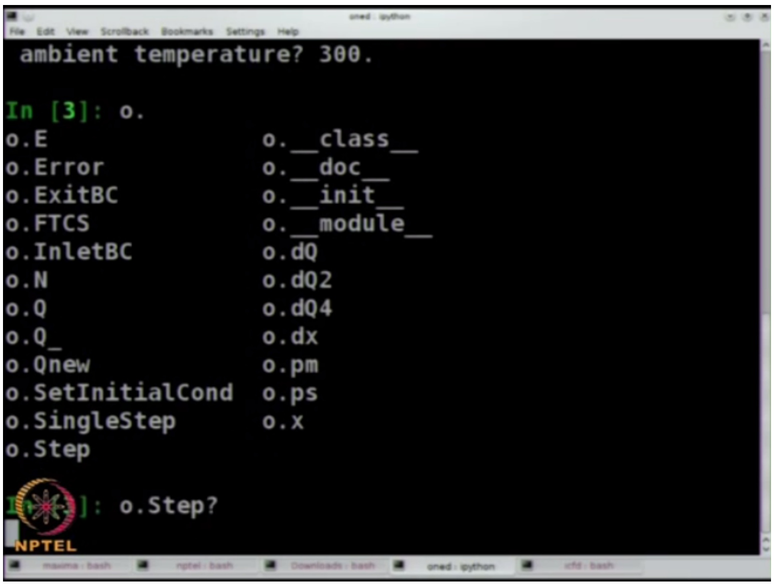

What I can do now is this solver which I called o for whatever reason has various things built into it. You can see the one that is actually of interest to us is this item called step and what step does? What step allows you do?

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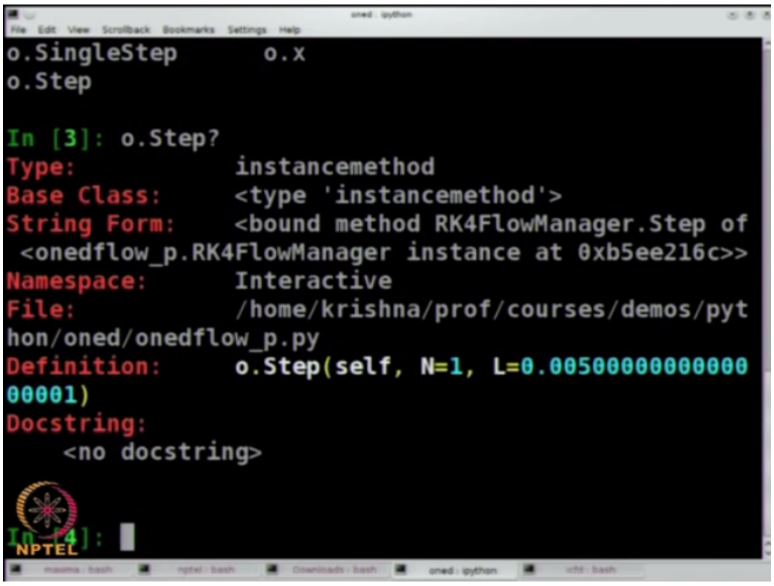

Step takes the number of steps that you take and L it is not lambda, it is L, it is delta T/delta x okay. It is not sigma okay. This is not the CFL, this is delta T/delta x so I want you to remember this. In this case, I am prescribing delta T/delta x okay. So we need to have a discussion as to how come I am not prescribing the CFL. How did I decide to pick this delta T/delta x?

And I seem to have taken 005 delta T/delta x is 005. What is my delta T? If my pipe length happens to be 1 meter I have taken 1000 intervals each interval is 10 power -3 meters which is 1 millimeter and this is basically a 5 microseconds okay. So I am going to be advancing my solution in 5 microsecond time steps fine. Is that okay? That is what we are planning to do.

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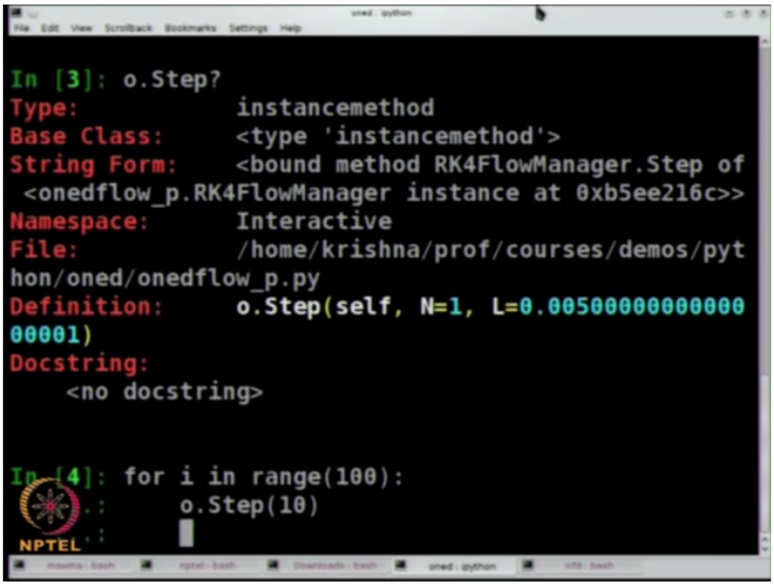

So what I will do is I will again remember this for loop just to remind you basically says that do this 100 times. What we want to do 100 times? I want to take steps and I will do 10 steps at a time right because it is going to trudge along at 5 microseconds or whatever it is. I will take 10 steps at a time. We will see where this goes okay and we will see what happens with this. Is that fine?

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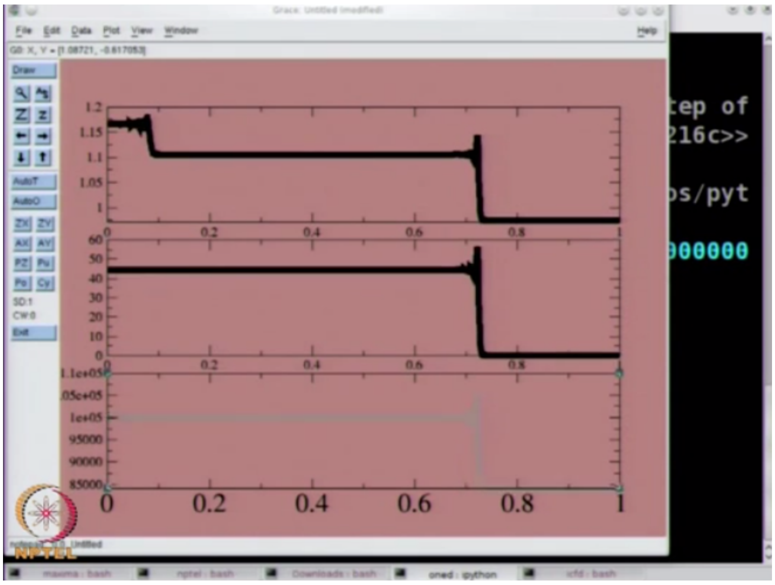

Let me get my graph backup so that the graph is clear and if I do enter and one more enter oh very noisy curve, this is not a sanitized simulation in the sense that I have not cleaned it up and all of that kinds of stuff. Am I making sense? So you have to figure out what is happening here. So question is what happened? Actually, I should have taken may be 50 time steps and I should have looped to 50 may be instead of 100 but it is okay.

