Introduction to Computational Fluid Dynamics
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Lecture - 24
Shock speed, stability analysis, Derive Governing equations

So that is a shock. They pick 2 points on it.

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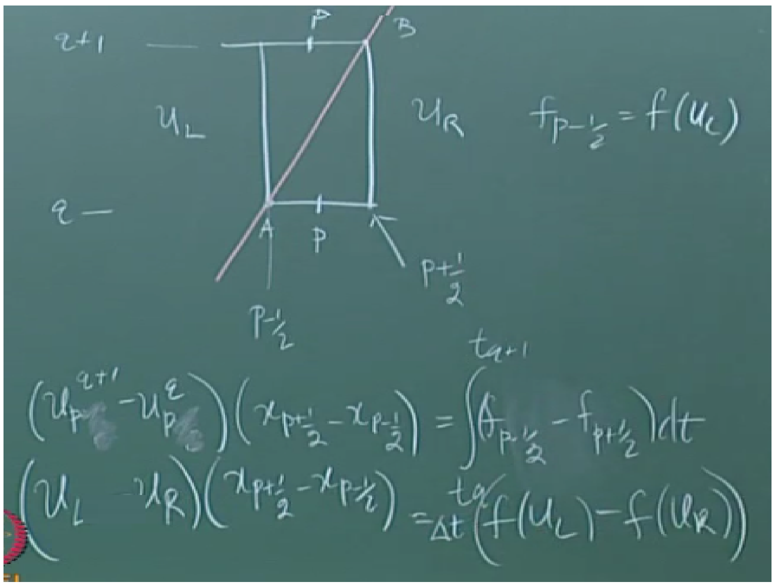

One was A and the other was B and they picked these points in such a fashion, that I was able to take a control volume. In fact, they took a control volume in such a fashion that they pass through these points. It is deliberately chosen, with the proposal that I am going to limit A go to B or B go to A. This point is not only identified as A, but in our finite difference scheme. This is a time level q+1. This is a time level q and in our finite difference scheme this point is p-1/2.

That point is p+1/2 and this midpoint is p, is that fine. So it turns out that that midpoint is p which is the critical thing that I need. What was the equation the governing equation that we had? What was the equation that we had you remember up+ 1/2? I will put the q on the superscript and I am not sure if that is what I have been doing so far – up-1/2 *xp+1/2 – xP-1/2 and what was this equal to that was equal to f of there was an integral.

Let me write the full thing tq to tq + 1 f(x + xp-1/2). That is not the way I did it last time. So I will remove that p-1/2 – f(P+1/2) dt, is that right and what we said was that we are going to consider a situation where to the left of the shock you have ul. The state is ul and to the right of the shock the state is ur. I am introducing extra notation you know I have got A, B and I have 1/2, 1/2 because these are ul and ur are something that you will see in standard texts and papers and so on.

But it is also important that so to the left of the shock, I have the state ul and to the right of the shock I have state ur. So the question is what is the value at q+1 upq+1, it is ul. So this becomes upq+1, what did I do you are not on top of things, fine. So this gives me ul–ur xp+1/2-xp-1/2 = this flux is a function of u and therefore if I look at p-1/2 on that face f(p-1/2) = f(ul). So this gives me f(ul)- f(ur)* I will write it just as delta t here just so that I can squeeze it in here.

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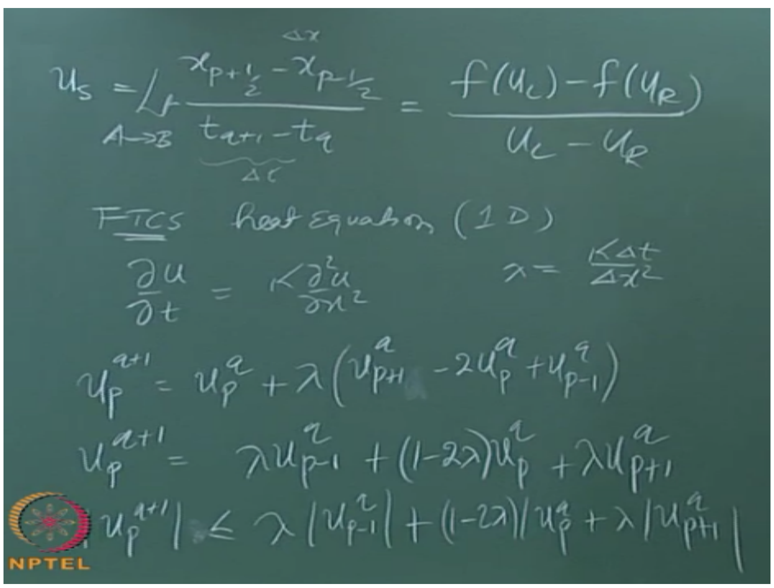

Time is delta T which of course turns out which gives us if I divide through the shock speed us is xp+1/2-xp-1/2/delta t which is nothing but tq+1-tq and that equals f(ul) – f(ur)/ul-ur. Is that fine? So this is whatever delta t that we have and that is whatever delta x that we have and this we get this because you get it. It is the speed us if you take the limit A going to B right limit A going to B, but this still holds. The shock is a discontinuity and this still holds.

