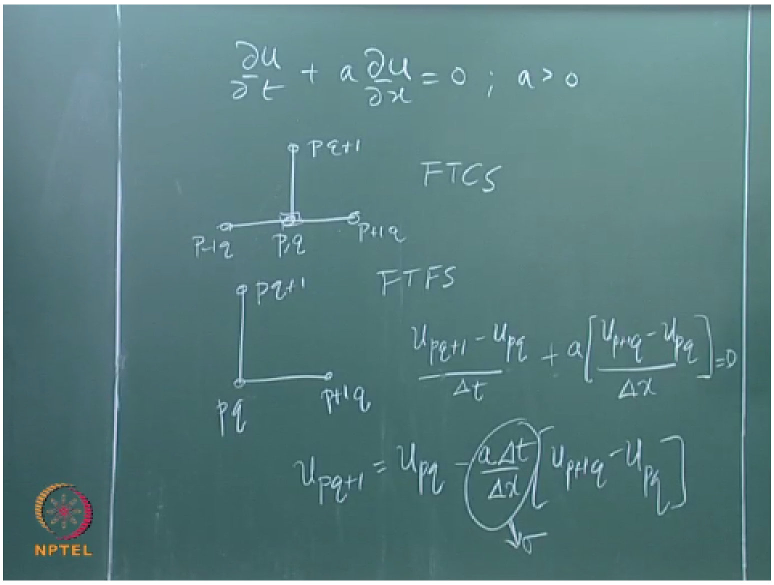
Introduction to Computational Fluid Dynamics
Prof. M. Ramakrishna
Department of Aerospace Engineering
Indian Institute of Technology – Madras

Lecture - 17
Generating a Stable Scheme & Boundary Conditions

Last class we looked at linear first order one dimensional wave equation, right and how to solve it we thought we have to solve it. We tried forward time central space, okay. And forward time central space turned out on analysis on doing stability analysis, forward time central space turned out to be unconditionally unstable, okay.

(Refer Slide Time: 00:39)



So, we are now going to try something else. So, the equation that we are solving is this. With the appropriate initial and boundary condition we have had a discussion earlier about initial and boundary condition, right. With the appropriate initial and boundary condition we decided that this is just a recap. That we seen FTCs, which means that the differential equation is represented at the point Pq.

q is an induction time, P is an induction space, right. P-1q, p+1q and pq+1. And we handed up with the scheme which was unconditionally unstable, okay. So, we decided the argument, I mean now we tried, right we just tried central difference it worked in Laplace's equation. With the same idea we just tried doing the discretization here and it looks like when we did an analysis that it is not going to work, okay.

We are doing forward difference in time. The proposal last class was why not do forward in space and see whether that helps or not. So, this is FCTS, Forward Time Central Space. Now we will try FTFS which is Forward time forward space. So, we will use the point, again we will represent the differential equation at the point Pq, right do not forget that. We are representing the differential equation at the point Pq, right.

P+1 we will involve the points p+1q and pq+1 and this would be the scheme FTFS, Forward time forward space, forward difference in both. So, what does that give me? So the dou u dou t term you give me a Upq+1 – Upq/delta t. The dou u dou x term would give me Up+1q – Up-Upq/delta x. That is the forward difference in space. So, if we solve again you will notice that except for one term which is in terms of at the time level q+1 all the others are at a time level q.

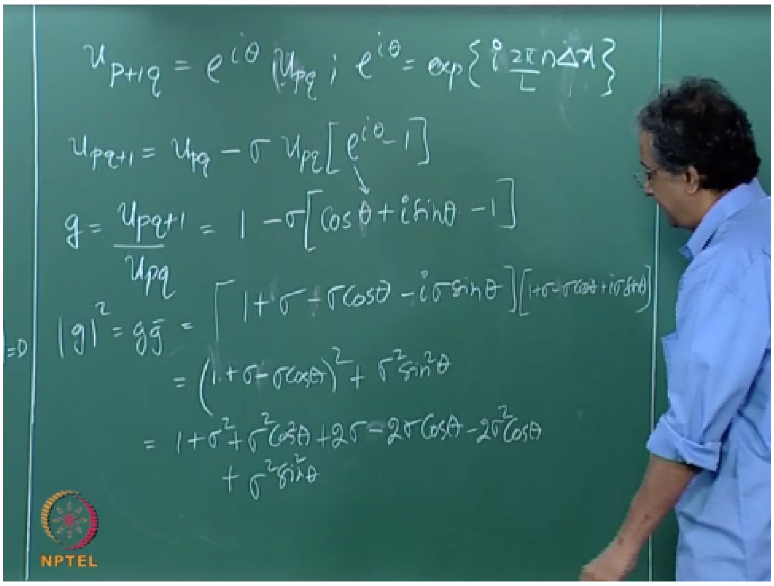
So, we will retain the q+1 term on the left hand side and take everything else to the right hand side. That is, we can write. The q+1 term explicitly in terms of the remaining terms, okay. So, this is also an explicit scheme in that sense. Upq+1 is Upq, when I take this over to the right hand side this of course = 0, I am sorry about that that = 0. When I take this over to the right hand side I get a negative sign. So, - a delta t/delta x times Up+1q – Upq, okay.

