Introduction to Computational Fluid Dynamics
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Lecture - 26
Derive Eigenvectors, Writing programs

Good morning, so what we will do is, so we derived the one dimensional equations Euler equations and we have seen that it is possible to diagnose the jacob flux jacobian matrix and there by decoupling the system of equations fine that is where we are. I was sort of deliberately loose when I was talking about how we go about to dilate diagonalising the matrix today maybe we will actually derive it may get a little dreary.

But I want you to know that these are not things that you flip open books and just check in whatever is in the book you take it for granted you should you should be able to do these things and you should be able to cross check whether the book has the data put in or the expressions put in right or in case you go on for you encounter a problem later on in your career where the equation is not something that is sort of a standard text.

Everything you know the process you know how to go about derive it. So, what we will do today is we will actually look we will actually derive the Eigenvalues and show you get the matrix right. So, if you get a little tired please bear with me as I said these are these are things that one should do it the other way is it the easiest thing for me to this to come each time with a little cheat sheet saying that well these are the entries right I will do it you take it down.

But at some point you have to see that it can be done right so the flux jacobians itself I hope you have tried out evaluating the flux jacobians expressions it is a good idea if it is a good exercise you should actually do it and it is just calculus. But you should actually do it to make sure that everything is clear and that there are no issues okay. So, first let me get the what was that let me get the expression right the way I want to do it.

It is sort of set it up last class and then we will derive the Eigenvalues we have already derived the Eigenvalues I will derive the corresponding Eigenvectors I will leave one part to you okay right.

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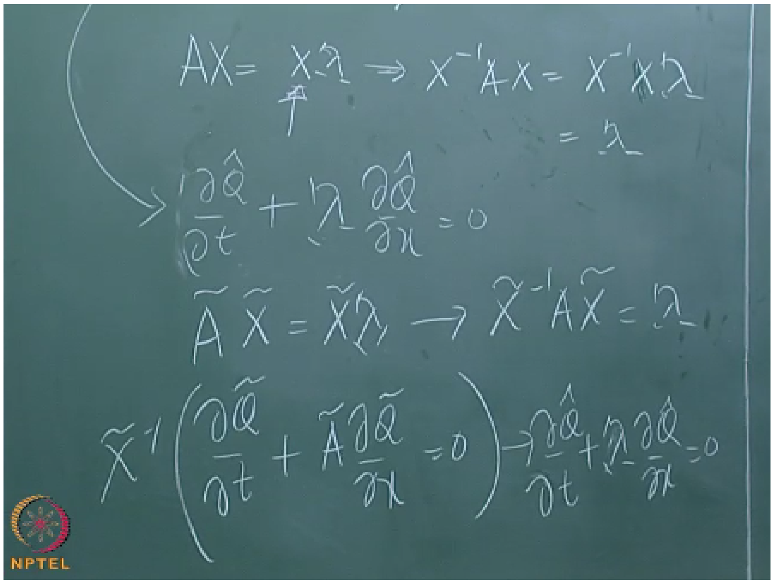

So, what did we said the general problem were looking at in the non-conservative form but in terms of the conservative variables this is the equation that we looked at and we asked ourselves the question when can I write how do I go about getting the diagonal form of this matrix a and you do what is known as the similarity transform. So, if you get the matrix of Eigenvectors and this is what I had written last night.

You get the matrix of Eigen vectors this is the matrix of Eigen vectors this quantity here is the matrix of Eigenvectors, lambda is the matrix of Eigenvalues along the diagonals. It is very clear that I am sorry it is very clear that if I pre multiply this by X inverse it is very clear that you will get X inverse AX=X inverse X lambda=lambda right because sometimes just like I did the FTCS first and it did not work.

You know that there are certain things if you look at the course the way we have done it so far just to give you an idea there is an underlying philosophy that I have here. So, initially we did finite differences representation and so on and then we just did Laplace equation no analysis we just jumped and it worked right but by the time we came to FTCS I of course deliberately chose FTCS because I know it would not work.

Right we did we did a little analysis now if you notice we have come to a point once we were talking about this 1d flow. I have not said anything about computing it we are only doing the analysis you should get used to this right so before you jump that is oh I have the equations questions let me start solving it you should sit down try to sort of digest what is happening what is the scheme, what are the features, what is likely to happen, what is likely behavior.

It is a good idea to do it right so you do not waste your time doing some kind of useless computation or whatever it limits the amount of time that you waste and you get a better understanding which is why we are going through this process as you will you will notice that as we have gone along we are spending more and more time before we get to computation right okay.

So, what does this mean that means is that in reality supposed to what I had written in the last class reality what I want to pre multiply this by X inverse the way I have written this. So, there has to be a connection between the 2 that is the point that I am going to make it is not enough just to put down the expressions. So, I really want to pre multiply this equation by X inverse right.

I want to pre multiply this equation by X inverse so that okay is is what then what I have written in the last class is it is just that rotational consistency is important okay. So, that you get dou q caret /dou t+ lambda dou q caret to X=0 correspondingly for A tilde there is an A tilde X tilde=X tilde lambda because a tilde and they are related through a similarity transformation lambda are but the Eigenvectors need not be the same that you will verify.