We are able to divide by ul/ur because the states are different because there is a discontinuity, is that fine right. Is there any questions? So I am going to do one other thing that I promised earlier right I said I will do the stability analysis in a slightly different way. I promised to do it for heat equation. So I will just do it for heat equation and then we will get on with the rest of the class. So if I apply FTCS forward time central space to heat equation that is one-dimensional heat equation dou u/dou t = kappa dou squared u/dou x squared.

I am going to skip ahead to something that I had written earlier. We have already done this upq+1 = upq + lambda * up+1q-2upq, I guess I should stick with either subscript or superscript +up-1q. So if I combine terms remember what was lambda just to remind you, lambda is kappa delta t/delta x squared. So I can rewrite this as upq+1 just combining terms is lambda up-1q+1-2 lambda upq + lambda up+1q. I had already suggested that you take these as ABC and work it out.

I do not know whether you have had a chance to look at it but in this particular case, you know that if lambda is <1/2, this is positive. So if you are given lambda is <1/2, this is a positive quantity and what does it tell us? We can use triangle inequality. You are familiar with triangle inequality. You have the sum of these 3 and you have that and triangle inequality basically tells us that mod of upq+1<= all of these are positive quantities lambda mod up-1q+1-2 lambda mod upq+ lambda mod up+1q.

Triangle inequality gives us that and then we can use for example the maximum value. You can use the infinity norm which I have not really introduced here but basically you can take it.

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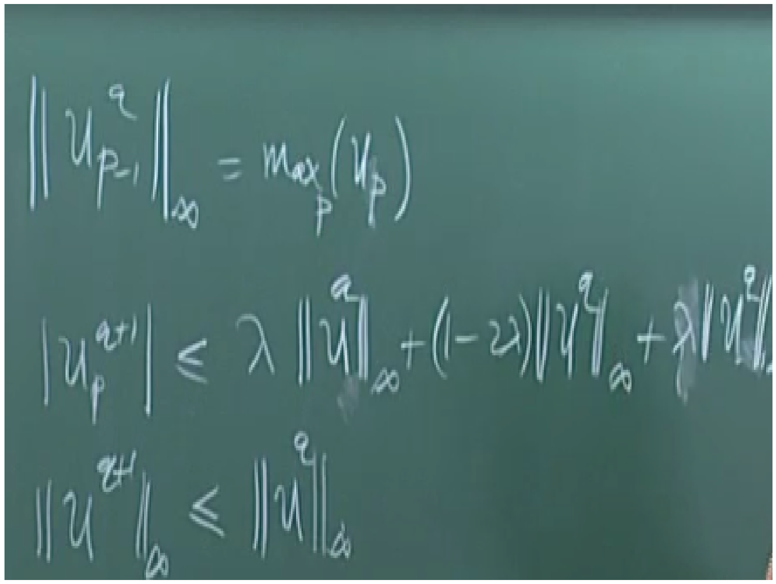

That if I say up-1q that what I mean is max of u, do you understand what I am saying? Max of u/p, is it fine? The maximum value, so again I rewrite my triangle inequality upq+1 <= to lambda u. I just write it as u+1-2 lambda. This is that way; you have got rid of the whole point was to get rid of that lambda. These are all at time q, that is what I need. I have eliminated the P. So each one of these at time q+1 each one of these satisfies. The maximum also satisfies this.

So I can replace this by mod u at q+1 infinity <= mod u at q. There are different ways to do. You do not have to take that exponential substituted that is one that is the von Neumann stability analysis. There are different ways by which you could do this. In this case I am talking about the function over the whole region, talking about it over the whole domain. I am not talking about it at a point. That is what I promised to do earlier I have just wanted to make sure that.

So the analysis of these schemes, there is a lot of scope for you; there are lots of things that you can learn from the analysis of the schemes. It is outside the scope of this course but there are lots of things that you can learn in the analysis of this scheme, is that fine? Are there any questions? So we are now formally what I will do is, we are out of the part of the course that I call simple problems. We have looked at Laplace's equation. We have looked at wave equation.