Again we have an automate value. Again we have something you can just given the time given U at q you can go on to q+1. Given the value of U at q you can go on to q+1. As I indicated in the last class we are going to see this so often that we give it a symbol that is called sigma, okay. It is called sigma. And it looks like we have similar simple scheme just like we had for FTCS, right.

So, but again there is a concern is it going to work or not, okay. So, let us try to do this stability analysis for this. Now, remember when we are doing this stability analysis we are involving only the points that are, that are involved in this computation. We are not talking about the whole computational domain. We are not talking about the whole region. We are not talking about the whole problem, right.

We are not talking about the stability as in a global sense, right. So, this is locally at this point if I go from point P point Pq to Pq+1 what is the gain? If I go from Pq to Pq+1, what is the gain? That is the question that we are asking. Is that fine, everyone, okay. So, what do I get?

(Refer Slide Time: 05:28)



Just to remind you Up+1 q is e power i theta Upq, I do not need the brackets, where e power i theta was actually exponentially i 2 pi n/L, i square root of n-1, fine. That is the only one that we need. So, Upq+1 is Upq-sigma times already you know I am going to factor out the Upq * e power i theta – 1. Is that okay? Just like we did last time we will compute the gain going from q to q+1. I can divide through by Upq.

So, the gain g is Upq+1/Upq which = 1 – sigma times using there is a delta x and oh I am sorry, thank you, yes and delta x, that is very important, there is delta x that and delta x, okay theta has delta x defined and its definition. Because you were shifting by a delta x, okay thank you, right. So, what do we have here this is cos i theta, cos theta + i sin theta that is Euler's formula for e power i theta – 1.

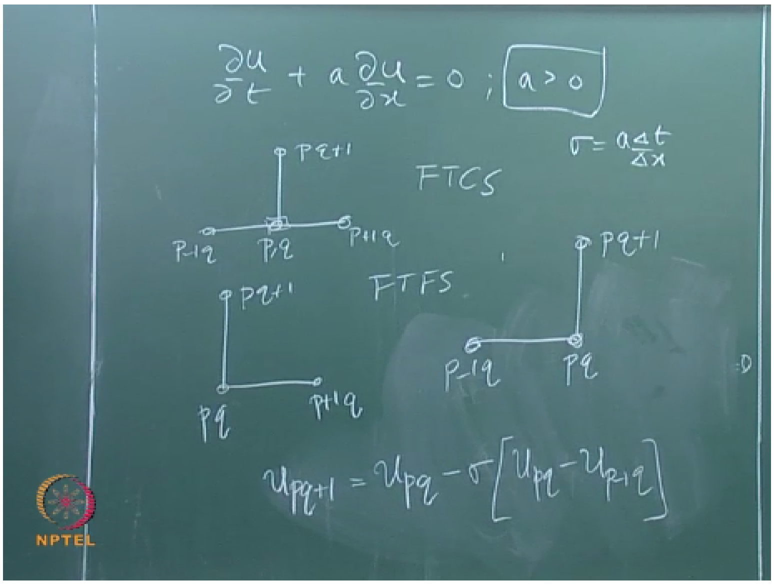
We want the modulus of this to be < 1 you are saying < 1 the square is < 1 give us the same thing. So, g square is g times g bar, g times its conjugate which = we could have lifted that way but I am going to expand the source. So, it will get little messy. So we keep tracks of what is happening, so that I do not make mistakes, okay.

So, what do I have maybe before I do this let me, before I do let us just expand this out. So, that I really do not make mistake. So, I have a sigma here. That is 1+sigma – cos theta – i sigma sin theta. Is that fine, okay? And its conjugate is, the conjugate of course is 1 + sigma + sigma cos theta + i sigma sin theta. I making the letters small but it is basically the sign flips on that. There is a – and – that is a + and the – sigma cos theta.

And the – and – that is a +, fine. So this is 1+ sigma + sigma cos theta there is I am sure each of you has a way by which you can do it, I am just going to do by brute force. + sigma squared sin square theta standard expansion. This gives me 1 + sigma square + sigma square cos square theta + twice sigma + - looks like that sin I am going to keep missing, okay. As long as you keep me honest it is fine, right, okay.

2 sigma – 2 sigma cos theta – 2 sigma square cos theta + sigma square sin theta sin square theta, that is the last term. So, I have a sin square theta + cos square theta that is a going to be give me a 1. Is that fine, everyone? Okay. So, what does this give me?

(Refer Slide Time: 10:38)



This gives me 1+2 sigma squared + 2 sigma – 2 sigma cos theta – 2 sigma square cos theta. And this has been < 1. A stability condition requires that it should be < 1, right. So far I am not, I have just been evaluating the left hand side. The stability condition requires that g squared < l, I am saying g square < 1 because we want g < 1, okay fine.

So, let me knock out that 1. This makes life a lot easier because now this quantity has been < 0, so it in fact I can divide by 2, okay. So, I get sigma squared + 2 sigma – 2 sigma cos theta – sigma square + sigma sigma squared cos theta < 0, okay, I can add I can subtract 1 from both sides, fine, okay. Where are we? So, now we can, you can obviously see that the sigma squared can be factored out, sigma can be factored out.