Right what I will do I will find the Eigenvectors okay for the matrix A that is what I will do today. I would suggest that you make sure that you are able to find the Eigenvectors for matrix. I will find out for A tilde you make sure you are able to do it for you okay right. I was exactly in the place that you are right now when I was taking this course and I did not make sure and I found out in my quiz that I had a difficulty.

So, I do not want you to be in that situation so I make sure that make sure that you are able to write it make sure that you are able to find the fine okay. So, what we have here so corresponding to this if I pre multiply my x tilde inverse I have X tilde A X tilde=lambda. Okay fine so what we will do is well find and of course just to just book completion so if I were to multiply.

I am sorry not this dou Q/dou +A tilde dou Q tilde dou X=0 1d Euler equation written in a non-conservative form in terms of non-conservative variables okay variables for rho u and p. If I were to multiply this by X tilde inverse I would again get dou Q/dou t which is the same equation+lambda dou Q caret/dou X=0 the same equation okay, we are transforming the dependent variable transforming the dependent variable.

Let us find so the Eigenvalues of A tilde or A.

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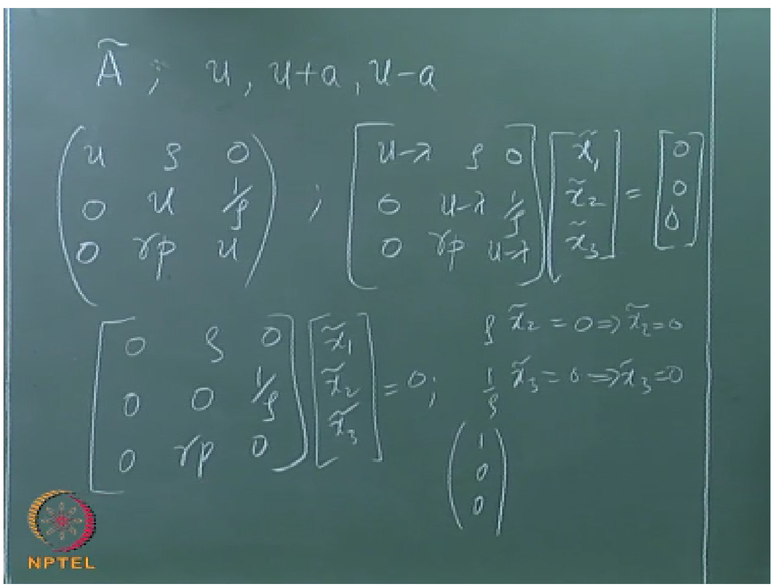

We will find the Eigenvectors A tilde as Eigen values I just put up you know A tilde has Eigenvalues u u+a and u-a and what was A tilde you remember it is relatively simple matrix you remember u rho 0 0 u1/rho 0 gamma p u actually this matrix I really do not have a sort of a difficulty remembering it because I know it is use along the diagonals and I remember the gamma p/rho that is here and there is a rho there you understand what I am saying right.

So, this matrix I did not have as much difficulty remembering okay fine that does not matter. So, now we need the what do we want we how do we get the Eigen vectors Eigen vectors satisfy the equations. So, let us find the Eigenvector corresponding to u okay. So, Eigenvectors satisfy the equation. So, I will have the characteristic equation is going to be u-lambda rho 0 0 u-lambda 1/rho 0 gamma p u-lambda acting on x1, x2, x3 right.

I should really have tildes here do not hold me to it if I forget the tildes but anyway this equals=0 vector right. So, if lambda is u this gives me 0 rho 0, 0 0 1/rho 0 gamma p 0 this is the system of equations on x1 tilde x2 tilde x3 tilde=0and the corresponding equations are if I just write it out what is the first equation rho times x2 tilde=0that is nice x2 tilde is 0 then x3 tilde is 0.

So, this is an easy one right x3 tilde is 0 so of course it gives you if you really take maybe x1 x2 x3 x1 0 0 or whatever. So, basically the first Eigenvector is 1 0 0 fine what about the next one.

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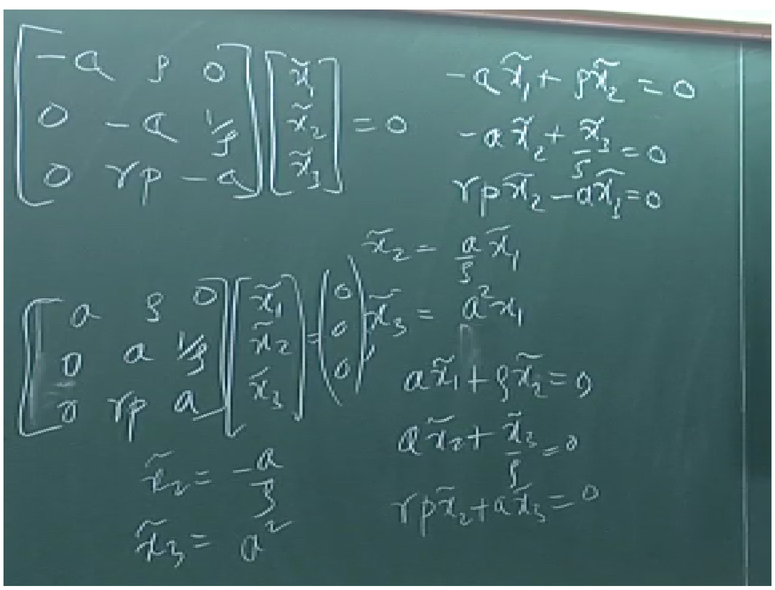

-c rho 0 0-c 1/rho 0 gamma p-c x1 tilde, x2 tilde, x3 tilde=0 what does this give you –cx1 **“Professor - student conversation starts”** sorry thank you very much yeah so it is very clear that sometimes I use c sometimes I use a thank you sometimes because I do not want confusion between this and the dou u dou t + dou u dou x sometimes switch to see if they mean it is okay it is like in my book.

