Introduction to Computational Fluid Dynamics
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Lecture - 21
Demo - Modified equation, Wave equation

So, has any questions? I have not been starting my classes without asking any questions. Some of the students might think okay. There are no questions. What we will do today is, we are going to look at, I am going to do a couple of demos. We will see how much we are able to get through in this class and depending on what we are able to do this time around, it will spill over to the next class fine. The 2 sets of demos that I promised was one using a symbolic manipulation package.

It is called maxima, this is a useful package, I use it as and when things get a little messy and I need to use something. I typically learned, relearned whatever I need I do it and then since I do not use it for a long time, I tend to forget. But, there is a reasonable logic to. So, I will just show you how this package works. I will spend a few minutes showing you how this package works and then we will look at the modified equation right.

For first order linear one dimensional wave equation. May be even for heat equation. And we will see what happens if you add terms to it. How to go about, how would you use this package and how would you get all those terms right. The second demo if we have for time will be the actual solution of the wave equation using FTCS, FTFS and so on. There is behavior that we have predicted saying that, how the code is going to behave.

We will see whether it actually behaves that right. What do we expect, what do we get? So, let me just start with maxima. I am running a version of it called wx maxima. This is public domain software that came out to a maxima, that was a commercial package earlier, gives us the wx frontend, which is reason why it is called wx maxima. What I will do is I will start that of.

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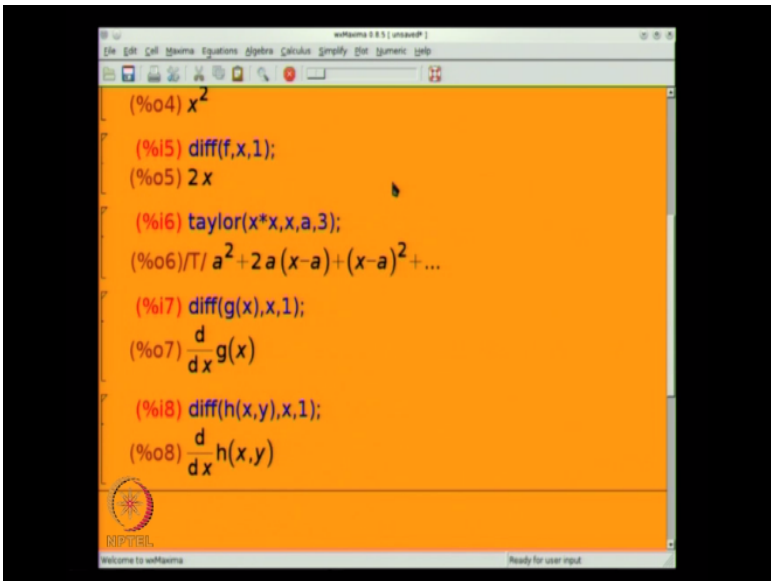

And for the sake of these demos, of course I have given it an orange background. Normally, it would be a beige background. The menus and all that are not important okay right. So, I think all of you are able to see this reasonably one okay. So, what can you do with this, what is a point here? You can essentially do some kind of manipulation right. So, you can x*x+2*x+1, you can manipulate in many ways.

So, for example see, I am just doing these so that you understand what was the percent basically means the last result right. And all the results as you can make out, they are numbered. I do not know how well it will come out on the video but, they are numbered. But, you can say factor percent, which is the last result and indeed, it gives you x+1 whole squared. Do you understand? These are trivial things. This is all school algebra right.

It is not a big deal. So, you can also define functions, the way you would normally define functions. And what we want here is, you can differentiate, differentiate f with respect to x ones right. You can differentiate, you can integrate ways. You can of course, there are other things that I am not going to talk about right now, which we may talk about in the future demo. You can basically do algebraic manipulation and calculus okay.

That is as far as I will restrict it to that as far as we are concerned. There is lots of other nifty things that you can do, that you can find out for yourself okay. You can do Taylor series expansion for instance, see use can say Taylor, x*x and you can expand with respect to x about a how many terms do you want? 3 terms. And it will give you Taylor series expansion right. Whatever it is function that you will get. Which is a kind of thing now you see where I am going, modified equation.

Taylor series is very important. So, yes, you can define functions. So, you can just basically say g of x so, if I take a derivative of and this is a indeterminate function in the sense that I have not defined the function right g of x and I differentiate g with respect to x ones, it gives me dg dx fine okay. So, you may say what is the big deal, it is just notation but, its ability to manipulate notation, which is what algebra, calculus is all about right. It is the ability to manipulate symbols.

So, because this d/dx is a bit cumbersome right. I mean for example, the reason why I say so, if I say if I differentiate, what shall I differentiate, I will differentiate h of x, y with respect to x ones right. So, it should be a partial derivative. Of course I could differentiate that result one more time with respect to y. So, I can say different that result with respect to y ones right. And the notation clearly gets messy. So, this is a package that will make it simple.

I needed because, I am going to use this right in my little code that I have written for. The package called p different, partial differentiation, which uses a much more compact notation. So, effectively something like h of x, y. if I go back to this derivative, it will write the derivative as h and this 1 indicates this is differentiation with respect to x. you understand what I am saying. And the 0 indicates, it is not differentiated with respect to x.

It is like the subscript notation, u sub x, u sub y, h sub x, h sub y. it makes life a lot easier. So, in the same fashion, if I were to use this, then the second derivative becomes h1, it is more compact. Notation is more compact and that is the only reason why I am using that package right fine. But, as I said, the whole game is about notation, the whole mathematics game is about notation and manipulating the symbols.

So, now we are set. So, what is it, what kind of, what do you want to do, you want to look at a modified equation. You can look at it for FTCS. I will show you a fragment of the code that I have written. It is not the cleanest maxima code but, I will show you a fragment of the code that I have written for FTCS modified equation right. The code can actually be, you will see, you will understand what I am talking about. We will go through it.

You just try to get a feel for. I am not saying that oh you have to go back and write maxima programs or something. That is not what I am saying. Just try to get a feel for it so that you understand the demos. That is all I want okay right. So, I go back here and in directory maxima, I have you can see various files that I have created called .mac files.