We have looked at heat equation. In the last class I had indicated to Laplace's equation falls into the category of problems called elliptic problems. Wave equation falls into the category of problems called hyperbolic problems and of course heat equation falls into the category of problems classified as parabolic problems. So these are the right now you may have seen it already in your partial differential equations course, but we have looked at simple problem, typical prototypes of problems that belong to these classes.

Now what we are going to do is, we are going to switch. Though we have been talking about propagation and in a sense diffusion those are 2 phenomena that we have been looking at propagation and diffusion it is still not fluid flow. So what I will right now do is, I will quickly derive the governing equations, but my objective in this course is to restrict myself to one dimensional flows.

So if you have had gas dynamics already, you are familiar with the solutions there are standard algebraic solutions to these equations that you have played around with at least in simple cases. In fact cases that are more complicated than what I will look at. So the idea is to put it in a familiar setting. For those of you who are not familiar with gas dynamics, the parts that are necessary hopefully I will cover, otherwise we will have to go look up whatever it is that does not sound familiar, but first the governing equations.

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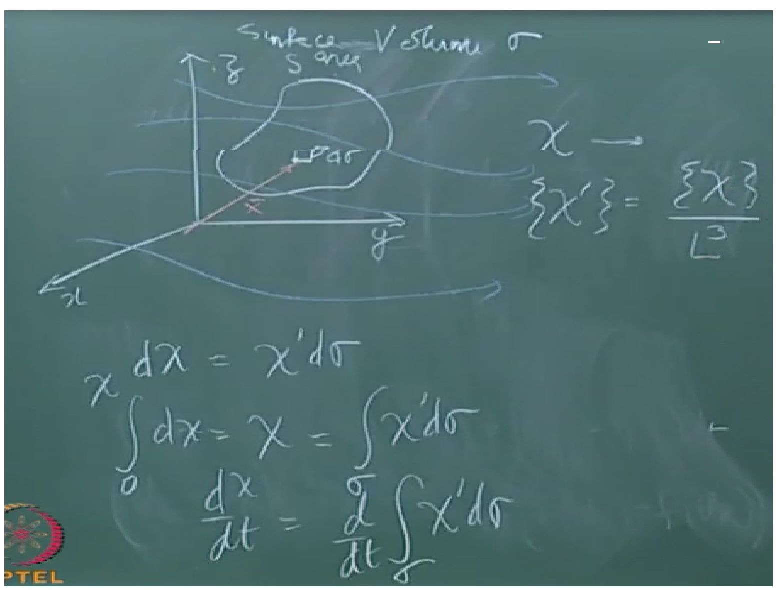

So to derive the governing equations, I will set up my coordinate system. This material is actually should be available on my website. The derivation of these governing equations in greater detail. There of course, I also use tensor calculus to do the derivation. Here I will stick to the standard Cartesian coordinate system. So before I derive the generalized conservation principle, we need to find out something, so we will look at some terms.

So the question that I have is what is the field property? Are you familiar with the term field property? I am not asking for something that is mathematically precise right now, but what is the field property? It is a function of position and time. Any field property is a function of position and time. Can we make it a little more what do I need to give you. If I have given you specified a field property, what do I need to give you in order to say that I have given you a field property.

I put it in a sort of a what do you call it funny way. So the field property is basically a region you are saying function of position. So there is the region, you have to give me a region, maybe even a period but let us just look at a region, because we need not look at an evolution in time. You need a region that is the field as we call it and you needed a property that is defined on that region, is that right? So if I say that I have a cornfield then I specify the region which is field.

I specify a property that is defined on that field, which is corn or it is paddy field or whatever it is. So the idea would be that if I go, look at that region any point in that region and I look at any point in that region. I am supposed to find the property, is that right? But in reality you know that that does not happen. In reality you go to the corn field, close your eyes and stick your hand on, you are likely to it. So there is a property. In that case, it is discrete.

We want to treat it, you want it as a function of x and t. We want to treat it as a continuum right. So we will convert it in some fashion to a continuum, we will come up with the density. So how does this work? So if I give you a region, say a country the size of India or something of that sort and you want to find out how much rice are we going to grow this year. So you could take a satellite photograph of the region, with the appropriate filters.

You figure out that those red colored spots there correspond to paddy fields, where rice is growing. The intensity of the red may tell you what is the ID density rate from a satellite photograph, it may be of the order of paddy density per square kilometer. We know that if you go to the paddy field, as I said that if you stick your hand in you are going to hit water and mud. But from seeing from there, it is a continuous distribution of red. The saturation may be changes.