In fact, I think I use c it does not matter as long as they are consistent notationally it is good thank you "**Professor - student conversation ends"**. But it is important right –a x1+rho x2=0-ax2+x3 tilde/rho=0 and the last one is gamma p x2-a x3 tilde=0 suggestions we can solve in terms of x1 right this is basically what you do so x2 a/rho x1 okay and x3 is a squared x1/ that right or just a squared a squared fine okay the second vector r therefore.

So, there are different ways to make it orthonormal you will see that the x1 will go away or you can set x1=1. There are some people who basically say why go through this headache make the first fact first element entry 1 anyway if there are different parts that you can follow I am not sure what you are familiar with okay so we will just right now say okay factor out x1 or whatever.

The second one is 1 a/rho a squared. Can you tell me what is the last one a rho 0 0 a 1/rho 0 gamma p a x1 tilde x2 tilde x3 tilde=000 right? All that is happened are some signs are flipped this corresponds to ax1 tilde + rho x2 tilde=0 ax2 tilde+x3 tilde/rho=0 and gamma p x2 tilde+a x3 tilde=0 again you can say x2 is= **"Professor - student conversation starts"**-a/rho and x3 tilde is a squared 1-a/rho a squared.

So, this this is x tilde it is x tilde okay instead of just saying x tilde is the matrix of whatever I wanted you to see that is x tilde what is x tilde inverse. I promised you would be a little really so just bear with me what is x tilde inverse the different ways to do it I prefer to solve a system of equations especially when it is 3/3 that way I know I am not going to make any mistakes. So, x tilde inverse "**Professor - student conversation ends"**.

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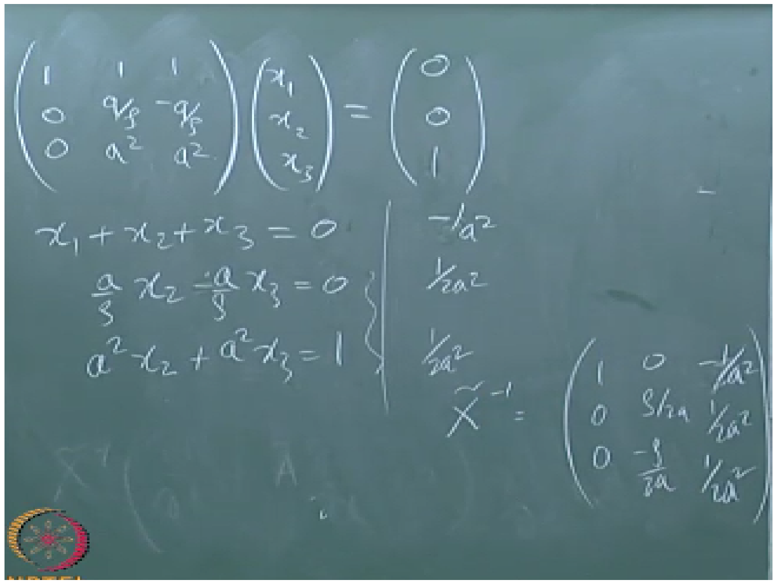

Basically I have what do I have 1 0 0 1 a/rho a squared 1-a/rho a squared right this is for the inverse now just for the inverse now I am going to just call them as x1 x2 x3 right x1 x2 x3. I want to find these entries effectively I am going to take columns from the identity matrix so this is going to be 1 0 0 fine. So, the first equation is x1+x2+x3=1nice second equation is a/rho x2-a/rho x3=0.

The third equation is a squared x2+a squared x3=0 fine what does this tell you x2and x1 of course will be 1 right so in the inverse in the inverse where shall I write this. I will write this somewhere here the inverse. That I am going to have 1 0 0 is one of the vectors okay what is the next one we find entries for the next one by making this 0 10 the second column of the identity matrix right.

In which case that becomes 0 that becomes 1 that becomes 0 right I will wait for your to catch up the easier for me to do this right so what does this tell me x1 is 0 x2+x3 is 0. You substitute it there x1=0 that is thing that we get here x2=x3 x2=rho/2a x3=-rho/2a is that right. So, then I have 0 rho/2a –rho/2a and the last one you will have to keep an eagle eye on that.

I may make mistake 0 1what is that going to give me 0 1/2 a squared 1/2 a squared is that fine because x2+x3 is 1/ x2+x3 x2+x3 is 1/a squared right. So. What about the others 1/2 a squared 1/2 squared what is this this is x tilde inverse okay? So, what we can do is we can use that so x1 has to have -1/a squared okay what about the others 1/2 a squared 1/2 a squared what is this this is x tilde inverse okay.