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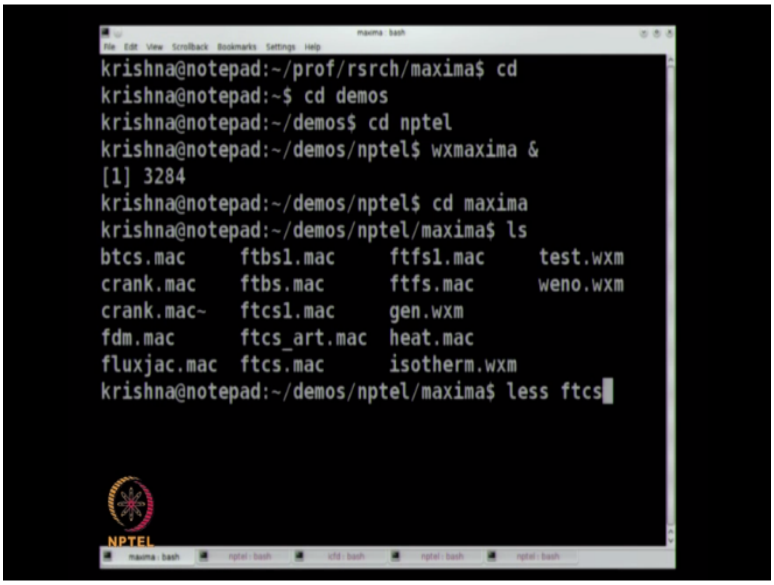

And what I will do is, I will look at the ftcs.mac okay.

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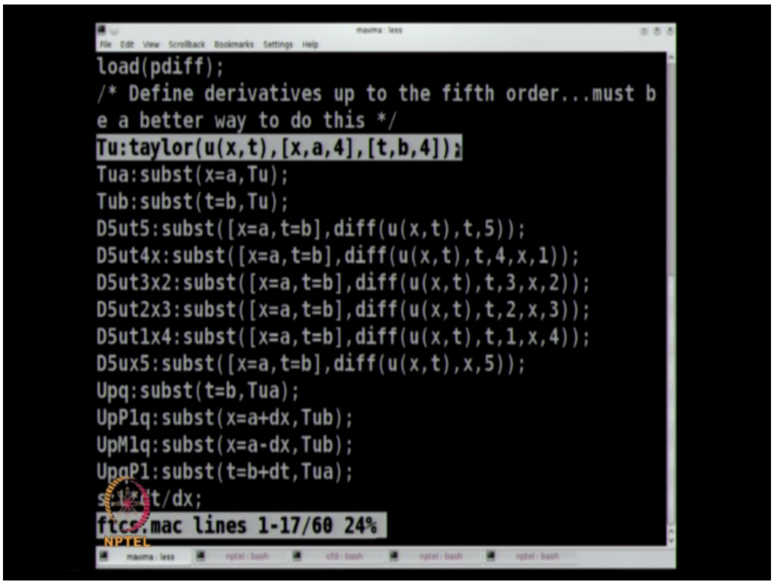

So, you notice, I start of, I load the pdiff right. Then in line 2, I am setting up. So, I will do these may be one by one and see what it does right. So, I have defined something called Tu, which is the Taylor expansion for a function u of x, t or wave equation as in space and time y of x, t. I am asking for 4terms in x and 4 terms in t right okay. Up to the 4th derivative a x and 4th derivative and t. basically I want up to the 4th degree.

I want to retain terms up to 4th degree. You have already seen that that 3rd degree, 2nd degree, 3rd degree, 4th degree they have an effect right. We have already seen that. So, let me just get back here stick that in here and see what it does. So, we will get the Taylor expansion right for this and it gives a mess.

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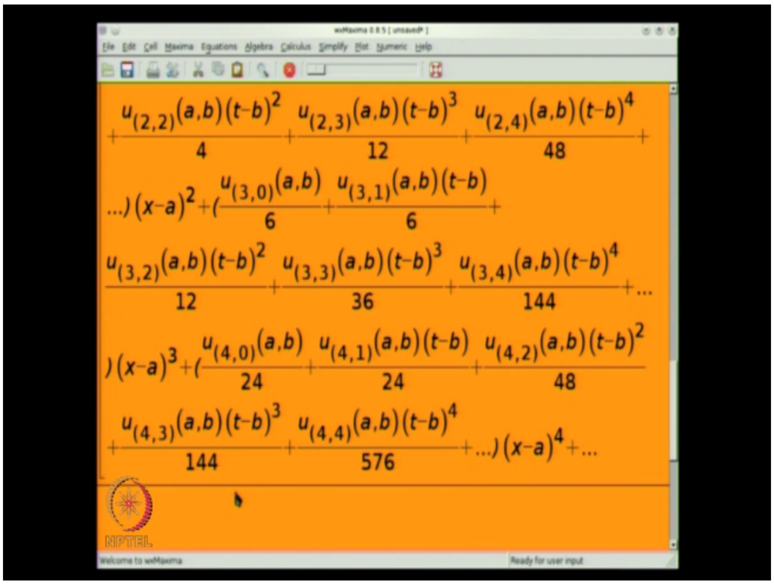

But, it is not that bad. Means, since you know Taylors expansion and Taylors series expansion in 2 dimensions. So, I am expanding about the point a, b. So, I have u of a, b time derivative times right t-b, the 2nd time derivative times t-b squared, 3rd time derivative times t-b cube and so on. 4th time derivative times t-b to the 4th okay. 1st spatial derivative times oh the x-a is outside.

1st spatial derivative 1-time derivative, 1st spatial derivative 2-time derivative, 3-time derivative, 4th time derivative whole multiplied by x-a. And it is doing it in a systematic fashion. This is not how we would write it. This is not how it was introduced to you when you did Taylor series in multiple dimensions. But, this is a program right. So, it is doing it in x0 first, and then x1 derivative 1, derivative 2, derivative 3 in a systematic fashion okay.

So, as I said so, this is a kind of thing that you could manipulate manually. You can do it manually right. And I have actually done it manually. But, it is also convenient to be able to do it in a automated fashion. And once you have confidence in that automation, you can at least sort of check out. Exploratory stuff you can do. So, I have defined something called Tua, which is convenient term which basically is the Taylor series expansion in 2d with x=a substituted.

Do you understand? Right. So, you are expanding only in time, Taylor series expansion only in time. X is substituted as a. So, as a consequence, you get something that is very small. Basically get a Taylor series expansion as one variable okay. I do not want to sort of vary out with this. But, there is a Tub similarly. May be I will just do Tub also just for fun. Tub and x-a right. It is the same thing. You get Taylor expansion about the point right a, b purely a x in this case okay in the x direction fine.

