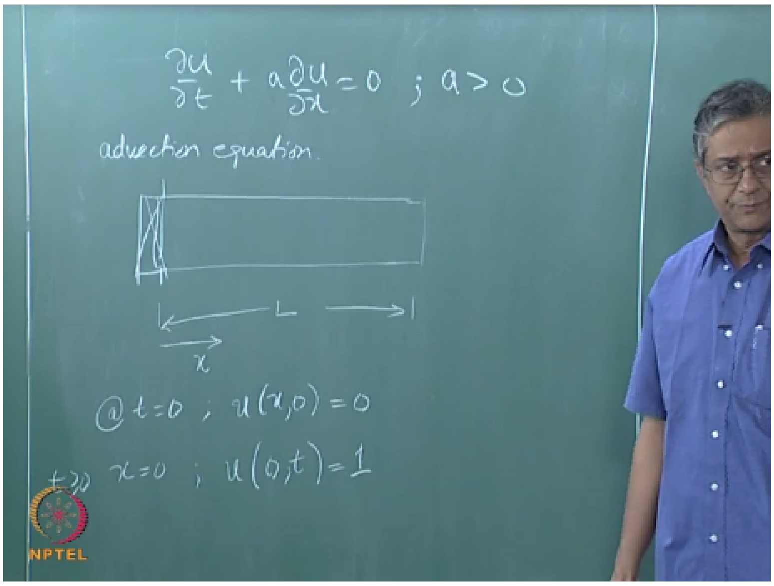
Introduction to Computational Fluid Dynamics
Prof. M. Ramakrishna
Department of Aerospace Engineering
Indian Institute of Technology – Madras

Lecture - 16
Linear Wave Equation - Closed Form & Numerical Solution, Stability Analysis

So, last class we started looking at the linear one dimensional first order wave equation right. That is where we left of, we were looking at a problem.

(Refer Slide Time: 00:21)



Let me just write the equation dou u dou t + a dou u dou x = 0 a is > 0, okay and we saw we call this we also call this the advection equation. So, it was basically it capture the idea of this pure propagation. The propagation speed is a, right and we are looking at the problem a sort of started talking about the boundary conditions but I did not quite state what the problem was, so let me just state the problem, okay.

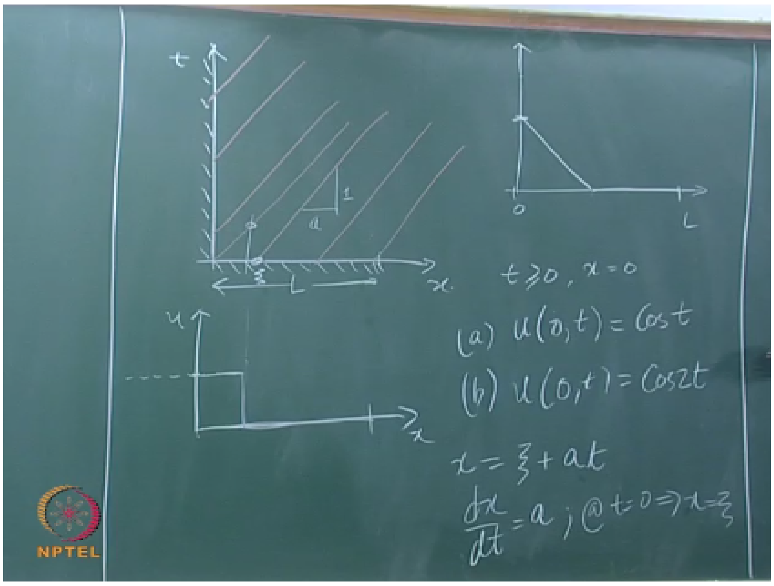
So, we could have say a pipe of length l at the this of course is the x direction. At the left hand side there is a valve and maybe there is hot water to the left of that, okay or there is hot air to the left of that whatever, fine. One dimensional problem. This pipe may have gone forever but we are only interested in the length of pipe or length l, okay. The pipe could have gone quite some distance. So, the idea is that t = 0 we are going to open the valve, okay right.

So, at t =0, so what are the boundary conditions that we have and the initial conditions. So, at t = 0 we have an initial condition at t = 0 the u throughout this pipe has a certain value right?

It can move it, that you understand. So, we can move our reference temperature or whatever it is property that you are talking about, so at t = 0 u (x,0) that is x, t is 0, okay. So, we are interested only in this length, keep that in mind.

And for all t at x = 0, right for all time t > 0 like we do not know how long the water is being get hot, so we do not go there, right. So, for t > 0 > or = 0 if you want. That x = 0 u (0, t) = we always scale the problem = 1, is that fine? So, this is the simple problem that we are talking about.

(Refer Slide Time: 03:31)



If you were to xt plane, so that is the length l. We have prescribed conditions from 0 to l on the x axis and we have prescribed conditions for all positive t, that is basically what we have done, okay, is that fine and we know that our characteristics go out in this direction. So, the characteristics that goes out here, I do not care for, right it is going out. So, we know these characteristics north in this direction.