So you can look at what is the density, you can infer what is the density and typically we would break up the country into small, small squares, multiply the local density in that square into the area of that square, add up all the squares and you have a good estimate as to how much rice you expect at the end of the year, fine. So we have basically looked at an idea of quadrature. We have introduced, we have thought of, so as I said I do not want to, in the fluid mechanics class.

Let me spend more time on it, but here we will rush along. So we introduced the idea of a continuum. So the words discrete we are looking at we want the continuum, we want to define a density. We want to define a density. So if you have some property, you need a region on which that property is defined. So I would basically say that in my region of interest, let me take an arbitrary control volume.

The volume that it occupies is sigma and this is placed in some kind of flow field. My region of interest of course is that I have this flow field. So it has a surface area. It is bounded by a surface whose surface area is s, volume is sigma, is that fine? At any given point on the interior of this volume and this volume is chosen in an arbitrary fashion. I could have chosen any volume. I take a small elemental volume at the point. I need a position vector at what point.

At the point x, I take a small elemental volume d sigma. So if the property that we are talking about, we will just pick some arbitrary property, if the property that we are talking about is. Since I pick Greek symbols, I will stick with Greek, chi. This is the property that I am talking about. As I said, it could be corn, it could be paddy, it could be anything. So the property that we are talking about is chi.

The corresponding density chi prime is basically, what are the units that it has, the units of chi prime or the units of chi/L cubed, whatever units you are using to find because this is 3d. If it were 2d, it would be l squared I had get an areal density and so on, fine. Given this, so now we can answer a simple question. What is the amount d chi of the property chi, that is there in the volume d Sigma that is chi prime d sigma.

So that is like taking one square out of that satellite picture multiplying by the area. Now we know this is the standard game that you can play now. So now you integrate 0 to Chi, the total amount. It just gives me the amount of that property that I have in that volume and this is integral over sigma chi prime d Sigma. So it is integrated over the whole volume, fine, is that okay everyone? I just want to make sure that this idea of conservation is fine.

We will apply it to conservation of mass. This is my plan for today and then I will quickly write out momentum and energy equation. As I said I do not want to spend a whole lot of time on this. So at this point this is like an analogy that I like to give for, this is like a bank account, chi prime is like let us say we go the bank, you go to the teller and there is currency, you have an account there. Each account they actually have a box. Let us just say they physically have a box.

So you give them money and they put that cash in. So they are sitting there and they are adding up all the 1 rupee notes, all the 10 rupee notes, all the 100 rupee notes, all the 1000 rupee notes, do you understand. They are adding up all of it and they are saying this is what you have in your bank. This integral is the total amount of money that you have in your bank account. The control volume is the little box in which they have kept all your money.

Our mind will imagine that there is a little bit, so as far as you are concerned the bank is this control volume right and this control volume is the account and you have just added up all the money in your account, but we would like to know what is the rate at which this our money is changing. We would like to know whether my bank balance is growing or decreasing. So what you do is, you take the time derivative of that.

So the amount of money in my bank balance d chi/dt is d/dt of the integral over sigma chi prime d sigma, is that fine. Now you know there are lots of rules regarding how money gets in and gets out. You have gone to your bank and you fill out forms right, deposit money. There is a set rules by which things can come in and come out. So the set ways by which money can show up in your account. So you have to look at what are all the possible ways by which it can happen.

So one is that somehow inside you have some way of creation. In the case of a bank account that could be in the form of interest that the bank gives you. It could be a source term or there could be a sink term. You go and you say I want to demand draft for 100 rupees or 120 rupees that disappears, because they take 20 rupees as a service charge, but as far as you are concerned you may have withdrawn only 100 rupees. So there may be a source term, sink terms.

There may be charges that they charge you and so on. So there are possible sources and sinks within creation of money and at the boundary, you can have inflow and outflow. So you want to know what is the net inflow and outflow, is there any other way by which there can be changes? Well as far as we are concerned with what we know if it were currency and I can talk about interest and so on, but if there is no currency all I have is chi, some unknown quantity.

Then there can be sources or sinks inside and there can be flow in and out. So in order to find out what is the flow in and out is that was a that is an outward unit normal. In that outward unit normal is on a little elemental area. It determines the little elemental area ds on the surface because now I realize that I have to do some integration to the surface. So the rate at which the amount of chi prime that I have in the volume is changing.

