Introduction to Computational Fluid Dynamics
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Lecture - 29
Flux Vector Splitting, setup Roe's averaging

Okay. So we have been looking at 1-D Euler equations.

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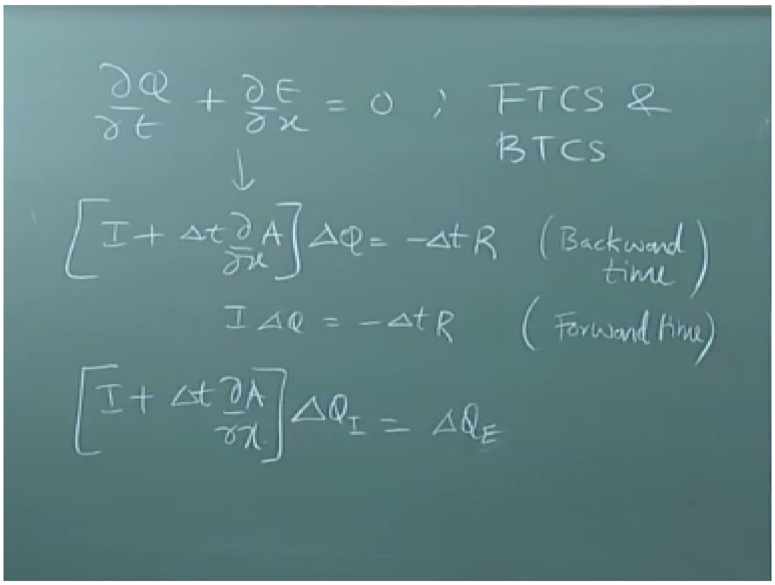

One-Dimensional Euler equations, dou Q dou T + dou E dou x=0. We have seen FTCS, we have seen sort of FTCS applied, FTCS and BTCS applied to this and BTCS applied to this. Okay. Right, we have seen the BTCS applied to it Backward Time Central Space applied to it delta form. We also seen that FTCS can also be interpreted as been in the delta form right, that is basically.

So this equation, you could write it as I+I, I just want to make one observation, delta t dou/dou x A acting on delta Q is –delta t*R, and we have noted we have registered right, we have noted that I delta Q = -delta t*R, so this is Backward time, that is backward time and this is Forward time. So in fact if you use central differences to discretize R, right and central differences to discretize that then it becomes Backward time central space and Forward time central space.

So I delta Q which is delta Q is nothing but R, - delta t*R. So another way to look at it, I just want to just point, another way to look at this would be that BTCS as I indicated you can delta Q and now I will say I stick an I in there say implicit=delta Q explicit. Am I making sense? It is like we are calculating the explicit correct, the correction explicit correction and from there getting the implicit correction. That is the different way to look at.

So any algorithm you should try to see the look at the algorithm from different points of view, it may give you some insight, right it is always a good idea to try to look at it from different points of view, we are just wanted to point that out before we left this form. In the last class, we had looked at application of boundary conditions using characteristics, right. And I promised that in this class we would start of looking at finite volume method.

But before I leave the finite different classes scheme let me just do one last thing that comes from what we have been doing in the class so far. Now, instead multiplying this by the X inverse, X being the matrix of Eigenvectors, I am going to go back and multiply this by X inverse which is what we have done right in the beginning, right because this business of splitting the waves based on which way they are travelling.

It could give us the hints that how to generate a new scheme, that is the idea I just want to point out. So every little thing that we do, see typically what happens is you are trying to solve a problem, you are trying to solve a problem you have a local issue that you fix, let us say I have to apply boundary conditions, I want to do it better, how do I do it, and you are thinking imagining ways by which you can fix that solve that particular problem.

But you should also realize that you may come up with something or there may be clues by which you can generate other techniques, right other techniques. So for instance if you take this and if you have to multiply it.

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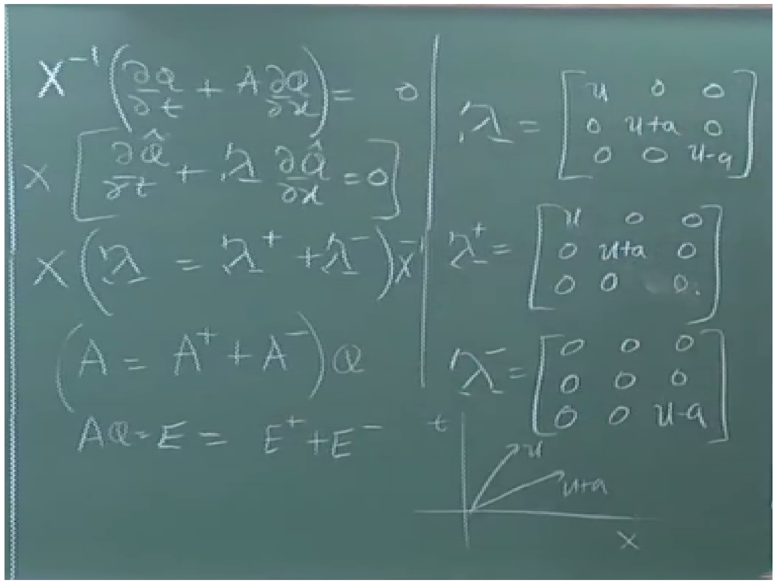