So, this is the orientation of the characteristics, okay, is that fine, everyone? So, whatever is the value here on the axis is the value that is going to be propagated along the characteristics at the speed a, fine. So, yesterday I call those points x not, I will call it psi today for my own reasons. So, what happens to this function now, so the initial function? If I plot the function u versus x for the same length l, right it is a rather simple function.

Just to the left of it which we are not interested in apparently the function value was 1, right and then it drops to 0 and there is indeed a discontinuity at that point, okay. And I do not actually care the fact that there is a discontinuity. If it bothers you, you can choose some other, we can choose something else, right? But let us start to this, let just start with this.

So, this valve is open because this is the problem we have been looking at so far, the valve is open and what you would expect is the water propagates left to right. So, whatever value you have here is going to propagate out, whatever value you have here I have not drawn the characteristic going from that step. You have a step there. That step is going to move along that characteristic, you are going to propagate along that characteristic, is that fine?

So, end time as you go along this step is going to come out. So, at some time t this step is going to propagate out. So, what you would expect is this were an animation you would expect this step is going to travel right, left to right, is that fine? right and if you think about it you have a tap you open the, right you open the valve and hot water is going to travel and you would expect something like this to happen, okay.

So, pure propagation that is all it picks up. No diffusion nothing else, no other physical phenomenon, pure propagation. There is no decay in the size of the step there is no smoothening out of the step nothing, okay. It is just going to propagate left to right, is that fine?

Okay, so this property is important, so if you had instead of your initial condition instead of being a step some other function you can imagine now that you can choose other functions. So, you can choose a function the initial condition for example to be to go off from some value, right go down, some value. It can be anything, I mean you can just pick right, you can pick any initial condition that you want and that condition will be propagated it will just basically flow out of the pipe.

What I am trying to say is that if in your pipe whatever it is that you have, right there is a temperature distribution that water is going to flow out, right. So, if turns out that the pipe because it is being sitting there as warmer, there is a region where it got warm due to conduction through the valve or whatever then when you open the valve it is all going to flow out, okay. So, this function is just going to propagate left to right, fine.

So, you can try out a few different functions the other thing that you can do is at you can try this out for $t >= 0$ you can replace the function at $x = 0$. You can replace the function 0, t instead of being constant which is what I have done so far pick a function see what happens. So, you can take that to be say cos t or you can take this along with cos t or sin t, right or try another one.

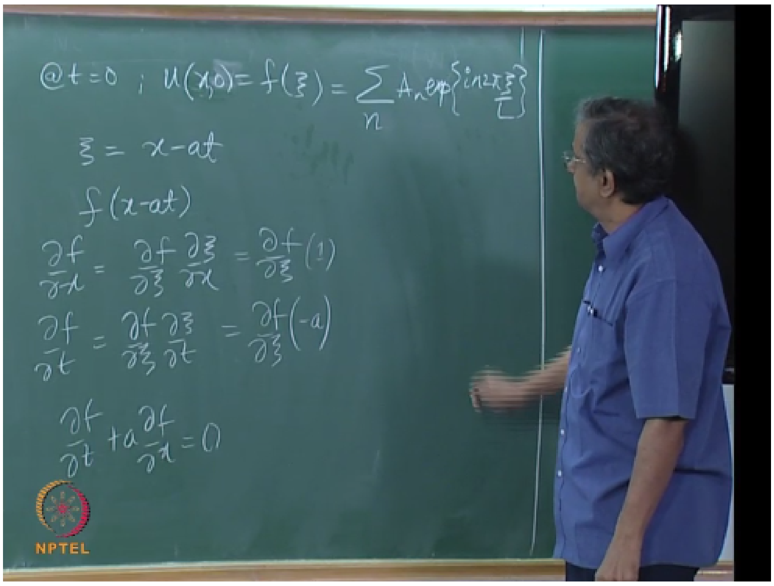
So, this is one function try a different function, right 0, t = cosine 2t. Try various functions and see what happens, okay. Try various functions, these are varying in time see what happens, is that fine? Okay. Let us get back to this picture, so this characteristic a typical characteristic intersects the x coordinate, right at the point psi, yesterday's class, I called it x not, today I will call it psi at a point psi.

So, what is the equation of this line? Right, remember that this line you are going to propagate a distance a in unit time, right. The property is being propagated at the speed a units per second. So, it is going to be, x is going to be psi + at in fact the reality of the differential equation. The differential equation that we are actually solving is dx dt = a @ t = 0 that tells you x = psi, that is what we are doing.

You understand? and I am integrating that to get essentially x = psi + at because a is a constant this integrated out but I want to make that statement a little more precise, right. I want to be little more careful with that, okay because I will use this fact later, a in this case is a constant everywhere. But what is more interesting to me is a is a constant along that characteristic. a is the constant, right but I want to say, I want to put in a peculiar fashion.

a is a constant along that characteristic that it happens we constant everywhere else I do not care, right. Okay, so a is a constant along that characteristic and in fact it is just turns out that x = psi + at, is that fine? Okay, so that is the equation of that line. Now, so what does this equation do, what have we discuss so far? What this equation does is if on this length l you prescribe some function this equation will propagate that function. Because a is > 0 it propagates left to right, okay.

