Introduction to Computational Fluid Dynamics

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Lecture - 28
Implicit Boundary conditions

We were looking at One-dimensional Euler equations, right in the last class. And equation, just to recall is dou Q dou t + dou E dou x=0.

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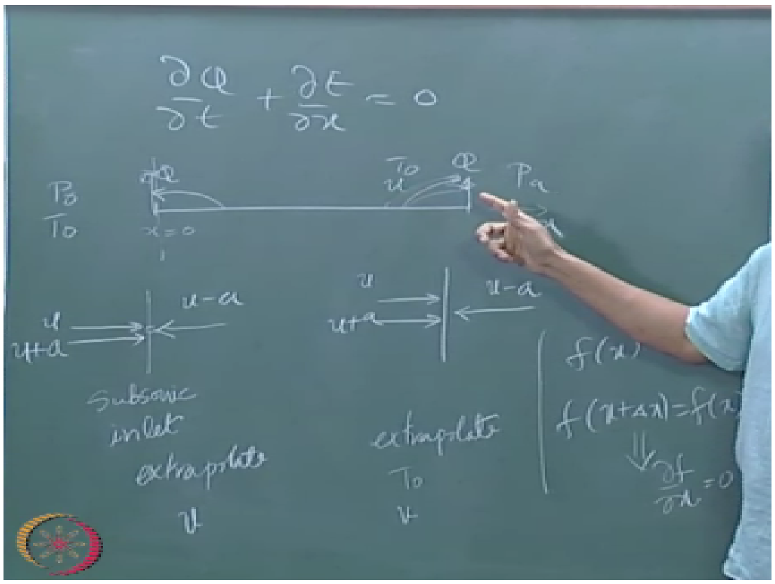

I just want to make an observation from a—this is the continuation of a comment that I made earlier, so we will write it out in the delta form and so on. I think we will write the discretization out in the in the delta form, the semi-discrete version, right. I will make a few remarks and then we will look at applying boundary conditions that is where we were. I think I ended by saying that I will show you how to apply boundary conditions in a proper fashion that is where, am I right? that is where I basically ended.

Now what we have seen so far, as far as boundary conditions are concerned we have seen—I will just draw a line, we have seen that this is the x-axis, t is of course the direction which we are marching. We have seen at x=0, we prescribing total pressure and total temperature and we are doing this looking at it from the point of view of gas dynamics that we know, right. So I have a reservoir or something that from there I will typically know total pressure and total.

This is the classic problem that you see in gas dynamics. And on the right hand side you have P ambient. And if you recollect – if you use the scheme like FTCS or BTCS you actually need, you need q at this point and you need q at this point that is you need the total state at the boundary, okay which you are not going to have which the physics neither the physics nor the mathematics require, the differential equation is require.

So it is our scheme that requires, we need to generate, we have to somehow generate those boundary conditions. And in the last class the point at which we sort of left is we were basically saying that if I look at this interface at that point and I look at the characteristics at that interface the direction in which—see now I am only in space I am not drawing time at all, the direction in which these characteristics are propagating some information, we know it is Q1 hat, Q2 hat, Q3 hat.

But I use the word information because this side we know we are prescribing P0 and T0, right not actually Q1 hat, Q2 hat, is that okay? So, Q1 hat and Q2 hat characteristics propagate inward information propagates inward then in fact propagating Q1 hat and Q2 hat, I am just using information as a general expression. So this corresponds to the characteristics u and u+a, okay. And to the same interface we have coming from the within the domain one characteristic which is propagating to the boundary u- -- the u-a characteristic, okay. This is for a subsonic inlet.

In a similar fashion we saw, right so I am just recapitulating what we have done earlier. The similar fashion we saw that here towards the interface on the right hand side at the exit we have u and u+a, okay so remember these are not characteristics I am not drawing them in the xt plane, right I am only indicating the direction in which propagation is taking place okay so that you do not fall into the trap or drawing this and saying they are characteristics, right they are not characteristics.

And propagating in the opposite direction is the u-a. So Q3 hat is been propagated into the domain from the exit and Q1 hat and Q2 hat are apparently being propagated out of the domain. Okay. So at the interface I have, I am going to see Q1 hat, you understand what I am saying I am going to see Q3 hat coming one way and I am going to see Q1 hat and Q2 hat coming the other way the exit, that is basically what effects, standing at the interface and looking at, that is what I am going to see, the characteristic coordinates. Is that fine? Okay.

We just use these direction, we use these directions to rationalize sort of a hand waving waiver applying boundary conditions. We use these directions to basically say, why do not you extrapolate—see we get away that is why we use the word some information is being propagated right, I use that vague term because then it allows me to say extrapolate some quantity, right. So we extrapolate, we used that physical intuition and basically say we will extrapolate u, that is I propagate u from within the computational domain to the boundary.

So that I am able to get I am able to determine all the conditions, right at the boundary, I am able to determine q. In a similar fashion at this point you basically say there are 2 quantities that should go from the interior to the boundary, we extrapolate. **“Professor - student conversation starts”** What did I suggest? T0 and u is it? T0 and u, okay. **“Professor - student conversation ends”**.