So, what we can do is we can use that you can verify right so even though all of this looks good you know you should always you should always check so once the first time I derived this I sat down I make sure X inverse was identity matrix. I make sure that x is I made a verified see there are remember this is a really critical part of any derivation that you do you make a sanity check on the results that you get.

That you make sure that these actually do give you the Eigen values and this is an interesting exercise because there should be a relationship between how we stack the Eigenvectors and all the Eigenvalues come okay. So, you should go through this process just to make sure that you understand what it is that what is actually happening okay.

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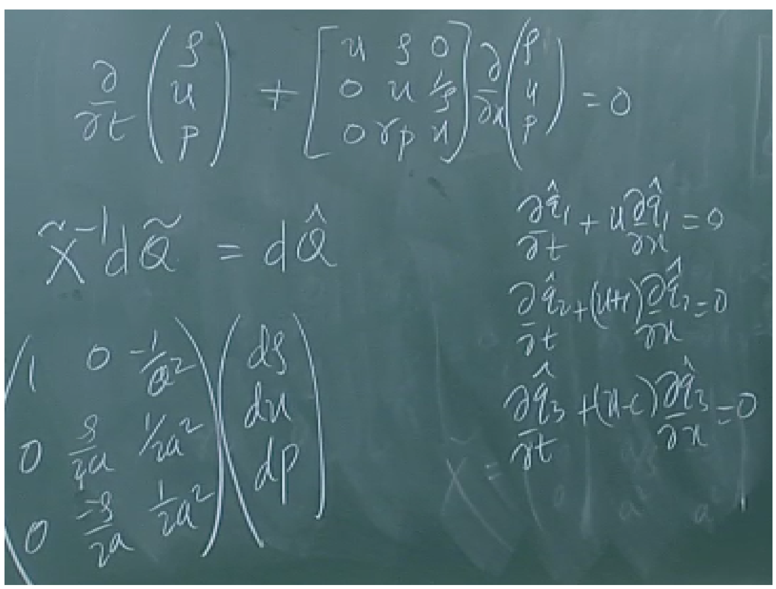

Because as I said it is easy to just manipulate the chalk thus saying that oh I multiplied by x inverse right and I should get lambda check it out that is matrix and there is a critical component you know there will be small connections that you will make it you will actually do this. But it is a good idea always anytime you do a derivation like this it is always a good idea before you proceed start coding.

And all of that stuff to make sure that what you have got is right it is supposed to be the inverse check that it is the inverse it is supposed to make those diagonal check that it makes the diagonal right okay just to be sure then you can go on fine okay. So, what was the deal what we basically got was what we basically got was if we multiply this if you were to multiply so the equation dou q tilde equation is dou/ dou t of rho u p+a tilde dou rho u p d/dx=.

So, if you were to pre multiply this whole equation by x inverse it is supposed to diagonalise right. So, if you were to pre multiply this equation by x inverse what you are going to get here what you are going to get here this is corresponds to if I want to do it in terms of differentials right what I am basically saying is dq tilde if I pre multiply by x inverse it is going to give me dQ caret okay.

And this equation should then reduce this equation should then reduce to if I were to do that this equation will then reduce to some form that is like I am going to do it there so I will write it here dou q1 hat dou t+u times u1 hat dou x=0 okay dou q2 hat dou t+u+c times dou q hat dou x=0and this is what I meant how do I know it is u+c because the second Eigen value I substituted to get the Eigenvector was u+c right so there will be a relationship between all of them.

In the last one will be dou q 3 hat dou t+u-c dou q3 hat doux=0 these are the 3 equations that we expect to get okay looking at this looking at this what is x x1 x inverse dQ so that is this matrix I have here 1 0 -1/a squared 0 rho/2a 1/2 squared 0-rho/2a 1/2 a squared acting on d rho du dt is that fine and what will that give me what will that give me.

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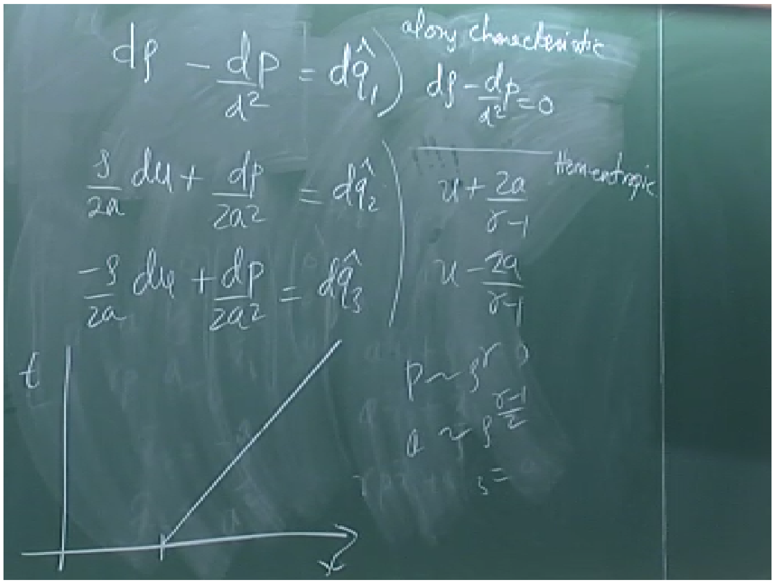

It says d rho–dp/a squared is dq1 hat did I get that right okay what is the second equation rho/2a du+dt*/2 a squared=dq2 hat and the last one –rho/2 a du+dp+2 a squared=dq3 hat fine and it is possible that we are able to it is possible that were able to integrate these equations and find q1 hat q2 hat what they correspond to of course what we know is along this look at this one equation right nothing back to the wave equation.

