Introduction to Computational Fluid Dynamics
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Lecture - 22
Demo - Wave equation / Heat Equation

Right now what we have done is in the last class we saw a demonstration of package called maxima that I used to derive basically the modified equation okay and amongst various things so we have already seen the effect that the extra terms that you get in the modified equation. We have investigated in earlier classes what is the effect of adding the second derivative term, what is the effect of adding the third derivative term, fourth derivative term and so on.

And in the last class we actually saw that by adding appropriate terms to the differential equation, it is possible for us in a targeted way to eliminate certain terms in the modified equation okay. So now what we will do is we will look at a demo right solving only the linear first order one-dimensional wave equation okay. We will look at a demo. I started a little of it in the last class but we will just go and we will look at the demo.

And see whether all the properties that we have predicted everything that we have predicted so far. See as opposed to Laplace's equation, Laplace's equation we just solved right you had to went ahead and you solved it. It is a relatively what should I say robust problem right. I will explain to you maybe towards the end of the semester when we do a little calculus of variations where that idea comes from.

I have already given you the essence of that but it is a relatively robust problem, so they had no qualms about asking you too just go ahead and solve it numerically. You understand what I am saying writing a program but by the time we came to a wave equation where we did a little analysis and the idea was that you know we will find out whether it works, whether the schemes work and so on and we found that there was stability related issues right.

And there could be what do you call it the fact that the equation is modified and so on and based on these things we have made predictions as to how the code will work. Now we will actually run the code and see whether those predictions how they work out, to what extent are they true, how do we interpret the analysis that we have done. Am I making sense? Okay right.

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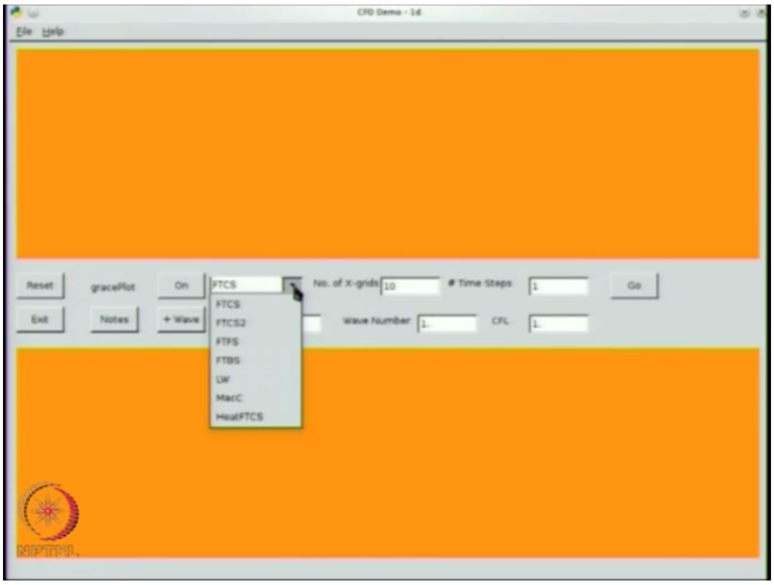

So let me run that, this is by the way as you can see it is a program written in python and the user interface is built using a package called python card which is built on wx windows. So what we will do is let me just start that off okay. I will quickly get rid of that. I have made a few small changes since the last code as an evolving code, few small changes since last time. For example, if I say reset I have eliminated the reset anyway okay fine.

So right now if you cannot make out it is right now set so this window that pops up here allows me to choose various and sundry schemes, it is set by default it is FTCS. There is FTCS2 a variation that we will look at later which is basically FTCS with the second derivative term knocked out okay. So that according to us should have been a stabilizing influence we will check that out.

There is FTFS, FTBS so all the schemes that we have looked at. There is a Lax–Wendroff scheme, MacCormack scheme which we have not looked at, you can go check that out and there is FTCS applied to heat equation, so all of it is wave equation except for the last one which is heat equation. So these are the schemes that I propose to look at in this demo. We have already seen what happens when you apply FTCS to this equation.

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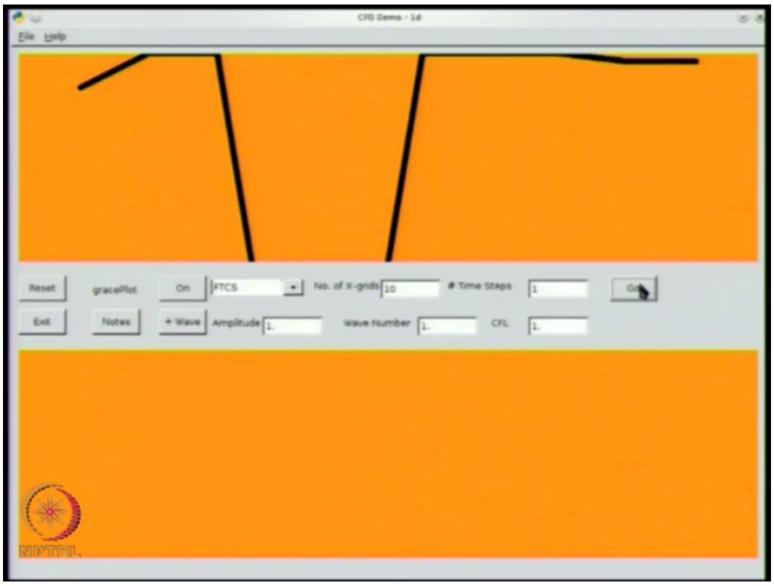

So basically it looks like you know it is supposed to be unstable and indeed it does behave as though it is unstable right, it is diverging. I have not bothered to rescale simply because we just want to know whether it diverges, once it goes beyond out of the screen right it does not make any sense anymore.

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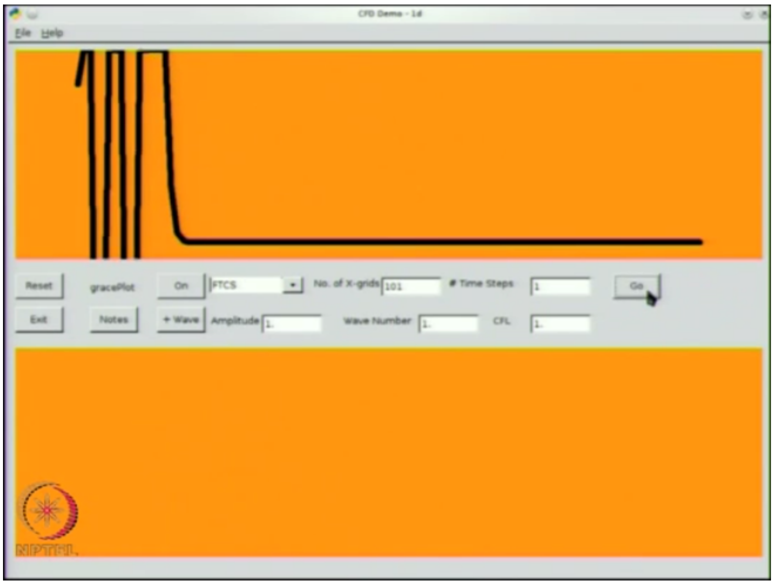

We can try increasing the number grid points to see if that makes a difference. All that does is that ramp looks a little better but wherever the ramp is it starts to oscillate and remember the notion of high frequencies associated with the grid size, so we can see that it is a high frequency is that a diverging right. So this is near very close to the highest frequencies that can be represented on this.

There is still propagation. The basic feature of the equation is still there. It is still propagating in the x direction. It is just that it is eventually going to diverge okay so since I do not want to get into trouble with that. The other possibility that we thought about was would it work if I lower my CFL, so instead of 1 if I made my CFL 0.1 would it work?

So this is going to 0.1 so this is little arithmetic that we have to get right, 0.1 given that the speed is the same and the grid size is the same, lowering the CFL to 0.1 means that the delta T has become 1/10 right. So we really need to take 10 times the time step.

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I will increase the number of time steps, the little window here is the number of time steps you really need to take 10 times the time steps to effectively get the same time step and then you see that the behaviour though it is smoother than last time, why it is smoother, it looks cleaner, it is not diverging as much so the growth is not as fast right. The growth is not as fast because the size of the mu2 term has gone down.

The magnitude of the mu2 term has gone down. The magnitude of the coefficient of the second derivative has gone down but it still seems as though it is going to diverge right and there is nothing that we can do with this right unless you want to try something smaller if there is hope and sometimes this happen. We fall into this traps and let me make it a little smaller maybe it will work for that right, there is a little hope.

So if you want to make it smaller since I made it 0.01, I will take 100 time steps at a time and we reset and who knows maybe it will actually work, nope there it goes. It is not supposed to.