Now, what I proposed to do, so, let me stick with this for some time. I have defined a whole bunch of higher order mix derivatives here. I will tell you why I have defined them. I proposed to retain only terms up to the 4th derivative. I do not want to track higher derivative terms okay. The 2nd thing is, you remember from the modified equation what you basically did was, you substituted, you took time derivatives of the modified equation and substituted for those temporal derivatives in the sense that, you are going to end up if you keep taking time derivatives of the modified equation you are going to end up with mix derivatives right.

Modified equation has terms that are in terms of x and t and if you differentiate it purely with respect to t, and you will get mixed derivative right. And I want to just set my objective is to use the modified equation itself and derivatives of the modified equation to eliminate all higher derivative terms and time whether they are mixed or pure derivatives in time. So that I have only spatial derivatives on the right hand side.

The only temporal derivative I have is dou u/dou t that is what I want. That is the objective of the modified equation okay for this setup. So, all the other derivatives I want to set them =0, these are various powers that you get from mixed derivatives. So, I am defining them so that at a later date, I can say if this term occurs set it to 0 okay. That being done.

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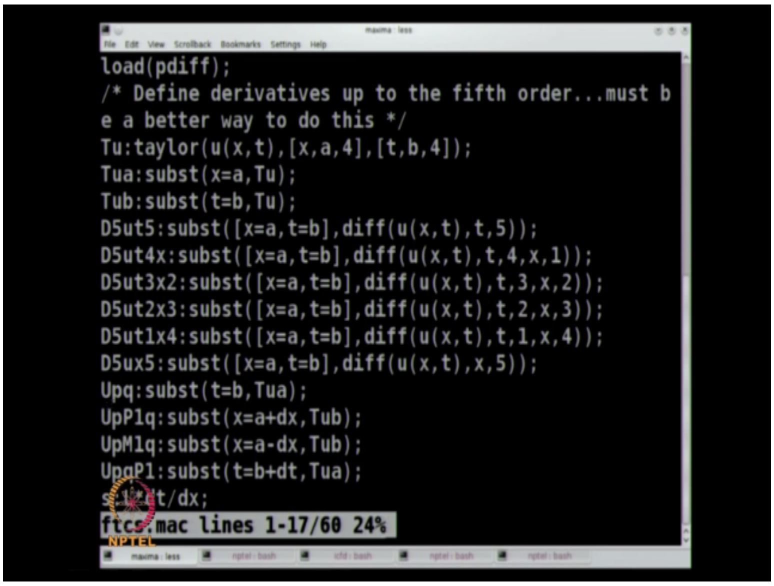

See, all of these is just what I would call set up time. Then I define 4 quantities upq. So, just read it the way it is written upq, I substitute right upq. Does that make sense; do you want me to show you what upq is? Upq, I will run that. So, the point pq is at the point ab right okay. So, upq is basically uab. Is that fine? We are approximating the differential equation at the point ab right. So, upq is uab okay.

So, up+1q so, I have a funny notation here up+1q. p+1 means x is a+delta x right. P-1 means x is a-delta x that is next one. P-1 means x is a-delta x right. Upq+1 is b+delta t fine. So, this looks familiar. This looks like what we have been doing so far. Sigma, that is s is the speed of propagation in this case is l delta t/delta x, that is sigma. And go to the next page.

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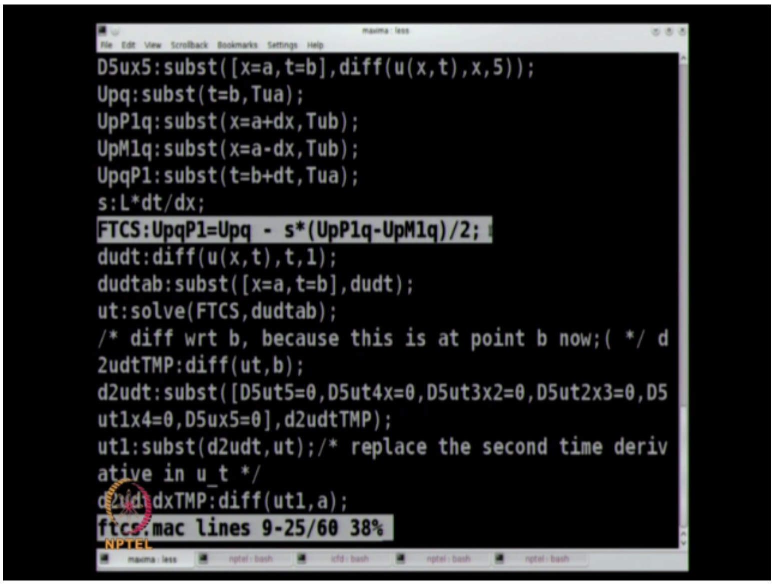

I have a lot of messy stuff. Let me see if I can get you something that you can see. Yes, that is the one. And this is what we have. So, upq+1 FTCS is upq-sigma times up+1q-up-1q divided by 2 fine, that is FTCS. And after this, it is all a matter of substituting Taylor series that is what I am doing now. When I define this FTCS, I substitute Taylor series into FTCS. So, what I will do now is, instead of paining you by cutting and pasting, I am going to run that batch file.

I will just run the batch file and will get the modified equation right at the end. And then I will tell you what it is that we have done. So, FTCS I open it, it runs through my script and comes to the end okay. This needs a little explanation because I made some substitutions and all of that stuffs.

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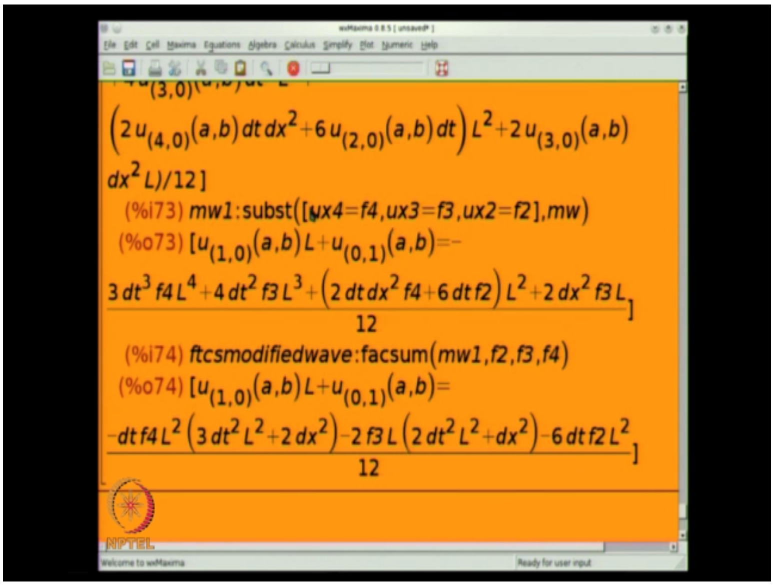

In fact you can see that, up here I have substituted for the 4th derivative, 3rd derivative and 2nd derivative I have called them f4, f3, f2 that is an operational reason. It makes it more compact. So, I have just called them f4, f3, f2 right. So, the 4th derivative with respect to x is f4, the 3rd derivative with respect to x is f3, the 2nd derivative is f2. Is that fine everyone? Okay. So, here we have it. What is this? L dou u/dou x right. I hope you understand now that, this is l dou u/dou x, this term is l dou u/dou x.