(Refer Slide Time: 11:14)



So, I repeat @t = 0 if you prescribe u (x,0) = f (psi). This is going to be propagated how? What is going to happen to this f? So, something of the form f (psi) but what is psi? Psi in fact from our equation is x – at, okay. So given the initial condition, so now we make the jump. Given the initial condition f(psi) in fact given any function f right, which has the appropriate derivatives as far as we are concerned.

Then f(x-at) is the solution of this equation. Is that fine, okay. Great and you can verify that what is dou f dou x? Dou f dou psi dou psi dou x chain rule**,** right. What is dou f dou t? Dou f dou psi dou psi dou t substituted into that equation which is how you would verify whether it is a solution or not. What is dou psi dou x? Dou psi times 1, dou psi dou x is 1 and what is dou psi dou t? times - a, is that okay, everyone?

So, this is indeed a solution. So, you can just substitute there and you see that dou f dou t + a dou f dou x = 0. So, its satisfy, the equation is satisfied, okay. So, any function of this form equation is satisfy. So, you remember where we started this? Earlier when we started off I said can we guess the solution? right so now we are proving, we are looking at there is a way by which we can construct the solution.

There is a way by which we can construct, the geometrical way by which we can construct the solution using characteristics, right. Those lines were called characteristics and using those lines we can actually construct the solution geometrically, okay and from there may be we can get some. But now what we have done is we have seen from that just using a little analytical geometry that we are able to say that any function of this form is a solution as long as these derivatives makes sense.

Okay, you can complain that I took a step function, what is going on, right? Okay, right we will see. We will encounter a lot of those situations but as long as these derivatives make sense, right something of this sort is going to be a solution, everyone, right? So, how do we do? Where do we go now? I will not still be able to guess the solution we have a general form. I want something a little more specific, okay.

So, we will repeat what we did with Laplace's equation. You can of course if you give me a function f (psi) it is possible, it is on finite interval of length l to give me a function f (psi) then I can use possibly Fourier series, I can use periodic extension, okay and I can use Fourier series to represent this function, okay. Why do I use Fourier series? May be I am getting a little ahead of myself.

If I look at this function dou f dou t, so see, please remember I am now, I am trying to explain the process of how we go about guessing. See, this equation is simple. You can easily, I am pretty sure you can sit down but the process that we go through is very important is more important, okay. So, I see first derivatives here, so from my differential equation I am thinking exponential, right?

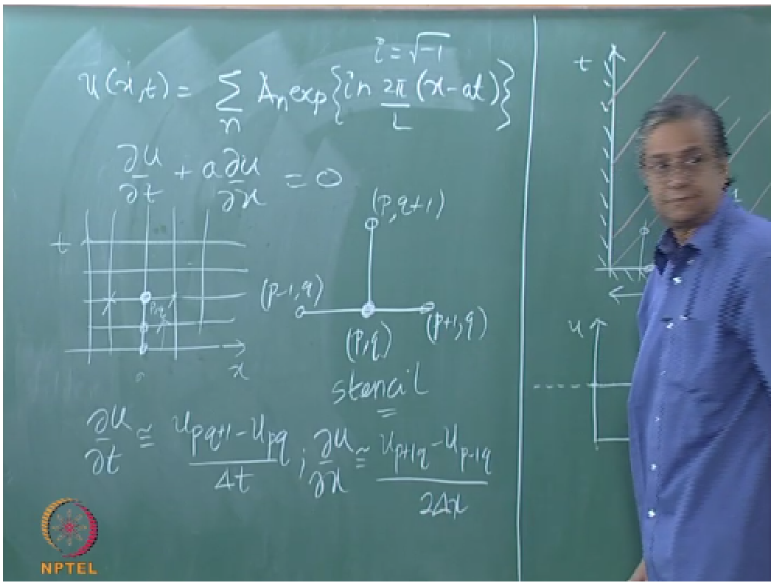
I see first derivatives; I am thinking exponential. So, okay I am going to get an exponential but an exponential can go 2 ways. An exponential can go 2 ways. In this case the function value did not decrease, right? So, I do not want an exponential and the form of an e power – x kind of a thing. Because the function value will decrease, I want only pure propagation because that is what this equation represents, fine.

Which means that so, if have an oscillation it is going to continue oscillating that is what that oscillation is going to be propagated. That is all that is going to happen, okay. So, the minute you say oscillation not decaying, we have something that looks like a Fourier series, is that fine? So, what we will do is we will represent if you give f (psi) I can write this f (psi) in terms of the Fourier series, okay.