See the point is it is not supposed to exceed 1. This is like having a dam that suddenly breaks right. So I expect that the water to flow out and right now I mean you will see of course in the next class I will show that it is possible that crests and peaks form in real life, it does happen right but right now the simplistic model that we have, we expect the water to just translate, we do not expect anything else.

And it does not look like that is going to happen right because I am doing more work, my program is taking more time because I am taking a 100 time steps each time but we are not getting anything out of it. It is going to eventually diverge. I mean it is there, all the symptoms are there. Is that fine? Okay so the next thing that we do is we look at FTFS. Let us look at FTFS and go back to CFL 1.

I will reset it. What do we expect? I do not want a 100 time steps, maybe I will take just 1-time step. What do we expect? FTFS what do we expect?

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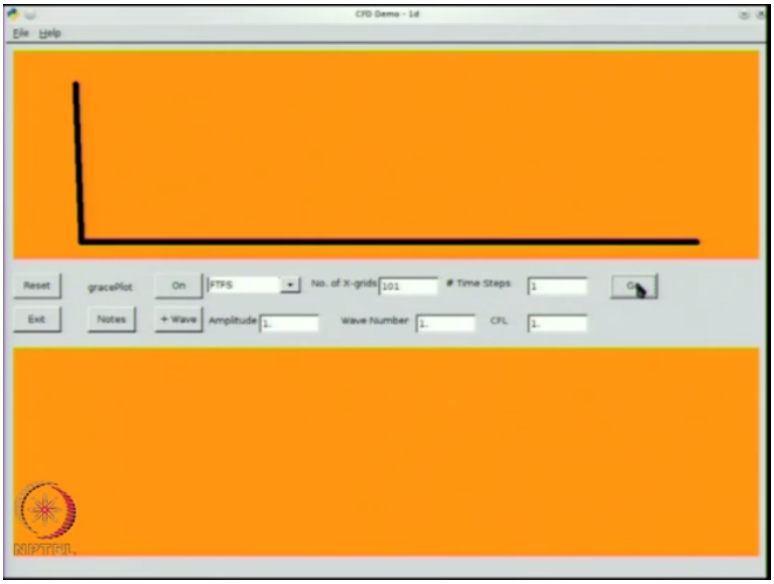

It is supposed to be unstable, going to blow up, why do not I take 100 time steps, what is happening, I am actually clicking on that button right. I thought it supposed to blow up. I have taken at least 400 time steps so far. The trouble is everything is 0. The only point that is nonzero is at the left boundary. Am I making sense? The only point that is nonzero is at the left boundary and it is not participating in the computation.

FTFS is only using points to the right of the first point and only the first point is a nonzero. All the other points are 0, so the computation just is not starting off. There is nothing to do.

Am I making sense? You are just manipulating adding and subtracting 0s right okay. So there maybe times and it is possible that we look at this you have to be very careful.

So one thing that you remember from this is you have to have a sense of what is the answer that you expect especially when you are developing these codes you have to have a sense as to what is the answer that you expect, what is the behaviour that you expect okay. Yes from the numerics point of view we expected it to diverge but from the physics point of view, the physics of the equation if I may say I expected this wave to propagate.

The fact that it does not propagating right tells us that there is an issue. The fact that it does not diverging does not mean that oh we are happy saying that oh I have a steady state solution, I managed to get the solution. It is very important the fact that it does not diverging does not mean and it is not changing you understand, the change in the solution from one-time step to another time step is 0.

So it is very easy to look at this equation and look at this and basically so I have got the solution, I have a steady state solution okay, so not true. Let us try FTBS, FTBS of course I will take 1-time step and I reset that just for good measure.

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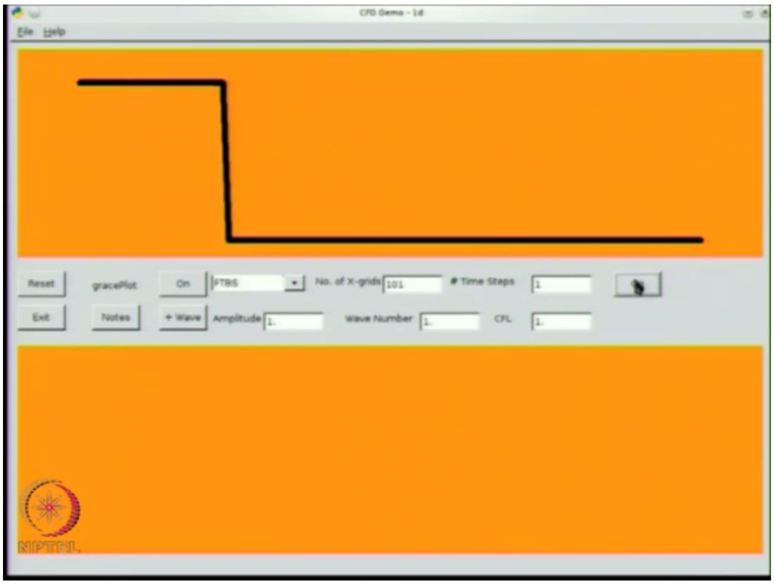

And as you would expect FTBS with sigma=1 right behaves the way we expected to behave. In fact, if I reset it and let me take 50 time steps okay. It makes it half way, I take 50 more time steps and it goes, it is actually just at the exit, it has just gone through the domain. Am I making sense? Okay. The propagation speed is right I mean it looks reasonable that the propagation speed is right.

But of course the ramp is still there, so if you want to represent that step the closer you want to represent the step the finer your grids will have to be fine or you will have to do some fancy grid generation okay, which is sort of outside the scope of this course but you will have to do some fancy grid generation okay. There are 2 ways we can go now right.

Since FTFS we have already seen the stability condition did not quite workout the way it supposed to workout, what I am planning to do is I am going to run CFL>1 right. I do not know if you guys have tried this. I am going to run a CFL>1. So I will run 1.1, 1.1 is supposed to diverge okay. So I guess I could just take 50 time steps in one shot, so that on the other hand it may blow up okay fine.

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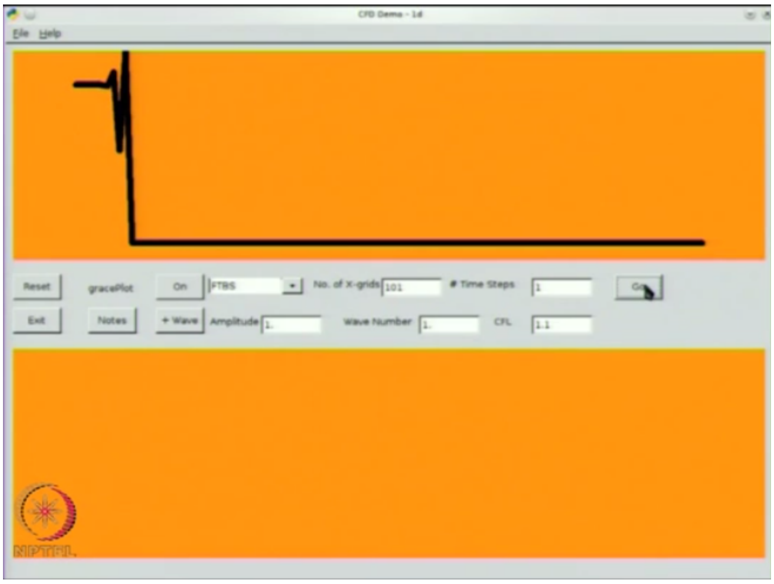

I take 1-time step there is that peak, so it looks like it is going to go right. It looks like it is going to go. You notice they look sharper than what we got with FTCS. It looks a lot sharper than what we got with FTCS okay. So is it worthwhile going you think or should I just stop? Let me take 10 time steps at a time and see what happens.

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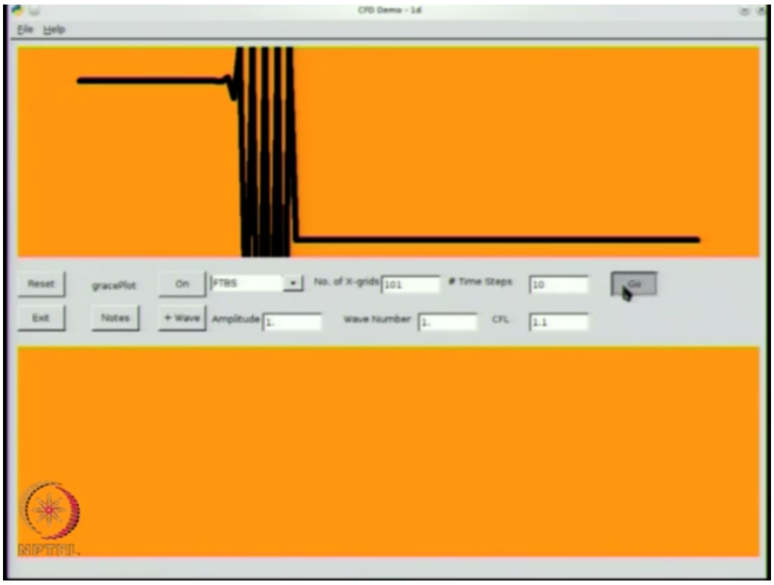

Clearly, it is diverging but it is also being propagated out and it has gone okay. So this is an important thing. So you can turn out and say wait a minute, it is supposed to be unstable this program had no business working when it depends on what you are looking for. If you are looking for the steady state solution, it actually works okay and why did it work because we are looking at a finite domain.