This I dou u/dou t. so, what I have on the left hand side is my linear wave equation dou u/dou t+l dou u/dou x. What I have in the right hand side is the terms that I have carried so far. There is 4th derivative term, there is a 3rd derivative term right, this is the 4th derivative term, this is 3rd derivative term and this is a 2nd derivative term. There is a divided by 12 so, you have to appropriately take care of it. Is that fine? Okay.

So, the cheat sheet that I used that day and wrote it out was basically from here right okay. So, I just basically took it from here and wrote that out. Let us look at but see, how did this come? How did we get here? So, if I type if I go to FTCS, which I had defined right in the beginning right. So, this FTCS is basically a substitution from the various terms right in our discritization and from there I have created a whole series I have solved for dou u/dou t because I need dou u/dou t right.

I have to get dou u/dou t differentiate it ones I get dou squared u/ dou t squared substitute back, differentiate it one more time get dou cubed u/ dou t cubed substitute it back right and I systematically go through trying to eliminate and each time what do I get? Each time I get a dou u/dou t, which does not have certain terms in it okay. So, I called dou u/dou t, I call it u sub t o tut okay. So, I solve for ut from the previous equation and this is what I get fine okay.

I am going to skip a few steps here. Because otherwise I wear you out. You will become very tired right. This is too much deep, there is a lot of detail right. So, this is u sub t I just want to show you so if I go to ut1, which I generate in between. So, from u sub t my very first one, what does it have? It has a 3rd spatial derivative with respect to x, 1st derivative with respect to x, 4th derivative with respect to t, 3rd derivative with respect to t, 2nd derivative with respect to t.

When I come back by the time I have come to ut1, something funny has happened. The 2nd derivative with respect to t is gone. That is what I did. I eliminated it. The 0,2 term is gone. If you stare at it long enough, you will see it right. The 0,2 term is gone. But, I have been gifted in exchange for that a 1,1 term a cross derivative term. So, now, I have a headache that is the reason why this is a pain. You have to be systematic, you have to be organized right.

I managed to get rid of the time derivative, the 2nd time derivative but, I have got a mixed derivative. So, now I have to take the dou u/dou t differentiate it with respect to x and eliminate the mixed derivative. You understand? So, you have to go through this in a careful fashion. It is like solving a system of equations. You have to do the elimination process in the systematic fashion. I will show you one more, sometimes it is not quite an improvement right.

So, you could sit down, so you can see that I have the 2nd derivative, I have a 3rd derivative which is next, a 4th derivative which is mixed the other way around right xt cubed and so on. Then, I have all these other terms that I had. So, I just eliminated. What did I eliminate? I eliminated the 1,1 derivative right. So, you can see that you go through systematically, eliminate all of these and every time you differentiate, you get a 5th derivative term, which you do not want.

You set it =0 right. Every time you differentiate, you will get a bunch of 5th derivative terms. But, you know the nature of those terms. There are only so many terms. So, you set them all systematically to 0. That is what I have done. And finally got the modified equation. Is that clear? Right. If you want to do it manually, that is what you would do. You have to be very organized right. In fact, I apologize for my last quiz but, it was a dreary quiz.

Because, there is a lot of key strokes involved right. So, if you look at the time that you have 3000 seconds and the number of key strokes that you had to do, you will see the key strokes per second was quite large, you do a fraction but it was quite large. That is the key. So, it gives you an appreciation for it is an anticipation of demos like this. It gives an appreciation for why we use these kinds of things okay. It is a headache.

We cannot sit down systematically do right, a whole set of these calculations manually right. It takes a lot of effort. It takes a lot of care fine okay. So, that is basically what we have. In fact, I was just thinking whether I should narrate an experience of my own as a student I was given an assignment to teach me the same lesson. Of course, you had it sort of in the quiz. To do the determinant of a 7 by 7 metrics by hand. I was supposed to do it.

The long way it was an assignment 1 out of 15 assignments. Suppose to do it the long way and show all calculations. 5040 calculations. I learnt a lesson. I cannot do 5040 calculations in a rho without making mistakes. I did it once, I got a number. I was not sure. I did it a 2nd time, I got a different number. So, then we do the standard engineering test. Do it a 3rd time, it has to match one of these. It did not. Not only it did not match one of these.

The numbers were diverging right. So, at that point I stopped saying that so, my assignment basically said which apparently what the teacher was looking for. The conclusion I had was I cannot do 5040 calculations in a rho without making a mistake right. So, there is but when we do symbolic manipulation, you can be a little more careful because they are patterns. These kinds of things we can do actually. There are patterns but, package like this will get into drudgery right.

But, I would always cross check. Is that fine. So, we have FTCS if you want to see that I can show you a few more of these, the penaltmut.

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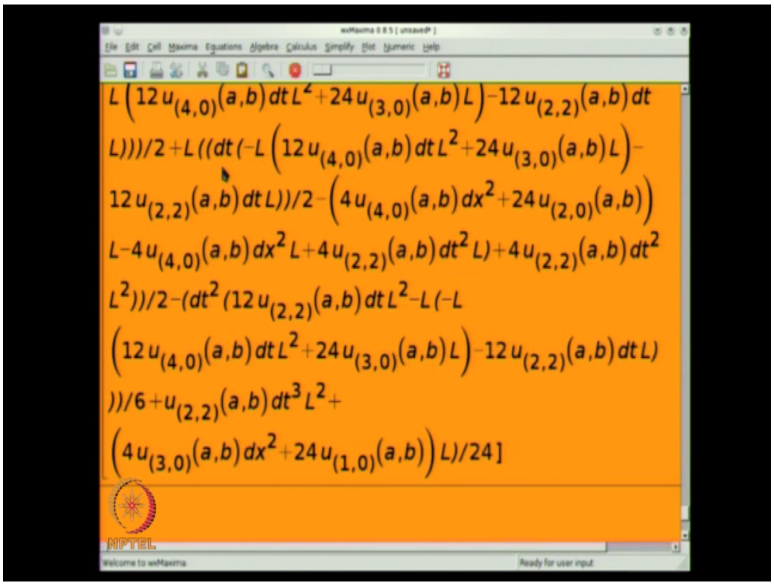

So, you go to ut8, you see my God before it simplifies it gets really bad right. Here now, I have made substitutions, differentiated, made substitutions, you try ut9 right. So, there are a lot of these but this is just near that. And finally let me see if have mw. See, that is a modified wave equation I have already simplified it a little right. And then I decided that I wanted to gather terms, collect them and put them in a little more easier fashion, which is why I created the way f3, f2, f3, f4 so, that it came out in a compact fashion fine okay.