So you can extrapolate T0 and u, all from the same point T0 and u from within the domain. Okay, and we justified it as I said in hand waiving fashion we justified it saying well there are 2 quantities that are going towards the boundary, let me specify this and if I am able to determine q it is fine. Okay and we will you can run this and see what basically happens, you can use FTCS add that artificial dissipation that I was telling you about. I will tell you way to get around but, right now you can do and everything is fine. Is that fine?

So there are 2 questions that pop-up. Clearly the boundary conditions that we are generating a not in this case they are not Q1 hat and Q2 hat, clearly. The second thing is we are extrapolating and we are using first order extrapolation, this is the question that some students asked me after the class last time. We are extrapolating and if you think about it think back when I said approximating things in the beginning of the semester in the beginning of this course.

I said that if you have f(x) and if you approximate f(x)+delta x as =f(x) then that is the first order approximation. The term that you dropped off is delta x times f prime, okay. So just to copy values when I said extrapolate just copy values from one-point inward, this is the first order. So does it make, is it a good thing, after all we are using FTCS is the second order, is it a good thing to be doing, this first order? Okay, right.

So we need to think about that a little because after all what differentiate one problem from another problem, there are Euler equations, they are all in the pipe, in our case in this particular case, the only thing that differentiate one problem from another problem, right could be either the length of the pipe one possibility we have that degree of freedom or it is a boundary conditions.

You notice even the length of the pipe basically shifts the boundary conditions that is what it does. So primarily it boils down to right boundary conditions are at the edge of their length where and what, okay. So if you do, if you approximate your boundary conditions using lower order approximation then it is likely that, that will determine what is the order of your scheme. Is that fine?

However, remember the sigma=1.1 demo that I did for wave equation, it started to diverge but then what happened all of it just went out of the domain. So in this case these are propagating to the right, so if you do a first order extrapolation here whatever error that you make is likely to just go out. Seems to be seems to be possible, seems to be possible. Okay. So downstream boundary conditions it may not matter, upstream boundary condition it may matter. Okay.

But, the fact is if you look at this you can translate this when you say you extrapolate that is f(x) +delta x=f(x). This is like applying what is the mathematical boundary conditions we are applying what is the equivalent mathematical boundary conditions we are applying, that is first order accurate representation dou f dou x=0, right. We are basically saying dou f dou x at the boundary is 0. Is that fine?

You can use 3 points and set dou f dou x=0. You can use, there is a 3-point representation for dou f dou x, you can use 3 points and set dou f dou x=0 then you you get second order extrapolation, am I making sense, that will use 2 interior grid points. It will use 2 interior grid points to determine the third point that is at the boundary. Is that clear? Okay, so that is the one issue I wanted to clarify.

So remember whether we are talking about solution Laplace's equation whether we are talking about solution any equation, right just giving the equation which is what I did here, I start that gave you the equation, did the analysis and I really only provided the boundary conditions when we started to solve the problem, right and in fact we even talked about what was the problem but in a sense that is the point, boundary conditions are what are going to determine what is happening, okay they determine the problem.

Once you have pick the equation, the equation is known, right one differentiate one flow problem, Euler equation all those constraints, from the other is the fact that changes the boundary condition, okay so the all Ls being fixed, the application of boundary condition completely should determine the solution. But then of course there are deal, there are issues of upstream, downstream and so on. Try out second order extrapolation, fine. Is that okay?

If the exit were a subsonic exit all 3 characteristics will be propagating outward, I have to use just first order, okay I have to just use first order, these boundary conditions they are called Shift boundary conditions.

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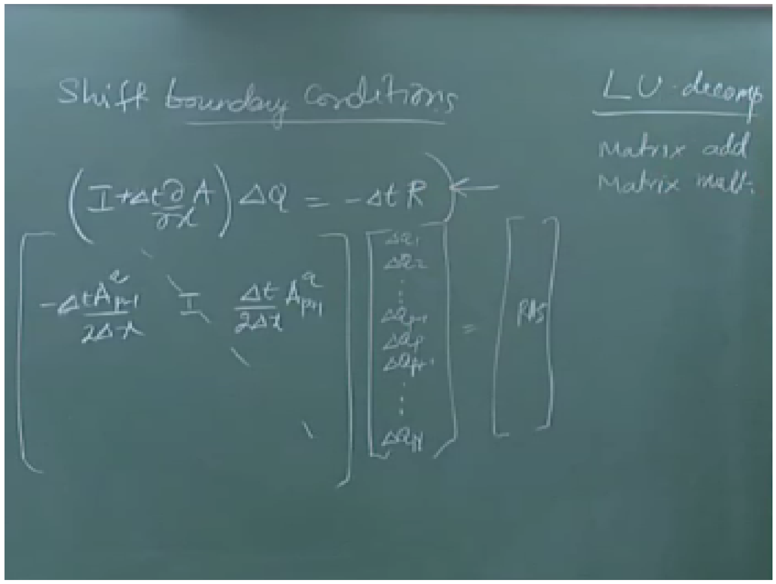

Shift boundary conditions. I would just use first order. You understand what I am saying? It is supersonic it is all flowing out, nothing is coming back, I just use first order. I am not going to put in the effort to do second order. Is that fine? Okay. Right, so little awareness can help you out. Okay, so that is as far as boundary conditions go, the approximate hand waving boundary conditions. Now they come to the point where I promised you that we will do it more precisely.

And I would prefer to do it using BTCS it because we know that BTCS that matrix on the left hand side if you replace by the identity matrix you will get FTCS, so we will do it with BTCS in general setting and look at how to apply boundary conditions exactly, right exactly more precisely more exactly, okay. Fine.