May be I will run that through again one more time, you have seen it once, second time you can actually observe. So I want you to observe certain features what you saw okay. We will run through that again and instead of doing a 100 I will do 50 okay. I think confuse my FTCS part which I am going to do later with my Runge-Kutta part which I did now. So what you saw was some feature travel left to right seem to possibly reflect off.

I do not know what is happening and something coming back and then there is this one solitary step here that is propagating left to right okay. So we will try to do the following. We will try to estimate what speeds are these things propagating at and what are these? What do we expect? What is that we expect okay? What are we expecting here? So let me just I will just kill this.

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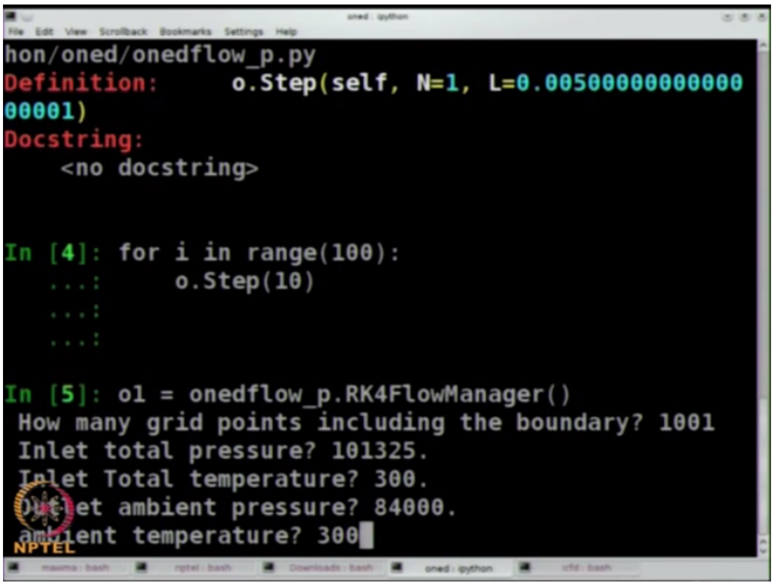

I do not want to run that so maybe I will just create another o1=onedflow_p.RK4. I will just quickly create another one. Okay there we go, centered up again, that is fine done and this time I am going to be little more sensible and instead of doing 100, I will do 20. I am getting conservative yeah I will do about 20 that is fine. What happened how did I disconnect the pipe? Here you go.

At this point you can most probably hear we think fine here we go. Let us try it one more time, good show okay.

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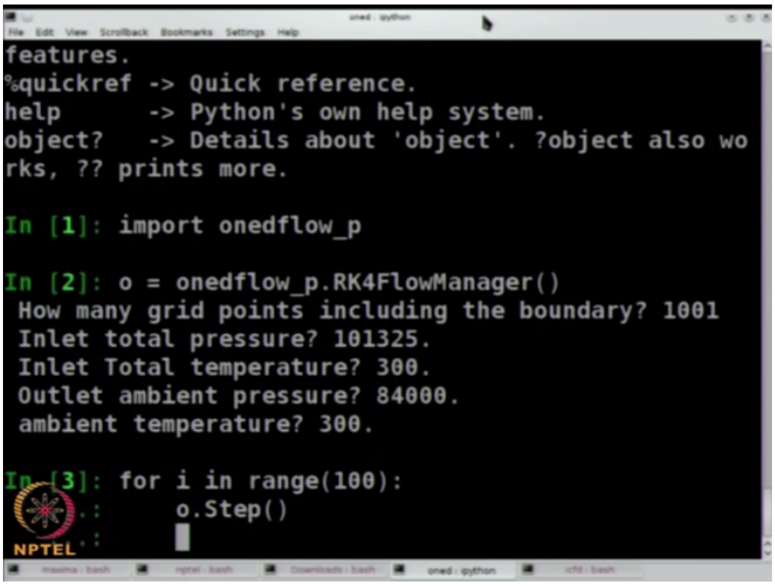

So for i in range may be because I am taking sufficiently large time steps, I will do 100 and just do o.step, I will change that later and taking a 100 time steps.

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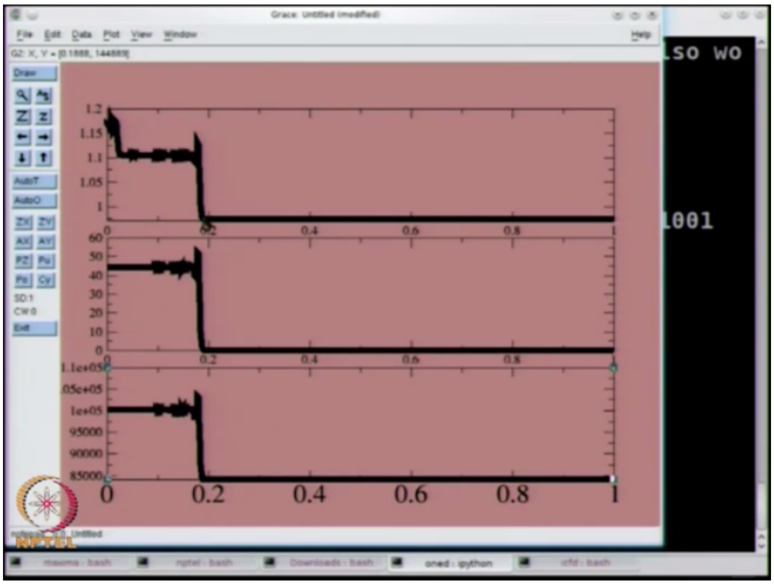

There you go, taking one time step as the time as I indicated is separately painful thing. All those little sharp features that you see are the solution oscillating. We saw this in the demo for linear wave equation. So one of the problems is it is possible for me to tune it so that I eliminate those sharp peaks whatever right when the control that we have on some parts in fact I could have tuned the parameter.

I could have hunted around, look for a parameter to find that I will eliminate these okay right but I actually want you to see even at this screwed I actually want you to see what was the. Let me take 100 more time steps because the step here has not traveled that far. This has travelled quite far so in a 100 time steps where has this come can you give me an estimate about 185 or something or that is what 180, 185.

So if I take a 100 more time steps, I expected to come to about 360 this step and these 3 seem to be aligned. So that is some feature that is traveling forward. This is going at a slower speed okay. This is moving at a slower speed. So there it goes again. Now as I said the value that is out here I hope this flickering is not too uncomfortable. The value that is out there the magnitude of the step get you an idea as to what is happening the magnitude of the step.