So, that is the sequence basically. You sit down, you systematically eliminate okay. And as I said I am sure there are better ways to write this program than the way I have written it here. Are there any questions? So, this is a pretty straight forward demo. There is nothing spectacular about it. I can try if you want me to try something else we can look at FTFS okay. And FTFS gives you.

I have shown you these terms so, this is the 2nd derivative term here right. 2nd derivative term is multiplied by sigma-1 right a sigma-1 in our notation. I used L here, because when I was doing this derivation I was using lambda, dou u/dou t+lambda dou x. so, you can see that the terms, in this case I am using the modified equation to eliminate the higher order temporal derivatives okay. So, you could also do it using the wave equation itself.

So, there is something that can try out fine. So, that is the 3rd derivative time. So, you know that any scheme that does this is going to be dispersive, it is going to be unstable because there is a negative sign in front of the 2nd derivative term FTFS is unstable. It is going to be dispersive because it has a 3rd derivative term right. The existence of the 3rd derivative tells you it is dispersive and the existence of the 4th derivative with a negative sign could stabilize it but, we do not know.

Or analysis showed that it is unstable. We have to actually run it to see what happens. Is that fine. We will move. Shall we look at something else? What would you like to look at FTBS I think is something that we have done. so, there is FTBS.

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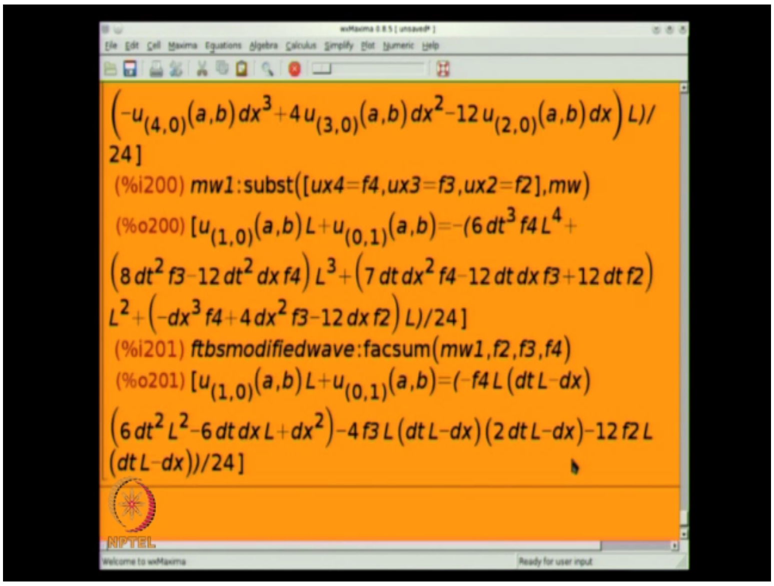

And again just to bother you, one last but one time I am going to do this one more time. So, you have the wave equation there dou u/dou t+L dou/dou x, FTBS as we suspected has a 4th derivative term. It has a 3rd derivative term it is dispersive and yes it has a 2nd derivative term there should be a negative sign outside somewhere here fine. That is sigma-1-12 where is there a negative sign? Unless I have made a mistake. Yes, that is something that you can check up.

Half hand I cannot seem to make it out right. You can look at that file. So, FTBS.mac upq-up-1q right and I am substituting and solving for this term there. I am actually solving for dou u/dou t right from the FTBS term. Is that fine okay. So, indeed I am doing FTBS it is a long painful program. But, you finally get here. Let me look at mw1 just to make sure that I do not have a problem. Yes, this is one of the reasons why I do the factoring because, you get these terms that are.

So, this is positive, there it is negative. This is sigma-1, it is not 1-sigma sigma is<1 there is negative sign in there somewhere okay fine. For a minute there I was a bit concerned. So, if you look at the earlier one, see you should not let me get away with this right. If you look at the earlier one, if you look at the one we had. Let me do FTFC just to convince you that I am not cheating you okay. The sigma-1 is what does it.

So, I go through do FTCS and here it is a sigma+1 that is the thing that sign makes a difference right. Sigma<1 so for stability. So, that is where that condition comes from okay. So, yes, you can get the modified equation what else can you do with this? Well, I have one file that I have created just for this reason and if you want we can clear around with that file or we can. So, I have got something where I have added artificial dissipation.

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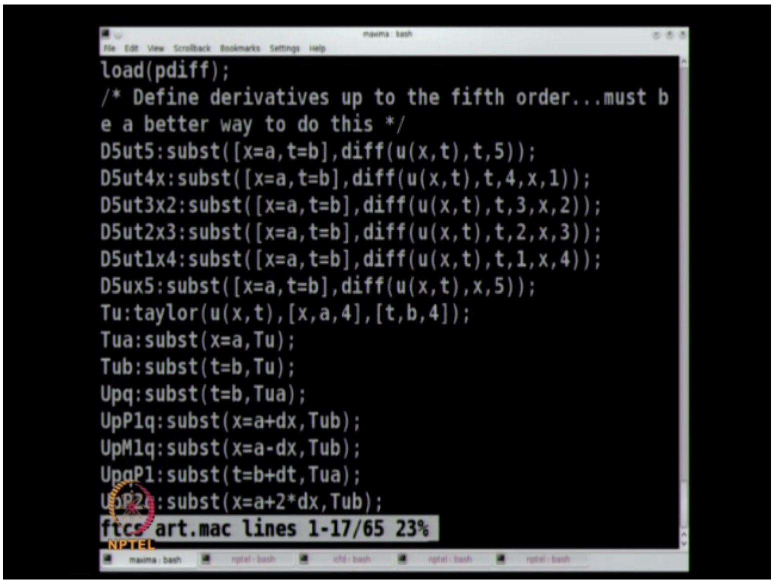

I have soothing of where I have added artificial dissipation. I am doing the same and you can see that what I have done is, I have added a sigma squared up+1 q I will just select that. Sigma squared up+1q-2upq+up-1q divided by 2 fine. I am just doing FTCS, I just add that extra term and then you can ask the question, what happens to that. You can actually ask the question what happens FTCS if I add that extra artificial dissipation term.