“Professor to student conversation starts” So what was the—remember you remember the delta form? “Professor to student conversation starts”.

Delta form basically is I +dou/dou x A, I always forget this delta t, delta t, acting on delta Q is – delta t*R. Okay. We can use central differences to discretize this and here and here at this point I want to make a remark because last time I sort of made a remark and let it go I want to make a remark. If I use central differences and discretize this a typical term will look like Ap-1 q delta – delta t – 2 delta x, I am sort of doing it in my head so it is great.

And there is an I and there is a delta t/2 delta x Ap+1 q, I do not remember the last time where I used superscript or subscript that does not matter. So this is the typical matrix. There are I's along the diagonal, there is the –delta t whatever in the sub-diagonal and this matrix acts on delta Q1 or delta Q0 depending on where you start counting delta Q2, delta Q t-1, delta Qp, delta Qp+1, delta Qn. Am I making a sense, okay?

And this is delta t times at right hand side so I will just say right hand side because that is the little more, we will see that the right hand side it will become little more involved because you have to incorporate the boundary conditions and so on, right. I sort of casually made a remark but – I was hoping that you would see this but just in case I wanted to make sure that I pointed out that to you. If you use Gaussian elimination, right these are all blocks.

What I mentioned at that time is you have to do LUD composition and so on. But remember in this particular equation the diagonal elements the pivot elements are all identity matrix. So Gaussian elimination for this is relatively easy, you can actually if you actually try it out you will see it is relatively easy. Okay as is Gauss–Seidel because the identity matrix is on the diagonal. However, in those list of functions that I suggested that you implement right upfront--

I would suggest that you implement LUD composition for a 3/3 matrix. Am I making sense? 3 implement LUD composition for a 3/3 matrix, if you want metrics addition and multiplication, matrix multiplication. Am I making sense? Okay. I would suggest, I would suggest that you implement this I would suggest that you implement this beforehand because if you do change something here the useful functions to have. Okay.

So in your base that I was telling you earlier thing that you know implement how to allocate q, how to allocate E, A all of that kind of stuff you add few of these functions the matrix operation functions, right. If you are going to do it for 2-dimensions all those metrics would be 4/4; if you are doing it for 3-dimensions all those metrics will be 5/5 and you would have corresponding right, corresponding vectors these are become 5/5 blocks in 3-dimensions and so. Is that fine? Is that okay? Right.

So now let us look at how we do, we will use this equation okay. So I just wanted—this was the second item I wanted to point out and the need to implement this anyway despite the fact that you may not find the need for them right now. Okay. Now the second item that I have is well that is done, what I want to do now is I want to use this equation to apply boundary conditions. Okay.

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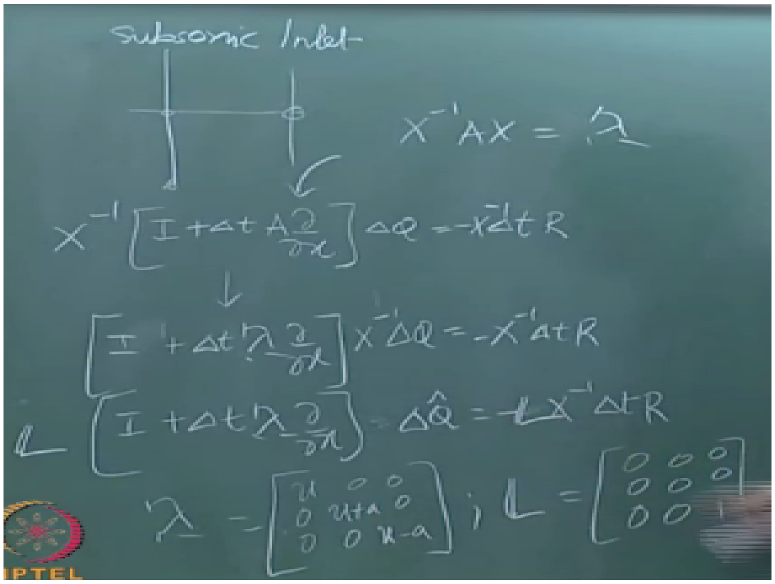

So if I look at the left boundary and I look at a subsonic inlet, so these are the subsonic inlets and subsonic exists are the interesting one, inlet, I look at the subsonic inlet, so this is the interface at the inlet. So at this point, at this grid point I have one I have this equation I coming in from the left hand side + delta t, now what I am going to do is over this small interval that I have between these few grid points.

To keep life easy, I am going to assuming A as the constant. A is not going to be within the operator, okay I am going to assume A as a constant. We will take the A out, dou/dou x. Why do I want to take the A out? Because I want to multiply by X inverse, right if I am going to do it exactly then I have to make sure Q1 hat and Q2 hat are what are been propagated or Q3 hat is been extrapolated. Am I making sense? So to that extent I want to take the A out.

So this operator acting on delta Q=-delta t*R. So remember that I have done this, I have take that A out. Okay. A is constant but only in that neighborhood. So then I will pre-multiply by X inverse on both sides; I pre-multiply by X inverse on both sides. X remember is the inverse of the matrix of Eigenvectors. What does this give me? This will give me an X inverse + delta t, the matrix lambda dou/dou x acting on X inverse delta Q.