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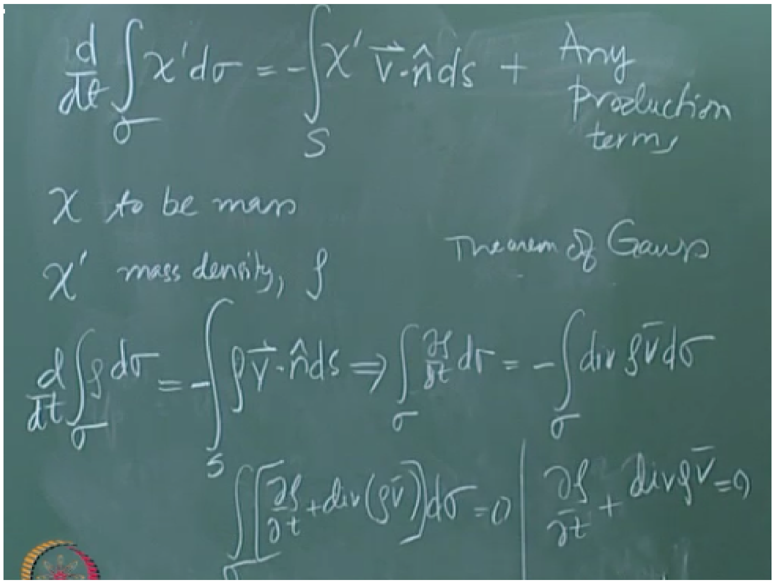

What is it if the velocity vector here is v, v dot n ds gives me the rate at which something is either coming in or going out. In this case because n is output normal going out of the volume. What that thing is, this chi prime is being carried out of this volume or the whole area s and because it is going out, it is negative. It is going to create a decrease because we are looking at how much is going out. Therefore, do you understand?

So you can go to the teller and you can stand in front of that person, the guy or the lady at the teller and as long as there is a barrier there is a little hole through which you can put your money, you can do anything you want with the currency side ward. It does not hurt. If it does not go through, she is not going take it or he is not going to take it. There has to be a normal component. So I can stand here, watch students walking past the corridor and somebody.

The student that does not have a normal component when he comes to the door. He is not going to enter my class. You need the normal component; do you understand what I am saying. The tangential component does not really help me. Tangential component gives you nothing, right because you are just going past. You are not getting in or getting out. So the v dot n you have to look at the normal. It is not enough to look at v, you have to look at v dot n, is that fine.

Plus, any other source or sinks, any other production terms, production comes and this has to come from the physics of whatever chi is. So we will quickly applied know that this is the idea of conservation, have any questions? So this is a generalized conservation principle. I will take chi to be mass, chi prime is mass density, which we encounter so often we just call it density, mass density and we have a special symbol for it rho.

So we substitute it there. Now know that we have this, we just have to turn the crank every time we do this. So you have rho d sigma is the amount of the mass in that d sigma integrated over sigma gives you the total amount of mass in the control volume, d/dt gives you the rate of change. So there is a little thing that goes with it. You have to repeat it in your mind sometimes it is worth it. So what does it v dot n, ds is the rate at which something is going out.

Rho is going out over the whole area and because it is going out whatever I have in the control volume is going to decrease. So I need a negative sign, is that fine? There are no sources and sinks. So we are not looking at either relativistic effects or you know all we are not looking at any of these games of no sources and sinks as far as far as we are concerned. That is what we have, is that fine. So this is the integral form the most general form that we have right now.

Of course, I have taken a rigid sigma, you can derive an even more general form with the control volume itself deforming and so on, but we are not interested in that. We have this, what next? What is the next step that we do? From here, I can take the time derivative and I want to get to a differential equation. We have been solving differential equations so far. This integral form is useful. You will see it in a different class, but in this class we have been looking at differential equations.

So I want to get it into a form of a differential equation. So this gives me the integral over sigma dou rho/dou t d sigma, I have taken the time derivative in all the necessary properties continuity related properties we presume that we have. It becomes dou rho/dou t because rho is a function of xyzt, whereas here you have integrated it out and therefore it is d rho/dt, you understand d/dt. I have taken it in, it becomes a partial derivative, what can I do to this.

I should convert it to volume integral using theorem of Gauss. So that becomes minus integral over sigma divergence of rho v d sigma, is that fine. I use theorem of Gauss and of course combine these 2 and you get the integral over sigma dou rho/dou t+ divergence of rho v, d sigma = 0 and now comes the critical assumption we made earlier. We chose the control volume arbitrarily. So this is valid for any control volume sigma.

So I want an integrant here whatever the control volume you pick, I have to guarantee that the integral is 0. That is possible only if the integrant itself is 0. So this tells us this gives us conservation of mass. I do not want to write it there, I will write it here, dou rho/dou t+dou divergence rho v = 0 and I am going to now constrain myself to one dimensional flow and say dou rho/dou t, I will have only the x coordinate + dou rho u/dou x = 0, is it fine.