So, this looks like sigma square + sigma * 1 – cos theta < 0, right. Theta = 0 is the worst case scenario where it is 0, right. I want strictly < anyway theta = 0 again corresponds the dc component, right. So, except for that, so if theta is not 0 I can just divide through by 1 – cos theta, right which says that for any other value of theta this quantity is going to be positive. For any other value of theta, theta > 0, 1 – cos theta is >0.

So, I can divide through by 1 – cos theta, fine. This gives me sigma * 1 + sigma < 0. So, we have actually have a stability condition, okay? When does this work? No, it works. Sigma just figure out what is the quantity. Let us see what it gets? Yes, sigma is < 0 this will become negative, right. If sigma is < 0 this become negative whether you agree with sigma < 0 or not sigma < 0 there is a condition under which it will work, right.

Sigma is < 0 but then we want to make sure that this stays positive, right. So, sigma as we >-1, so and sigma has been > -1, what it basically says is -1 < sigma < 0, okay. We have the condition now we can look it at and complain about, right. So, what is the problem may be we go back there. What is the problem? a > 0, sigma is a delta t/delta x and I want that to be negative, right.

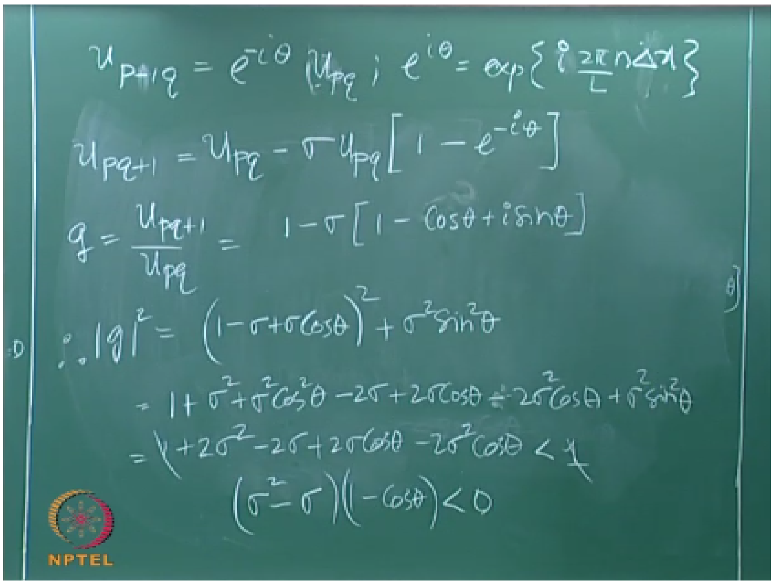
So, I do not know I look at it and say we can from that their condition will be stable, right that the scheme will be stable if delta x is negative. We do not want to go back in time, you do not want to go backward in time that is not our intent, right. Clearly, if we go backward in time this scheme will work you do not want to do that.

The other possibility is it seems to tell us delta x is negative in my mind, I would say well we are doing a forward difference is that telling me to do a backward difference, right. Or a could be negative we have chosen, we have chosen the problem a > 0 which means that a is negative it would work, right. a is negative it would work. So, what I will say is well you know I get that an incline that may be instead of forward space I should use backward space.

The condition seems to tell me to go in that direction, okay. This condition seems to tell me that instead of using FTFS that I should use FTBS. So, instead of using forward time forward space I will use a stencil that looks like this. So, this is Pq, P-1q, Pq+1 and we suspect from the analysis that we have done right now. We suspect that it should work, okay. We now more hopeful that this will work. So, what does that give me?

By now you should be able to write it out, okay, right. So, Upq+1 = you all have the same form Upq – sigma times Upq – Up-1q, okay. Just look at the FTFS and you will see that this is what you get. The sigma s got the delta t and delta x from the derivative. Is that fine everyone, okay? So, as a consequence we just quickly redo go through this process. I do not want to get into trouble with signs. I will get a little lazy here and make that P-1.

(Refer Slide Time: 17:05)



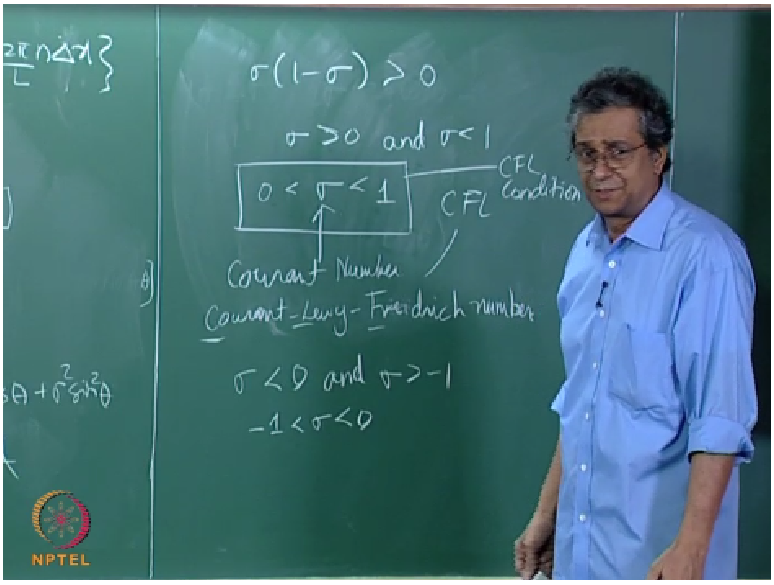
So, it is e power -i theta, okay. So, e power i theta of course is till defined that way. That is fine. So, what do we have? Upq+1 = Upq – sigma times Upq * 1 – e power –i theta. Is that fine, okay? So, all 3 of these schemes let me got our explicit and here we have again Upq+1 in this fashion, okay. Divide through the gain g is upq+1/Upq which = 1-sigma e power –i theta because this is I written it last time as I just expanded it out – cos theta + i sin theta, is that right?