Yeah it will stop now right so they are propagating at seemingly a constant speed right. They are propagating at a constant speed. This lower part the floor here is 84,000 that is the right hand boundary condition. That was our initial condition. This here is one atmosphere 101325 so that 101325 from the left hand side is now propagating towards the right that is basically what is happening.

This air at 101325 and that is propagating from left to right and as that pressure is the question as what is propagating I said pressure propagates so along with this so you have this feature that is propagating that is sort of a pressure is sort of like an acoustic wave if you think about it. So I would expect that this corresponds to u+A and traveling in the right direction.

And there are 2 features traveling left to right, 2 traveling left to right what is this other one correspond to? Most probably corresponds to the characteristic u. This speed here is about 44 or 45 meters per second. This reading if I stick it here you cannot make out, I can see on the scale there are numbers on the top that tell me. So this reading it could be anywhere but if I stick it in the middle right this is around 44 or 45.

So this distance that it has traveled you have to look at the x coordinate, the distance that this has traveled right corresponds to that speed times how much what is the time we are taking 5 micro seconds*200 right which is one millisecond. Am I making sense? So the numbers sort of workout so this is basically propagating. This entity here is propagating at this speed okay. So this is most probably the contact surface.

Am I making sense? Is that fine? Okay that is most probably so the pressure is gone and the fluid is coming along great, the actual molecules that were inside the stagnation chamber right inside the reservoir the actual material that is inside the reservoir is now traveling through but that is traveling only at 44 meters per second. It is traveling at a slower speed. Is that fine? Okay so that is coming along.

This is actual material that was inside the reservoir that is traveling okay. All of these are occurring because of the compression wave because there was compression fine okay. So you have already seen this. What do you expect when this goes to the other end? When this pressure pulse reaches this end you are going to come to this end and it says and there is a declaration that it should be a 101325 but it is not it is at 84,000 right.

So the pipe is completely at 101325, the boundary condition there is 84,000, that expansion where is going to start propagating back, it is an expansion wave okay that is what we are looking for. So let me get back here.

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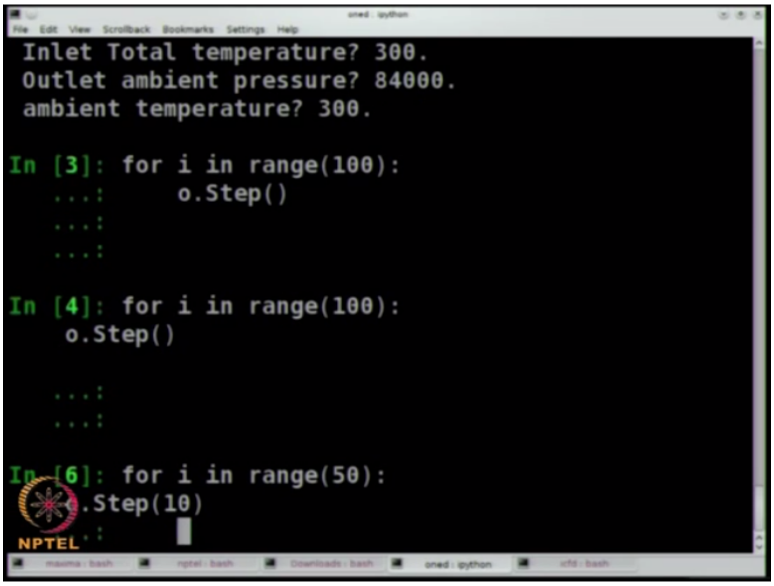

Now we will run it maybe what you say if I run this 50 time steps 10 at a time so I do not want to pain you know. Now we already know where we are going right. Let me get back there.

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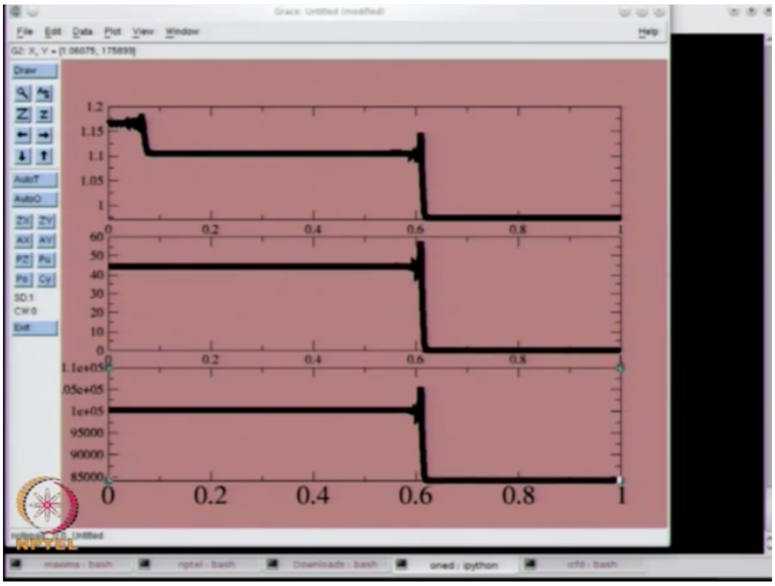

We have our picture back. Yeah so 10 at a time it really moves fast, 50 time steps 10 at a time is 500*5 micro seconds. Now I want you to pay attention to this slope. What is happening to this slope as it propagates right to left? The slope it starts to tip; the curve starts to tip right because it is an expansion fan. Am I making sense? It is an expansion fan right so that the slope starts to right.

Does it make sense to you? And what is the speed you expected to be propagating at? Well we will find out okay. We will run it a little longer and we will find out. Now remember it is very easy to look at this feature propagating right to left and think that things are happening right to left. No, the flow itself is still left to right, do not forget. This 44 or whatever 45 meters per second is left to right.

What I see here is 90 meters per second is left to right, which is why this is propagating to the right faster than the bottom is propagating to the right which is why it starts to tip okay. That is why it is starting to tip it is an expansion fan okay. I hit an extra enter last time so just to make sure let me get back, sorry about that. We are likely to cross this, it is not quite a step right.

This expansion fan is likely to cross, this should be a sharp step, it is not a sharp step right. We are likely to cross this. Do you think anything will happen when they cross each other? Any predictions? You think anything will happen when they cross each other? Do you think this will get destroyed, its magnitude will change, the step size will change? They correspond to different characteristics remember.

This definitely corresponds to a characteristic of its own right okay let us see.