A is constant so X inverse is also going to be constant. That was important assumption so I could take X inverse into the derivative, you understand what I am saying, this is an operator so by doing this I have taken into the derivative = -X inverse delta t times R. I do not remember last time whether I retain the delta t or not it does not matter. Is that fine? X inverse times I is X inverse. Is that fine? Now I see a few physical questions.

So here I have multiplied X inverse then in here I put X inverse. You understand what I am saying. So X inverse AX gives me the lambda matrix that leaves the X inverse which I have taken out that is basically what I have talked.

"Professor - student conversation starts" Yeah then there should be I+A if I am going to factor it out I should have an I+A., great. Thank you very much, yeah. Yes, fine. Because I factored out X inverse from the right hand side which is a rather need that right, multiplied if from the left. Okay. **"Professor - student conversation ends."**

So what does this give me? Of course this gives me familiar form lambda dou/dou x acting on delta Q hat, oops acting on delta Q hat is –X inverse delta t times R. And this lambda has – see now I have got in characteristic form. And this lambda has, what is this lambda, so depending on how you written the vectors right, depending on how you have written the vectors, it will turn out to be that, okay.

Of which from this equation I want only one, I want only a third equation, I do not want the first 2 equations the first equations are coming from outside the boundary. I do not want, I do not want the third equation I mean want to the third equation, I do not want the first 2 equations, so in order to do that I define a what should I call it the matrix, define a matrix, okay, I will call it L let me written on the left hand side or you can imagine whatever.

I will define a matrix L which is mostly 0s. Fine. Okay. And I will pre-multiply this by this L, if I pre-multiply this by L and I pre-multiply this by L, now this has only a third equation. Am I making sense? Everyone, it is fine? Now we have to go to P0 and T0. Is that okay? Now we have to go to P0 and T0. We have P0 and T0 here coming here. We have to go P0 and T0 and see what we can do.

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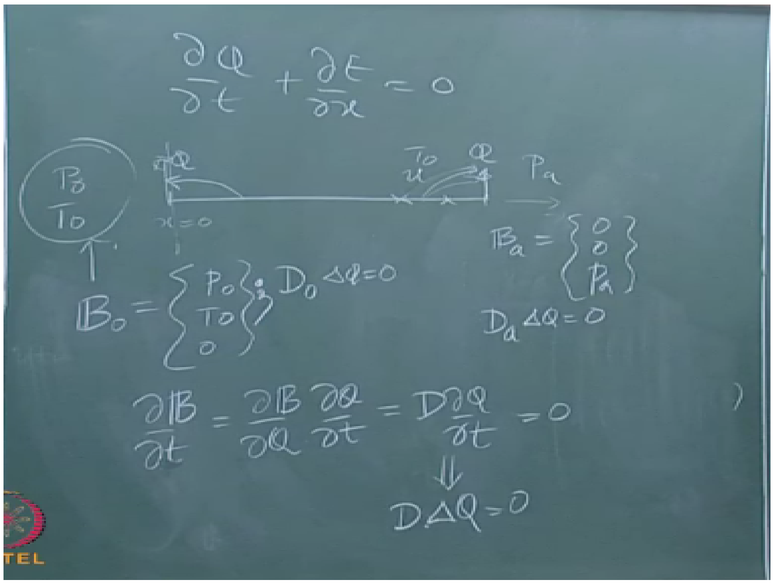

So these boundary conditions, right I want to put it in the same delta form. Okay. My objective I will tell you what I am trying to do now. What I am trying to do now just take this P0 and T0 and integrate that into system of equations. So I have a matrix equation, I want my boundary condition to be one of the equations in that system. So that I do not have to worry, you know I do not have to deal with as – I mean of course I am dealing with it is a separate thing.

But there was a point at which the all essentially look the same and I have incoming outgoing they are all in the form delta form in the form of a delta Q. Okay. So first we have to make up a vector because it has to be block, 3/3 block everything has to be 3/3 block. So I will make up a vector here, I will call it B sub-zero because these are all something sub-zero. Okay. P0, T0 0. See now I have a straight vector like thing for the boundary.

And I will similarly call this B sub a and create a B sub a which is you can think about whether it matters where I put this and consider at a later time whether it is matters where I put this, but I will stick it on the third. Okay. Fine. See B could change with time, actually you know that you know the reservoir pressure, if instead we having a huge reservoir, if I have a bottle of air and it start bleeding air, P0 will drop, you know from Gas dynamics.

Because expansion is occurring even the T0 will drop the temperature, the bottle temperature if you hold the bottle you will find that the temperature is actually dropped—P0 and T0 is a function of, right are a function of time. Normally we assume the reservoir is very large. But it is possible that this drop okay, bear in mind that I still want to look at look for only the steady state solution, I am only looking for the steady stated solution but still there is scope open there varying in time.

Let me look at what dou B dou T is. I have not given it a subscript I just give dou B dou T boundary condition using chain rule. This is dou B, dou Q, dou Q, dou t. Right. Why did I do this now? Because I have a Jacobian, I have a dou B dou Q, you understand what I am saying? So I can actually write this as some D times dou Q dou T. Is that fine? Okay. And if this happens to be 0 there are lots of way by which we can do it. Here I just decide to use this.