That we careful a lot with all the negative signs and therefore g squared = is going to be a 1 – sigma + sigma cos theta, okay. 1 – sigma + sigma cos theta in sort of skipping a few steps here. I think you can keep track to make sure that I am not making any mistakes. – sigma that gives me a sigma squared sin square theta. That is fine everyone? So, that = 1 + sigma square + sigma square cos square theta looks very similar.

Here is where it changes – 2 sigma that + 2 sigma cos theta what else + -2 sigma square cos theta + sigma square sin square theta. sin square and that cos sign square again will again combined. So, this will give me a something very similar 1 + 2 sigma squared – 2 sigma + 2 sigma cos theta – 2 sigma squared cos theta < 1, okay. Subtracting 1 from both sides that gets knocked out, so right hand side becomes 0.

What we get here? We can of course divide through by the 2 also, okay. So, what do we get now? So, we have a sigma squared sigma squared which is a sigma square * 1 – cos theta and the - sigma * 1 – cos theta. So, it will be sigma square – sigma to 1 – cos sin theta < 0. Is that fine? And this should do it for us.

(Refer Slide Time: 21:36)



So, using the same argument divide through by 1 – cos sin theta, so you get sigma * 1 - sigma < 0. Am I got that right? And here because this is 1 – sigma you saved. So, this tells us that sigma is > 0 and sigma is < 1, okay. So, 0 < sigma < 1. Is that fine? This is called the, am I make mistake? Yes, sigma is > 0 and < 1. Did I make a mistake? Sigma *, I do not see it. I am sorry, okay this is > I flipped it around, okay sorry.

It should be either be a sigma * sigma – 1 or deal I made it 1 – sigma actually that is true. I do not how make that jump, okay that is fine. Yes, it should be > 0, fine otherwise it will not work. Is that fine? Sigma is positive 1- sigma is positive, so the product is positive, okay. So, Sigma – 1 of course, then it has be like < 0, that is fine. Thank you, okay. So, what does this tell us? The sigma value is positive, so it works, okay.

So, this is called the, remember I remind you again this is either called Courant number or depending on who you talk to whom you talk Courant-Levy-Freidrich number, okay. Very often just referred to as the CFL, named after the three very often just called the CFL, okay. So in computational fluid dynamics this number figures so often, right.

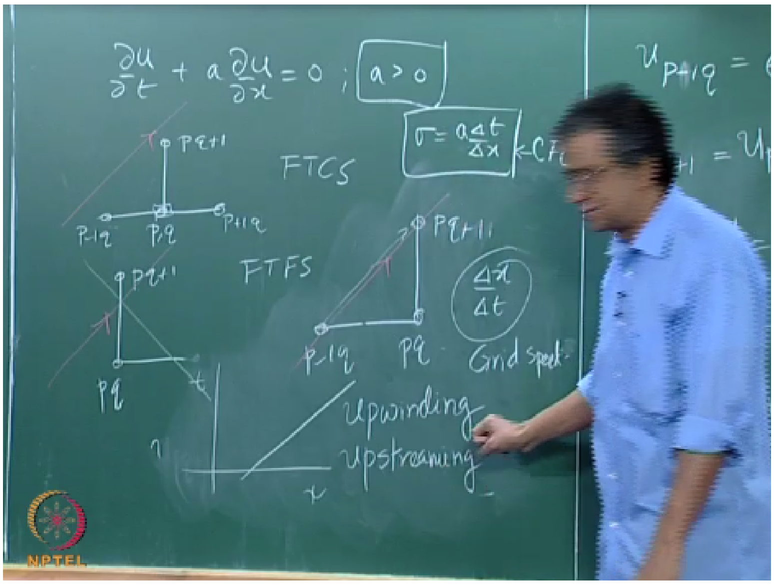
So, and the usual jargon will be, so if you say oh my God I am having some difficultly. People may ask you what Courant number are you running or what CFL are you running, right. They are not talking about these conditions which itself is called the CFL conditions Courant- Freidrich- Levy condition. So, this condition itself is called the CFL condition, right but very often we do not know. See, we are just done this we have got this for what?