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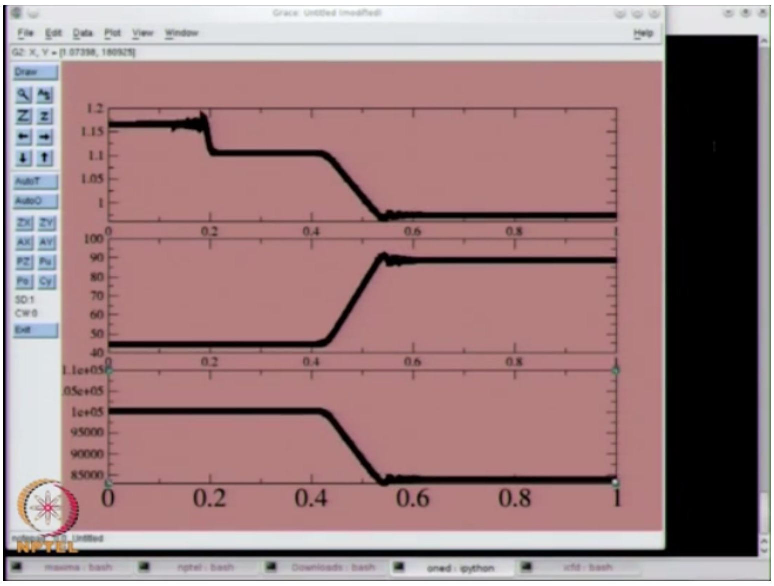

I do that, it is propagating, some tiny oscillations here and there. So this is another 500 time step so we have so far run 1200 time steps. There it goes and what happened? It is like the water level you know the water level going up or coming down that is it. That step did not go away because remember that is physically still the air that is from the reservoir that is traveling and it is traveling at its own speed.

But when the expansion fan came something interesting happen now. Now the speed at which it is going to be traveling is 90 meters per second. You expect it to travel faster, u has increased, I expect it to travel faster right. The expectation is that it will travel faster okay and from this point onwards it will get a little noisy, we cannot really track because of all these characteristics.

See now what is going to happen is imagine the expansion fan consists of lot of characteristics and each individual characteristic is going to go hit the right hand boundary and bounce off it. If your pipe is long enough they may again coalesce into a shock. Our pipe is not long enough okay right. So we are not going to see, we are going to see some funny sloshing kind of a motion that is what we are going to see right.

But I want you to think that at this point what I want you to pay attention to is propagation speeds. How fast is something going left to right and how fast is it going right to left okay?

Any feature how fast is it going left to right, how fast is it going right to left okay? I am constantly rescaling which is why if you had it on because I want it to squeeze 3 graphs in my scales are changing okay that is why there is some peculiar behaviour.

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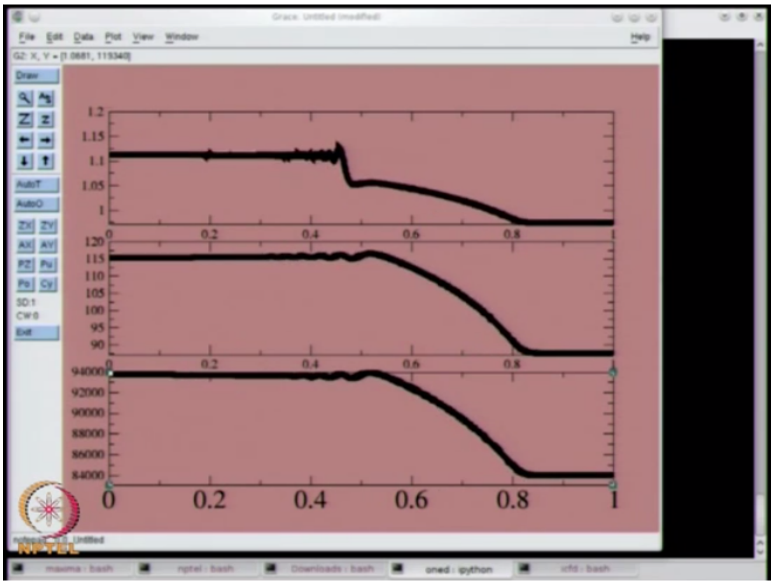

Look at this, look how fast this thing fellow came back here. How come it came back so fast? It is traveling at u+A and u is increasing right, left to right it is traveling u+A and u is increasing, the magnitude of u is increasing, it is traveling at something that is similar to u+A at least and u is increasing okay and look at this right, this was something like pottering around right at a very slow speed.

And if you make a calculation if you are keeping track right if you want you can note down the times or whatever it is the time stamps or you can run it yourself right. This is about 550 millimeters okay. Note the propagation time right to left now here we go one more interaction, that step sinks, the step identity is not lost. That is it, it did not even make it across to the end of the pipe okay.

Because the return line is that u-A and if you are going to be coming down once you get down to 150, 170 see as you get closer and closer to the solution, the expansion waves coming from the other side are going to take more and more time okay and if you want the steady state, you already have an issue here that is seemingly a problem where we have a potential problem here right.

What is the issue? So if I want the steady state, I want basically the right hand boundary condition and the left hand boundary condition to be in equilibrium and there you go. So now you should have a compression wave coming from the other side and watch how fast that travels, that compression wave of course will be it is not a compression wave, it is a series of waves that are reflecting off okay.

So we took 2 500 time steps to go to one end, I will just do this one last time and then maybe will either choose a larger time step or will either choose a larger number of time steps before visualization or will switch okay.

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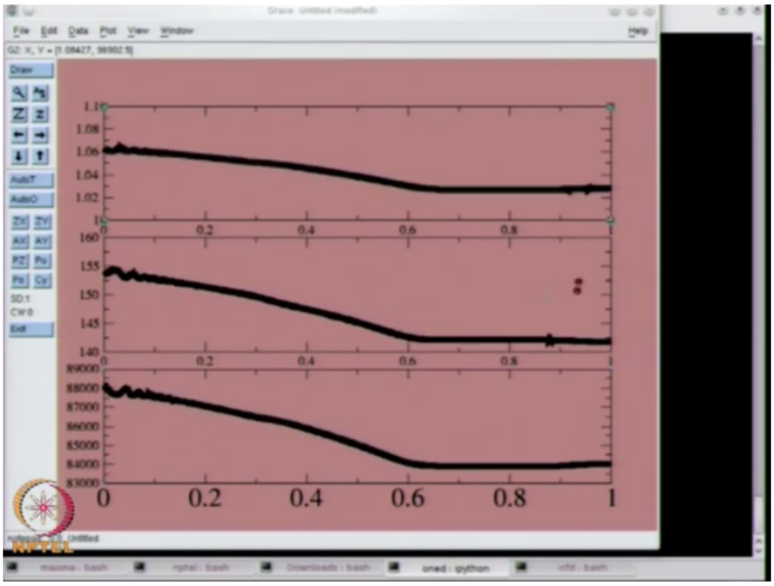

So the leading edge is already here, the leading edge of that compression wave is already here and it is already at the right boundary right. So already at the right boundary and there you go. So now the expansion fan is going to again and here now I am using these terms loosely because you have to actually a mess off characteristics right like characteristics going in 2 different directions.