dou Q dou t + I will write it in the non-conservative form in conservative variables A dou Q dou X=0 and if I were to pre-multiply this by X inverse, X is the matrix of Eigenvectors, this gives me dou Q tilde dou Q caret dou t+ lambda dou Q caret dou x=0, right where the matrix lambda is u 0 0 0 u+a 0 0 0 u-a. Is that fine? So you have already seen that in the problem that we were looking at the problem that we were looking at, right.

That, these 2 are basically propagating left to right and this one is propagating Q1 Q3 hat right to left, okay. So this lambda can be partition into lambda+ + lambda -. Okay. So lambda+ will have right running characteristics and lambda- as you would expect will have the left running characteristics. See I have just, so characteristics that propagate right to left I am calling the left running characteristics, characteristics that propagate right to left, I mean left to right I am sorry, now getting my lefts and rights confused.

So characteristics that propagate from left to right, I will phrase the word left to right as seen on the board, okay which are like this a right running characteristics—I make sure that I do not get that confused, so that is t that is x, u so that would correspond to u+a and that would correspond u, fine, right. So I will go from left to right. Thank you. See the more I keep on explain left to right till you point out that I made some other mistakes, otherwise I see question marks on your face.

And that is lambda-, okay. Are we set, fine okay, so these have only the right running characteristics and these have only the left running characteristics okay a sort of okay, my left running characteristics that means characteristics that propagate right to left. So if I wanted to go back now, if I wanted to go back now to the original coordinate system, what I would basically do is I will pre-multiply this by X, you understand what I mean.

So we transform from the primitive variables Rho, Rho u, Rho et using X inverse, so Q1 hat Q2 hat Q3 hat okay. Now I am transforming back, I am pre-multiplying by X and transforming back. So this lambda I am going to see what happens if I pre-multiply this by X, because matrix multiplication distributes over and if I pre-multiply—post-multiply it by X inverse, matrix multiplication distributes over addition.

So this gives me A = some A+ + A-. Is that okay? Everyone? And in this particular case I have an easy way to get out for these equations, if I were to multiply this equation by—if I were to multiply this equation by Q. What would I get? AQ was E which is E+ + E-, right I split the fluxes. Of course the other thing is you could go back there and you could multiply by delta Q, okay that is the different scheme. So what you basically get here is—

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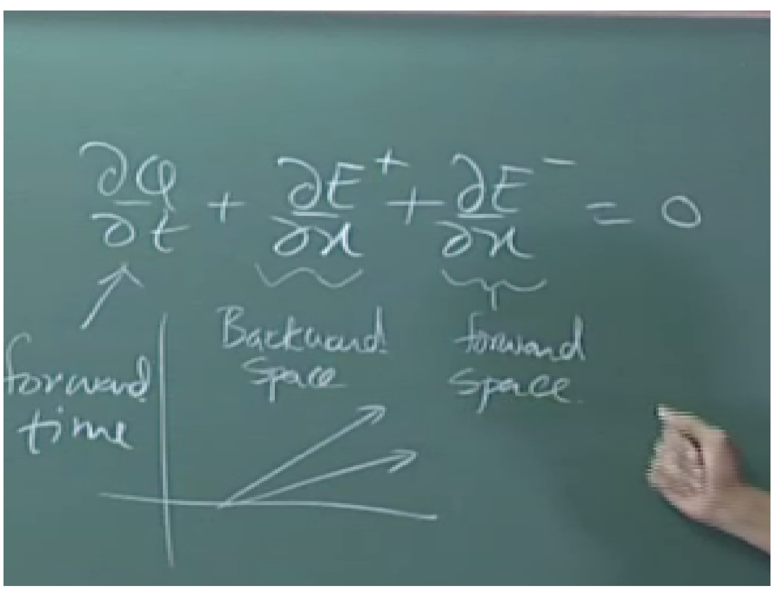

i.e, dou Q dou t + dou E + dou X + dou E – dou X = 0. Okay, where I basically split, I have basically split the fluxes in 2, 2 parts. I will let you do the-- the algebra for this is not, it is not that difficult, right so I let you do the algebra for this, I am not going to do it. So what we are manage to do here? What is the consequence of this? We hope that you could, this corresponds to, what is the—this corresponds to Eigenvector that are, Eigenvalues that are propagating in this direction. Okay, right so we should use, what kind of spatial derivatives?

Backward Space, right? We should use Backward Space here. So you use Backward Space here and use Forward Space here. Is that fine? Okay. Right, so you could in fact try instead of using just 2-point backward space, if you still want second order representation for your spatial derivatives, you can try using a higher order backward space, you can use a 3 point backward space, right.