And that is the need thing here. Now, we can do this what if kind of right. You can play around. And what happened? F2 disappeared the 2nd derivative disappeared. So, I figured out the term that I added was exactly the term that I needed to add. Sigma squared/2 2nd derivative, discretization of 2nd derivative. But, that is not the term that you get with the modified equation. There you get some sigma-1 right.

We just saw that -12, sigma-1/2 what is the deal? Am I making sense? See, if you add a seconds, so, you have to realize this. If you say that I am going to add a 2nd derivative term, you look at the modified equation and you say this is a 2nd derivative term. So, let us look at the modified equation again. I will go back and I m going to just rerun the modified equation because this just generates lot of. May be I have this is FTBS modified equation is that what it is called? Yes, there we have it okay.

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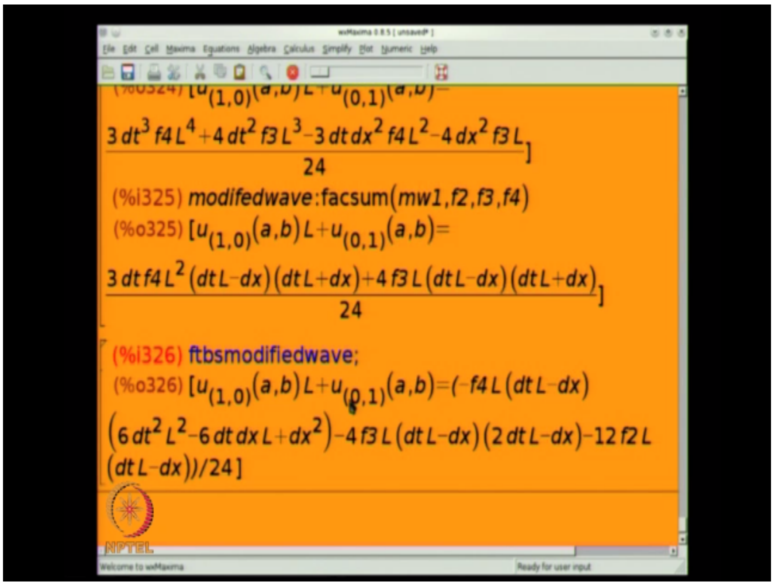

So, there is a 12, remember that is divided by 24 so, it is a 1/2. So, you can say wait a minute, why do not you add a times sigma-1 divided by 2 dou squared u/ dou x squared that is what you should add to eliminate the term. Why do not I add that? Why am I adding something else? Does the question make sense to you? Do you understand what I am saying? I want to eliminate this term.

In order to eliminate that term, I should just add that term to the modified equation. In order to eliminate this term, how come I am adding some other term? The key is, in my discretization I never add dou squared u/ dou x squared. I do not add the f2 term. I add a discretization of dou squared u/ dou x squared. Now, I have a new discretization. I find the modified equation for that. Am I making sense? I find the modified equation for the discretization of I do not add dou squared u/ dou x squared.

I add a discrete representation of dou squared u/ dou x squared. I do not add dou squared u/ dou x squared it is important I add the discrete representation of dou squared u/ dou x squared. I get a new improved modified equation which will still have the second derivative term okay. And if I go through a systematic layer say okay I will add that, add that, add that and I keep on adding extra terms. All of those terms will combine into sigma squared/2 dou squared u/ dou x squared.

That turns out to be the modified equation term that you get if you would use your original equation. That is what I was trying to say. I wanted to, you need to think about that. You need to think about this. It is not that difficult but it is because there are so many different equations that are there you have to able to knock it out. I mean you have to figure out how you knock out that term okay right. So, my suggestion is, if you try this out you know how to knock out the 2nd derivative term.

See, if you can figure out how to knock out the 3rd derivative term right. I have knocked out the 2nd derivative term see if you can figure out how to knock out the 3rd derivative term. Is that fine okay. Are there any questions? So, yes this is basically what we have as far as this is concerned. There are no questions may be then I will go on to the solution of the equations and see what is the effect that these modified equations have on the solution of equations okay.

So, to that end unless you want to try adding other artificial dissipation terms to see what happens. You want to modify this thing and see if it makes a change? If you want to add any other term you can add any other term. May be we will do that. Why do not we do that before I go on. Let me just edit I will change this okay. So, I will copy that ftcs_artificial dissipation to FTCS I will just say NPTEL. It looks like I do not have entered the last term.

What do you want to do? You want to make this sigma-1 and see what happens right that is what we had. Make it sigma-1 and see what happens. May be I should have done that first. But anyway it is okay. Next time I do this. Insert open bracket 1-s because I have a + sign in the front okay. Fine everyone. That is what it is. Speed*1-s divided by 2. I would not save it I will just write it in case you want to make a change let me reload that batch file and see what happens. I called FTCS. Yes, it got lot messier.

Modified equation got a lot messier right. There is the 12 f2 term that is still there. But, this is what happened. And that does not look like anything is going to cancel there. Am I making sense? So, as you can imagine, as you go through each time you made so, you can now potentially turn around and say no no I will add this term and back and say I will add this term to it fine. That is what I meant. So, in order to get rid of it, you have to find out the actual term that you need to add in a discrete form.

You want to find the actual term that you need to add in a discrete form. That is involving p+1, p, p-1. What are the terms that I need to add? Linear combination of p+1, p, p-1 so that the modified equation does have a second derivative. That is really the question we are asked okay. And it turns out that it comes from sigma squared/2 dou squared/ dou x squared for the wave equation. For every equation we will have to figure out what it is. Is that fine? Okay.

There was one other thing that I promised to do for you. Unless you guys want to try something else out I will go head I do not know. What was the difference between central difference and forward difference, do you remember that? That was just a sigma/2*delta x actually right. So, let us see. So, we have a sigma*dx/2 that is what it was okay. If you go back and look at it that is what it was. I will write that file. See what it does. So, I run the batch file. I will run it again. And this is what you get.