This happens to be 0, dou B dou T happens to be 0. See otherwise you discretize this, Am I making sense? You discretize this using forward time or backward time or whatever it is, but dou B, dou T we are looking for the steady state we assume P0 and T0 are constant, if that is 0 then in delta form this will come become D delta Q=0. Is that fine? Everyone? Is that fine?

So on the left hand side, the equation that I have in the left hand side where I have this subscript 0 I would have D naught, put a: delta Q=0 and on the right hand side I will have Da delta Q=0. Is that fine? Now my boundary conditions are in delta form, very similar to the rest of the equations. Fine. What do I need to do, tell me?

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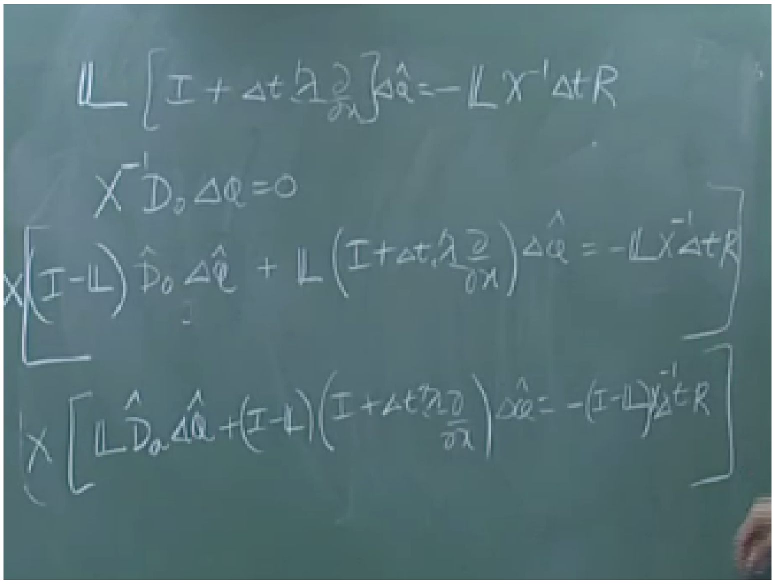

On the one hand I have L times I+delta t lambda dou/dou x is –L times X inverse delta t*R I wish I could have observed that delta t in that R but anyway it does not matter. And at the other hand we have left hand side, this is at the left hand side remember we did this for that left hand side. D naught delta Q=0. But this D naught delta Q, what was the mistake we were making earlier when we were prescribing P0 and T0?

We do not know whether we were propagating we were prescribing only Q1 hat and Q2 hat. We were most probably prescribing conditions in Q1 hat, Q2 hat and Q3 hat. Am I making sense? Okay. See this is equivalent, I will give you an equivalent example. If this is like, if you think back your engineering mechanics you have a simply supported beam, right, there are boundary condition, in fact if this were on rollers you definitely know that there are boundary conditions here that basically only known.

This does not generate a boundary condition that is tangential. Am I making sense? So if you prescribe a tangential boundary conditions there is an issue then, all that if you-- and if you prescribe a force this way of course it will be taken by the pin joint, right but the boundary condition itself cannot have a tangential component. Am I making sense? So if you are solving this problem somebody, you know then you throw away the tangential condition, it does not there, it does not exist, it is not relevant to the problem.

Just because right you say why do not you consider a vector like this, you say, no that does not work so I am going to take only the vertical component. In the similar fashion this has all 3, P0 and T0 have Q1 hat, Q2 hat Q3 hat but we will take only the Q1 hat and Q2 hat component, we will throw away the Q3 hat you understand, it is basically analogs. So how do I get that? How do I get the first 2?

I have well which picks the last one so I-L will pick the first 2, okay. So I can make I-L times that, oops I made a mistake. What is the mistake? First I have to get it into characteristic form. So pre-multiply by X inverse. I have to project that onto the right coordinates system, that is in this case if I have a coordinate system that is like this which is most probably why you have got, so I have to first projected at the right coordinate system.

I need to get the coordinate system that is aligned along the direction which my forces are acting, Am I making sense? So I have some Hot-ball coordinate system, I need to project that on the right coordinate system which is what I am doing here. Okay. So you do this, this equation then becomes D naught hat delta Q hat=0.

“Professor - student conversation starts” Delta Q hat. Oh, I am really making a lot of mistakes, okay. Thank you. **“Professor to student conversation ends”.** And pre-multiply this by I-L. Okay. And for the very first grid point, for the very first grid point I combine this equation and that equation. And again now I have 3 equations everything works. Am I making sense, I combine this with that. Everybody fine with that? Combine this with that. Fine into one equation I can just add them up. I can just add them up. So what will I get if I add them up?

I will get -- I do not know whether to factor out D I-L or not or I just add them up I can say this and we just add + L times I+delta t lambda dou/dou x delta Q hat = -LX inverse delta t*R. To the right hand side, I have only added 0 from the boundary condition, I have not added anything, because this right hand side is 0, I have not touch my right hand side. So a right hand side will only have that, only have that third equation, form third equation.

What now? You have 2 possibilities. Sometimes I just solve it in the characteristic form, sometime I just, I will be honest with you I just solve it in the characteristic form. Mean I talk about integrating it. In CFD we very rarely assemble, CFD we very rarely assemble that matrix, the big matrix that I was talking about. If you talk to your friends who are doing finite element methods and structures and so on, they will talk about assembling the global stiffness matrix, actual assemble the matrix, put it put it together.