Last class when we are talking about boundary conditions more accurately we have already talked about. That is what I am saying, you have all the bits and pieces now, right it just a matter of coming up with the scheme with all the – taking the combinations. So very often it is a matter of being alert. The individual ideas—all the ideas are there, now it is just a matter of being alert and making match saying that oh, I can try to do this and see what happens. Okay, right.

So you could do use backward space here, forward space here and forward time if you want, right and forward time. Is that fine? Any questions? Okay so with this—this of course when I am saying backward space, forward space I am talking still in terms of finite differences. So as promised in the last class what we will do is we will try look at finite volume method.

We had talked about it when we are doing the one-dimensional wave equation, the Quasilinear wave equation. We had talked about it when we are talking about the one-dimensional Quasilinear wave equation.

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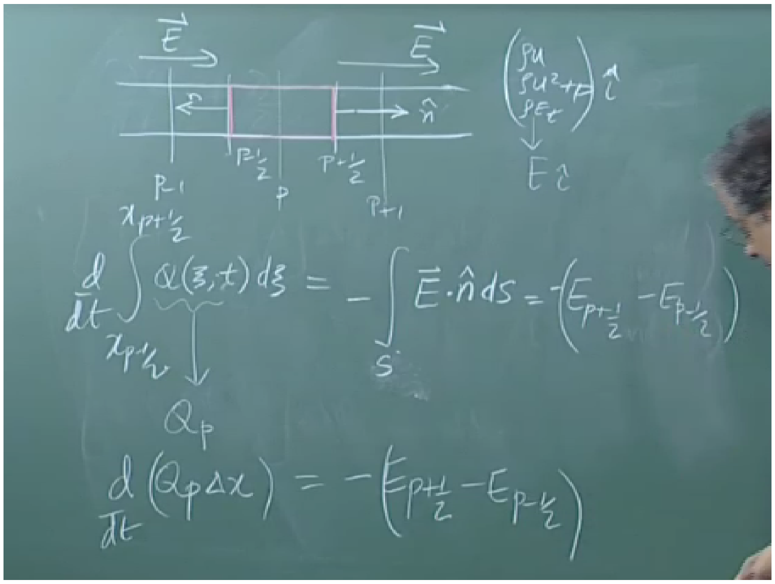

So – since it is an one-dimensional flow if you want I can draw that tube again that I had a pipe last time, so I will draw the pipe. So that is the pipe. So at this point we have the grid point P at that point we have the grid point P, at this point we have P-1 and at this point we have P+1. This intermediate values are P+1/2 and P-1/2. Is that fine? P+1/2 and P-1/2. Okay. Is that okay? Just like we did in our derivation, we have an outward normal n here, outward normal n there.

So the control volume that we are talking about, that is the control volume we are talking about, that is the control volume we are talking about. Is that fine? So if you say you have—what was the integral form of the equation? Do you remember Integral form of the equation? Integral Q, if you want Psi, t d Psi over Xp-1/2 to Xp+1/2, Psi is some kind of a running coordinate d/dt, time rate of change of the amount of Q that you have in, right.

We will take the area to be unit area right now. That is the control volume. This is the amount of Q that is there in the volume. This is the rate at which it is changing time rate of at which it is changing is the integral. What is it? E.n, E is the flux; vector E.n ds, over s with the – sign because n is an outward normal. S in this case goes from Xp-1/2, is that right? over S. So S is the unit area. It has to be this – that.

So this turns out to be Ep- -- Ep-1/2 Ep+1/2— **“Professor - student conversation starts”** Sir, E is the vector… E—well E is the vector but E is the vector that either pointed at this way that is the vector E. It has only one component, it has only x component. Is that fine? Yes, sir. E is the vector which has only one component, so E.E here will be the E that we know, right which is the matrix, remember that E itself is a matrix.

It is a Ei—if you want it in Cartesian Coordinates where this E is Rho u, Rho u squared 2+p, Rho Et, right and this is the unit vector I. Am I making sense? That is what I mean. Okay. Is that fine? By doing this I am basically doing all the equations mass momentum and energy conservation in one shot, okay. **"Professor - student conversation ends"**. Yeah, so what do we get? I have a – sign Ap+1/2-Ap-1/2. Is that fine? Okay.

I will replace this, I will replace this by an average value in the volume—this is what we did even there, right I will replace this by an average value in the volume which I will call Qt, okay so this becomes d/dt Qp delta x = -Ep+1/2 – Ep-1/2. Right and you can see that you can see one of the reason why I want to go through that is that you can see the equivalents between the finite volume method and finite difference method here, okay, that you get the same equation.

As I said, we can get into all sorts of discussions about what scheme where but sometimes quite a few of these will come out to the same kind of equations, right but they are not quite the same. Similar equation but not quite the same. What is the difference? Here we have the fluxes at, here we have the fluxes at Ep+1/2, we have P+1/2 and P-1/2. Am I making sense? We have the fluxes at some intermediate value.