We have got this for forward time backward space for linear first order one dimensional wave equation with $a > 0$ propagation speed > 0, right. This is very restricted. So, it is not like FTBS, this is the condition, right for that problem. So, we do not know what conditions you would get otherwise, right. We do not know what are the other possible stability condition. So, this condition is called the CFL condition.

And this number is called the Courant number, CFL number, okay. So, it is normal for people to ask the question what CFL are you running or what Courant number are you running, okay right. So, at a later date some assignment you have some problem or whatever it is and maybe a questionnaire ask what Courant number are you running, is that fine? So, this since number seems to be so important let us just sort of take a look at this number.

See, what it is all about and see why FTBS works, right. There are 2 things that we need to. First that we have this number, so I get back here, okay maybe I leave this here just so that you can see what is happening.

(Refer Slide Time: 25:54)



We just look at what we got here. So, a is > 0 sigma is a delta t by delta x, right. So, this is the Courant number or the CFL. what are it dimensions, physical units? It is non –dimensional, okay. So, first of all we see that it is non-dimensional, it has no dimension. So, a is a propagation speed which has units meters per second. So, it seems that delta x by delta t is also some kind of propagation speed, right.

I mean, I am dividing a divided by some propagation speed. This is how we would normally think about it in fluid mechanics. So, delta x/delta t comes from our grid. So, every time we take a time step we are every time I take a time step and sort of propagating something that is at a distance delta x across the distance delta x.

When I go through a time step delta t I am propagating something that is p-1 whatever property I have at p-1 and causing its influence to propagate to p over a distance delta x. So, this is the speed delta x/delta t is the speed at which my grid is propagating the property is that I am right, that my equation is trying to propagate. Equation is trying to propagate you this is what we found out.

This differential equation is propagating the property u at the speed a. That is what this differential equation that is what we saw when we looked at characteristics. What the scheme does is the scheme propagates the same property u, right at a speed basically delta x/delta t, effectively delta x/delta p. That is what it looks like, okay. So, the ratio a/delta x/delta t is the CFL number and this is called the grid speed.

I give this to you only as interest because very often this is not going to crop up that often in conversation, right the CFL will occur more offer in conversation this is by way of just information. So, delta x/delta t we will call the grid speed, okay. So, it is non-dimensional, that is fine. So, that looks like right now we will settle with that, right. We will come back to this propagation speed and what does this means.

What does it mean a little later there is only one of the thing that I want to tie with this. So, central defenses did not work, forward defenses did not work a backward defenses and space worth, okay. So, if you look at again the fact that I am talking about the ratio of propagation speed to grid speed what is the speed with which it propagating. It is not just speed, right.

There is also direction and what direction is being propagated. So and that seems to be the problem here. So, in this case we were propagating with this grid in both directions, right. So, whereas our characteristics I will just draw, I will simply draw the characteristics just for our reference. A characteristics are for this problem with a positive slope basically that means that the characteristics are, I will use a different color chalk.

A characteristic is oriented in this fashion, okay. So, the physics wants to propagate along this direction. The physics wants to propagate along that direction. In this case the numerics was also propagating in the opposite direction which is not what we want to do. The numerics again wants to propagate in the opposite direction that is what we are trying to do here when we do a forward space.

Whereas, here the numeric and the physics are propagating the same direction. This idea that the numeric captures, right that the numerics captures what the underlying physics is doing, right, is basically called referred to us up winding. So, this is given a name up winding or if you are more interested in that water stream flowing that I talked about earlier or up streaming, okay wind as an air flowing up winding, okay.

Up winding or up streaming, fine. So, there are we will get back. I have something to say about this at a later date. But right now, I just want you to be aware that the fact that we went through this process, I want to know what is the meaning of this sigma and where it comes from. This is a relationship, right. The fact that the sign change, the sign change a changes sign the slope changes.

If a changes sign, if a were negative then you will get a characteristic like this, okay. In which case forward space would work, that is what it tells us. That is what the clue that we got. And we use that clue to get to this point. Is that, okay great. So, before I go on there, there are some remarks I want to make.

So, the first thing that you have to see is that just because Laplace equation essentially it means the Laplace equation we will sort of got a little lucky, right. We took Laplace's equation, we did central differences, I mean it was not coincidences, I obviously pick Laplace's equation as a problem because you remember this came under the category of simple problems.

And I obviously very deliberately pick Laplace equation because it would work, right. But if you were trying it out you may try Laplace's equation and say oh it works this is very easy. I know how to do methods, right. I am set. So, it does not work that way. So, you could take an equation.

You can use you could go through, right, you could go through a legal process, saying I will do central defenses I will do forward defense. We went to the discretization and we came up with the scheme that will not work. If you have not try to implement FTCSs yet please implement FTCS, right. That is what I am trying to tell you is just because we know that FTCS does not work. We know because we have done some analysis.