And how does that go? That goes over a summation over n, we are not going to bother with the limits right now, okay. Go from – infinity to + infinity because I am going to write it in or because I will take it from 0 and- l to + l. It depends on the lengths of the interval; you have to be careful. So, I will just leave it vague, I will just leave it as n, right depending on what exactly we pick the n range will have to be pick appropriately.

Whether, it goes from 0 to infinity or – infinity to + infinity. Okay so, we then we have An exponent in wave number 2pi psi/L, is that fine, everyone? Okay and then you can do, you can take ls 2pi, the 2pi is go away all that kind of stuff, that is fine. So, you can have a function of this as a solution, so let me swing over here to the other side. So, what we have is we will get back to the …

(Refer Slide Time: 18:09)



So, what it basically says is that may u in general, I can write it as summation over n An exponent in 2 pi x –at/L, fine, okay. So, we started off looking at this equation right. So and by sequence of investigations trying to find out what it is, we were able we were lucky. It is not always possible that we are able to do this but we were lucky. We are actually able to guess a form of this solution, okay a general form of the solution, fine.

Okay, so we can construct geometrically using characteristics or given the initial conditions you can try it to see whether you can use Fourier series to actually construct the solution, okay. Right are there any questions? This course of course is not about analytical solutions. We are interested in analytical solution only because we want to know how the solution to the equation behaves.

So that when we do the numeric we are able to compare it with the solution, right and where the numeric does not work that I am sorry, where you do not have an analytical solution, right. Generally, then we will appeal to some theory of differential equations, so that we can figure out, right.

Theory of differential equation for example when talk about Laplace's equation having a maxima or the minimum on the boundaries that is the kind of result that would come out of theory of differential equations, right. Then we would appeal to theory of differential equation saying this is the problem that I have does the mathematical theory tell me anything about the solution before I even solved, okay.

That is what the theory does for u. It will tell you something about the solution before you even solved, okay. It is very important, right. So, the theory part is very important. So, then we do the numeric and as a first cut then you can compare to see whether what the theory has told you as satisfied by the numeric, okay.

In this case we have constructed a solution because this is actually a simple model equation for the kinds of equations that we are going to solve at later point and we will run it to some of the difficulties that you would run into let just say the full Navier-Stokes Equations or whatever. We will run it to them with this equation, right. There are certain elements of this equation, so we start with the simpler equation for that reason, okay.

So, we have a solution but now to the basic point of this course. I do not want to solve this analytically, if I did not know the analytic solution how would I solve it numerically? What could be the method that we would use to solve it numerically, fine, okay. The usual progression would be well we use central differences for Laplace's equation that seem to work why not your central differences here?

That is you use for the equation dou u dou t + a dou u dou x = 0. You use a central difference representation for this and a central difference representation for that. But I look at the boundary condition and at $t = 0$ I have a condition but I do not have anything beyond that, I have a concern, okay. And for the central difference here, yes on the right side does used to be some issue but I will ignore that.

But it is clear that I do not have, if I am going to in time and give a condition t = 0, I have nothing behind that, right. So, if were to draw grid lines like I did for Laplace's equation if I were to draw grid lines, I really do not know how to take, I really do not know whether, I do not know this value, I do not want to guess that value the future value. That should be like guessing the future value, you understand?

This is the issue here and I do not want to start here because then I do not know anything behind it. If I represent the equation here I do not know anything behind, right. I am only giving you a complaint. I am not saying that it is impossible to use central differences, right. I am sure we can think of ways of getting around it but my objective is, that is not my objective right.

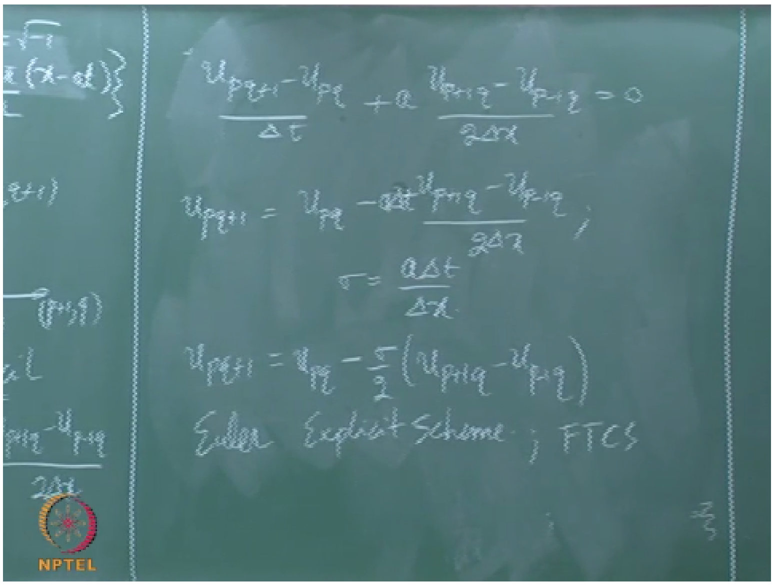
So, I have central differences here, I want to keep life easy so I use the forward difference here. Is that okay, is that fine? That seems reasonable that I use a forward difference I use these 2 points, I use a forward difference at that point. I use a forward difference at this point and I use a central difference for the spatial derivatives, okay. So for any arbitrary grid point, for any arbitrary point, okay.