Just forget this equation what does this equation say about q1 hat. So, it says on the xt plane on the xt plane if you have at a point right if you have a value u you take a slope a line whose slope is 1/u basically right and along this line this characteristic q1 hat is a constant q1 hat is a constant because the right hand side happens to be 0 remember this characteristic equation is very strange if there was a source term than q1 had to actually change right.

You would have to integrate along that fine okay so q1 hat is 0 along this that means along this line dq1 is 0 along this line dq1 is 0 along this line dq1 is 0 along that line along the characteristic along and just say characteristic d rho-dp/a squared=0 right dp/d rho is a squared that is the definition dp/d rho=a squared when is the dp/d rho=a squared on a p rho curve on a p rho graph right.

If I draw a path and say I find dp/d rho on that curve there is a particular curve where it is equally a squared right that is on the isentrope okay right. So, this basically says that since dp/d rho is a squared this cells us that entropy is constant along that line dq is constant along the line it is almost as though the entropy is being propagated along that line so if you had different values of entropy at different points.

Along this line entropy is constant is that fine okay no integration of this I do not know whether you have seen all of this in gas dynamics you have you done this. So, you have done this in gas dynamics that is fine. So, you can assume the flows from entropic can so on and then there are ways by which you can integrate this right. So, I will just write the typical characteristics that you get are you remember u+ you can just check 2a/gamma -1 and u-2a/gamma-1.

You can just go back and check right I am just writing this from memory you can just go back and check right you may have seen this in your gas dynamics class right that comes from assuming it is home entropic if it is home entropic then p is rho power gamma and a is like rho power gamma-1/2 okay I am sort of doing the calculation in my head fine right square root of anything of that sort you can you can work it out.

You can actually go back I am sure you have done it in gas dynamics you can you can derive these expressions right but we have made an assumption beyond just I made an assumption that the flow is home entropic first form in topic right it is not just constant along this line it is constant everywhere it is a strong assumption right may be valid with a lot of time but it is a strong assumption from where we stand is that fine okay.

Normally as I had indicated in the in the in the last class normally if you say that you have differentials related in this fashion and you want to perform an integration and you want to find q caret right normally this works only if the curl of this vector column vectors is 0 okay so normally you would want call them x inverse to be 0 in order to be able to invert it fine right but that we would not go there.

So, if you were to encounter a general problem you may not always be even here I had to make assumptions with the flows home and it may not be always a you may get an integral that you are not able to evaluate that is one thing the other thing it may not be even be integrable right just because you have a velocity field it does not mean that the potential exists. So, there is a constraint on the velocity.

So, in a similar fashion it is possible that you can integrate it may not be integral is that fine okay right we have now all we have set up we are able to get this we know we can get the characteristic form if you were to solve the equations actually in characteristic form of course we do not know if you do not always know q1 hat q2 hat q3 hat we may not be able to find the flow may not be home entropic right.

So, we but if you were to do it in terms of q1 hat q2 hat q3 hat you would then be solving what we normally call the method of characteristics. We will be using the method of characteristics right that basically it is not always viable especially if you go to multiple dimensions. So, now let us see how you would actually solve this how would we actually solve this equation and what I am going to suggest is.

I am going to suggest that before you start coding maybe just a suggestion they could have made earlier so that you would have started doing this before you start coding there are certain functions that you should start implement before use before you start coding right. So, before I tell you how were going to go about solving this there are certain functions that you should implement these are base functions that you need.

Whatever solver you are writing right. So, the idea is that you write simple functions that will do one thing right and I will tell you each of those one thing what each function does and you test the function okay to make sure that it is doing what it is supposed to do right and once you have tested you can set it aside saying okay this works fine so if you have a collection of functions base functions that you require for most solvers.

And you have tested and when you are confident then it is a matter of implementing the solver of course there may be other headaches that come down but at least you are not worried about the base functions being right or wrong okay you get what I am saying right so that that ensures a certain degree of success right and guaranteed at some level certain degree of success so before I get to this all I would suggest you first implement and allocation.