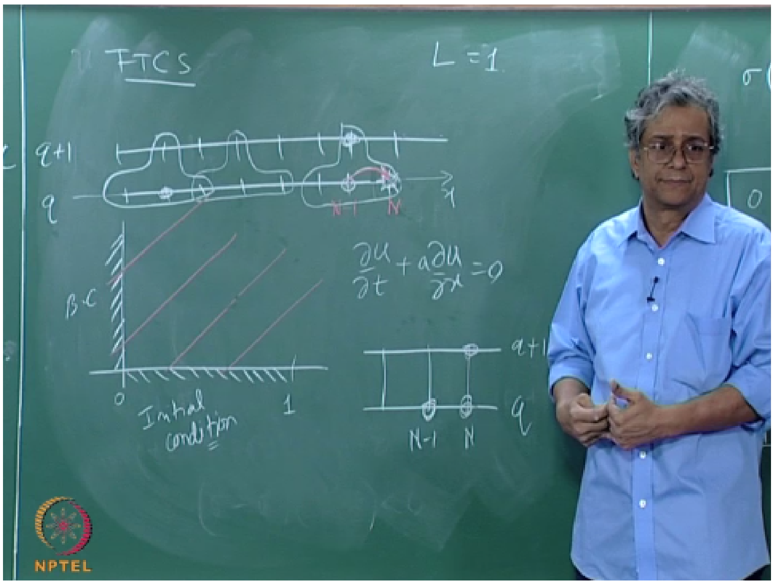
My first question is, do you trust that analysis? So, check it out to see whether it works or not, right? If it is going to fail, you have to have an idea is how it fails? I want you to see your program not functioning that is some, because you will encounter these errors. See, you may go you may discretize an equation at a later date, right and I want you to be familiar with how these programs fail when they fail.

It is not enough do not just implement FTBS, saying that I know this works the other do not work I ignore the other two. Implement all three of them, try all three of them and see how it works, right. And there is a little issue about boundary conditions as to how apply boundary conditions. So, I want you unless you implemented you do not face that, okay. So, you have to try it. I want you to try it, right.

I want you to try it, so that you will see that you can try this, this and this all three stencils, okay. You can try all 3 stencils. Now, as I said normally when you go through this is what is going to be happen. Conclusion is just because we have a discretization that does not mean it works, okay. So, now we have a scheme that actually works may be I will say something about the boundary condition and boundary conditions and.

So, we are doing up winding, we are doing up streaming. I will suggest that you try FTCS but is there any issue with FTCS as I said normally what I would do is I would wait for you to try it out and come back and complain to me that there is a problem, right. But because we are in a different setting let me go ahead and tell you what is going to happen. So, what is going to happen let us try FTCS.

(Refer Slide Time: 34:22)



Let us say you do FTCS, okay. So, this is your x axis, remember this is the pipe. This pipe was of length l we can take without loss of generality we can take 1 is 1, right. So, the length of the pipe is 1 if you go back to the problem that we picked and we can pick grid points along that length. This of course, is a time level q and that q+1 you would have similar set of grid points. That is q+1 and what does FTCS do?

FTCS uses these 3, the 4 but the 4, the middle one does not actually play a role in, right because of the central space first derivative the middle one does not show but it does not matter. It uses these 3. So, this is the stencil, it uses these 3 here and the 1 there in order to march forward, to step forward in time, right, fine Where have we prescribe boundary conditions?

Remember for a > 0 what was the problem where have we prescribed boundary conditions? We have prescribed boundary conditions at t=0 or rather initial conditions t=0, right. Initial conditions t=0 and at x=0 we have prescribed boundary conditions. I will just say boundary conditions. Is that fine, everyone? It is okay? And we gave a reason, we said look the characteristics are from left to right. We gave a reason.

The characteristics are like this and therefore the boundary conditions are prescribed on that side, right. We gave a reason for it. Does it work with our notion of what differential equations are all about? It does, because you have a first derivative in time that tells me that I need an initial condition and you have a first derivative and space, right.

So, in mind when I say I am going to integrate with respect to time I will get one constant of integration that needs to be determinant. And if I am going to integrate with space, I am going to get one constant in space that is going to be determinant, right. So, I need a condition along this and a condition along that. They satisfy. Mathematics is satisfied, the physics is satisfied, right. But what about the numeric's?

Look at this, if you were doing forward time central space what is going to happen? If you were try to implement FTCS what is going to happen. So, when you are calculating this grid point, yes that works. This is known, this is known, this is known. What happens at this end? When you want to calculate this grid, for this grid point you need this. You need the value on the right hand grid point. Am I making sense?

So, the mathematics we have differential, we have boundary conditions that came from the differential equations on the mathematics. We have boundary conditions which we are rationalized. We agree that this is what the physics is, right. We gave this reason saying oh this stream is flowing down, I put chalk dust here, once I put the chalk dust here sort of committed.

You cannot at a later date, right that the chalk dust flowing down should be something other than what you have put now, right. Once you put or chalk dust or whatever it is, trace your element into the stream you cannot then say no, I do not want any chalk dust at the other end, it is there, you have no choice. You cannot, once it is in it is in, right. We are assuming that you are not able to grab it from in between.

There is no source or sink in between, okay. So, therefore the physical condition, the physics of it are also satisfied. But here the numerical scheme requires a boundary condition. The numerics requires a boundary condition in order to implement FTCS, I need a boundary condition. I will admit the fault is mine. If FTCS is not stable but if FTCS was stable I would use it. I mean it is central defenses second order accurate in space.