So we can use central differences, so I will zoom in on this. So, for any arbitrary point Pq and as we did in Laplace's equation maybe at least in the x direction we will take equivalent roles, right. The t direction also we can take equivalent roles but we will see what happens here. So, Pq this is P+1, q this is P-1, q this is Pq+1, these are the points, right. I have gone with Pq because I already used i for square root of -1, right.

Up here we have already used i for square root of -1, so I do not want any confusion, that is why I have gone to Pq, okay. So, this very often in books, text books and so on you will see this referred to as a stencil, okay. This is just for you to get the jargon. You will see this referred to as a stencil, right. So, this stencil occurs everywhere. So, you can take these 4 points and presumably solve wave equation.

And what we are doing here very important what is proposed to do is we proposed to represent the wave equation at the point Pq using Pq and this 3 other points, okay. So, dou u t dou t forward difference Upq+1-Upq/delta t + the truncation error which I am not so, I should actually write an approximate there, it is the truncation error. Dou u x dou u dou x is approximately. U p+1q – Up-1q/2 delta x, is that fine?

(Refer Slide Time: 25:36)



So, do we have substituting into our governing equation. So you have Upq + 1-Upq/delta t + a times Up + 1q – Up-1q/2 delta x, should be approximately 0 we set it = 0. And this is the objective, this is what I am trying to get. This is the objective. The objective is Upq + 1 then we written as I take everything else note this is only one that is the time level q + 1. All of these are our time level q.

So, all the ones that are a time level q and going to shift over to the right hand side, so I have Upq – a times u + 1q Up – 1q/2 delta x. I miss something? a delta t, is that fine, everyone? So, this is what we have, so in fact this expression a delta t by delta x is going to be appear so often. We are going to give it assemble, we going to call it sigma. This appears so often that we are going to call it sigma.

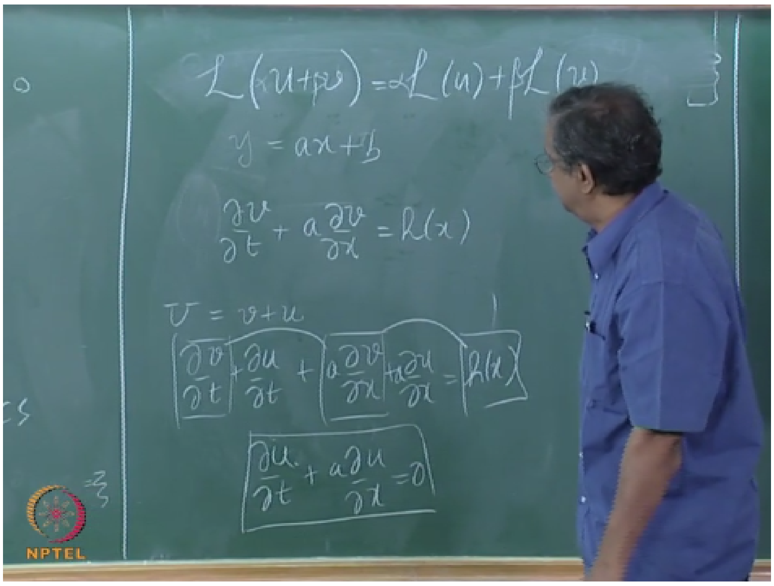
So, this is going to be Upq + 1 is Upq – sigma/2 Up + 1q – Up – 1q, okay. We have what I would call that, right. So, given something a time level q you can then incremented to the next time level. You understand what I am saying. You can find out what happens in the next thing. Given at q + 1 you can go to the next time level, right. So, at each time level you can move forward. We have a solution.

So if you have given an initial condition, you given an initial condition you can march that initial condition forward in time, right. q + 1 is given explicitly in terms of expressions that are recurring at time level time q, okay. So, this scheme is called an explicit scheme. Simply because you have q + 1 occurring and this is these are all a time level q variable to solve for it. It does not occur in an implicit fashion, is that fine?

It does not occur in an implicit fashion, so this is called an explicit scheme. Some time you will hear it being called a Euler explicit scheme. We will call it forward time central space, FTCS, forward time central space is that fine, okay, everyone? Right, so you could actually quote this, you can go ahead and quote this, right but this is different, this is little different from Laplace's equation.

So, we are going to do a little analysis, right. There are 2 things that we have got going for that. One is when we did the stability analysis for Laplace's equation we substituted exponentials for the error term and the error check that is the error term decays or not what happens to the error term, okay. In this case we have a similar set, in that case that was, they were if you think about if you remember back they were Eigen functions of the exponentials where Eigen functions of the Laplace. In this case they actually represent the solution.