So what does it have to do with? What was the motivation for doing finite volume method? Why did I suggest this yesterday? What does this have to do with application of boundary condition? Well, what we said what instead of applying, we know that we can apply boundary conditions.
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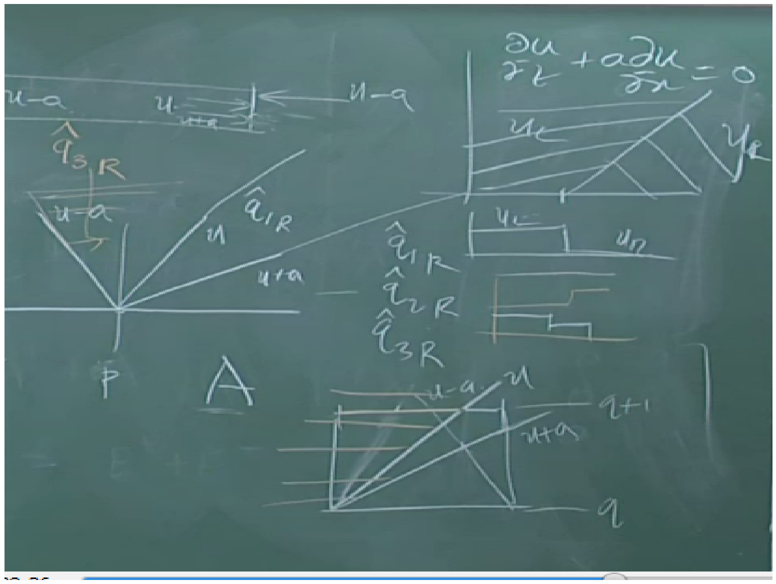

We can apply boundary conditions, we now know how to apply boundary conditions on the whole domain, right. So you have 2 incoming characteristics one outgoing characteristic. So for a large domain we use this idea, right this correspond to u, u+c, u+a, u-a, u oops—u, u+a, u-a, okay. This corresponds to those characteristics. The proposal last time was why not shrink this down, why not shrink this down to a small volume.

This whole pipe that you have, we can shrink it down, so we break this big pipe into small pieces like this, I am just separating out the individual pieces. You break it down into small individual pieces like this, small volumes and look at the areas that we generated for boundary conditions and see where that takes us. Okay. Since, here this material I am only going to be I am only going to be giving you the motivation, you understand what I am saying.

That is we will, I will show you that yes it is possible to use this characteristics, it is possible, there is a possibility that we can reconstruct, right there is a need to find the value of the flux at the intermediate point at which we have already seen. What is the way by which we can estimate? Okay, and there is a whole gamut of schemes, huge number of schemes on estimating the flux, right at that interface. Am I making sense?

So I am not, as usual I am not going to get into it, I just want to present the basic ideas, okay. Is that fine? So let us get back. Let us look at interface. So I have this, I have an interface, that is the interface. So that is P or P+1/2 or whatever it is, that is the interface. So the label itself is does not matter. So that this interface what do I have? Or at this point, let pick P at the point P what do I have?

We will look at the interface later. At point P what do I have? I have 3 characteristics. One corresponding to u+a, one corresponding to u, one corresponding to u-a. Is that fine? On the left hand side, I have the state, you can call it uL, so I have I have uL Rho L, PL these are the state variable that I have, right or you have Q1 hat, this is going to get messy L Q2 hat L, Q3 hat L. Is that fine?

So write here I have Q1 hat R, Q2 hat R, Q3 hat R. Across this I have a jump d Q u-a, across that I have a jump D Q3 hat, right across this I have a jump d Q3 hat across that line, fine. So the question is what happen to d Q3 hat? d Q3 hat is propagated along that, that is what our equations says, right Q3 hat. So what we basically have here is when you cross the first characteristic, when you cross the first characteristics, what does this characteristic say?

See remember, let us go back to the linear wave equation, dou u dou t + A dou u dou x = 0. So if you had a step, if your initial condition was as step, right so this uL, this is uR, the characteristics was one over, propagated with one over A. So to the left of this is the state uL to the right of this is the state uR, that is uL. Am I making sense? Okay. So in a similar fashion here when you—across the characteristic corresponding to u-a only Q3L changes to Q3R.

Only Q3 hat changes. Does that make sense? Only Q3 hat changes. So in this in between region Q3 hat in this in between region Q3 hat become Q3 hat R. Okay in a similar fashion, so in this range in this range, in this range from here you have Q3 hat R. What happens when you cross the u line? Q1 hat will change. Am I making sense? So then beyond this you get Q1 hat R. Okay. And beyond here of course the jump will be in Q2 hat R. Am I making sense?

So it is actually possible for us to use this characteristic to figure out what is it change in state as you cross each characteristic. If you know the conditions on the left hand side—so it is like saying that if I have a small locally if I have a small shock tube, the small jump right, so what, how is that jump going to propagate, how are those, what are the characteristics corresponding, right. I have a small shock tube; I have small diaphragm, small pressure difference. I break that diaphragm and we have a shock tube problem.