Why would I not use it, right? And if I will not do it, the fault is mine that I am going to use FTCS but once I decide to use it, I need a boundary condition, right. I want emphasis that neither the mathematics nor the physics in that. So, we have to generate this boundary condition, okay. So, how can we do it? What do we do? What are possible ways by which we can do it?

So, one possibility is that you extrapolate, right, extrapolate. So, you could take, you can take whatever value here when its initial condition it does not matter but that is why I strategically called it q. I want it to make sure it is != 0, right okay. So, we do not know, there is no condition prescribed here.

So, what you can do is you can copy maybe I will use, you can copy the value at if this is the grid point **N,** the last grid point from N-1 you can copy it to N. Then you will have one there, right. Why I am copying it from N-1 to N because that is the direction in which my equation is propagated, okay right. There is another possibility, this problem of using requiring a condition here will clearly be there even in FTFS.

But it will not be there for FTBS, right. It will not be there for FTBS, in fact in FTBS we can solve the differential equation right here at the last point, okay. So, in FTBS, I am going to draw only this portion. In FTBS these was the points that would be, these are the points that take part in this, right. So, this is N, this is N-1 this is q this is q+1 and in the case of FTBS you can actually solve the differential equation at this point, okay.

At this point, right and you can actually get the value here from the values at time level q, right. So, one possibility is that you just copy the other possibility is just for this one point you use FTBS, is that fine? Just for this one point you use FTBS. Is that clear? So, there are different ways, see now you can see why do we have these options? We have these options because this is a problem that we have created by picking the particular scheme.

It is our need that we need the value of U at that point, okay. Is that fine? I cannot emphasis that enough, so I will not you to be aware of this. There are boundary conditions that you have physics that are available from your physics, right. So, when you solve the problem when you are, I will not say solve the problem when you pause the problem you study the physical problem that are certain boundary conditions that our availability, okay.

Then you go through a process of modeling and come up with up a mathematical model. The mathematical model will require boundary conditions, right. There are problems in which the mathematical model may require more boundary conditions than the physics provides, Am I clear? The mathematical model may require more boundary conditions in the physics provides, okay. I can give you instances of such problems.

Third possibility is given that you have the mathematical model and the physical model when you get to the computational model the computational model may require even more boundary conditions than these 2 require, okay. So, in both cases the physics is what availability, right because our physical what tell us what we need to do.

So, when you get to the mathematical problem requiring more then you have to figure out the way by which you come up with those boundary conditions, okay. That is a little more difficult because have to be compatible with differential equations and so on. In a similar fashion when you have the numerics, right you need these boundary conditions you have to apply those boundary conditions in some fashion.

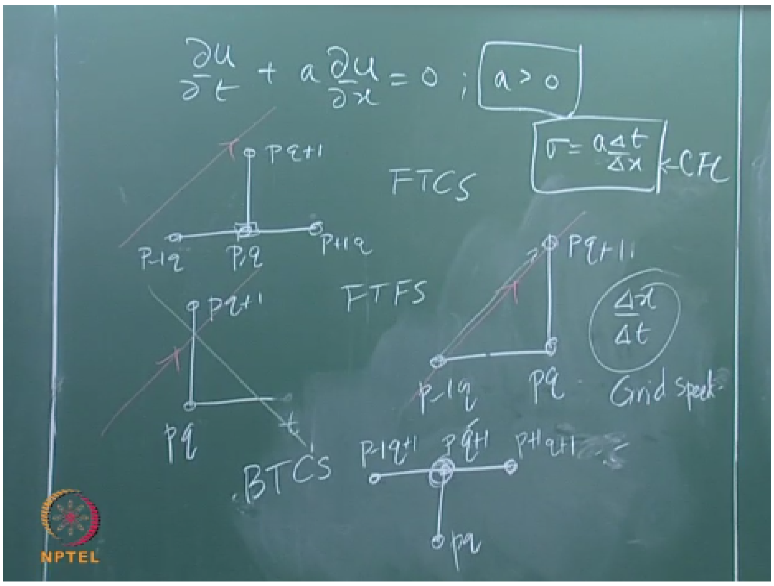
You have to generate those boundary conditions in some fashion, fine? Okay, are there any questions? So, now we have one possibility, what we will do is we have done forward time central space, forward time backward space and forward time forward space and backward space, okay. We have done all 3. Since, forward time and backward space work, see this is you have to look at the process that I am going through right.

Remember yesterday's class I said well we are sort of groping around trying to make sure that we come up with the scheme that works. We tried FTCS, it did not work that is some amount of shock. Then we are left with let us figure out what to do. If you were doing this for an equation for the first time that is how it would typically work. What you try first of may not work.

So, I say well backward space seem to do it, why not backward time, right I mean, after all I tried forward space because I was doing forward time, right. And it got us somewhere, so I asked the question why not backward time? Why not take a backward difference in time and if I am going to do a backward difference in time, greedy fellow why not go back to central defenses in space unless I want to do second, I want to do that, right.