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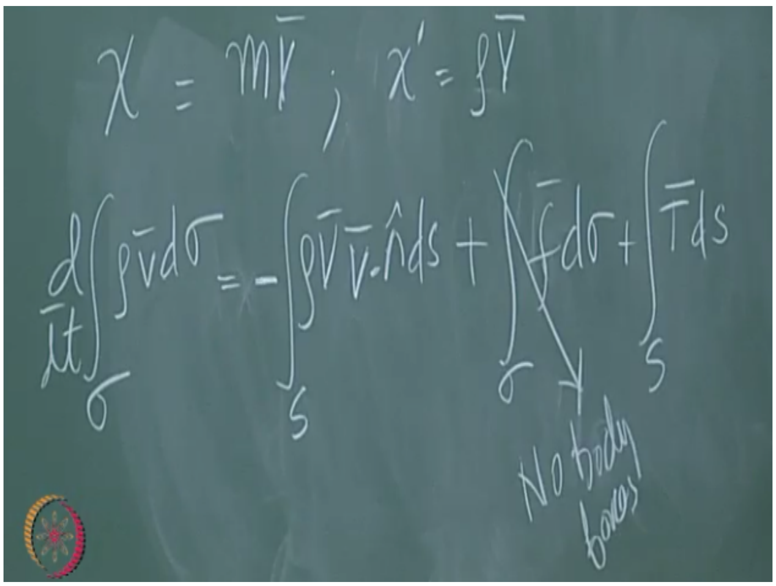

This is my one dimensional conservation of mass. I just write CM there conservation of mass. You will herewith call it mass balance, conservation of mass variety of names that you will hear. Are there any questions? So you have seen all of this before. Let us do conservation of linear momentum. For the same equation now I am going to substitute, what is momentum. It is M times v, so momentum density will be rho v.

Again just blindly substitute it into that equation, rho vd sigma, is what we have in that elemental volume whole volume, time rate of change is v dot n ds rho v is being carried at that rate, integral over the whole surface negative sign because it is an outward normal plus. Now we come to the point that I was talking about you have to consider the physics of what you are looking at. So what are the rate ways by which you can change the momentum in that control volume.

You have to apply forces. Forces come in various layers. The forces that we are used to come in various layers. You have the action across a distance where you can reach into the volume, you are talking in terms of something like electromagnetic force or gravitational force. You can reach into the volume and apply of force, that is basically one way and the other that you have are surface forces, forces that are applied presumably at all length scales as.

That is what I said I did not want to get into the idea of continuum and so on, but at the length scales that we are talking about the continuum level surface forces. The fact that I am pushing happens the mechanism actually happens with contact. I cannot keep my hand here and expect to apply on this bone a compressive load, but if I come in contact yes then I can apply that force. They are making sense that is the surface force.

So the surface forces or traction forces these forces are there you call traction forces. When I am walking around between the sole of my sandals and the podium, there is a force, a traction force. So if I am standing erect it is likely that the traction force is perpendicular to the surface, but when I start to walk or I am leaning forward, the traction force is actually at an angle. So we have to take all of these things into account.

So at this point there could be a traction force the figure is getting messy but it does not matter. There would be a traction force t, through the volume at any given point, there could be a body force F, as any line forces we finished volume, we have finished surface, anything that corresponds to a line force not that you are familiar with. I do not know maybe surface tension or something of that sort, but we are going to ignore those.

Even at this point we are going to ignore those. So F d sigma that means that I have considered F to be a force per unit volume Fd sigma is the volume on that element integrated over sigma and gives me the net body force, volume force or body force on that volume. Similarly, Tds is the net force on that element and that force can be a shear component. It need not be normal and the integral over s gives me the total force that is acting on the surface, is that fine.

For the sake of this course, I am going to ignore body forces, no body forces; we are going to ignore them. In fluid mechanics anyway we have already studied when you need to take gravity into account and when you can ignore it and again for the sake of this course, I am going to ignore viscous effects, which means that I am not going to go through that full game of (()) (36:22) theorem and coming out with defining a stress tensor and so on.

Instead of which you have already done, instead which I am going to just write ts, what is the direction of t in this notation. I can hear something what is t. If you have only pressure if there is no viscosity –p*n. So we are back here maybe I leave that just for your reference. So what happens here so I get the integral over sigma dou rho v/dou td sigma. I have taken the time derivative inside equals integral over s or over sigma if you want divergence of rho v d sigma + integral over sigma gradient of pd sigma.

Pnds again you can go through the same process and convert it to a volume integral, is that fine? We have minus sign. So this tells me that dou rho v/dou t + divergence rho v v + grad p=0. I have merged them all together the integrant has to be 0 because it is an arbitrary control volume. In the one dimensional case which I will write here is dou rho u/dou t + I will combine the 2 terms dou/dou x of rho u squared + p=0.