But just to give you an idea the right hand end is about 165 meters per second, the left hand end is about 155 meters per second okay. So it is traveling quite fast. Please remember left to right, it is traveling quite fast left to right. This pressure here is about 88,000 right. This is a static pressure. The total pressure here of course taken the 155 into account will be 101325. This pressure here is still at 84,000 fine.

And the process that you have seen is what is going to happen is these waves are going to keep traveling back and forth but critically the 84,000 is something is going to travel up stream causing the flow to speed up and it is going to keep happening. Right now it looks like it is going to keep happening till left hand side and right hand side are both 84,000. So we have a pipe that is communicating is 84,000 up stream right.

That is basically what is happening and the difficulty that we have with this is as you get closer and closer to the solution, this communication up stream becomes more and more difficult, it becomes slower and slower okay. This communication actually gets, is that fine? Okay look at this we are running the same 500 time steps right and it is not even made it. So you can imagine.

Now you understand why I ran the condition Mach 0.5 right. So if I am running 0.9 right if I am running transonic speeds, we have a problem. Transonic speeds we have a problem right, u-A is almost 0, low subsonic speeds we have a problem because u is almost 0 okay right. There are 2 situations, u+A is not a bother, u+A is never a bother right so we have 2 situations where the u-A characteristic can turn out to be a problem that is transonic flow.

And if you think about either from your experimental aerodynamics or any other course that you had transonic flow is difficult to compute, is difficult to handle, is difficult to perform experiments and so on right okay. The other extreme low subsonic flow is an issue here okay. This class of schemes if you say I want to compute Mach number 0.01, we need to do something special fine okay.

So what I will do maybe just for the fun of it, I will make that 50, I do not know anybody has been keeping track of the amount of time that we have spent on this.

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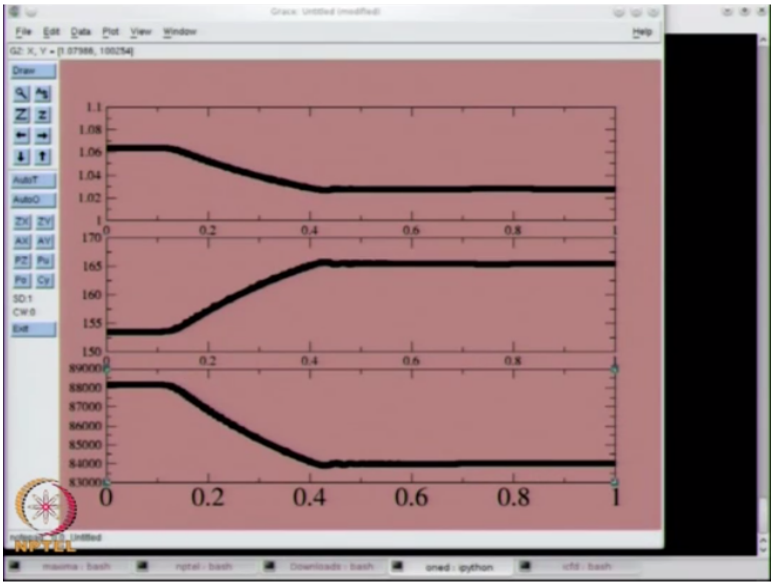

But let me see so of course I am spending 50 time units before I graph right. So it seems to move relatively better but it is not running any faster really except that I am not wasting time graphing. The return as expected is a lot faster in fact it will just go through. So I just want to show you where this goes okay. This is fine may be what I will do is just for the fun of it instead of doing this maybe what I will do is I will run a coarser grid right.

Then we can rush to steady state quickly instead of doing this. I will run a coarser grid after this. Do you have any feeling for how much the residue is going to drop or release the delta Q? See right now if you look at this, the speeds range between 174 and 173 right, 175 and this is little over Mach 0.5, it is about 175 and the pressures range from 84,000 to 84,500 so you could say to do if you are only looking for 2 decimal places is converged to 2 decimal places.

If you look at the residue right the residue may not be that bad. Is that fine? May be instead of trying to drive this to what I would call convergence right. Now there is an issue that you have to remember, you have an existing code, you have tested it right. It has been tested, you are running it in the flow regimes for which you have tested, it is well tested, it is a mature code then you can look at it and say for engineering analysis if I have one part and 1000 it is okay.

Am I making sense? But when you are developing the code, when you are actually developing the code, you would like to know that if I give the code you know derive it in terms of floats if I write it in terms of floats that it will converge to whatever machine position that you have with floats and if you give a doubles it is going to converge to whatever that is code converges you understand.

You would like to know while you are in the development process. When you are making production runs it is a different story. Once the code is tested right, it has been verified, somebody have certified that this is the code that right it works then it is a different story as to how you use it right but when you are doing the development, so just to emphasize that and not so you do not get the impression that this guy is just talking and let me just show you.

So I do not take too much of your time I am going to restart this because I do not trust right now.

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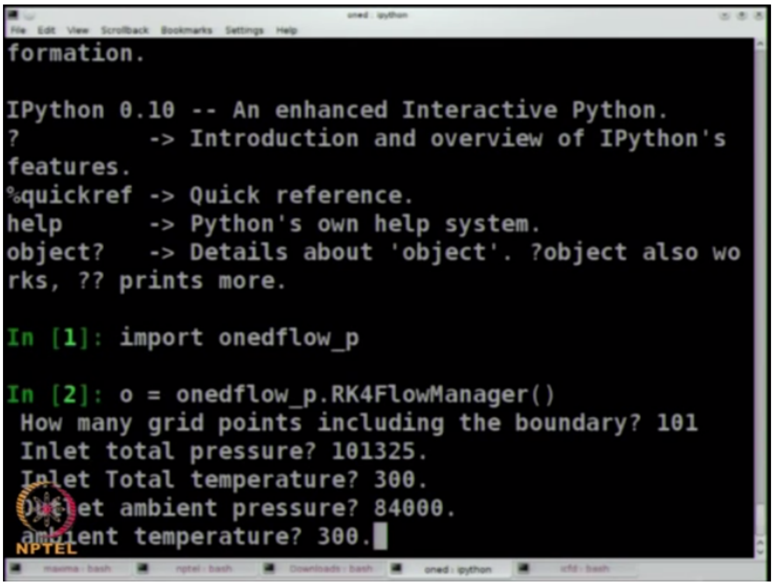

I import I forgot one thing that I should have done, it does not matter. I should have done the convergence plots it is okay I will do it for this right. I mean having done all of that stuff I have the convergence plot could have been plotted. I will use 101 grid points so that will be 10 times faster. I will run the same case, 300 kelvin. I would suggest that different temperatures they all have an effect, temperature will affect the speed or sound, they all have an effect right.