I am looking at an interval here which goes from 0 to 2 pi okay and now it is a matter of dynamics, it is blowing up, it is going out like it is like getting to a game that ends in a tie in a sense right, you do not know what is going to happen, you do not know which is going to go, whether it is going to blow up or it is going to make it out fine okay. So if you are looking only for the steady state solution, the transient is clearly wrong right the transient is clearly wrong fine okay.

See I know I am sort of going through there are some observations I want to make which are extremely important for the kind of work that you may do in CFD okay. So if you are looking for the steady state solution, right now what I seem to have said is I do not care what the transient is like as long as I get my steady state solution okay. The second thing is your computational domain is finite.

We cannot handle infinite computational domains right, so unless you do something interesting you cannot handle infinite computational domains. Typically, we truncate the domain okay and in that case even if your scheme is unstable, it is possible that any unstable mode if it flows out of the system that you still get the steady state solution fine okay.

So it is unstable, the scheme is unstable, 1.1 it should not have worked maybe towards the end right I will come back to this and run we will rerun this and maybe I will run 1.2 or 1.5 or whatever it is and see larger values. I do not want to just crash my program right now but right, it is not so robust that I can recover from a crash okay fine. What about smaller values of CFL? What do you expect?

If I run 0.5, any predictions, did I do this in the last class? 0.5 if I take 1-time step that looks the same and that looks a little different you may not remember it, so maybe what I do is I take 0.5 by half the time step right so maybe I will take 20 time steps at a time which is equivalent to. What is happening can you tell me what is happening?

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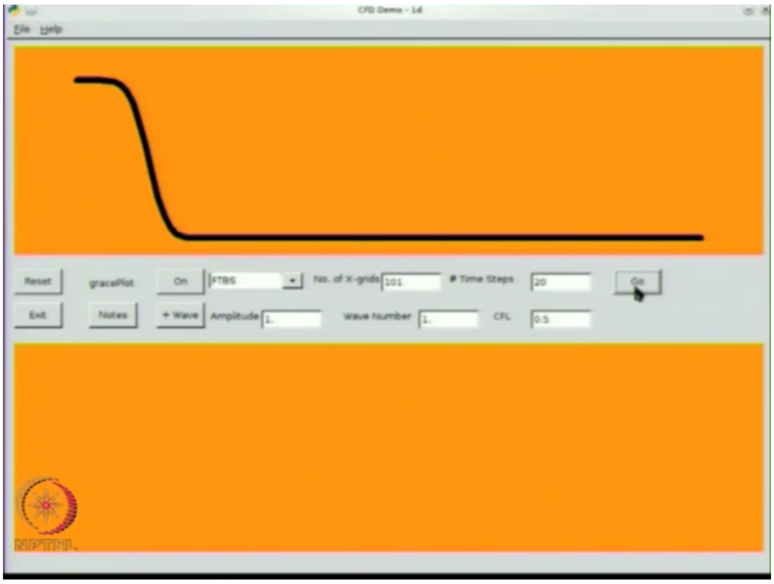

"Professor - student conversation starts." No, dispersion was what you saw when, no dispersion. **"Professor - student conversation ends."** What is this? FTBS. What did you expect? What was it when it was 1? It is run sigma=1 again. If you were to do a just say instead of a ramp it is actually a step okay, we know it is a ramp. So it is ramp, you do a series expansion using Fourier series right in order to get the ramp.

In order to get that ramp, all of those have to match exactly. They do not match exactly you will start seeing oscillations okay. So when we did 1.1 you actually saw that so if you do 1.1 it starts to oscillate at that point okay so it is not just diverging, there seems to be some but if you do 0.5 maybe I will reset it. If you do 0.5 what is happening?

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What was the other phenomenon that we had? It is clearly stable but there is something that is even 1.0 is stable. Just describe to me what has happened to the curve? The curve seems to have become smoother. In order to get that steep step what do I need? See all the high frequency terms have disappeared that is basically what has happened okay. It is dissipative; it is decay when I prefer.

The high frequencies are decaying right because at 0.5 basically what is happening is the high frequencies are going away. So the curve is getting smoothed out right. The curve is getting smoothed out. Am I making sense? So the high frequency terms are going away and the curve is getting smoothed out. If you are again looking only for the steady state, you do not care but if your steady state has a discontinuity in it then you have a problem.

Because this may not be what you want right. It supposed to be a step instead of a step you get something that so you could say that why do not we try a larger number of grid points and see whether that makes a difference. So if I go to I do not know 201, I will just double it just for the fun of it.

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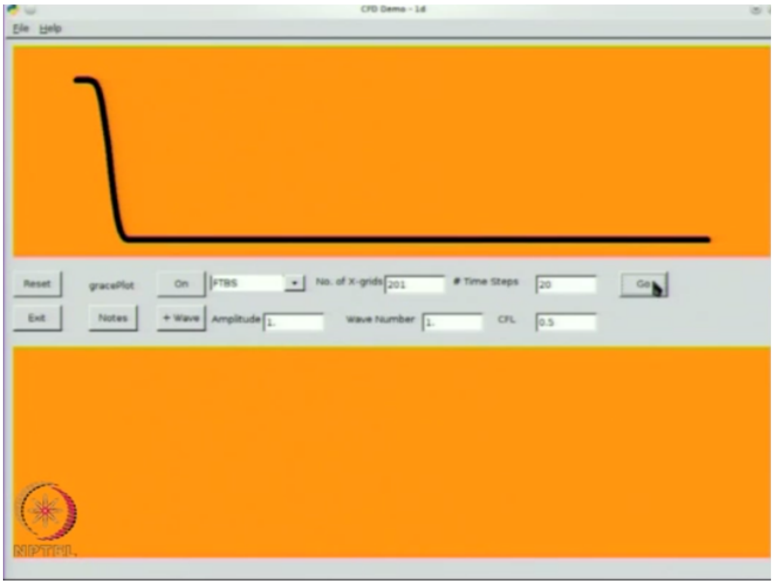

If I go to 201 yeah it is sharper but you can see that it is still getting rounded off right. Just that I can represent much higher frequency is because the grid is finer that is all. The definition of high frequency has changed by changing my grid but on that grid it is getting rounded and it progressively gets worse okay right. So it is very clear it has a second derivative term that is basically dominating all the other terms.

It also has the fourth derivative term and this combination is causing higher frequencies to decay right. So that you are getting a curve instead of getting a step or a ramp in this case of representation you are getting a nice smooth curve. Shall I try something smaller? You think it will get any better any worse?

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Yeah so I do not have that much hope for this because right off the beginning you can see that it does not seem to help, when I made the CFL smaller can you make an observation from last time to this time, is it more dissipative than last time? It looks more dissipative than last time right because it had a 1-sigma right. So as sigma gets smaller and smaller, the coefficient of the second derivative term is definitely getting larger okay.

I am stressing the code out here fine. What else do we have? There are 2 possibilities, may be what we will do is we will look at the heat equation and this idea of dissipation and dispersion will look at a little more carefully okay. So before I do heat equation let me do FTBS itself. I have a little thing here in the bottom where I can add waves. So for instance, I can add sin x okay and to this I will make the amplitude smaller.

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So I can add with an amplitude of 0.5, a wave number of 10. So I can add right so I am constructing. Basically, what I have done is I have done sin x+0.5 sin 10x okay.