Some of this from your gas dynamics should start looking familiar, is that fine. Any questions? We go to the last one which is total energy. Look at energy equation, so we looked at conservation of mass, we looked at linear momentum, conservation of linear momentum, now we look at conservation of energy. So we go through the same process.

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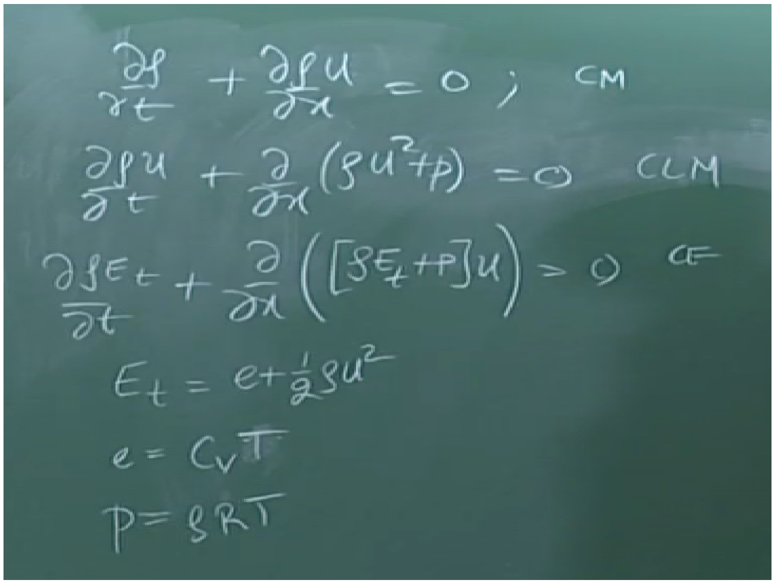

If I have total energy, I will write just the density here chi prime similar fashion. It depends on how you want write it. I will write it as rho Et because I have rho v and rho. I mean it makes me I like the pattern. So you could have just written it as Et and gotten away with it. We need to do a little worrying here, what is Et, I just introduced Et, Et is e+1/2 rho say v dot v, that does not help us, so what is little e then. Depending on what we are doing is cvt.

Every time we do something we need closure, what do we get closer up typically you take a constitutive model or an equation of state. So we need an equation of state here which is p equals rho rt and now we have t in terms of the other 2 quantities we already know, so we have settled that, otherwise this could go on forever. So we have settled that, is that fine? So what do we have, let us write the thing again rho et d sigma integrated over sigma the time rate of change of that is v dot nds is carrying out rho et integrated over s with a negative sign.

What are all the ways by which we can change the energy in that control volume. You have 2 possible processes. Let me know right there are only 2 possible processes heat and work. Heat and work, so you could either have work of course can be done by these forces. We already know we have volume forces, so we enumerated the forces, we can deal with that. We keep it simple because you have already ignored certain forces.

What are the mechanisms of heat, you can have radiation, radiation conduction that is energy carried by convection rates, that is taken care of. So that is a different game. Now we are talking about, it is conduction or radiation. There can be source terms. You can have exothermic reactions of some kind, endothermic reactions of some case. There can be source terms. There can be other sources, so we will ignore chemical reactions.

You understand we ignore a chemical, nuclear all those reactions. All those things that are no sources and sinks. We will ignore radiation. Because we have already ignored viscosity, we will ignore conduction also and we have done heat equation, what more, but seriously we will ignore conduction, so what is left. Work done by body forces and we have no body forces. Body forces is gone. Anything else? You have pressure, work done by pressure.

So you can figure out how that is going to +/- you can figure out how that is going to, I will let you figure that out. I am going to come back here. I am going to jump now. I will let you know. Because I have thrown away, I have been a nice guy and thrown away a lot of things. So you can you can simplify that and figure it out. I am going to just write the final form here, dou rho et/rho t+dou/dou x of rho et+p*u=0. That should give you a clue.

You see the work done by pressure that is power, the rate at which work is being done p*u. Because it is the rate at which the total energy in that volume is changing. So you can work that out, it is just p*u*n whatever it is. You want me to work it out, you can work it out that is fine. So now this is linear momentum, this is energy, you have the 3 equations and then we have a whole bunch of auxiliary equations.

We have the whole bunch of auxiliary equation ets e + 1/2 rho u squared, this is for the one-dimensional case e=cv*T, what is cv specific heat at constant volume and then p=rho rt with which we have closure. We have spent so many weeks looking at linear wave equation. So you should be legitimately upset that this does not look likely linear wave. So we will try to make it look like linear wave. We will they take a little effort in trying to make it look like linear wave equation.