(Refer Slide Time: 29:45)



The equation is a linear equation, you understand remember linear equation basically means equation is linear basically means that if you have if L is linear right, what does it say L (u+v) = L (u) + L (v), right. L (u) + v is L (u) + L (v), fine, okay. That basically what, so again and usual stuff, I mean the L (alpha u + beta v) = alpha L (u) + beta L (v). And this was the usual description of something being linear, right.

Which is, why we know that ax = ax y = ax or ax = y, is linear right or the classical when we talk about the straight line, this is the usual trap that people fall into. So, if you take the equation y = this is an assign y = ax + b, this is not linear, right. Linear it necessarily passes through the origin; this is not linear. It is a straight line; it is not linear. So, if you say linear wait a minute it is a straight line does not mean that linear?

If that confuses you then think of curvy linear, right. Linear all it means is the line. Curvy linear means curved line. You understand what I am saying. So, but in this case linear, this is the technical term, right. The function is linear, the operator is linear basically means that L (alpha u+beta v) is alpha L(u) + beta L(v), okay. And it is a very precise definition as a consequence of which y = ax+b is not linear, though it is a straight line, okay unless b = 0, fine.

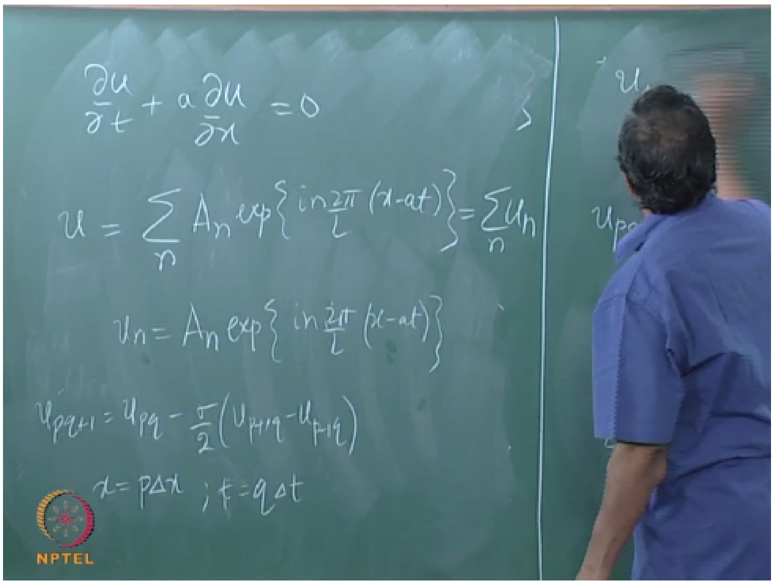
Okay, so that equation is linear we ask the question again, so if you have dou u dou t or just to keep this clean let me write a very general equation. So, if you have dou v dou t + a dou v dou x = sum h (x) the h (x,t) h (x). Keep it as h (x), right. The solution is v, okay but v is disturbed a perturb v, it has an error, right in Laplace's equation case I called it e, in this case I choose to call the error u, for obvious reasons that I already have an equation for u.

So, if the solution, if our candidate's solution is v + u, for candidate solution is v + u, okay and you substitute into that equation because this is a linear equation, what is going to happen. What is the equation governing u? Our original equation, you understand what I am saying. So, if I substitute this backend again Dou v dou t + dou u dou t + a dou v dou x + dou u dou x = h (x). I am sorry a dou u is very important, a dou dou x thank you, right.

This combination, this, this and this the combination of these 3 satisfy each other. They knock each other out. The combination of knock each other out us dou u dou t + a dou u dou x = 0 as the equation that governs that. Looks the same suspiciously, the same as the original equation, right and you would expect that because it is a linear equation.

Even in Laplace's equation case the equation governing the error was the same as the equation governing our original function, a solution is that fine, okay. Right, so what we have? We go from here.

(Refer Slide Time: 34:10)



So, I want to substitute now in dou u dou t + a dou u dou x = 0, which is now the equation for our error and I want to ask the question when does this decay for this particular scheme that we have chosen does it decay, okay for the particular scheme that we have chosen does it decay. I want, what was our u? u is like summation over n An exponent in 2pi by L (x-at). What you say?

Okay and because the equation is linear this = summation of Un over n, right we have that individual Un's are An exponent in 2pi/L x-at. Okay, we will play the same game we are playing for Laplace's equation. We will say for which wave number, is there a wave number is there a bad wave number, right? If we can say that for the worst for the wave number which has the largest gain as we march in time so this is what we are doing.