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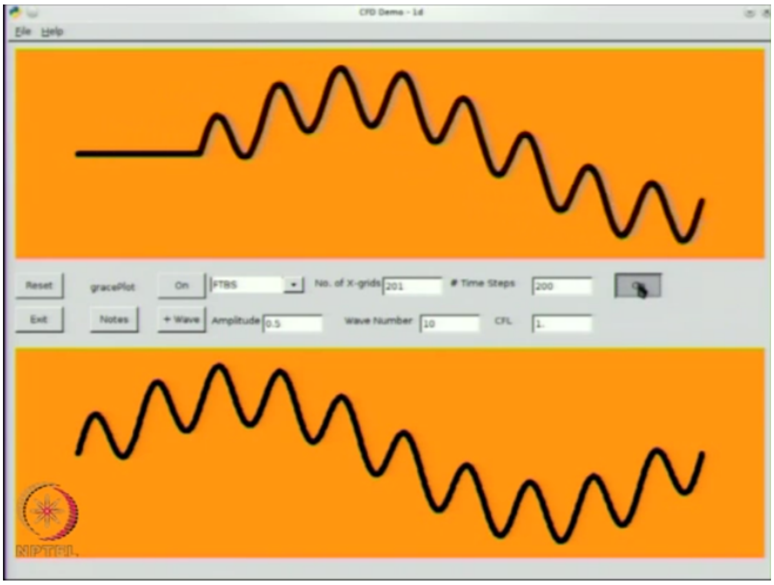

So if I run this at CFL 1 with FTBS it just goes, in fact because I am taking 200 time steps, it is too many time steps okay.

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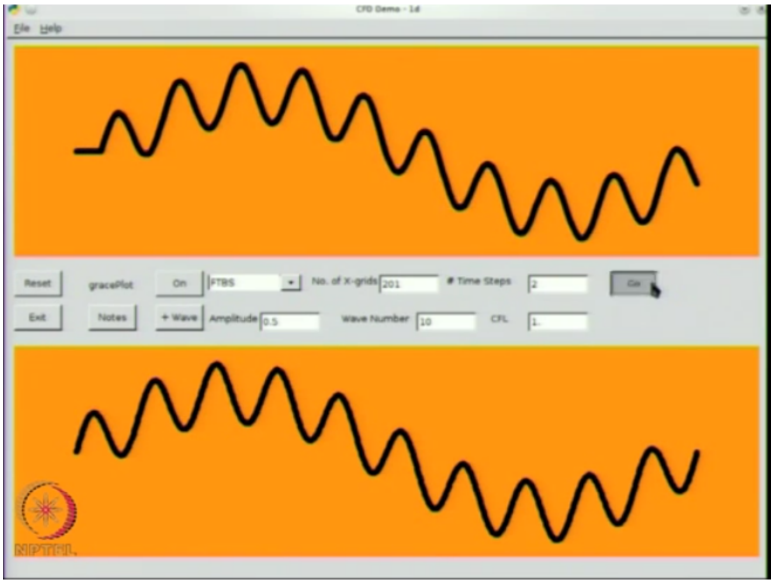

So if I run it with CFL 1 FTBS you can actually see that and that does not seem to be within what you can make out with your eyes that does not seem to be any change in amplitude, nothing is there, it is beautifully translating left to right. What happens if I change the same on to 1.1 CFL?

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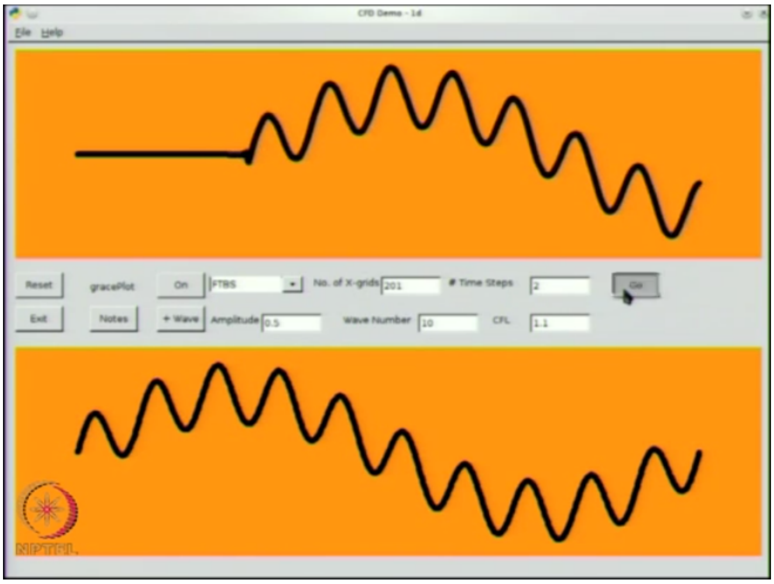

Yeah there it starts the oscillation. Where is oscillation starting? Why is the oscillation starting there? So that is where the high frequency content is right. I had a sin x but it is not continuing as sin x, what is coming from the left hand side is a constant function so you have a function if you think about periodic extension that you will do in sin x I mean for Fourier series right.

Then you have a function the derivative has the jump lot of high frequency terms that you show up there, higher terms that you will show up there. Once that are shifted, it is no longer just sin x+sin 10x because there is a little straight line segment that shown up right which is going to now include a lot of high frequency term, so you can see that is there. So it looks like if I were to go to 0.5, let us go to 0.5, I will reset it. We will start from the beginning.

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What do you say anything perceptible? It looks like the sin 10x term the amplitude is dropping. What about the sin x term? Not so much okay maybe I will add sin 20x or 40x or something of that. So let us add I do not know how a 40x will look, let us add 40x and see what it looks like I have not tried a 40x okay so that is needless to say that looks interesting okay.

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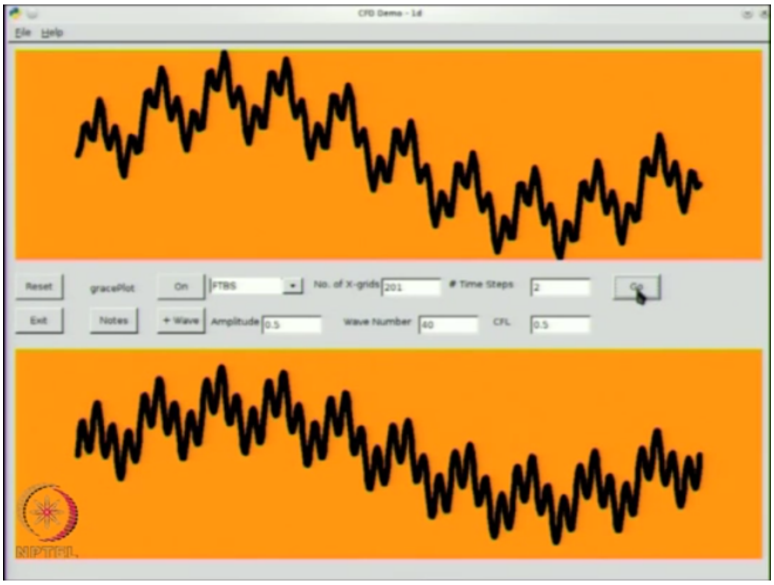

The graph on the bottom I am plotting using 800 points, the graph on the top I am sampling at 201 points okay. So obviously they are going to be sharper corners. You immediately see the quality of the function has changed and if I take you can see the 40x is decaying really fast right.

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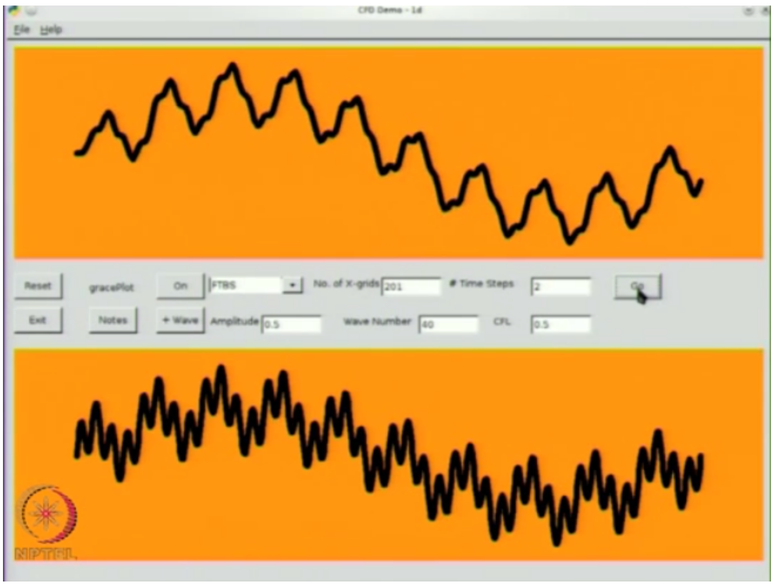

And before you know it, the 40x is gone fine and all you are left with is right so this sort of moves, this we can possibly explore a little more because this is also simultaneously translating there are high frequency supposedly being introduced from the left hand side, which is reason why I have implemented the heat equation because heat equation you have set A=0 right.

We have the dissipation term but we have set A=0. The only trouble with heat equation I mean just let me take one time step instead of 2 times, this is really dissipative okay. See 2 times steps is going to disappear so I think one time step that 40x is still there, 2 times steps it is almost gone, 3 time steps it has gone okay and in this case it is very clear that the high frequency is going faster than the low frequency.

It is really decaying faster than the low frequency. Is that fine? Okay so if you want to see how many I have done.