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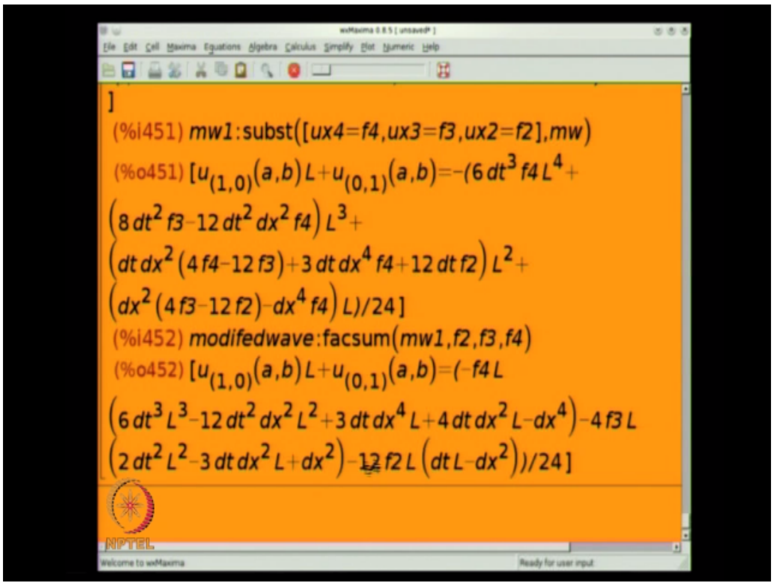

Hang on. Let me make sure that am I doing this to FTCS? Yes, I am doing it to FTCS fine okay. So, this is what I get. Does this look familiar right. You look at this. Let me see if this works. %-ftbs modified wave. I am not sure if I have done that right. Sometimes I may just look at ftbs modified wave. Ftbs modified wave should basically give me that and that were essentially the same okay. Here I seem to have some extra terms that I will have to possibly simplify. Fine okay.

So, yes may be I will get back. I will do this ftbs-modified wave and see what it is doing okay. So, if there are no questions what I will do is, I will just start. I am sorry there is one last thing that I wanted to do, which was something that I have not done in class. It was a modified equation for the heat equation. And there is a reason why I want to do this.

I am doing this simply because when we did the modified equation for the wave equation right, I connected up the linear stability analysis that we did for the wave equation, for ftbs, ftfs, ftcs I connected it up with the modified equation and the various terms appearing in the modified equation okay. And I basically showed that if the 2nd derivative term was negative, the coefficient of the 2nd derivative term was right what you are adding was negative. That you had instability that it corresponded right. There was correspondence.

So, I do not want to leave you with the feeling that on I just have to do the modified equation which is a mess but, I just have to do the modified equation and I will get the same stability condition that I get with the linear stability analysis okay. So, to do that I know an equation where it does not actually work out exactly that is the heat equation. So, we will do the modified equation for the heat equation so that you will actually see that does not work.

So, I run the modified equation so this is just sort of things streaming pass heat equation heat equation is very simple right. It does not have the odd derivative terms. It is nice dissipative system. Heat equation is very simple dou u/dou t that is the 1st term, dou u/dou t-l dou squared u/dou x squared that is heat equation=there is no 3rd derivative, no dispersion. There are not going to get dispersion. It is not there. And you can get a 4th derivative right. Am I making sense? So, you can ask the question, for stability, the 4th derivative has to be negative.

There is a negative sign in front. So, you can ask the question when is this positive. When is this thing in the bracket is positive? And it does not give you the same stability condition that linear analysis gives. The stability analysis that we did by substituting exponentials cosines and sines gave us a 1/2. This seems to give us a 1/6th. Am I making sense? Okay so, you have to have an awareness. So, if it <1/6th it will work. But, you can actually go up to 1/2.

The guarantee is still works. If it is <1/6th, yes it does work. It is just that you cannot actually go to a larger value okay. So, that in itself should give you a clue about these stability conditions what they mean, what is the analysis, what is the result that we get, what we expect from the behavior okay fine. So, I think I will leave it. Of course, this is the 4th derivative. So, you expect that, as it gets small, it is going to get really bad okay. And we will remember this 1/6th may be when we run our codes okay.

Let me now go to the. I will just start of the other demo. We will most probably not be able to finish it in this class. we will go on to the next class right. I will start of the 2nd demo. This is for the wave equation right. Over the years these demos have evolved and now finally I have added a small sort of user interface to it. Not a big deal. So, what I will do is, I will run the demo and we will see how far we can take it in this class. So, let me quickly get that bright thing out of that okay.

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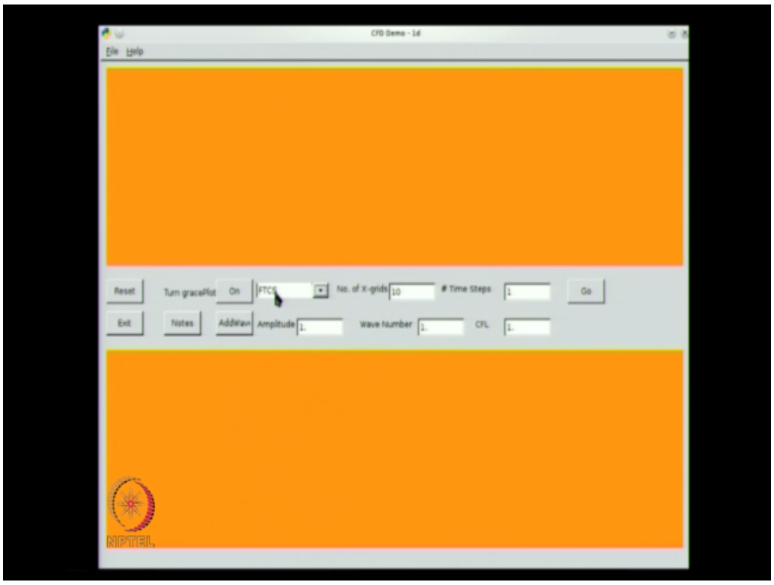

So, this is what I can do. So, some of this may not come out because the font is quite small. I was not able to figure out how to make the font larger right now before this demo. So, it does not matter. May be I will be able to fix it but, anyway we will see. Right now, it is ready to run ftcs okay. The number of grid points is 10. That is what is set here. The number of time steps that I take and every time I click this go button is one-time step.