Higher order scheme and unless it fails say this is I am just talking about pure right just from I want try to do the best that I can. Whether it is indeed the best that we can or not we have to see, whether it is the best thing to do. We have not judged; we have not even computed any of the system. We do not know how good these schemes are. Right now we are just as I said exploring or groping around to find out what we are able to do.

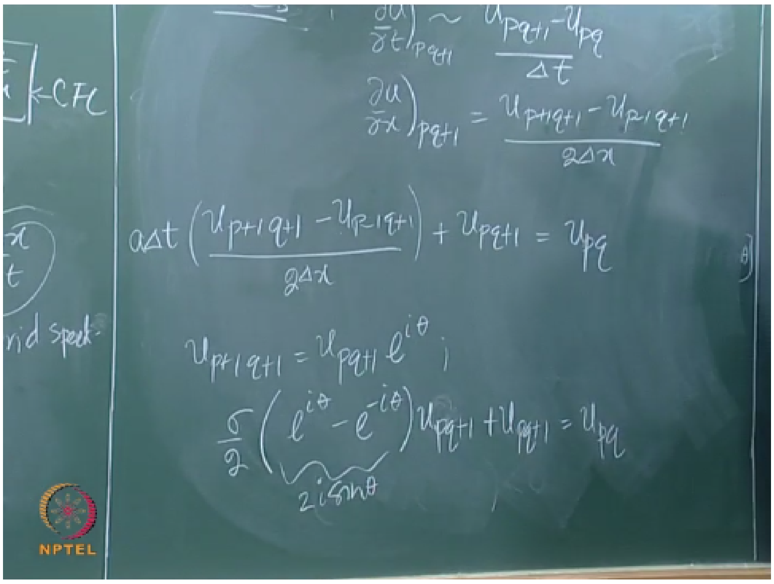
(Refer Slide Time: 45:33)



So, if I do backward time central space. If central space does not work then we will try forward space backward space most likely we will try backward space, right it is obvious, right. But as I said we will get a little greedy and see what happens. We may pay the price for it. So, what does that mean? That means the differential equation now is that p well we have a choice.

We can either represent the differential equation at Pq or represent the differential equation at q+1 may be we will do it at q+1 for a change, pq+1, in the sense this index is our choice, right. So, we will call the new grid point always q+1, p-1 q+1. P+1 q+1, Pq and we are representing or approximating or representing the differential equation at that point. Is that point at the future point which we do not know the answer, okay?

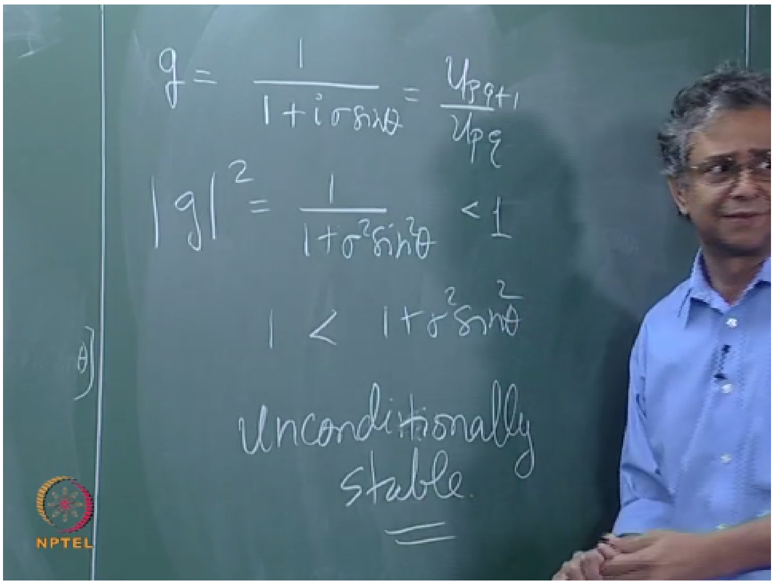
(Refer Slide Time: 47:03)



So, what we get. This is BTCS, Backward Time Central Space. What is dou u dou t approximately, I am writing it out explicitly because I want to tell you where I am doing. Upq+1 – Upq/delta t, right I mean, it is obviously the same thing. This is what we are saying earlier when we are talking about finite differences, okay. In dou u dou x at Pq+1 which is where we are approximating it is Up+1 q+1–Up-1 q+1/2 delta x.

Substituting it into the differential equation going through the same process, keeping all the terms that are in terms of q+1 on the left hand side we will get Up+1 q+1 – Up-1 q+1/2 delta x + Upq+1 a delta t = Upq. Is that fine? And again Up+1 q+1 is Up q+1 * e power i theta, right and so on. So, this becomes sigma by 2 e power i theta – e power – i theta Up q+1 + Upq+1 = Upq, is that okay? And what is this term? 2 i sin theta.

(Refer Slide Time: 49:58)



So, if I find g now, g = 1/1+i sigma sin theta. What is g? It is Upq+1/Upq. Is that fine? I quietly skip the step there. And therefore g as be < 1, right or 1 < 1+ sigma square sin square theta and we are very happy. Yes, we finally have a scheme that is unconditionally stable. So, in the next class we will see what this means. See, what we have done. Is that fine, okay? Thank you.