And we asking the question, the jump is very small, the jump is very small. So I seem to assumed here what I assumed here I am drawing this characteristic like this I have assumed A is constant. I seem to assume that capital A is constant, right I seem to assume that capital A is constant because capital A is constant propagation speed and the Eigenvalues are Eigenvalue are constant, right and therefore these curves have a constant slope.

So locally around here or if A is constant or if the jumps are very small, right so that the A is constant, so we can derive linearized equation, you have seen it in gas dynamics, I am not going to derive the equations here. But if A is comes from a system which is sort of a small perturbation system, right or A is essentially constant then these characteristics lines turn out to be straight lines, right. These characteristics lines turn out to be—even otherwise they are straight lines?

And when you cross them you have to think about it I mean if you cross them the Q1 hat, Q2 hat, Q3 hat depending on which characteristic you cross the state will change, right so it is actually possible for us to—it seems starting from here moving into the feature the predict what is going to happen, it looks like that, right there is a possibility here. Now we go back to the volume. Now if you look at the volume it is like you have the u-a characteristic that is coming in.

This is at time Q, this is at time Q+1. You have the u-a characteristic coming in. and you have the u+a characteristic again as I said just to give you trouble I deliberately choose so that it goes down, but anyway it does not matter. So this is u+a; that is u. So in this volume we actually know what – we can actually take you know what are the states at the various level, we can actually reconstruct, right we can actually reconstruct the function. Am I making sense?

If I tell you, if I tell you what from this side, what from that side, it is possible for me to actually reconstruct the function along this line, because I know what is the value, I know what is the value through past u I know what is the value that is coming their past u so I know the value of Q1 hat here. I know the Q1 hat, I know what is Q1 hat is doing there, am I making sense. In the similar fashion, I know what is the Q1 hat.

I look Q1 hat for instance doing like that, right. I know Q2 hat, I know Q2 hat is doing something similar, Q2 hat is basically just a constant, Q2 hat they are not to scale, right. Q2 hat, these graphs are not scale, right okay. Q2 hat is just doing something like that, right and Q3 hat again is the same thing. Q3 hat maybe somewhere in between Q3 hat is, right. So in the volume I know the variations.

I know the variation, in fact right through I know the variation of Q1 hat, Q2 hat and Q3 hat. I can actually reconstruct. Okay, so whole class of scheme is that, that basically based on reconstruction. Is that fine? Okay. So it is possible for us, it is possible for us to reconstruct and get. So using—it is looks like we are sort of using this idea of characteristics, we are sort of using this idea of characteristics, right.

It is possible for us to come up with the class of schemes. As I said what I want to do is I want to sort of bring this just to give you an idea of time where you are, we are most probably now by way of large volume of research somewhere in the mid 80s, okay. By the time the large volume of research started we are most probably somewhere it is. Okay. Back here.

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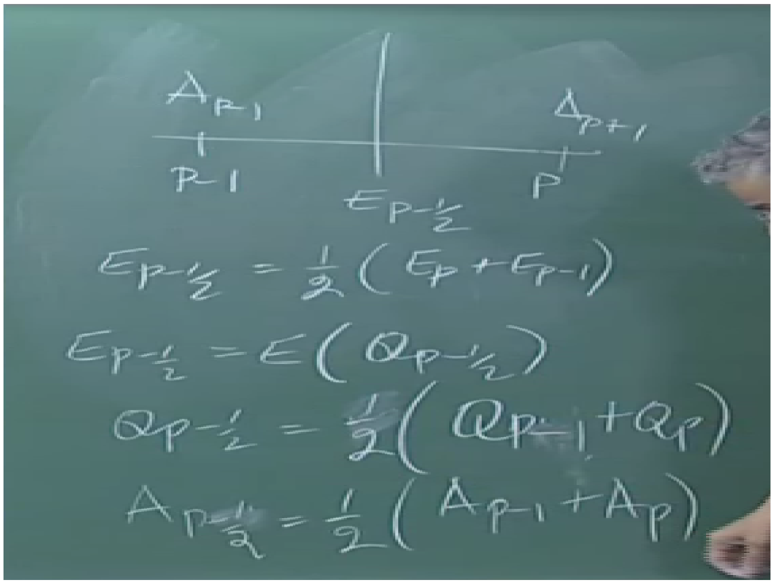

There is another issue; there is another factor that I want to talk about. So if I have P-1 and I have P. Okay. And we have the issue of finding E at P-1/2. We have the issue of finding E at P-1/2. Of course we can look at these characteristics and try to estimate the state at that point using these characteristics. The other possibility is in order to do that – of course there are other possibilities. The other possibilities, what are the possibilities we looked at right in the beginning?