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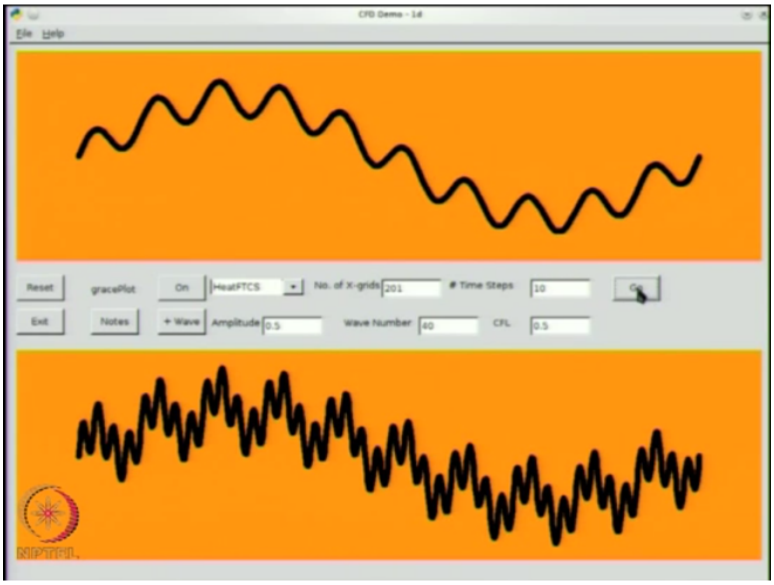

If I take 10 time steps at a time yeah right. The 40x lasts about 3 or 4 time step, it is not there at all. So that is 20 time steps, 30 time steps, 40 time steps, 50 time steps, 60 time steps that has gone right and now if you want to patiently wait, this is going to take a long time okay. So if you are looking for the steady state solution let me take a 100 time steps at one shot. This is going to take a very long time if you are looking for the steady state okay.

So I want you to bear this in mind, this is something that you will note I will recollect this behaviour at the end of the semester right because I need it elsewhere. So yes it is nice that high frequencies decay faster than low frequencies but if I am looking at the steady state solution right if I am looking at the steady state solution and I am looking for the steady state solution and this is the way that the error term is going to go the low frequencies are going to be a headache to get rid of.

If this is the error term, this is the rate at which my error term is going to decay, the low frequency I have taken you know the high frequency has gone, I am looking for the steady state. The steady state solution in this case is 0 because that it is heat equation, temperature is held 0 at both ends, the steady state solution in this case is 0. So now I am taking in 100s of time steps and it is not going right.

It refuses to go away, is that fine? Okay. So I reset let us try something different okay if I take heat equation 0.5 is the stability conditions as 0.5 we are right and right there so if I make it 0.6 will it make a difference?

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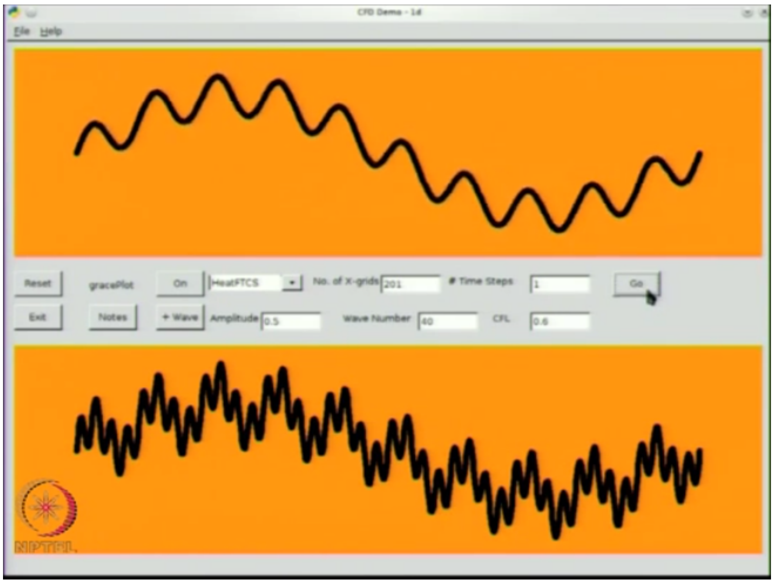

Let us take 1 step at a time. Well it still seems to decay, not clear, reset. Yeah, let us try 0.7, reset, so these first few time steps seem to work quite well and say wait a minute what is this they all seem to work. What is the problem? There is difficulty, you want me to make it 1.

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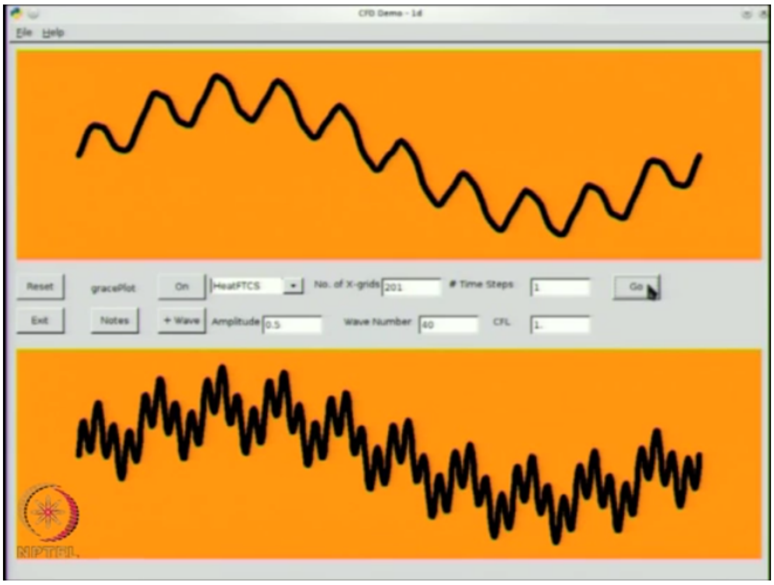

Was there a mistake in our stability analysis? So this is another one of those things that you have to be very careful with when you are doing CFD. It looks like something is converging and you may go along thinking it is converging till it starts to diverge right, say it is converging, it is fine, I am doing great, I have run 5000 time steps, code is converging, it has converged 3 orders of magnitude.

You understand what I am saying. So for engineering purposes that is good enough. This is a classic argument for engineering purposes that is good enough. No, when you are developing your code make sure it converges okay when you are developing your code. When you are making production runs, you have run it for test cases right you are making production runs and you have some parameter like A or whatever it is and you run it for A=1, A=5.

You want to try it out for in between, it works for A=1, A=5 then in between yeah let it you know you can say 3 orders of magnitude engineering accuracy it is enough but when you are developing the code when you are actually developing the code you want to make sure that it is going to work. If you give it single precision it will work single precision, if it goes down precision it is going to converge to double precision you understand.

If it does not you want to know why, it supposed to be stable okay and for those of you who may be interested in research maybe a research problem somewhere. So yeah this seems this is going to go and you can see and it is going fast okay. It is going fast, pretty pattern but it is going fast. So I say okay let me quickly try to recover.

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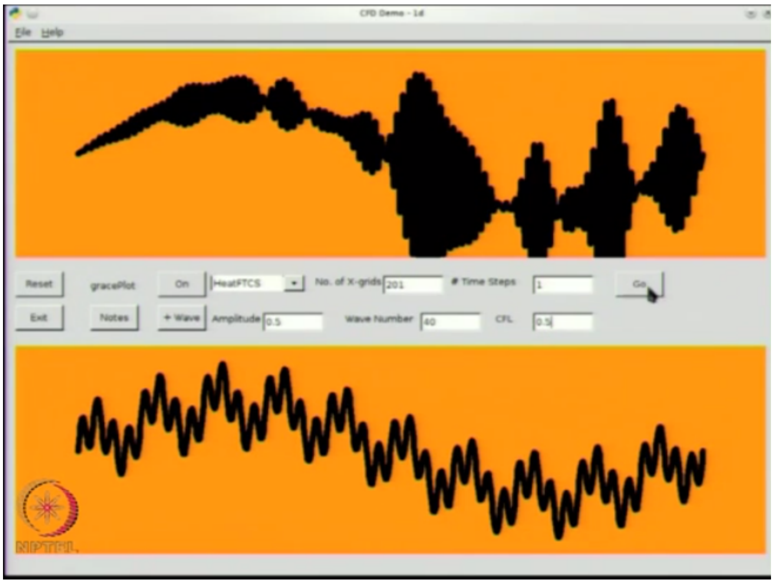

I go back to 0.5 and that seems to have held it and we take 10 time steps at a time. So this is like a small struggle going on right. So high frequency somehow they grew, they took time to grow and they also seem to take time to go back okay but I am taking 10 time steps at a time, let me make one shot of a 100 here fine. Let us try something different okay let me reset this. So that was it 1, I had to push it to 1.