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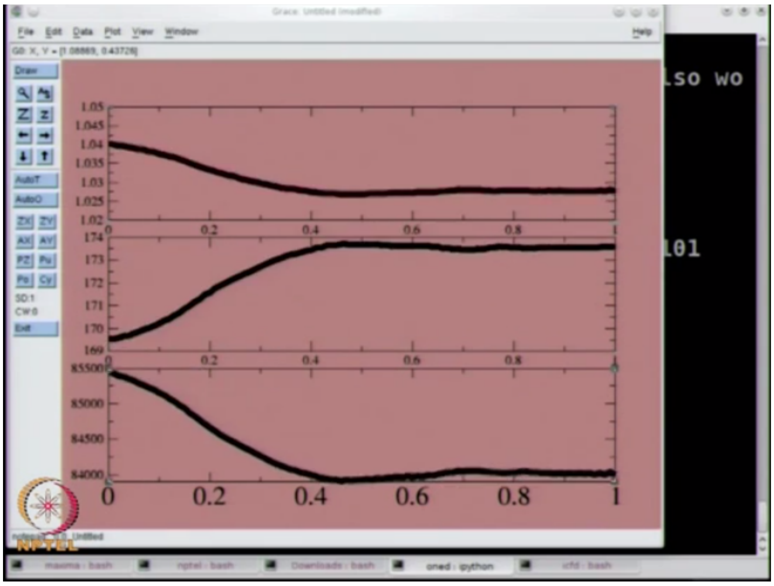

So I would suggest that you look at that, let me just quickly okay it is fine. So 101 time steps for i in range 100, 10 time steps at a time, we will just see what it does, that is a 1000 time steps whether that is enough or not I doubt if it is enough but this is going to move fast. There you go, so it is definitely faster maybe 10 was too fast.

This is going to move fast but I want you to note in this animation I want you to note that the waves coming back are slower than the waves go. See that is it, it just goes slow almost instantaneous, slow almost instantaneous. You understand what I am saying. It is very clear that left to right okay so that was a 1000 time steps. It is already at 84,000, the pressure is at 84,500.

The speeds are at 177, all of these will soon, see this density in fact is to 5 digits it is a same number, it has not changed. So it looks very noisy but remember the magnitude of that noise is the order of 10 power -7, 10 power -6 is that fine okay? So those of you that cannot see this all of this read 10275 on the scale because I am not going to add any. I can add more digits I am not going to add any more digits, it just messes up the display.

All of these read 177.47 right meters and all of these read 84,000 pascals. So you can say at this point why do you persist on running this? But as I said if you are in development mode, you want to make sure that it actually goes right all the way. So let us push it. See what it does and you can still see there is this propagation back and forth, it gets noisy, some high frequency seems to appear and disappear right.

It seems a lot worse than it actually is because I am not dynamically rescaling the y axis, so these wiggles are not really as large. If you were to plot them on the actual scale, these wiggles are not really as large. What do you think will happen now?

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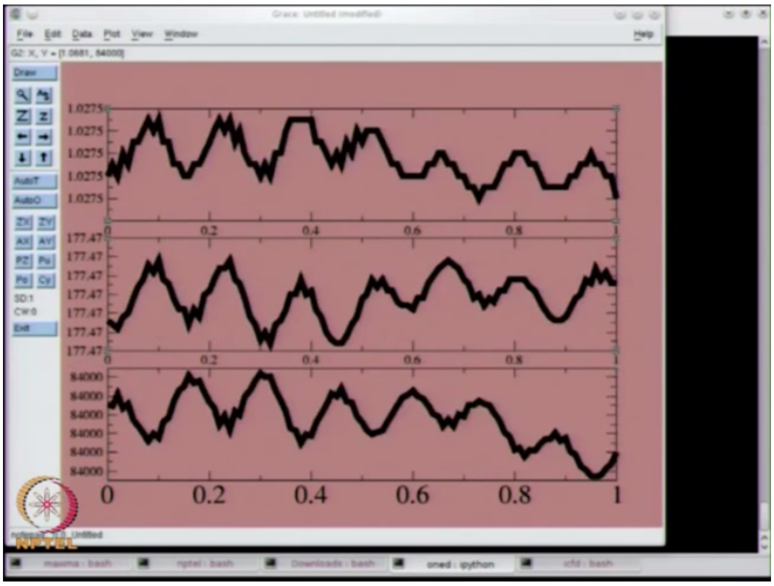

Look at the top graph, now we are at quantization error, we are at the last bit, we have a single bit left. You can only discriminate one bit. We are making sense +- epsilon. We are there. We cannot calculate this, we cannot calculate it any more accurately right. So if I run one last, just for the heck of it, if I just run that is it, nothing (()) (35:39) they are flashing right, it may not show off, is going to just sit there flashing.

We do not have sufficient bits, so now I feel good. Whether there are better schemes, I need not have added this dissipation the way I have added it whatever right but the code converges and if I give it quad position the code will converge okay and if I tested over range of temperatures, range of pressures, range of Mach numbers then I will turn around and say in this range this code works if you want to use 10 power -3 use 10 power -3 right.

If you want 10 power -3 you have a reasonable solution. So let us just look at let me import let me just plot that error.

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I will create another grace plot plotting and I will quickly excuse the color, I will quickly change that I think I had something called demo is not it last time which will fix that okay. So it is now relatively pleasant colors, g.plot and I have just to remind you I have something called error. I am going to plot that g.plot o.error. As usual before you look at it you should try to imagine what I will get and maybe that is not what you expected right.

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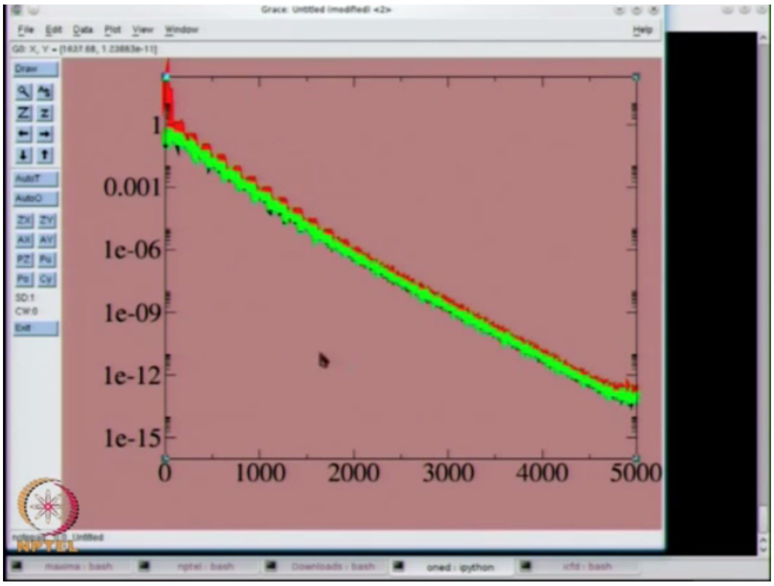

So remember the other thing that I told you. Let me get this so that you can see that scale there. The other thing that remember that I told you is that the y axis you plot on log scale, so will make this log logarithmic 1e-16 and I will show only every 1000 and it looks a lot better okay and there are 100 grid points and if I had actually integrated it. Here I am not integrating, I am adding up the residues right.