The CFL that I am running for, remember what I told you. So, people used to quickly ask what is the CFL condition for which, not condition, what is the CFL for which you are running. That sigma is 1 and this other stuff I will tell you what it is later. So, there are no scales because actually I do not care. I just want to see behavior right. There are no scales. I am not bothered with scales. So, that is it. The initial condition is the step right.

So, with that initial condition of course, if you have 10 grid points or you can only represent a ramp. You cannot actually get a step right. By increasing the number of grid points, you can make it better. But, this is ftcs. So, what was the expectation that we had? Ftcs is going to diverge. 2nd derivative was negative. There was a 3rd derivative. It is also might be dispersive okay. So, the question is, is it going to diverge? Yes, it is already gone above 1.

So, it is going to diverge and it is wavy, which is all I can say. Something that started off is a ramp. Now, has was waves in it. So, it looks like it is dispersive. So, may be that is true, that is a fact. Let us take more grid points and see what happens. So, instead of 10 grid points, I will take a 101 grid points. Let me reset that. I take one-time step. Because I have more grid points, my ramp is closer to the step. I have a step function, which is much closer to the step.

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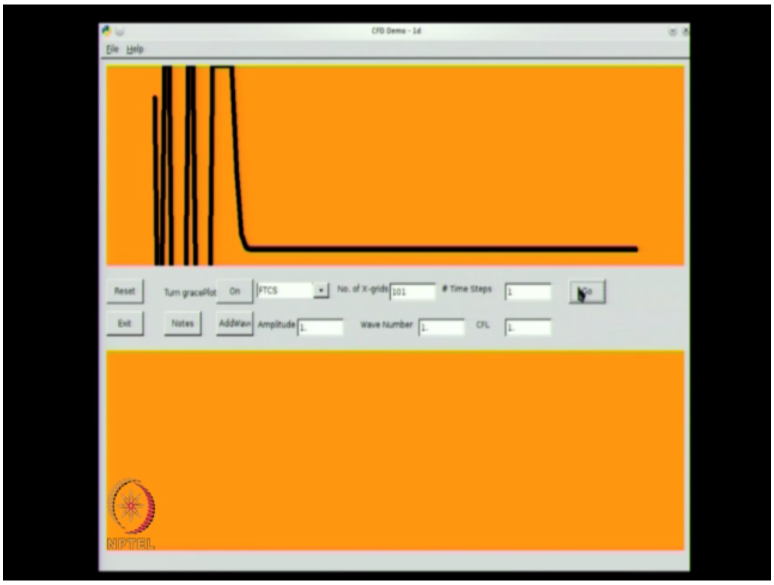

So, if I take one step at a time, after 1st 2 steps. It looks like it works but, afterwards it is not going to work and you can see indeed it is like you have a rope and that you are shaking that rope. It is really oscillating. I am not bothered rescaling, nothing of that. That is what I do not care if it is of the scale, when I start with 0,1. I already have problems right. So, indeed it oscillates. So, there is not much that we can do.

Can we take a smaller? Do you think may be if we take a smaller time step, it may work? Mathematics says, it is unstable but, do you trust it? If I take a smaller time step would it work? Right. So, let us try point 1. I will reset. If I do not reset, it will sort of continue with that, which is not what my intention is right now. That is point 1. Well, point 1 basically means I am taking 1/10th of a time step. So, maybe I will take 10 time steps at a time right.

So, well for a minute they looked very open but, it does not look like. There is one thing that is happened though. It is not diverging as fast. So, the dispersion is becoming much more clear right. Because the coefficient in front of the 2nd derivative term is small. It is going to diverge. It will eventually diverge fine. But, because the coefficient is small, it is not diverging that quickly fine.

So, but, it looks like the dispersion is very clear and you can actually make out that, what started off, what should have been the step function. Or what could have just diverged right, the values could have diverged. It is in fact oscillating. Is that fine? I will choose. As I said, in the next time that we do it I will choose a more what should I put it the next class when I continue with this demo. I will choose a more, I have a tuned set of, I can tune this.

And just for fun, there seems to be some sinusoidal something riding on top of right. So, what is that? It does have to with the fact that, this scheme is dispersive that that is happening okay. It is not an illusion, it is there. You may think, it is an illusion, it is not an illusion, it is actually there. Is that fine? Okay. So, that we do not end this day with a pure disappointment okay. I will run ftbs right.

We will run ftbs and from what I told you, ftbs with sigma=1 was supposed to have worked. Did I do ftfs? Oh I did ftfs. I will do ftfs in the next class. Ftbs with sigma=1, I want to leave ftfs, see whether you guys can predict what is going to happen right. So, before we work, last word for the day, we will run ftbs with sigma=1. First let me take one-time step at a time, reset it.

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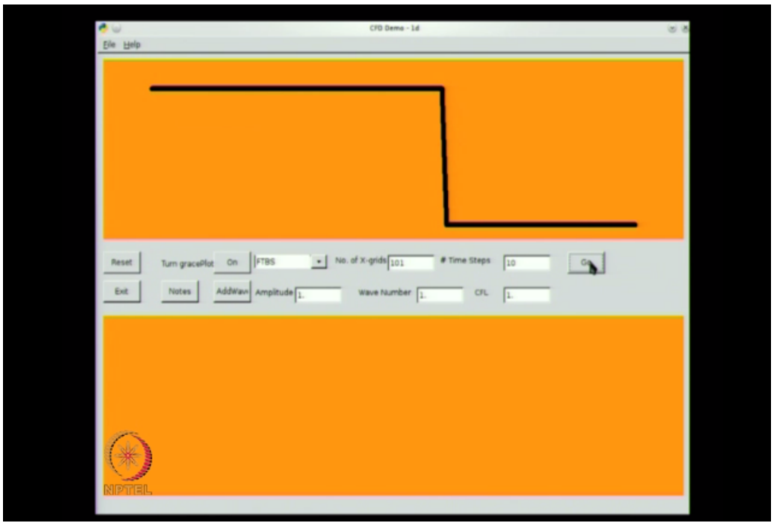

Oh, it is a ramp, but that is the way, it should behave. 10 time steps right. That step, no distortion in that ramp, but, it is a ramp right. You say, wait a minute, how do you that it is, let me run 50 steps. So, if I run 50 steps, it should come half way right. I will reset it, run 50 steps, yes, it comes half way goes right up to the end. Just to the end okay. So, I am picking up the propagation speed, I am actually picking it up exactly. Is that fine? Okay.

So, when we come back in the next class, I will run the rest of the, we will see, we will try to explore what happens with numerical solution to wave equation and the lots of nooks and crannies that we have to look at okay fine. Right thank you.