Would the same thing happen if I did it 0.6 or was that an illusion? After all 1.1 worked but remember here there is no advection, it is not being carried out, whatever you have is there okay.

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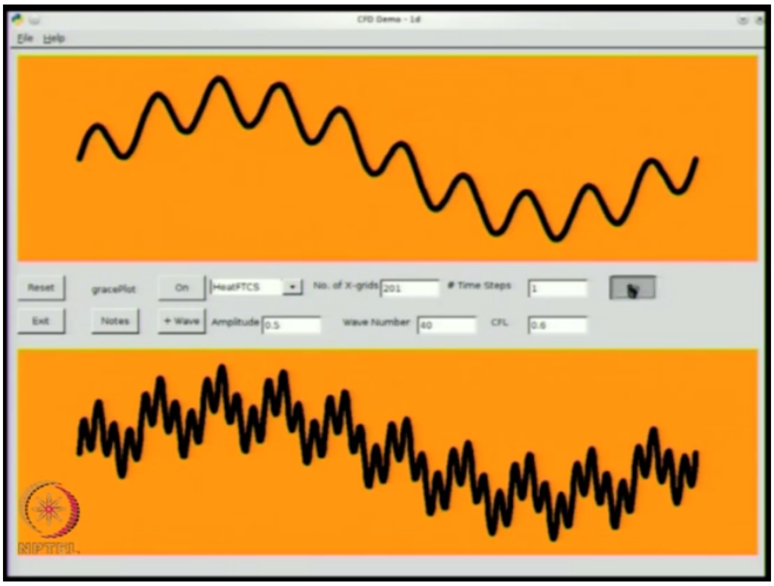

So that was it, yeah it seems to decay and just like in the case of what do you call it in the case of CFL of 1 you do not usually call it the CFL for the heat equation. I do not know, yeah there is something some activity right, it is really strange and then there is this temptation saying oh maybe just before it you know blew up, I could take that as a solution, all sorts of your mind starts playing tricks on you right.

So yeah go back to 0.5 and yes the behaviour is about the same.

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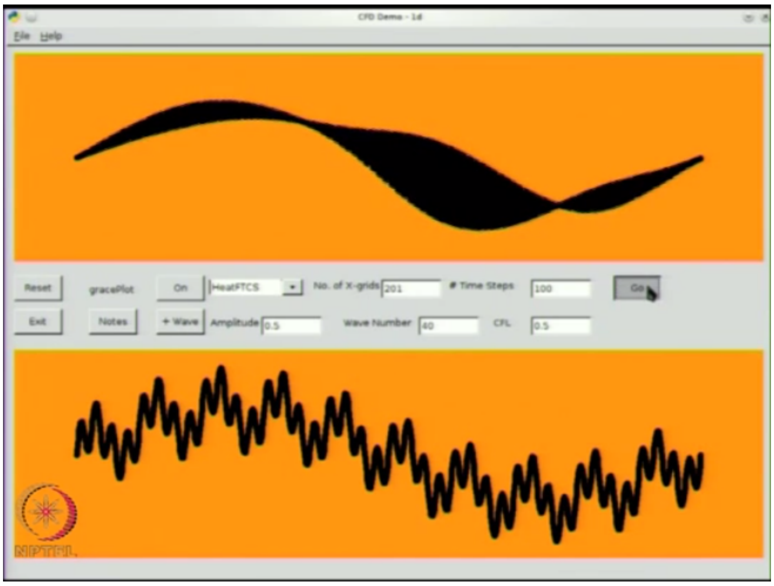

I will take a 100 time steps and yes there seems to be some kind of strange propagation come okay. So here we go what do you expect will happen if I try something smaller instead of 0.5 I try 0.1, you want me to try 0.1? I will take 1 time step.

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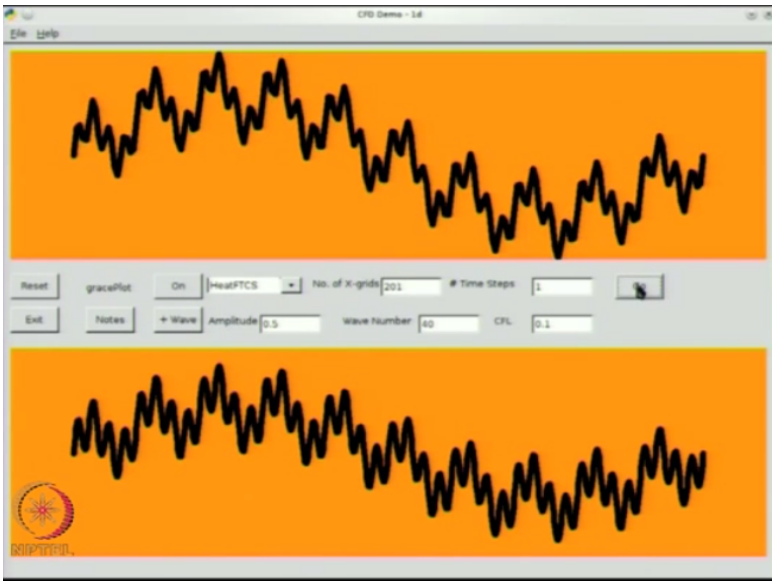

So as you would expect, it is fine. It seems to work okay. As you would expect, it is fine and seems to work, it is decaying. If I take a 100 time steps in one shot, it goes quite fast okay. I am going to do something little funny now. So what I will do is I will start off with 0.6, so I want to know is it decaying faster than 0.5? Is 0.1 decaying faster than 0.5? I mean after all it is heat equation, it should be the same.

So what we will do is we will do this 0.6 business okay, one shot we are there.

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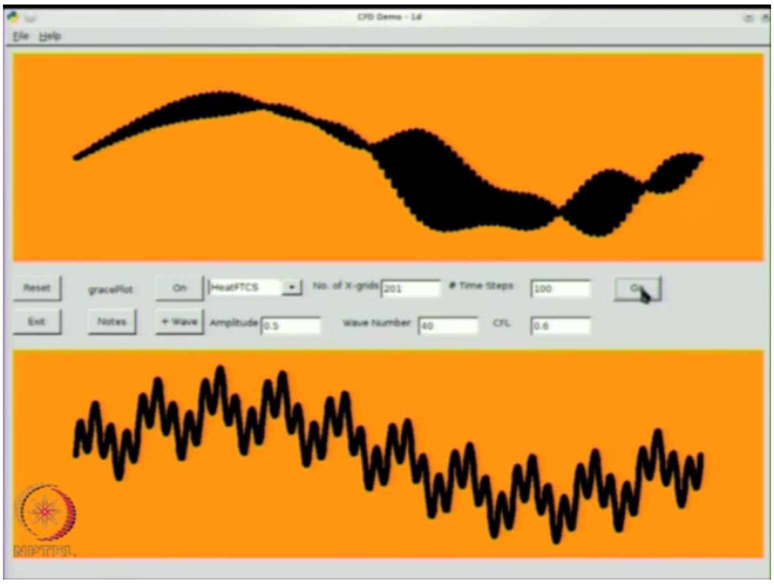

And then I will change into 0.1 and it goes quite fast right okay. So 0.1 is definitely decaying faster. You remember the modified equation for heat equation had fourth derivative, fourth derivative is really, really fast right especially on those high frequencies fourth derivative is really, really fast. So it just completely knocked it out right, whatever you had it completely knocked it out and now we are struggling with this okay.

The reason why I am harping on this thing is I really want to emphasize this, this is very important for us. Now we are struggling, our convergence it looks like we basically depended on especially if you are looking only at the steady state solution right, you understand what I mean by looking only for the steady state solution what I am proposing is what I am saying is just like we showed that in heat equation marching in time is same as sweeping Laplace's equation in space right.

It is possible that you are looking for the steady state solution right. So you have the time derivative and you start marching in time and you say I want to wait till I get to the steady state solution and there is a small error I need to get rid of the small error. Well this is the small error and the small error is supposed to go to 0, it is not going to 0 okay fine. Before I change this let me do one more.

Because I do not know if you remember this, I will do 0.16666. If you go back and look at the modified equation right that had a 1/6th right it had 1/6th so let us see what it does if I take 1/6th.