But actually what I should do is I should integrate which means that I multiply by 0.01. So this is at about 10 power -12, 10 power -13, that whole curve would have shifted down. You understand what I am saying whole curve would have shifted down and if I run it longer at this point it will just be flat right. In fact, I stopped it quite early normally when I am developing code, I let it continue run. I let it continue to run. Am I making sense?

I not only want to see that slope dropping and look at this is really neat right. You can see, I would like it to be steeper right, look at that. We will talk about that later. I would like it to be steeper and as I said I really wish I would have shown you that 1001 anyway it does not matter. So it is straight and then after that I would like to see the horizontal part extending out maybe the same amount I want it extending out right because fluid mechanics is a very funny thing right.

I will give you an example of a computation that we have done. A student of mine was doing 1-D flow premix combustion of hydrogen and oxygen and basically what happened was so you have this flame front that is traveling through this mixture right and it is possible that let me remember, let me get this right so it is possible that if hydrogen uses up all the oxygen and it depends on the mixture ratio right.

If it is stoichiometric, it is a different story but it is possible that the hydrogen has used up all the oxygen in the neighborhood. So you run it and the reaction rates are all in picoseconds, nanoseconds, the fluid mechanics is running in milliseconds right. So we take picosecond time steps and we march through and we say it is converged and we say okay just for some more time why do not we run it for twice the amount of time and see what happens.

And in twice the amount of time what was happening is the species were propagating down right at all over sudden the hydrogen encounters oxygen again and then again there is a rise. Am I making sense? There is again so you cannot just basically so what is all settle down, everything is fine, I have converged to a steady state solution may not be a fact. So you should run it for a little more time to see what is happening.

Is that fine? Okay right, so any questions? Okay so the things that we take from this right now are the propagations speeds are important. We need to do something therefore for convergence okay. We need to do something for convergences as far as these propagations speeds are concerned. The fact that the propagation speeds are disparate as headache, one way travels very fast, the other way it is very slow.

The faster travels this way, the slower it is going to travel the other way right. So we have to get some balance, ideally we like the propagation speeds all to be identical okay. So in the next class I will derive an expression for that. Why do not we just for the fun of it look at FTCS because that is likely where you will start your implementation to see.

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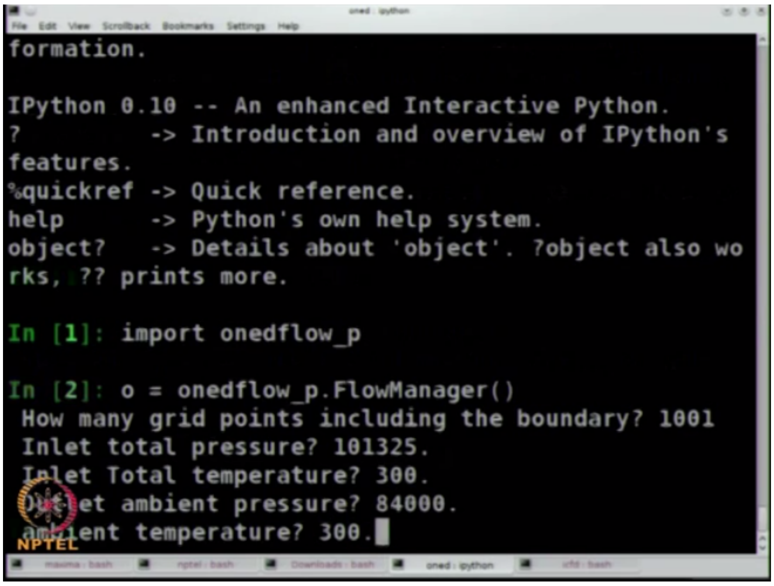

And the reason why I am doing this is I want to give you confidence okay this is what I am getting this is what he got so at least I am on the right track. As I said this is not a fully, I have not sat down and think out with this code but it is enough to okay so I will create a onedflow and in this case the generic flow manager which uses FTCS. It is not particularly surprising. I use this, I have the same interface.

Should I do 1001? Yeah I will do 1001, 101325, 300 kelvin, 84,000, 300 is the temperature, quickly change that screen to oned, that is done. For i in range what you want me to run? 100 okay and how often do you want me to show that? Every 10 and maybe I will take and look at this that is what I had 0.0005 last time, 0.0004 okay and we go back there see what it does.

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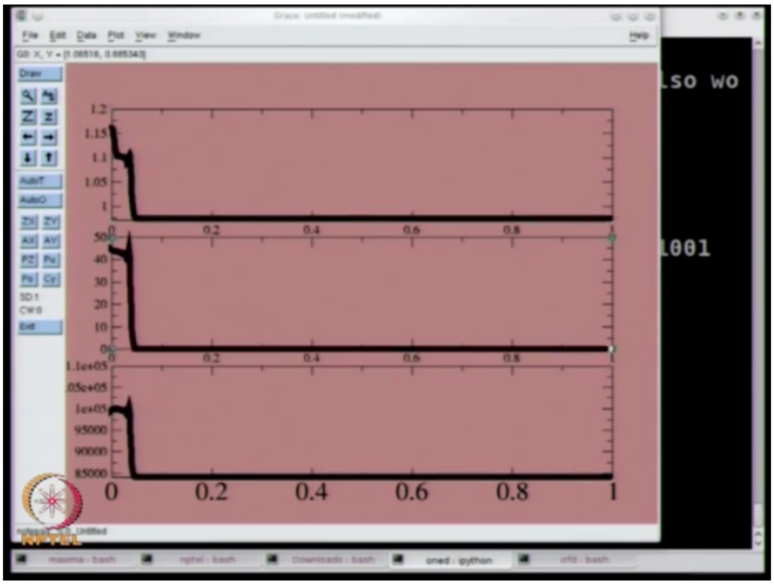

Now in this case may be the 10 was in that grid. In this case for this demo I deliberately added the dissipation added was a little stronger, so you will notice that it is oscillating very close to that step it is oscillating but they die out very rapidly okay and you also notice that this looks a lot smoother right. We will run it for little more time and you will see that I am not joking.

And in anyway at this rate it is going to take some time. So we may be instead of doing 10 time steps at a time, we will make it a little larger right okay because we do not have that kind of time. How much larger shall I make this? 50 time steps at a time well 50 may jump we will try it out. Let us try out 50 to see what it does?