We very rarely assemble the metric; we try to avoid assembling the matrix. We may compute the Jacobian as required, right depending on the circumstances when I compute the Jacobian as required. Or if you feel I mean if you turn out that it works you may assemble not assemble but you may store all the Jacobian. But typically will compute the Jacobian, we very rarely actually assemble that big matrix. And if you do you will only store the diagonals and so on. So you keep track of what is happening. Is that fine? Okay.

Now in this case if you wanted to—but if you want in your mind to have it in terms of Q which you may do then you pre-multiply this whole thing again by—how do I take it back to the original coordinate system so you rotated it to one way now you have to rotate it back. Okay, you pre-multiply the whole thing by an X. And you can actually you can actually analytically workout this equation. You can actually analytically workout this equation right, okay.

Maybe at the end of the semester if you have time I will give it to you, but you should do it. I know you should be able to do it. I will give it as an exercise. First of all, try to make sure you area able to find D naught and so on. Is it obvious that it would be something similar on the right hand side? P ambient, what would happen for P ambient? Can you guess? Can we just look at this and write out what happen for P ambient?

So instead of – we have only one condition coming, so on the right hand side it would be L times Da at delta Q hat. I am just looking at the chalk dust and doing a pattern match, you should just check to make sure that there are no issues. Okay + I-L times the equation, because there are 2 of them propagating to left, see from the left to the right. You just have to look at the interface what is coming from the left as an I-L on it what is coming from the right as a L. Is that fine? Okay.

I+delta t lambda dou/dou x delta Q hat is I-L delta t times R close the big bracket. Is that fine? Okay, X inverse. X inverse yes. Yeah, this is what happens I am very product that I remember the I-L, I forgot the X inverse, okay that is fine. Great, we have to be careful. Algebraic manipulation calculation you have to bit careful. It is very easy to get excited about something that you have done and make a mistake elsewhere, fine okay, we are back. What now?

So remember, this is the equation that you will use. So I will erase the top one now and I will write.

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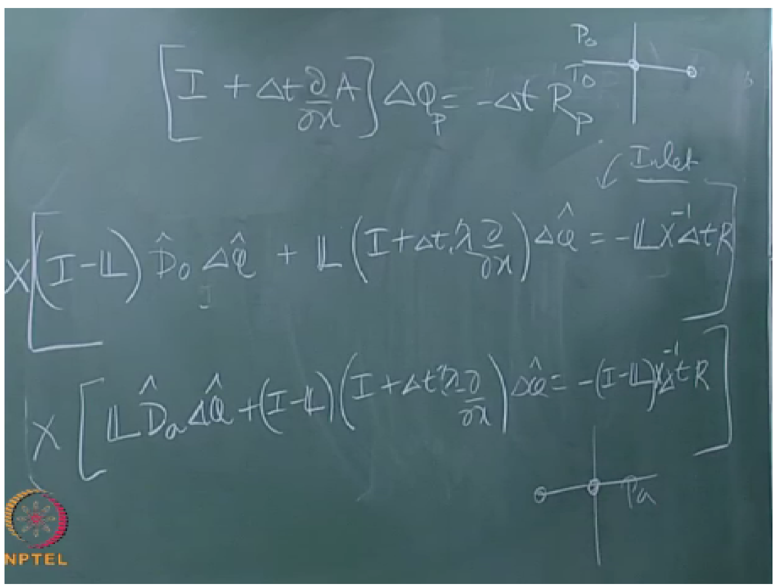

This is the equation that you will use at any arbitrary interior point, right at time level Q. So this is at point p, this is also at point p at some interior point. This is the equation that you will use at the inlet and only at the grid point only at inlet grid point. Am I making sense? Only at the inlet grid point that is this is this is the inlet at this grid point this is the outside this is where P0, T0 is coming from. This is the first interior grid point. This is applied only at that point.

And this equation is applied only at the exit point. Fine. Okay, so only at the exit point. So we have P ambient here at this grid point and you use the interior grid point. So obviously when you discretize the derivative using the interior grid point you are going to use backward differences here and you are going to use forwarded difference here. So the propagation will be in the right direction. Is that fine?

Okay so this sort of feel better than that oh, let us extrapolate you all of that changes but it is obviously a lot of work. It feels intellectually a lot better, but it is a lot of work. In One-dimensions, it is seemed relatively easy. These ideas right, see I am doing 1D, the One-dimensional flow the stability analysis that we have done, okay I just want to make, I want to make an observation here.

The stability analysis that we have done in this course extend to 3-dimensions and 2-dimensions that you can sort of squint at the equation and say that yeah this is the stability analysis. Am I making sense? Because we know now what do we know typically it is going to be like—see we need to know what is the CFL, right and that will be like propagation speeds some propagation speed.

So you have to figure out what is the propagation speed times delta t/by delta x, well it is not delta x there are 3-dimensions figure someway to come up with the length scale, right. Propagation speed delta t/length scale, that extends to multiple dimensions. I am rationalizing why I am restricting myself to 1D here I am justifying it. These boundary conditions, well when you come to the boundary when you actually come to a 3D manifold you come to the boundary will basically at that boundary you either have characteristics that are propagating information out of the domain or into the domain.