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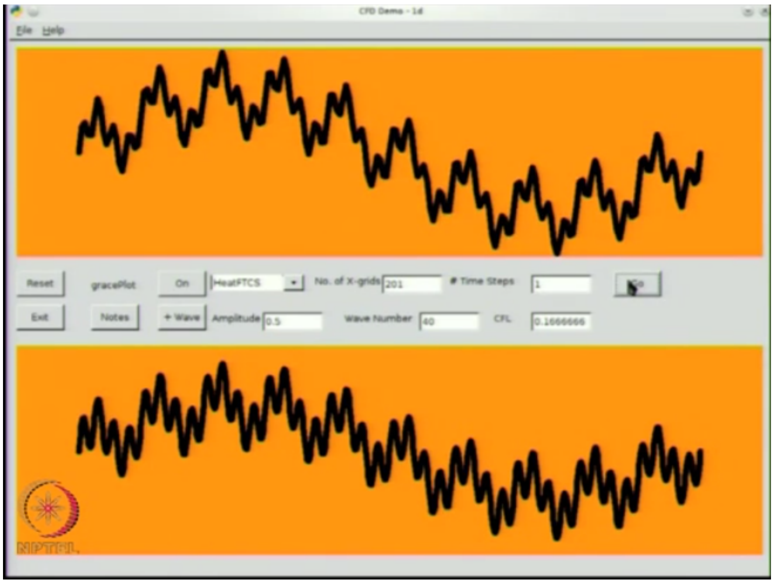

Behaviour does not seem to be that much different from what I can make out right. So there will be 6 derivative terms, 8 derivative terms. So there are higher derivative terms. So you just have to see at 1/6th as to whether it is converging faster, not converging faster okay. That is as far as let us look at one that I have not done so far which is FTCS2 I called it. So this is FTCS with the sigma squared/2 that term added on second derivative that term added on.

So that it explicitly knocks out, it explicitly knocks out the second derivative term and FTCS which we identified. We pointed to that and saying oh this coefficient is negative that is why it is diverging. So first of all, if I do this and I run for a CFL of 1, this should work, it should look like FTBS. What do you say? Does it seem to behave like FTBS?

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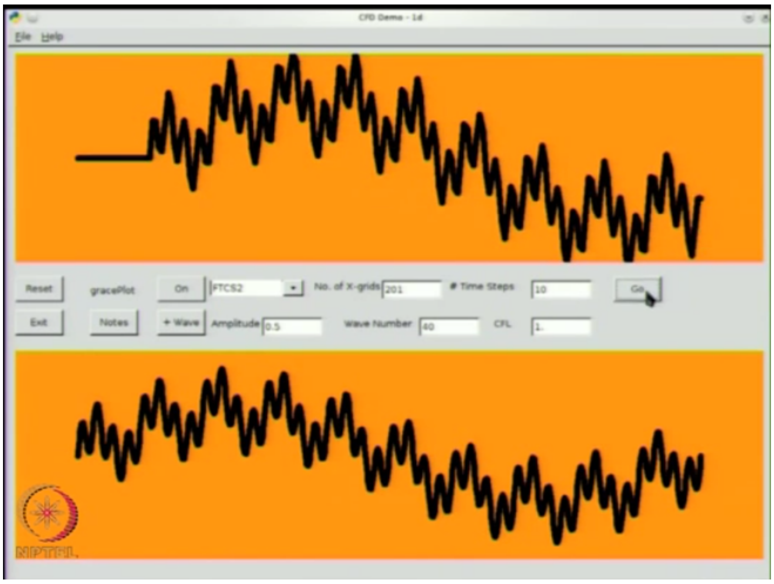

Yeah, there is not decay, no, nothing. There is no visible decay, nothing of that sort. It seems to behave just like FTBS. Is that fine? Okay. What happens if I lower the CFL? But you want me to raise the CFL and see what happens first. Let us take one-time step at a time, erase the CFL.

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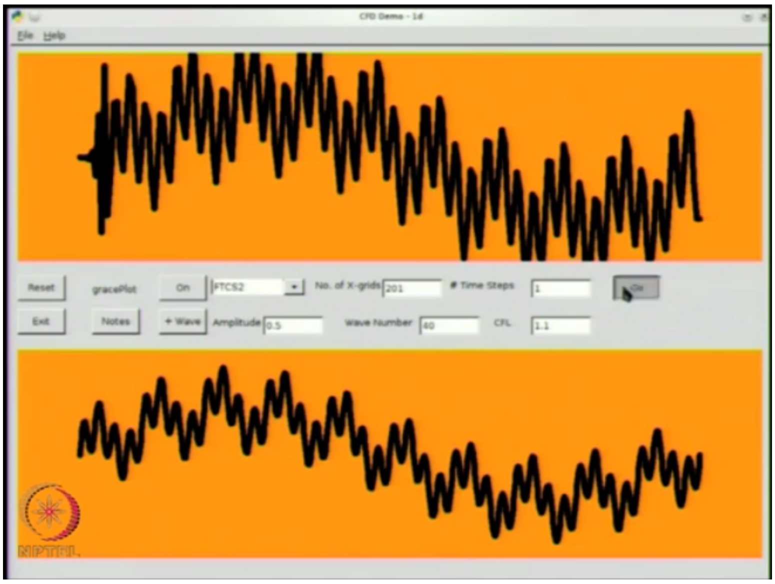

So yes, you can see that the 40x term first of all was growing but in a minute enough of it came in from the left hand side is very clear that enough of the right it is very clear that high frequency content is also growing. So okay so it seems to have that behaviour from FTCS and FTBS. What if I try at CFL 0.5?

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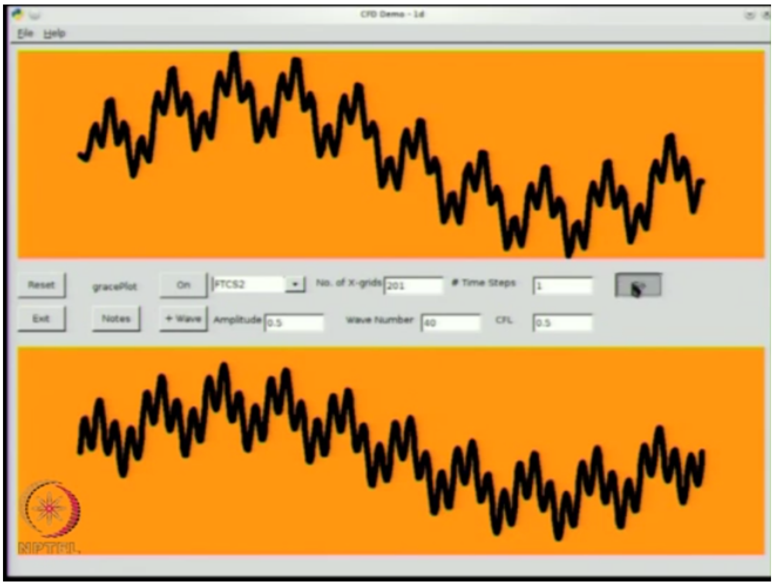

Look at what is happening carefully. I mean can you make out what is happening? There is something funny happening here. The shape of the peaks and curves, they are changing but they are changing in a unusually slightly different fashion from what you saw earlier. You want me to run it again. I will see as if we run our smaller CFL I think then it will become more clear right.

Because it is sharp there are now look at what happens to those sides? It seems to be some kind of rotation kind of a thing going on. Is not it? See I do not know. Can you make out? Okay maybe this does not make sense. So yeah, it is decaying, high frequency is decaying, it is also translating very much like FTBS. Let me try CFL 0.1 and see whether we are able to get something out of this.

CFL 0.1 so allow me to take 10 time steps at a time right, CFL 0.1 10 time steps.

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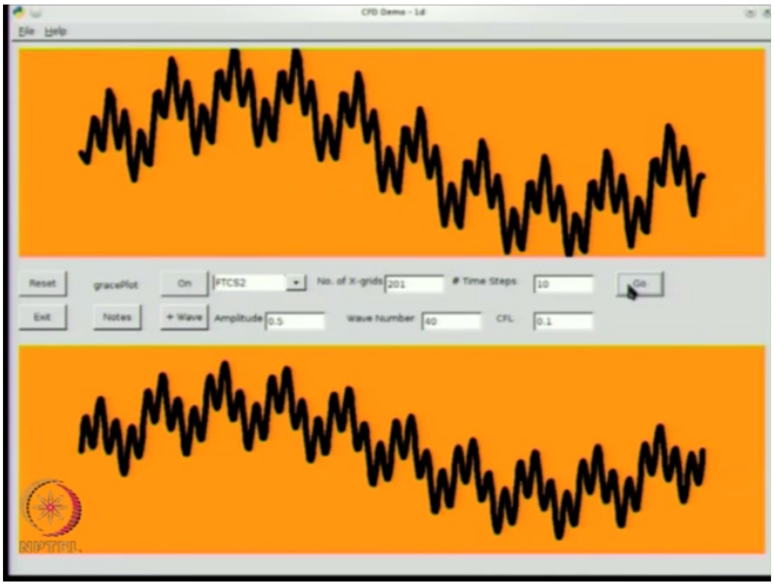

Can you see that kind of, it is not just translating, it is not just decaying, there seems to be some kind of a little rotation like thing going on. Is it apparent? Okay so this is a strange thing. What do you think is happening? This is true dispersion okay; I have knocked out the second derivative term. There is only a third derivative term and higher derivative terms. The third derivative term now has a coefficient that is quite large okay.