We can just take an averages, so we could say that Ep-1/2 let us get into the simple possibilities was Ep+Ep-1, right, we will just eliminate, first eliminate just recollect what was the other possibilities we had. Ep-1 is a possible definition; I will say Ep-1 it is a possible definition. I am not going to use different symbols for Ep-1, could be E of Q+ Qp-1/2, the Qp-1/2 is Qp-1 + Qp. Am I making sense?

Qp-1 Qp-1, thank you. Qp-1 + Qp, right. This is the other way to do it. These are 2 possible ways to do it. So basically what you say is you say that the E, the E the state there is determined by the, the value of E there is determine by the state of Q there, one possibility is that you say that actually I do not need the Q at the interface, the value of E there just the average of the Es at the 2. These are what we considered so far. This is what we considered so far.

What we have to realize is that we are trying to do an interpolation of some kind. Okay, what we are trying to do is an interpolation of some kind. We have seen that if you knew the characteristics and A was constant, we could actually do an interpolation of some kind using characteristics. Okay. The question is what is the A what is that propagation would be. You have a state A here, you have a state A there. You have an A you have an capital A, you have a capital Ap-1 here.

You have a capital Ap+ here. What is the A that you use? One possibility is you take the average of S. See there is no limit to this. One possibility is you take the average of S. Am I making sense? One possibility is that you take average of S. What else we can do? In order to do what I indicated earlier, you say Ap-1/2 is 1/2 Ap-1 + Ap, so that you know the A. Then what? These are possibilities, right these are possibilities.

But actually there are certain basic relationship that I want the A is, the E is and the Qs to satisfy, right and this satisfies one of them. This is basically saying that E at this, E at this point is E of Q at that point and we are starting with the Q. Am I making sense? This is just like as I said earlier if you extrapolate both pressure, if pressure is prescribed it exists and extrapolate both the density and temperature then it is possible that you actually have an inconsistency, you have to be careful, because there is an equation of state that ties all of them together.

So in a similar fashion we have generic equations and relationships, if you just start taking averages then there is always the fear that, there is going to be mismatch. Okay. So what we would like to do is, we would like to look at, get back here, we would like to look at, right we would like to look at some relationship between A E and Q and that we have in the definition.

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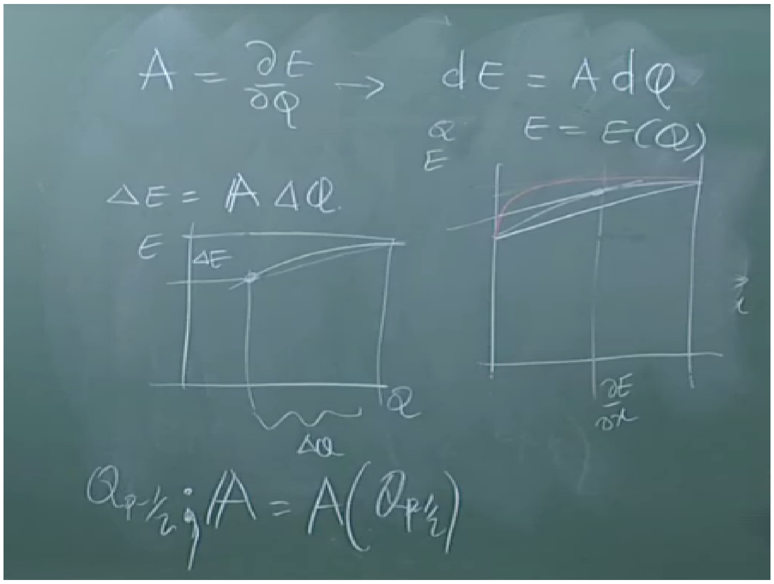

That is A = dou E dou Q or A as a derivative and therefore we know that the E is AdQ, right so the definition is derivative, fine. Definition of directional derivative. I am going to draw this as though I am doing it in regular one-dimensional dealing with scalars. But actually we understand that this is the vector of 3 elements, that is the vector of 3 elements, right and this is a 3/3 system which is 9 elements. Am I making sense?

But I am going to do the explanation using just scalars for now. What are the things that we have been doing so far? Okay, what are the things that we have been doing so far? Everything that we have done so far. This is the x-axis; this is either Q or E or whatever dependent variable. This is what we have been doing so far. So we have P-1, P and we want the intermediate values somewhere, right.

It is possible that this intermediate value, it is possible that intermediate value because we are taking equivalent rules is actually in the middle, is actually in the middle. If you are taking inequivalent rules it may not be in the middle, right that means whole other bunch of issues us to taking the averages and so on. So when we just take the average Ep+1/2 Ep-1/2 what are we saying.

So if you have a variation of E, if you have a variation of E or you have a variation of Q, by taking Ep+1/2, Ep-1/2 what are we saying? What is the value that we taking in between? You are taking; you are joining these 2 by a straight line and taking the mid value, right you are taking the mid value that is basically what you are doing and of course depending on how the functions varies that mid value may or may not be a value at that point.