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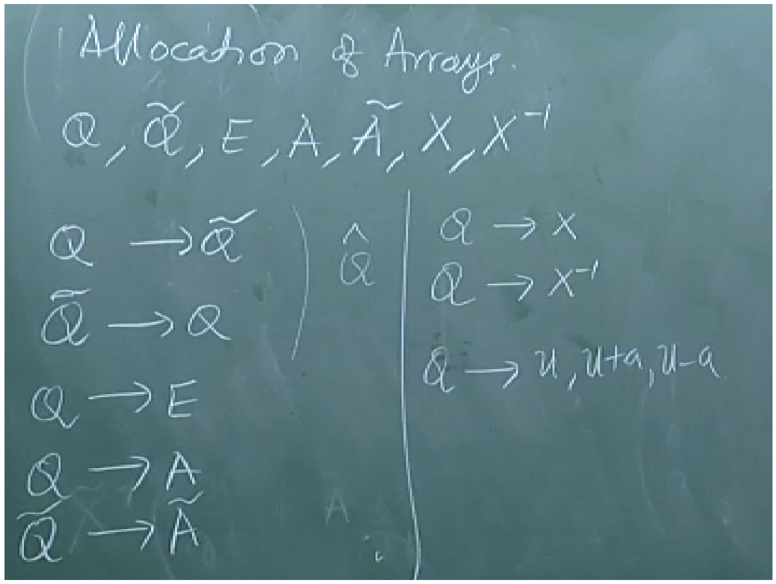

Allocation of arrays what do you mean by that depending on the language that you are using whether using Fortran, c, c++ whatever you make sure that you are able to allocate Q you are able to create an array of Q remember Q itself has 3 quantities it is not just a trivial thing. So, if you are done it you are into programming in a big time this may be nothing for you but I would still suggest that you do it.

Allocate Q allocate Q tilde allocate E these days we are not worried that much about memory. So, I will just tell you all the things for the one dimensional thing that you can allocate you can worry about memory and you know how much memory it takes when you get to multi-dimensional flows which is not part of this course anyways right A if you want A tilde X X inverse right very often what we do is be calculate them as required.

But first you allocate make sure that you are able to do this so that if at any point along the way it is not that you actually created but you have functions that will do this right. So, you have a function that allocates Q allocate Q right allocate Q tilde. So it will give you will have you will have something that will actually do it. I am not saying right they want big function that does all of these right.

I want functions that can that can create arrays of I am making sense okay fine. So, in Fortran for instance if you wanted 11 grid points and you wanted to allocate Q then it would be an array that has 11/3 right and see it really maybe depending on you may have a struct or in c++ you may have a class that has these 3 elements it has other behavior and then you would allocate 11 of them am I making sense.

That is how you would go by doing it and that would be Q alone and then you would do similar things for Q tilde also is that fine okay right. Now you write other functions given Q find Q tilde right because you have rho rho u rho et I mean very often we are not really interested in rho u and rho et we are interested in the speed we are interested in the pressure distribution right normally that but that is the kind of thing that you are looking at.

So, given Q finding Q tilde your input to the program may also be in terms of Q tilde but you may be solving it in terms of Q right. So, you have to write another function given Q tilde finds Q. If you are keen you can also find Q caret I will leave that up to you if you are if you are if you are if you are if you are if you are keen if you feel like you can implement what else given Q find E right so you have individual ones given Q find At you may see repetitive calculations and so on.

But you are in the process of developing code please do not take shortcuts please do not try to optimize your code right your objective always is to get every bit of your code functionally correct it functions properly then we can worry about the optimization okay find A tilde or if you want Q tilde find A tile either way I leave that up to you make a decision make a decision and what else you obviously have these given Q find X given Q find X inverse fine.

So, what are we where we are right now you are able to look at the memory you have individual functions that do that and given any of these quantities we are able to populate those arrays you understand what I am saying. So, as an initial condition if you are given Q tilde you can find the Q and from the Qs you can then find all of them you can populate all know your set right at this point to a set you have the array of fine okay.

You may find for instance if you are doing FTCS that you need 2 arrays of Q because you have a current time step and you have the next time step these things you can handle as you go along because it depends on whatever how we are going to what is the actual this is the base this you require a new 1 dimensional solver I would expect will requires any 1 dimensional solver I would expect will require.

You may there are auxiliary functions that we require okay now I just remembered because remember we had a stability condition stability condition required that we needed to find propagation speeds so maybe we need to find not just this but even the Eigenvalues. So. given the Q find u u+a u-a okay right good we are now set, given that we will do FTCS again it is not supposed to work but we will do FTCS again right okay.

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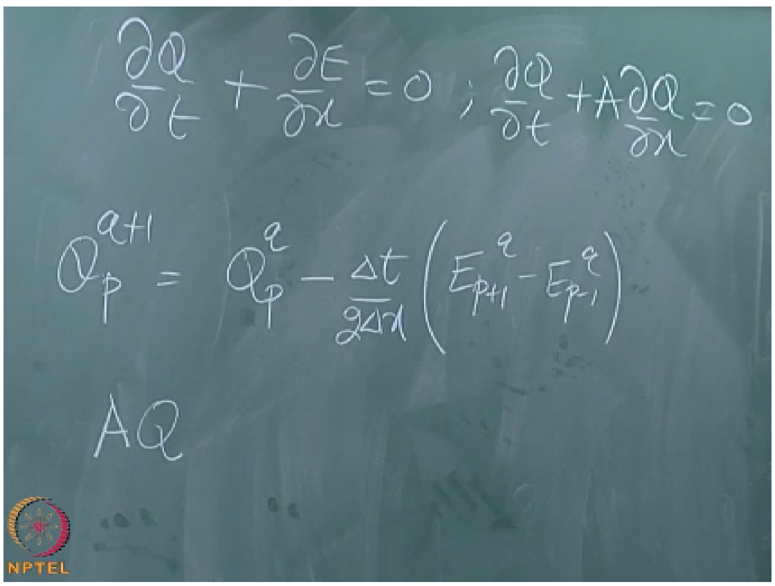