But it has a negative sign so basically what is happening here is the low frequency is traveling faster than the high frequency. The low frequency is traveling faster than the high frequency. So what you would expect and what will actually happen is as we progress with this you can see that here the high frequency is being left behind. The sin x term and the sort of sin 10x term are going and chugging along at their own pace right.

It is just that the sin 40x is sort of being left behind, of course it is also decaying, the magnitude is also dropping right because the term gets knocked out exactly only when we have fourth derivative term here, what we have? Maybe I should count, I will do a count. The high frequency there is completely gone and therefore even here it is completely going. I will do it with the count now so that 40x if you want I will add.

So by this point actually the amplitude of the 40x is almost gone and this is the 10x maybe what I will do is I will add a 20x also just something in between right. Let me add a 20x, I will reset that, we will take instead of 10 time steps at a time if I take 100 time steps what is going to happen with CFL of 0.1 okay.

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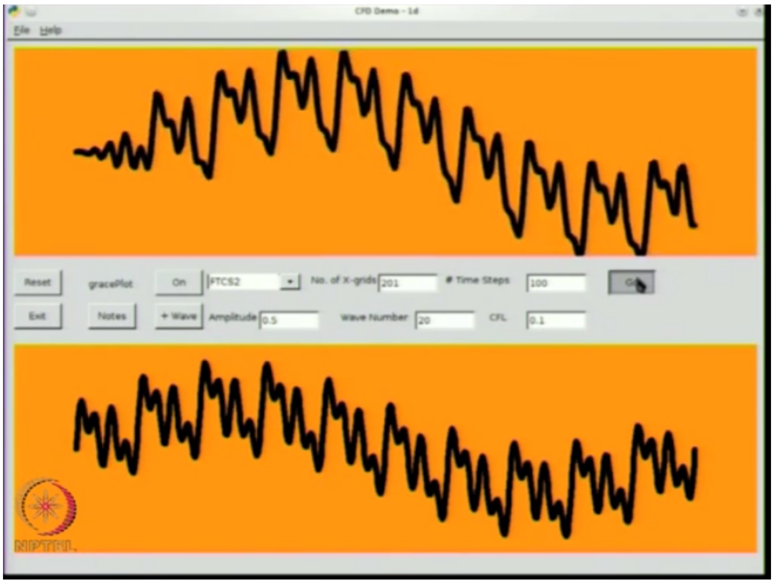

So that is half the sin wave that has gone, almost half the sin wave that has gone, half the sin x that has gone okay. The 40x of course decays quite fast and the 10x of course travels reasonably fast okay. So we have about a quarter of the original sin x term left okay. So by the time you go through and the sin x has completely gone, so then the sin 20x is left behind. Am I making sense? Is that okay?

Right so you could try it out see this is a case where it is dissipating, we could do the same thing with A of course, we could add a third derivative term explicitly and find out what happens right or you could do it with FTCS itself. If you take a small enough CFL it will turn out that it will grow. If you take small enough CFL, it will start to diverge but before it diverges you can see that the low frequencies are actually traveling faster than high frequencies fine that okay and any questions?

Okay so I think you could try this. Let me see what if I tried this with let me leave it with 401 grid points. See what happens here and I will take 200 time steps at a time, so CFL is 0.1 right. If CFL is 1, 200 time steps would take me half the way okay. It will take me half there but CFL is 0.1 so 200 time steps is going to take me, yeah it is going too small, you want me to take 400 time steps.

This may have other consequence, will see if it has other consequences, 400 time steps thing is definitely look a lot smoother.

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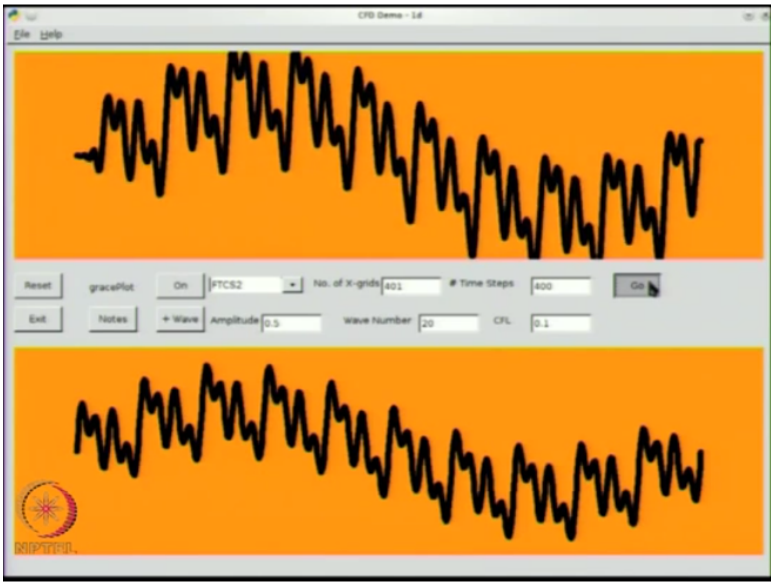

But here I do not know see I do not know whether in the video it is going to show up but here you can that something is traveling backwards. Your eye tends to pick up the big motion which is the sin x. So it looks alike because your eye is connected, your reference frame become sin x it looks like the rest of it is actually propagating backwards. So if you were to graph it, I do not know whether on the screen it shows right.

It actually seems as though something is propagating backwards because your eye tends to follow that sin x the big feature and the smaller features basically sent to travel back and they are in reference to the traveling backwards okay. So that I have lost count so I lost count the whole point of doing this for me to keep track of how many times I have done it but anyway it is okay.

So yeah the problem with of course smaller CFL's and larger what should I say grid size, yes your time that it takes to do anything is going to increase okay. So it is very clear that typically right now one lesson that you draw from this is if you are looking for the steady state solution, larger CFL is better than smaller CFL okay. That is one conclusion that we come to.

The larger CFL is better to run if you are looking for a steady state solution than a smaller CFL okay. If you are looking for the transient that is you are looking for a time accurate calculation right, then your delta t will be based on what is the accuracy that you are looking for within the stability limits right but bear in mind that if you just pick up a delta t you have to ask the question what is this stability?

What do I mean by the accuracy that you are looking for? That I am talking from the truncation error point of view but that is not enough, you can have dispersion and dissipation. I will give you a real life example okay. Just say your senses out there basically say oh there was an earth quake under the ocean somewhere right. You have used satellites come buoys to figure out what is the height to the wave that is there okay over the Bay of Bengal or the Indian Ocean.

As a consequence, you know that tsunami may arrive to the coast right. So if you have dissipation and dispersion that does not exist in the system. See now I am carefully wording it because the system itself may have dissipation and dispersion right remember the bag of potato chips right. There is always dissipation and dispersion. So if you have dissipation and dispersion that does not exist in the system then you may mispredict.

What is the size of the wave you may mispredict when it is going to arrive? If you mispredict the arrival time, it has serious consequences. If you mispredict the height of the wave then people may say oh there is this wimpy wave that came in and they warned me and they told me to come back, so the third or fourth time they may not evacuate when you tell them to evacuate.

Am I making sense? So accuracy is not a matter of saying that oh I have got delta t squared accuracy, you know I choose 10 power -6, I choose a microsecond or a nanosecond to predict, it is not enough, you have to make sure that there is not dispersion and dissipation because clearly dispersion and dissipation seem to affect the amplitudes that you are getting. They seem to affect right propagation speeds.

Propagation speeds which in that example that I gave you will affect arrival times. Is that fine? Okay so I think we will take it as it given now if you wait for this essentially what is going to happen is all the low frequency components were essentially gone and all that has left behind is the high frequency components which will by the way eventually propagate out in case you are thinking they will also eventually propagate out.

And you will get this correct steady state solution right. So if you are looking only for the steady state solution yes this scheme is going to work. It is just that in between you got oscillations that you would not expect. Is that fine? Okay so there are whole host of conclusions that I have come to from these demonstrations. They are all very important. They are going to show up at various times.

I am going to use this information at various times in the course as we go along right to either introduce techniques or introduce ideas okay. Is that fine? Right are there any questions? Okay so tomorrow what we will do is tomorrow we will look at the quasi-linear wave equation which is more closer to the kind of equations that we are used to right. I am not going to spend a lot of time on that.

Because I want to get very quickly to at least one-dimensional flow, so that something looks like the Euler equations that you guys are used to solving in your gas dynamic classes at least right okay. Is that fine? Right, thank you.