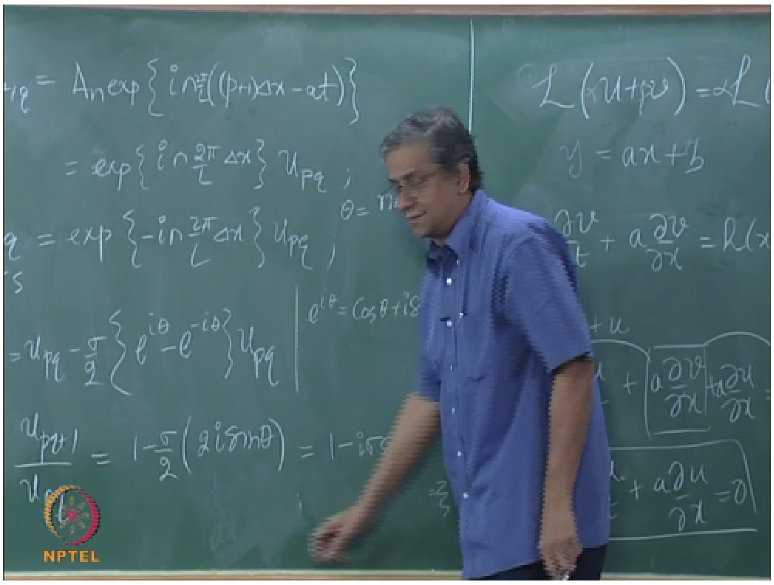
We have a perturbation or an error U in our solution, okay. We want to know that if march from time t = 0, t = delta t 2 delta 3 delta 3 so on, as I march. Is that error going to grow or is that error going to die out? Does that make sense? Is the error going to grow or is the error going to die out? That is the question that we are asking.

So, basically what we will do is we will pick one wave number then ask the question for the wave number which has the largest gain what happens? Right or is there a way number that has a problem that essentially what we are looking for is there some particular wave number for which we are going to have a problem, okay. So, because I already got queue right, so you please allow me to drop this n.

We understand between us we understand now that I am going to take look at the end wave number, right. So just, otherwise we just be carrying along a lot of these subscripts. So, let us just assume that we understand that I am dealing with end wave number. So, what do I have now? My view is Upq + 1 is Upq sigma/2 Up + 1q – Up-1q) and I said it earlier what is the relationship between Up + 1q and Upq.

So, if my x is P delta x, right and q could be we do not really need but q could be, t could q delta t. You could keep delta t constant if you want but it does not matter. If my x is p delta x and t is q delta t, okay my x is delta x t is q delta t.

(Refer Slide Time: 37:45)

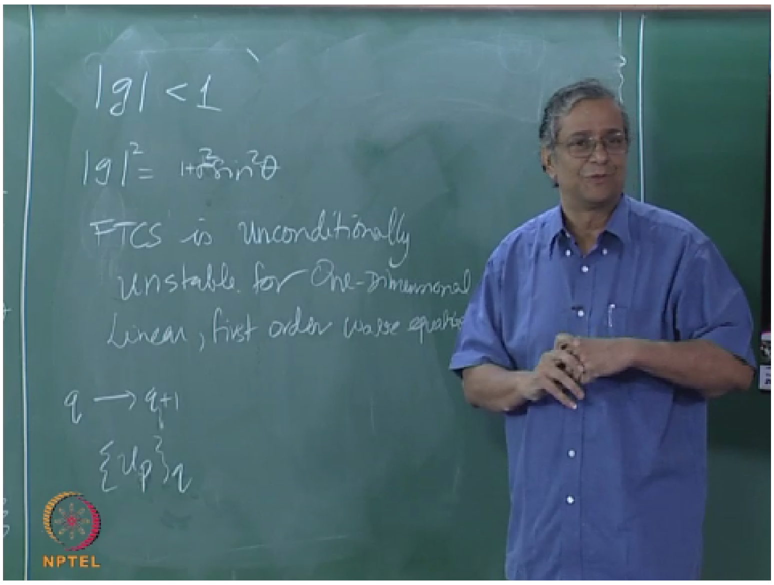


What we have, Up + 1q = where do I get this now, An exponent in excess P + 1 delta x – at at the time level, I leave this as t, okay. And this is nothing but An you can just check, so it is e power exponent in, I forgot the 2 pi/L in 2 pi/L delta x * Upq, fine. And in a similar fashion because there is a free delta x here which is x, this is basically express delta x, right. And Up – 1q in a similar fashion is exponent in the – in 2pi/L delta x Upq, fine.

So, I can now substitute into my forward time central space. Upq + 1 is Upq – sigma/2. Up + 1 is this one, this is a mess but these 2 expressions are the same. So, I will just redefine something, I will redefine that as theta = n delta x 2 pi/L, right. I mean, I could take L = 2pi n that will go away, it does not matter. So, this is e power i theta – e power – i theta * Upq. Everybody with me, okay.

I can divide through by Upq, and I get the gain g as Upq + 1/Upq 1 – sigma/2. What is e power i theta – e power – i theta? 2i sin of theta, remember I am using Euler's Formula. e power i theta is cos theta + i sin theta, fine. So, this gives me the gain as being 1 – i sigma sin theta. This a good news or bad news? What do you say? So, you want the gain, the magnitude of the gain to be less than 1, right.

(Refer Slide Time: 41:21)



We want g to be < 1, okay. But what is g? What is the magnitude of g? g is or g square be < 1. What is g squared? 1 + sin square theta. Sigma square 1 + sigma square sin square theta. See, the only parameters that we have to choose, right. Now, this brings this whole thing now we face the fact. You cannot change a, the only parameters that you have to choose a delt x and delt t, okay.