So there you can sort of locally do a 1D, you understand what I am saying 1D local coordinates 1D, and you can actually do something like this. Okay. Or you can cheat the way I have told you earlier extrapolate the appropriate your extrapolate the appropriate T0, it depends it is not that easy but right; multiple dimensions there are other issues. Is that fine? This is okay? Okay so now you have seen sort of the full spectrum of possible ways by which you can apply boundary conditions. Okay. And of course on top of all of the stuff you can now, this side how are you going to solve this, fine.

So instead of solving this equation using Gaussian elimination that the solving system of equation you can actually use say LU approximation factorization which I do not think I have done for block matrices. So you can actually do LU approximate factorization. There is a reason why I want to write this out. You can actually do LU approximation factorization, right.

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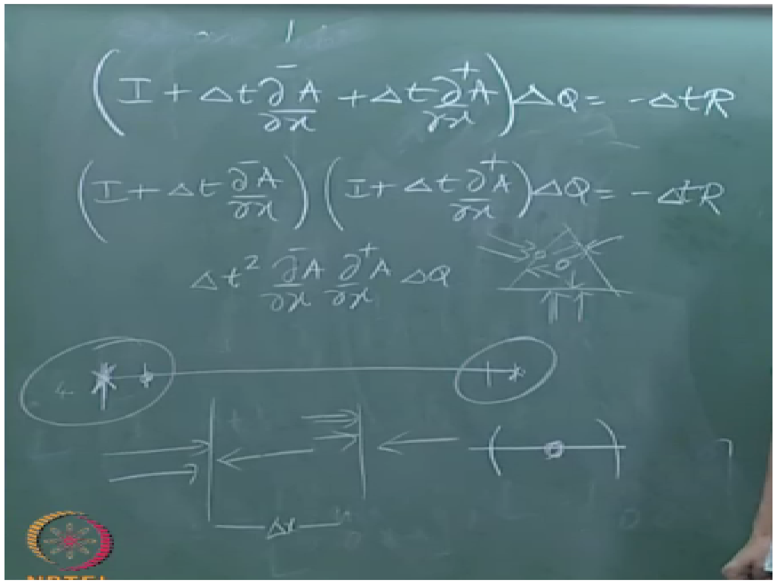

So the—if you remember LU approximation factorization we wrote it as dou/dou x – 2 operators, split it as 2 operators acting on delta Q is –delta t*R. And this can be factored, this can be factored as, this can be factored approximately as I+delta t A dou/dou x acting on delta Q=-delta t*R. Okay. So this derivative would involve the grid points P and P-1; this derivative would involve the grid points P+1 and P. Okay.

So you get a lower triangular matrix and an upper triangular matrix. And you can just again just do forward substitution back substitution, it is much faster than doing solving the system of equations. Okay. And the error is of the order of delta t squared. It is the product of what is the error term that we get the error we are making is delta t squared dou/dou x–A acting on dou/dou x+A, remember these are operators acting on delta Q.

That is the error that we are making on the left hand side. But if you are looking for the steady state this is going to go 0 anyway, delta Q is going to go 0 anyway, how do you care. Is that fine? Okay. Everyone. Right. Now here when you are doing this extrapolate—so you have to be a bit you can now decide that what which swipe you are going to do. When you are doing the extrapolation, so in one case in this case this involves what grid points? P and P-1, right.

So this will involve coming up, it will involve this point and this point. But if you actually try to if you were try to actually work, figure out what the value at that point, you do not have a P-1, you understand. This is not P-1. This involves a P and P+1. So this boundary condition here you are likely to apply it in the u swipe because it involves P and P+1 to determine the value at P. This is important. You understand what I am saying, this is important, right.

Because otherwise, LUD composition-- LU approximate factorization that is the thing that people always come backs how do where do I apply boundary conditions. We come to the other end so that will be applied in the L swipe, right because you are going to determine the value at P using the P-1 quantity. Am I making sense? Because you are using one sided derivative for the boundary condition, that is the critical part. Using one sided derivative for the boundary condition. Fine. Are there any questions?

So this is a relatively straightforward way to do it. Of course as I said when you get here, when you get there during the swipe you will obviously be using the equations that corresponds to the boundary condition and not corresponding to the equation itself. Okay. So we have now a method—and of course because we are saying that the R going to 0 tells us, determines when you have a solution.

R is something like a it is a predicate. You ask the equation; do I have the solution. You understand it, it is a discriminate. It tells us what is happening. Okay. So you want to calculate, you want to treat R carefully, R determine just like the boundary conditions determine the solution in that sense R determines when you add the solution, okay. This determines a correction on the left hand side. The left hand side determines the correction.

So the right hand side the right hand side—so in the sense you can look at this and say I can make the right hand side as accurate as I want because that determines the accuracy of the steady state. When the R goes to 0, if you are able to say R goes to 0 second order accurate then you have a second order accurate solution, right. Third order accurate, you have a third order accurate solution. The representation is that order.

On the left hand side you want acceleration, you wanted to go fast, right you can do lots of stuffs in order to get that quickly. Okay. However, I hope you are seeing this in your controls, you have to be a bit careful, right I am now talking about observations and corrections; think and controls lingo, now you have to bit careful. So if you have fine changes, fine changes that you detect which triggers your controller to make a correction.

But the correction is always very large then you will be hunting for the solution. If the delta Q is —this is really coarse, the difference between the order of this and that is so bad that small changes here create reasonably sizable delta Q. So this has to be well behaved. You have to be very careful with the choices. So as long as it is well-behaved it does not matter. You can make, you can make R right as accurate you want.