The function could essentially be flat and then drop rapidly. Am I making sense? Okay, right. So this is—it is a matter of representing, what is that, that we are representing. Am I making sense? Okay. So if you look at, if you look at dou E dou X if you look at dou E dou X, if you look at dou E dou X at this point, if you look at douo E dou X at that point it is the tangent to the curve.

I will draw a bigger picture. Let me just-- I forgot the coordinates; I will just draw a bigger picture, okay. So you can have a nice smooth curve like that where you could have a strangely behaved curve like that —also nice smooth curve. This is the chord that is the chord for both of them. At the midpoint this is the tangent that is the tangent for that. At the midpoint, of course this is the tangent for that is that fine?

And at this point dou E dou X, dou E dou X could either be that tangent or this tangent, if you were to take an average you would actually be representing it by the chord. Am I making sense? Okay, so the chord in some sense yeah it is Mean Value Theorem sense in fact for the white one you can see that it is almost parallel, right Mean Value Theorem basically tells us somewhere along the line it is going parallel to it.

It just so happen—in this case in the case of the red curve it somewhere there, it some other point, the point can vary, it can be at different points, right but it is possible for us to actually approximate, approximate this. In reality what we want to do is, we want to approximate this by a straight line, fine. In reality, what we are trying to do, in reality what we are trying to do is we want to approximate the function by a straight line.

If you think about that basically what we are trying to do, we are trying to approximate the function by a straight line. The normal 2 possibilities is that you have is either the tangent or chord. You either approximate the function using the chord or you use the approximate the function using the tangent, fine because we are actually performing, an integration finally. So you all trying to approximate the function. Okay.

The other way to look at it is we are trying to approximate this tangent using this chord, but typically we are trying to approximate the function using the chord. We are trying to find some linear approximation to our function, that I typically what we are trying to do. Okay then the question is what is that; what is the best possible approximation, okay. The 2 things that we looked at is we do a linear interpolation for Q or we do a linear interpolation direction for E.

Those are 2 possibilities we considered. The third possibility that we just throw out now is that now try linear interpolation for A which is the slope that we are looking at, am I making sense, which is the slope that we are looking at, fine. It is possible that the error that we are making, the reason why we have difficulty with this, the error that we are making is that we are interpolating these quantities, we are interpolating these quantities on the x coordinate axis, right.

But in reality, E is a function of Q, and this is really what we want to approximate. This is the tangent, this is the tangent. This is the tangent, not dE dx, not dQ dx, the tangent that we are talking about is here this is the tangent that we are trying to represent. Okay. Am I making sense? Or E as a function of Q is what we are trying to represent. In the cluster does the tangent better or a chord better? Okay.

So the chord typically especially when you look at functions like this red curve here, the chord typically is a on the average or better representation. Okay. So basically what you do is we will convert this increments and we will say we look at delta E = some A delta Q, delta E is some A delta Q, this A is some average A. Am I making sense? This A is some average A that needs to be determined, some average A not necessarily this A. It is not necessarily this A.

This A as some average A that is not necessarily this A, and it needs to be determined. So in the E Q, I write E and Q but I am only drawing one coordinate, I am still doing it as a scalar. In the E Q space we know that is delta Q and we know what is delta E. What we would want to fine? We want to find the A. We want to find that average A. So we want to find the average A. As I said, I draw one-dimension in one-dimension given 2 points the line through that is unique.

In 2-dimensions if I have a surface, right the E Q, E as a function of Q—let us now look at 3/3 let us look at in 2-dimensions, right. So you have Q1 Q2, E1 E2, so we even forget E2, let look at just E1, you have Q1 Q2, you have a surface. I give you 2 points, then infinity of planes that go through that point, those 2 points, right. So we need somewhere we need somewhere to make sure that we are picking, right we need somewhere to constraint.

So what we basically says, I want this A, I want a Q, I want a Qp-1/2 or p+1/2 or whatever I want Qp+1/2, p-1/2, I want an estimate of that. I want this A to equal A at Qp-1/2. Am I making sense? Okay. See the geometrical—we will get back, just in case I mean just in case you did not get the why there are multiple plane—look at directly here, what we have, how many components do we have-- this looks like an equation Ax=B.

This looks like; this is a vector, matrix vector, right. Okay. So normally what you do is you are given you are given that you are asked to find this. That is the usual problem that you use to in linear algebra-- matrix algebra. Here the problem is, given delta E or given delta Q you want to find A. How many unknowns do you have in A? You have 9. Well, if A is going to have the structure of is going to have this structure, you may not actually have 9, but in theory you have 9.

But you have only 3 equations. There are lots of solutions, there are lots of solutions. But fortunately for us this A the first few entries of this A are something like 0 1 0 or whatever it is, there is at least one line of them is just gone. Okay. One set of—by placing this constraint by placing this requirement which we can argue in physically institutive fashion. We have sort of eliminated, right that it has the structure. We are saying about, what does the structure look like, okay.