The one first obvious thing to do is combine them into a vector equation, at least the chalk dust will look very similar we hope. So let me come here we will combine all of that into one equation we do not need any of the stuff now. We are done with it all.

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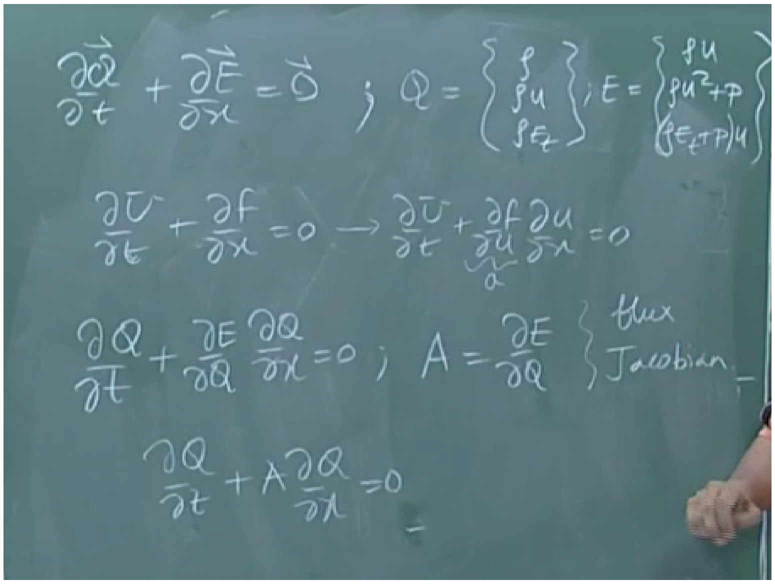

So the one-dimensional gas dynamical equations can be written as dou Q/dou t+dou e/dou x = 0 where Q is a vector. I should really indicate these as vectors or matrices, but I am not going to write. Even the 0 is actually 3 zeroes. I am going to drop them. It is very clear what it is that I mean. So this will be rho/rho u rho Et and E is rho u/rho u squared plus p rho Et + t*u. Close but not good enough and this looks like a generalized one-dimensional wave equation.

It looks something like that right I mean this resembles what we wrote as dou u/dou t. This u is not the same as that u + dou f/dou x =0 and there was a trick that we played here we could do dou f/dou u. In the last class, I have written this very light. I use capital U instead of little u because I have already. So we looked at this last class, looking at this equation a generalized one-dimensional wave equation kind of a thing and what did we do?

We could write this we could take the flux Jacobian. We could write it as dou f/dou u, we could write this as dou u/dou t + dou f/dou u dou u/dou x, using chain rule and this was the propagation speed A. At least it looks like what we have, we are trying to get it to that form. So I write these one-dimensional equations rho Q/dou t + dou e/dou Q dou Q/dou x = 0 and I will call dou e/dou Q I will give it a name A and like we said this is called the flux Jacobian.

Very often people will refer to it as the flux Jacobian, e is the flux term, look at this e comes from flux term. So this equation then is written as dou Q/dou t + A dou Q/dou x = 0. That is good news and it is bad news. It is good news because it looks like dou u/dou t+ A dou u/dou x = 0. So I have wasted a lot of your time working on that equation, is bad news because this is a system of equations. So we will have to do something.

So before I close for the day I want to point out this is again in the standard form that mathematicians call the divergence free form rate. It is first derivative some of these first derivatives dou Q/dou t+dou u/dou x=0. So it is called the conservative form. From gas dynamics, we have our own little reasons for why we call it a conservative form. What happens to these quantities across the shock? These quantities are conserved across the shock.

So we look at we have our own little game that we play, so it is a conservative form because rho u, rho u squared, in fact the derivatives in words because they are actually continuous. They do not jump. These quantities do not jump. So we have our own reasons for calling it the conservative form. I say this because you can get confused sometimes you may have asked, why is it called the conservative form and depending on who is doing the asking, the answer may have to appropriately change, is that fine.

This of course therefore is called the non conservative form. It is written in terms of what we call the conservative variables, these variables because this equation when written in this one, so these are called very often referred to as the conservative variables. It is written in terms of conservative variables q, but the form is called a non-conservative form, is that fine and Q is called conservative variables, simply because you can write the equation in terms of Q in this conservative form, is that fine.

So in the next class, we will make some effort to see if we can simplify this. We will find out what is the nature of A and how to find A and I can tell you know it is not going to be satisfactory. Then we will make an effort to try to make the equations look more and more like our linear one-dimensional wave equation is that fine. So that everything that we have done there, we will see what carries over from the one-dimensional equation, what have we done that carries over, is that okay. Thank you.