But you have to make sure that a small perturbation here does not create a large delta Q there and you can end up then hunting for the solution. Do you understand what I mean? The R basically triggers an idea, oh it has to be delta Q is a large is a positive quantity, you can get the largest delta Q which causes the corrections to be too large which causes R to say that feedback is slightly negative, then you get a largest delta Q because your delta Q size, coarseness is so coarse that your correction is slightly coarse.

Student will come back saying that oh it converges to 10 power -10 and then it seems to be oscillating a 10 power -10 it converges to 10 power –6 an it seems to be oscillating it 10 power -6. The corrections that you are generating were too large. Okay. And you are not able to pin on the solution you do not have that kind of resolution so it starts to hunt. Am I making sense? Is that okay?

So sometimes you have to be careful, you do not get carried away-- but typically, this is well-behaved—see I am being very vague now because I was equally vague when I say oh, you can equally make it anything you want. So as long it is well-behaved, right because now I am just waving my hand, I am saying R goes to 0 as long as this is not singular delta Q will go to 0. Okay. So R goes to 0, delta Q will go to 0, it does not matter with what we multiply delta Q.

And as long it is well-behaved you will get there. So I will leave it that in vague sense, fine. And something like this LU approximation factorization works. One last thing, if you wanted to add, if you wanted to add for whatever reasons you are getting oscillations and you attempted to add artificial dissipation, right. This is supposed worked this supposed be unconditional stable. You attempted to add, there are some oscillations that are not dying out, you are in a hurry and you will say I will just add artificial dissipation.

You can add it implicitly to the left hand side. If you can add it implicitly to the left hand side it is still the second derivative still gives you only a tri-diagonal matrix, second derivative still gives you only the tri-diagonal matrix. Am I making sense? The second derivative still gives you only a tri-diagonal matrix. Fine. So you do not have to add it explicitly you can add it implicitly. If you add, if you want to add forth order forth derivative term then that will change the bandwidth, that will change this.

It will not be in tri-diagonal anymore so I would suggest that you add that explicitly, it will contaminate your solution. But if you add the artificial dissipation on the left hand side especially second derivate it is not going to contaminate your solution because it multiplies a delta Q that is going to 0. Is that fine? So it gives –you know if you get tremendous sense of freedom feeling of freedom because there are lots of thing that you can feel around with knowing that the delta Q.

If as long as you do not mess up the fact that you are going to the solution that the delta Q is going to 0, so the amount of exploration that you can do is quite large. Okay. Okay, this is done. Now at closing remarks I want to say this will lead basically into our next class. What we are going to do is, you have a pipe that is of length L, you have a pipe at length L and we applied boundary conditions on the left hand side.

We applied boundary conditions on the left hand side using characteristics. And we used applied boundary conditions on the right hand side using characteristics. Instead of pipe being of length

L what if the pipe was a length delta x? You imagine that you have a pipe that is only have length x, so you have 2 boundaries. You have something going on inside, you want to find out what is going on inside and it looks like we have a mechanism now.

You understand, we have a mechanism now to determine what is happening at little volume of size delta x just by applying compatible boundary conditions the right boundary conditions, after all the boundary conditions came from the equations. Okay. So at some interior delta x I would basically say oh, there are characteristics coming from the left hand side at the interface, right. There are characteristics coming from the left hand side to this interface because it happens to be subsonic.

The characteristics coming from the, that propagating in that direction. This is delta x and you have 2 going this way and one going that way. So from within this little volume delta x the flux term here is determined by these 2 characteristics coming from within this volume and one characteristic coming from outside the volume from the neighboring volume. See, now you see scope for a scheme.

It is actually possible that we were talking about boundary conditions but it is actually possible that we can cook up the scheme using this because remember when we talked about finite volume method earlier what was it that I said? You have the state at this point and you do not have the flux, you do not have the state where you have the flux.

Now we are saying Ah, ah, wait a minute. I have the state in the interior and I have some way by which I can figure out what is propagating to the boundary. So it is must be possible for us to come up with scheme based on all of these add – see then you would be doing method of characteristics, right and the fact that we can do this kind of manipulation. Here you can go into the characteristics coordinates you can come out to the characteristics coordinates.

We have an easy way of getting into the characteristics coordinates and getting out of the characteristics coordinates. And after all we are only talking about an interface. So even in multiple dimensions I will just, though we are not going to look at multiple dimensions in this course even in multiple dimensions you have state in interior this is the volume I pick a triangle because it is not a quadrilateral you are not thinking Cartesian Coordinates, and you have the same things.

So you can figure out what is the, what is coming in, what is going out, right. What is coming in, what is going out; I do not know. Am I making sense? Right, I cleverly put to going out there because otherwise Q1 hat and Q2 hat will be accumulating inside, there is a problem if everything is coming in, fine. Is that okay? But we are not going to be looking at this. This is just, now I am dreaming just based on what we have done for boundary conditions.

And the little realization that oh, I can take a delta x and apply boundary conditions there, I am sort of in my mind jumping saying oh, I can do stuff like this, neat stuff like this, okay. Is that fine? So in the next class we will whether we can use these ideas of applying boundary conditions to actually come up with the class of schemes, fine. Okay. Thank you.