By insisting on a certain structure already eliminated whole bunch of unknowns but the difficulty that we have is that this, when it given this vector and that vector we need to determine A, that is the problem that we have. Is that okay? What is delta E? I will just set it up and see what difficulties we have, what is the problem and then we will see whether we are able to—okay what is delta E?

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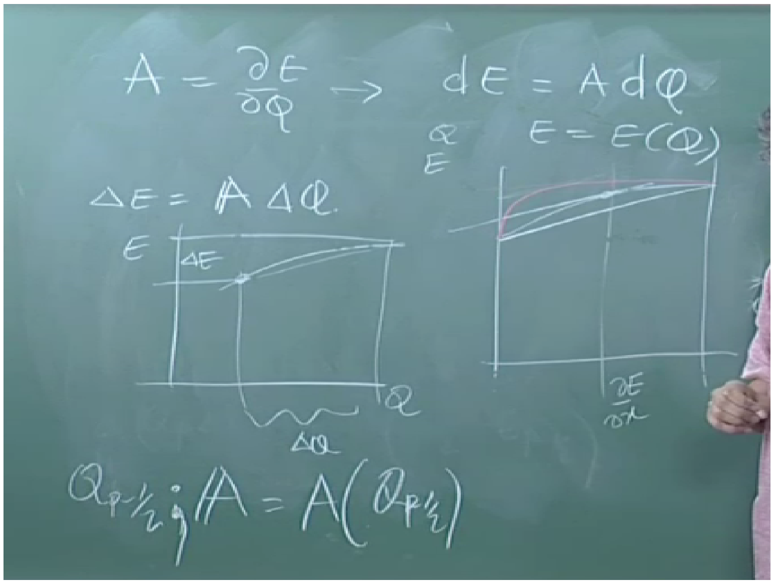

Delta Rho u, delta Rho u squared +P, delta Rho Et. **"Professor - student conversation starts"** Rho Et+p. Rho Et+-- I am thinking of delta Q. Rho Et+p times u. That look little too easy. Okay. **"Professor - student conversation ends"** And of course you can expand this out but we leave the delta Rho u as it is right now, that is delta Rho u squared, this delta p, this is delta Rho Et—delta Rho Et times u+delta p time u, fine.

What is delta Q? delta Q is delta Rho, delta Rho u, delta Rho Et. Okay. Do you remember the values of A? Does anyone have the values of A? Otherwise I will, we have the values of A, **"Professor - student conversation starts."** gamma-3/2 u, 3-gamma, gamma-1; gamma-1 u cube – gamma Et*u, gamma Et – 3/2 gamma-1u squared, gamma u. Okay that is what we have. Actually there is a reason why I expanded this. Normally I will not go through this headache; there is a reason why I expanded.

What we have here just by inspection because this is we are setting up for, right we are setting up for next class basically. What do we have here? **"Professor - student conversation ends."** We have Et, we have P, really if I look at this in terms of the kinds of unknown that I am going to have, I have Et's and I have a P, right I have a Rho and u. Is that it? Right, okay. Now you think back to your thermodynamics, there is a combination vt and p that we have a term, right. If you switch instead of total energy if you switch to total enthalpy, then we can observer the P. You can potentially get out the P from the equation. Am I making sense?

So we will retain, that is—the reason why I am writing this out is, we will retain the A—remember if you did a change of variables the Jacobian changed, we are going to retain this equation, to this equation. So the Jacobian equation we are going to do a change of variables. Am I making sense? Okay. So what we propose to do now is tomorrow I will write out the equations in terms of enthalpy, in terms of enthalpy so that I will have a total enthalpy here instead of the pressure term, okay.

So we will try to write out the equation in terms of the total enthalpy. See if we can get rid of the pressure in some form. Okay. So that all the pressure terms will be written in terms then we will have only, we will have only the enthalpy Rho and u, Rho u. Okay. And see where we can go from there. Is that fine?

So in tomorrow's class what I will do is I will start off with that, I will start off with the enthalpy part and we will go back and we will try to solve this equation, we will try to solve this equation—we will see what this gives us when we try to get in terms of delta Rho, delta Rho, delta Rho u, right and delta Rho ht or something of that sort, fine. And why are we looking at deltas because in one level those would be the jump across any given interface.

They would be basically, so we look or you can look at it as, if I have 2 points either spatially distributed or distributed in our state space so to speak. So if you have u left and you write and you are trying to figure out what is the interpolant that I should use given that have you left you right, we can actually use it do it here instead of doing it there. Okay, especially if you are talking in terms of oh I have a I have a small jump, right then this kind of a discussion does not make sense.

Even this graph does not make sense, right. Whereas a discussion herein the function space may still make sense. Is that fine? Okay. That is the idea. Right, so in the next class we will basically derive the averaging process, we will derive the averaging process, after that on the subsequent class I will try to do a demo for you, okay. And then we will move on to other—we will leave 1-D flow equations at this point, right and go on to other schemes. Fine, thank you.