You Have dou Q dou t+ there is a reason why I am writing this array because dou E/dx=0. So, the first thing that I want you to notice is I am writing it in the conservative form the a douq dou x was for the analysis right I am not going to use the a douq dou x form. I could also write this as dou Q/dou t+a dou Q/dou x=0 there they are there mathematically identical right but this involves a lot of computation it involves finding a right.

Though I said you have all those functions you can choose not to allocate a you can choose not to find a okay this involves finding a this does not. So, I do not have to do this I am not going to do it we wanted a form because we had done dou u dou t+a dou u dou x=0and we said I have done all that stability analysis for it. I want to get it in that form that is why we struggle with this to get it in that form because that stability analysis all of that stuff will carry am I making sense.

So, what we have here how I am going to do FTCS here. So I say Qp time level Q+1=Q at p time level Q –delta t 2/ delta x Ep+1 Q-Ep-1Q fine is that clear. So, from the scheme that we have talked about you can allocate the Qs you know the Q at the current time given the Q you can find the Es you know the Es at the current time you can find this difference therefore you can find the Q at the new time step okay right we are set right.

Do you think this will be unstable is this going to be unstable for wave question was that it was unconditionally unstable you expect to be unconditionally unstable how can we find out is there a way for us to find out what do you say there must be some way for us to do the analysis. So, we can see whether whether you have a choice we can see whether we do the analysis or not I want you to verify something I want to go to okay.

So, will the analysis part itself it will turn out that yes this is indeed unstable okay so you we will show it okay but I want you to find I want you to evaluate I have we have derived I hope you have derived I have given you the flux jacobian I want you to evaluate what is a times Q right you have the vector Q you have vector a and Euler equations you know Euler there are lots of nice things that happened with Euler equations yeah a lot of beautiful symmetries.

So, I want you to I want you to just try this out to find out a times Q and see what you get okay right may be in the next class we will come back and see whether were able to do stability analysis with this you are of course in a position now to actually implement the stuff but as I said before you do this please please make sure that you implement all those functions and test each one of those functions okay.

Test each one of those functions so I would normally when they say test each one of those functions I would say rho as 1 u as 100 p as 10 power 5right then you know what should be the values of rho rho u rho E t so I would check the individual functions to make sure that they are giving you the okay so test test each one of them make sure they are functionally correct right.

Okay we are set if this were stable you just march and march in time right.

If this was stable were just march in time is there anything which we have to worry about end point boundary conditions end points boundary conditions. So, we have to worry about how we are going to play boundary conditions just like we did for FTCS in the wave equation cases you have to worry about what happens to the boundary conditions. So, before I start talking boundary conditions see it all of the same.

I managed to carry you guys late all this time without actually talking about a specific problem I have just been writing equations right at some point you have to say what is the problem that were going to solve. I even written a discretization but when we come to boundary conditions I have no choice. Now I have to come out and say this is a problem that were going to actually solve so the problem from gas dynamics.

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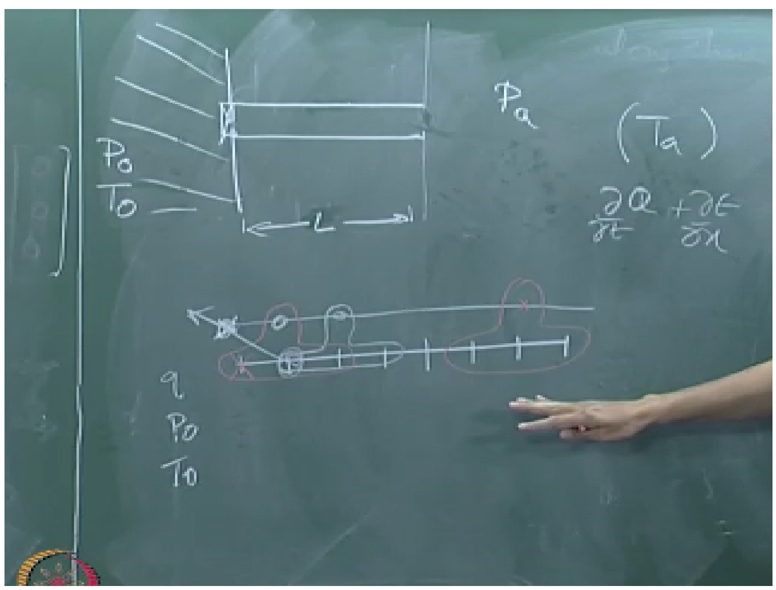

So, the typical gas dynamics problem would be 2 vertical lines indicating a huge enormous reservoir right enormous reservoir. We can decide where we want to put the valve when you put the valve here put a valve there and I sort of tied myself into a corner because I am taking one dimensional flow right. So, I cannot I do not have area variations right at the end of all this analysis will say okay we will let us do it let us look at the quasi 1a dimensional equations.