So, sure I mean theta depends on delta x and so on but there will be the wave we know that there is a highest wave number that we can represent, right. So, that all of that is very clear. We have no probably issue with that. So, the only parameter that you can chase change the sigma, right. It is not complex. I mean it is real. So, sigma square we are stuck with it am I making sense? We are stuck with it.

So, sigma square there is no sigma, sigma value for which the sigma is going to be < 1. So, this does not work for us. Theta = 0, yes = 1, right but theta = 0 is the DC component essentially. So, theta = it does not help us. We are not getting anything, right. Is that fine? So, forward time central space, FTCS is unconditionally unstable for, be careful now when apply to one dimensional linear first order wave equation, fine.

You have question? Why should g be < 1? See, when you go from time q to q+1, if you go from time q to q+1 if g is > 1, right. So, we are, you are basically look at this as a sequence. So, you have, you are generating a sequence, I will leave out the p part you are generating a sequence which is Uq. You are generating a sequence of use indexed on q. And we are asking the question does this converge and we in fact wanted to go to 0.

U in this case is the error, right. They are original solution U in this case is error. You are asking the question does u go to 0? U is the error in v. U can have which is non 0. U can be a solution which is no but it does not satisfy our original equation. Is this satisfying our original equation including the boundary condition? Well it has homogeneous boundary conditions and homogeneous then you can add it, right.

And then the question is what happens in the initial condition. So, it has be 0 at the initial condition and somehow it magically came out. You understand, okay, right. What we are basically saying is even from our numerics typically see where these things come from is even from the error. It is not that there is an error that you have injected into the solution and does not decay.

There is a source of the error which comes from our say from our round off**,** right. From various reasons just a source of error that you have. What happens to that source of error? Right. It is no different from the whistling that you hear in an amplifier. The whistling that you hear from an amplifier it is not that there has to be an input just thermal noise in the resistors in the circuitry.

The thermal noise is enough, so the issue was, the question that you have is it is not that you have the disturbance is there. You have the thermal noise which is equivalent, equivalent here would be I have round off error at every step I have these errors and making these errors. What I want to know is errors grow or those errors do not grow. So, in a sense I am not actually answering the global question.

What you are talking about is the answering the global question, right. So, the stability analysis that we are doing both what we have done here what we are doing now and what we did in Laplace's equation. This is a local one. We are only asking at a given grid point what is happening to the what happening to the solution, if I were to integrate it out but not for all x. Am I making sense? Okay.

Say I see where you are coming from you are saying it is a homogeneous equation. This equation is homogeneous equation it basically does not disturb you can add any amount of this homogeneous equation solution like you want. Some constant this homogeneous equation but normally in your differential equation if you look at it the way it works this, the homogeneous part actually takes the boundary conditions.

In this case even the boundary conditions are 0. So, in the sense this id does not, this is truly sort of in the null space of that operate. It does nothing, okay. But potentially if it grows it can grow up. The problem that you have is if I, the difficultly that I have is if this Uq if the gain is > 1 the sequence that I get is diverging. So, I am going farther and farther away from my v that satisfies the original equation.

No, there is no issue of boundary condition because I am only working at a grid point. No, I am not looking at a solution that is whole point. No, we are not looking at it, the solution is v. The only question that we are asking is there a wave number that is unstable in a sense, is there a wave number that is going to grow. That going to become unbounded, if I disturb that wave number or there is an error in that wave number is there a wave number that is going to grow. That is all.

That is the only thing that is happening, Is that fine, right? So, that is basically, so we are generating at this point p. So, though I removed it I will stick it back there at this point p we generating a sequence indexed on q. And they are asking the question what happens at the point at that X location what happens as q goes. But it is a, the analysis that we are doing is a local analysis.

It is not a global analysis may be one may be at one of the other classes I will do a global analysis where I take the boundary conditions also talk about, right. That is really what we should do, okay. Here sort of I beg of saying well this we are engineers this is how we get this and this is already unstable. Forget doing the full thing this is already unstable, right. We are already out of luck. This is already unstable, fine.

And you can try to implement this and you will see that it is unstable. Try to implement this so, this analysis actually works, okay. This analysis actually works which is not justification. I cannot just say that it works and they work, right. But what I am saying this is analysis actually, so this gives you this q's this sequence of q's that you are going to generate using that automate on which it is FTCS if there is any disturbance it is just going to be that.

Is that fine? It is going to diverge. If there is a smaller disturbance that disturbance will grow that is the key, fine. Is that okay. So, this is where we are FTCS did not work. So, the next thing is, next obvious possibility is we did forward time why do not we try forward space? Now we are groping in the dark. If did not work now we are sort of groping around trying to figure out what to do.

So, forward time, use forward time let us try forward space. Is that fine? So on Monday's class we will do forward time forward space FTFS, right and let us hope that it works, fine. Okay, I will see you then.