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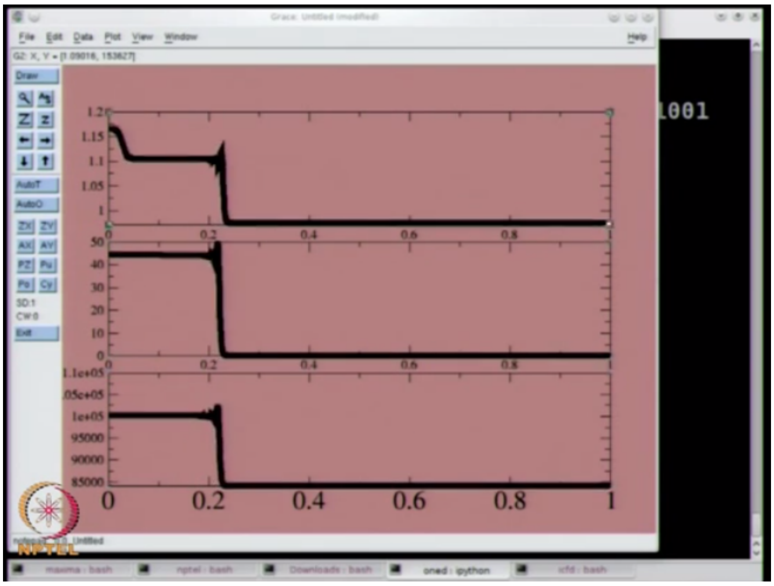

That is not too bad, so just like in our earlier wave equation thing it is clear near the step where the high frequencies are, there is dispersion and there is a problem but really look at that contact surface I mean it is nowhere near a sharp jump right. It is nearly smoothing out that is because I have added so much artificial dissipation that I have just smeared out the solution okay.

So the key getting a high resolution solution is determined where to add that dissipation and how much to add the dissipation right. There has to be a mechanism by which you can do it. The earlier one I told you had an underscore p right, one way by people do it is you detect the pressure, there is a pressure switch, you detect the pressure and you say the gradient to the pressure the pressure is varying very rapidly here and most probably in the vicinity of a shock.

And you add large dissipation there and the most probably in the way right you understand, you add second order this is not large dissipation, you are at the appropriate dissipation. I have added just arbitrarily as I told you right but you add the appropriate dissipation there and you can actually get a reasonably sharp shock. Is that fine? but you will notice that here it is smoother, there are some fine wiggles right.

But those artifacts are also because the line thickness is very large so even my graphics cannot catch, so the graphics basically here will show these artifacts are not real. The wiggles that you see there are not real okay. The compression wave is going to reflect off, expansion seems reasonably sharp. You can see it rotating now. It is going through the rotation process. This is smooth right. This is like there are really no wiggles.

As I said on the ramp, these wiggles are because it is drawing, we made up of horizontal lines which have a certain thickness that is only reason why it is there. This is really smooth. You want me to take larger time steps as well may be we have time and I will do 1 in a 100, 1 in a 100 may be large you know that is going to take 10,000 time steps. Your wave should not just let me.

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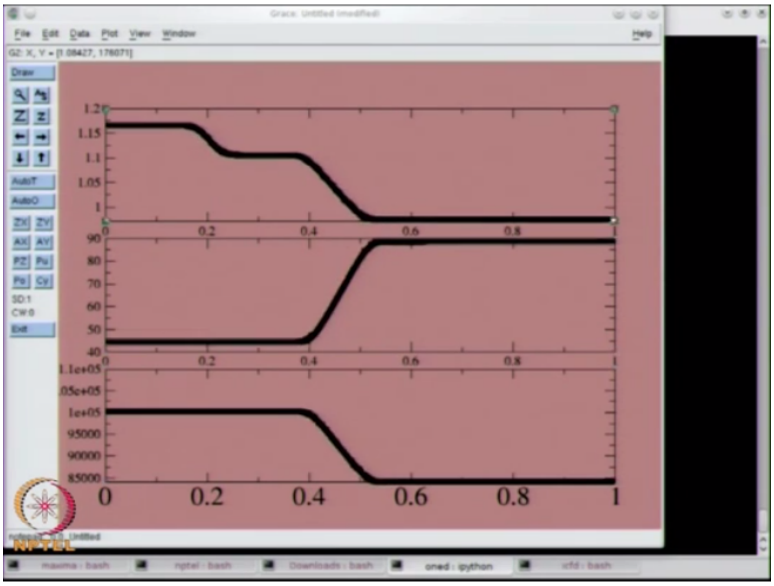

There it goes and it is like the ocean level rises and falls, so it is just the step that comes down. The step survives, the step is smearing because of dissipation, the extra viscosity that I have added is my fault and then the reflected fan they do get steeper but it is not long enough for it to coalesce. It reflects off the wall, is that supposed to be 10,000 time steps right because remember the individual time steps are extremely small okay.

It turns out the reason why I took Runge-Kutta is Runge-Kutta will allow me to take CFL that is close to 2.8 right, it allows me to go to a CFL that is quite large and that is the reason why I do not remember if I did up arrow yeah all I want you to see is that yes FTCS behaves the same way, so it did not have to do the scheme, the return wave is slower than the forward wave, the contact surface travels at the speed at which the fluid is moving.

If you add too much dissipation, the contact surface is not going to be a contact surface it is going to be a contact something region right. So it is not contact you think about it, very often you think of contact, you think of something that is sharp, so it is sort of a smeared overlap region kind of a thing. So you have to be a bit careful right. So even if you use what you detect, where you detect right so they have to be there are as I said please remember.

There are whole host of schemes. I will stop this here. Is that okay? Okay then we will just talk after this. The whole host of schemes that are high resolution schemes. If you want let me just show you the convergence plot for this and see if it looks any different FTCS okay. So from grace plot I import grace plot again, I create the plot, I will get rid of that glare from your eyes, something that I have started doing in my sequence that have started doing in my sleep now.

So then you have o.error plot oops, I am not accumulating error here okay fine, my mistake, I did not keep track of the error and I did not update the error anyway you can try it out. The behaviour is essentially the same right. I do not think we have enough time for me to run the 1001 for the other one. The behaviour is essentially the same. The thing is that I want you to take away from this for this course right that we need to do something about disparate speeds.

The propagation speeds are different and that is a problem especially at transonic speeds and low subsonic speeds okay. So we are going to I will suggest on Monday something to do so that convergence is faster right. At low subsonic speeds you can even get wrong answers because your problem is said to be ill-conditioned right such problems where you have disparate rates.

You may have seen in your differential equations course, they are called stiff problems is very ill-conditioned right. So if you take a system in which you want everybody is now hollering about. If you take a system in which it takes a million years for trees to collapse, beaker crushed and form coal or form oil and it takes you a few days to take that oil out and burn it.

Then this differential equation is very stiff if you are trying to do an energy balance, one is at time scale is in million years and the other is the time scale of days right and if you do it in days that million years will take forever and if you try to capture the million years the days you do not capture at all so there is a problem. These are stiff problems okay. So we will look at some way by which we can fix the stiff problem.

The other thing is here I have taken delta T/delta x fixed. I want you to think about that, Monday I will tell you which means at each point the CFL was different because the propagation speeds are different right so what does it mean to keep CFL fixed right. So in the next class, we will look at these issues. Is that fine okay? Thank you.