And they are not that different from here they are a little more interesting because you at least have area variations here you have no area variations at all right. So, the length of this is L take unit length if you want. So, in this reservoir I want to know I want you to think about the experiments if you have done an experiment in right gas dynamics lab or something of that sort. So, what are the conditions that you normally know in the reservoir?

You know the total rate you know the condition stagnation conditions that the total pressure and total temperature. So, you will know P subzero that is the total pressure and T subzero that is the total temperature anything else not much not really much else okay what about the right hand side what do you know you know the ambient pressure. So, you say I have a pressure vessel that is at 5 atmospheres or 3 atmosphere or 6 atmospheres or whatever.

There is a little pipe coming out of it and there is a value that is what I have drawn and if you open the valve the gas from that chamber is going to exhaust through that pipe and come into the atmosphere and the conditions here are P ambient per P atmosphere. Can you prescribe anything else you could in theory measure the could in theory measure the temperature in the lab right in theory you put measure.

I do not know from your gas dynamics you should realize that this is basically all that you will have that you could measure the T ambient what else can you measure that is it we are stuck. Now you just have to open the valve and let the let it run. So, you open the valve and the gas starts to pour out there is an initial transient and then it settles down fine and there is a flow field that is set up.

We are interested in that steady state flow field right now we are not going to look at the transient we are interested in steady state flow what does that steady state flow field that we get okay so if this is length L I break it down using numerous grid points right. So, if I am going to use this this is a time level Q. So, of course time level Q+1 I have an equivalent so I can at time level Q+1 I have equivalent points.

And we know that the FTCS stencil looks like that those are the points that are involved okay in more ok the only difficulty is that when you come to this first interior point you end up having to use you end up having to use a boundary condition that is a problem okay that is a problem and then there is the issue of what is the boundary condition that I am going to have here. So, what you have given me is only P0 and T0.

You have given me only 2 quantities okay how many conditions do we require from a differential equation point of view okay from the physics point of view I have P0 T0 and P ambient the physics point of view we are happy right okay and you understand that whether T ambient is at 300 kelvin rate or 320 kelvin or whatever T ambient as long as the pressure is the same.

You understand the gas dynamics is not going to change it does not depend on T we already know that it does not depend on T ambient. So, we already know that from the physics point of view we need P0 T0 and P ambient that is what we need that is what the physics tells us. What does the differential equation tell us differential equation has dou/dou t there is a Q. You need 3 initial conditions and dou E/ dou x a spatial derivative 3 conditions in space.

Well we have 2 here and 1 there so we have the 3 conditions the physics in the mathematics are jellick right, So there is no issue here the problem that we have when you do FTCS is in order to take this time set I need Q there not 3 condition 2 conditions here on 1 condition I need Q I want 3 conditions there and in a similar fashion when I come here I want to conditions 3 here. So my CFD because I choose FTCS here requires 3 there and 3 here.

What do I have? I have 2 here and 1 here so I have to somehow generate it I have to generate 3 here and 1 there is that fine 2 here 1 there we have to cut that out okay fine 2 here and 1 there is that fine okay right okay. So, now we have basically said okay we have to justify it starting to wave hands a little more we have to justify how this is going to happen okay so I ask myself the question what happens if P ambient is same as P0.

Let us assume that instead of being instead of there is some magical way by which I could fiddle around with this downstream they could be a pressure vessel here and I could figure out whatever P ambient I wanted. So, if P ambient goes the same as P0 you would have no flow you know the valve is open if I lower P ambient right I have a pump that starting to evacuate I lower P ambient what is going to happen?

I will set up a flow field the rho u m. will increase right gas dynamics we are thinking m. m. will increase if I lower it even further the m. will go down even further. So, this P ambient what I am able to control using P ambient is m. so if I lower the P ambient here the rho u entering here is affected by it am I making senses okay so it looks like somehow from here what needs to there is something propagating upstream.

I have to somehow take from within the interior I have to take this mass flow rate the P ambient is dropped more mass flow rate it has to be it has to be communicated upstream it is happening when you lower P ambient you know the actual physical fluid flow situation when I lower P ambient you know that the mass flow rate increases upstream. So, the fact that this P ambient is dropped right either somehow the P ambient itself is propagated upstream.

That is one possibility or the u or rho u increase in rho u the need for an increase in rho u is propagated upstream. So, in my mind if I do an experiment I say if lower the P ambient here well the immediate all of this if it is at P P0 right then the air here will get sucked out and it will propagate the wave will propagate upstream expansion fan or whatever it will propagate upstream right.

That is what I expect and propagate propagate upstream look at this propagate upstream all the way to the top all the way to the front fine. So, maybe what we will do is we will have to look at exactly explicitly how we are going to apply these boundary conditions you have the problem set up you know what is the problem we are solving you know what is the scheme that we are using we have 2 issues that I have left unanswered in this class one is that scheme stable.

If it is not stable what should we do the second is how do, we apply the how exactly shall we apply the boundary conditions we know there is a shortfall we need 2 quantities on the right and 1 quantity on the left. How do we generate is that fine? We will do that in the next class right thank you.