Introduction to Computational Fluid Dynamics
Prof. M. Ramakrishna
Department of Aerospace Engineering
Indian Institute of Technology - Madras

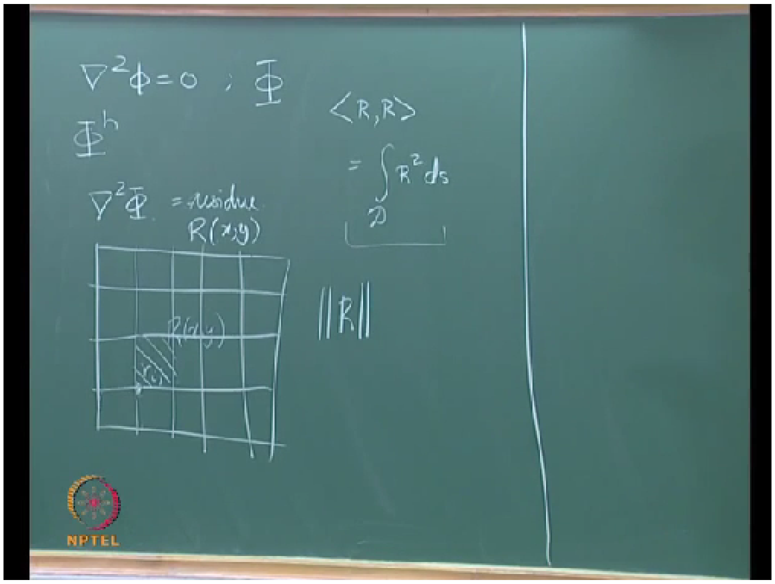
Lecture – 15
Laplace Equation - Final, Linear Wave equation

So we saw demos of Laplace's equation, right, last time around and what we will, there are few things that I left out, that I did not mention, which I will mention now. Then we will look at applying boundary conditions. I do not know if you remember before the demonstration, I was talking about applying various types of boundary condition, right. I talked about different problems.

So we will just quickly look at applying what are called Neumann boundary conditions or derivative boundary conditions, okay. We will spend a little time on that and if there are no questions after that, we will try to see where we leave Laplace's equation behind for now. We will come back to it later, right and go on to other, other simple problems. It is fine. So this sort of came under, I do not if you remember, this came in to the category of some simple problems.

That is right now just for you to recollect, we have shown that we can represent numbers on the computer, right, arrays on the computer, functions on the computer, derivatives on the computer and having done that, we said let us look at a few simple problems where these can be used so that we can represent differential equations on the computer and see if we can solve them. In the first simple problem that we looked at was Laplace's equation. It that fine? Okay? Right. So what we will do today.

(Refer Slide Time: 01:38)



Let me just first add a little so to yesterday's, so we were solving nabla squared phi=0 and the discrete version of that, right, we set the discrete version of that. As I had indicated earlier, so if phi h is the solution, right, is the candidate solution, right, or if you want to deal directly with the continuous equation, if phi is the candidate solution, this little phi is the actual solution. So if you were to substitute this in there, right, if it were a solution, then del squared of this nabla squared of this will be 0, okay.

But if it is not then it will leave a residue, that is you will get nabla squared phi=some residue, okay. You can call it R. So in facts in this case in 2-dimensions it will be R of xy, fine. So yesterday in the demo that one of the reasons why I did that 5*5 matrix again, was I wanted to actually show that one was to calculate the difference between 2 matrix and the other was to use the residue, okay and I would suggest that you do both without adding too much detail, right.

I would suggest that you do both. Especially in SOR you may find that there was an interesting difference, okay. Especially in SOR, you may find that there is an interesting difference. So I would suggest that you would find both of these. Now the residue of course if you have a domain, we have been taking a square domain so far so that the residue Rxy is defined within the domain, right.

On the boundary, capital phi and little phi satisfy the boundary condition. Therefore, it is 0. The residue is 0, okay. The residue is non-0. So if you were to take a grid on your grid, on your grid. At each point, you will actually have an rij, right. At each point, you will actually have an rij. Is that fine? Which is the residue at that point which you can find by taking Laplace's equation that is the average of these 4 quantities-4 times the middle quantity, right and if that is not 0, then you have, you have a residue at that point, fine, okay.

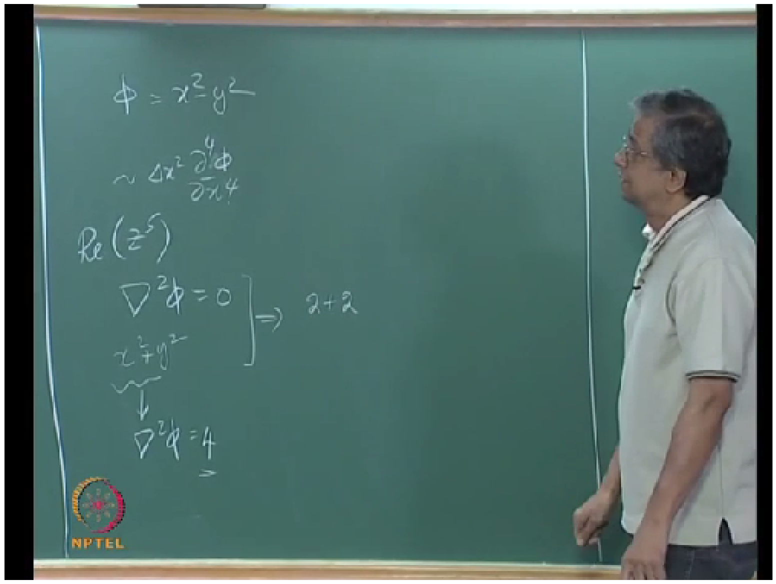
And if you say that I want to find out what is the magnitude of the residue, we will have to use the dot product again, okay. So the dot product be defined, so the dot product of R, R would be nothing but the integral over whatever the domain is. In this case, this is the domain, okay. R squared say ds or whatever. Am I making sense? Okay? It just very quietly extended the definition that we gave earlier, I just by making it D, okay, the domain of definition and it works.

It will work. So in reality, in reality, you try this out, you repeat the, you repeat the programs, whatever I wrote you, repeat them and you will see that the value of the error that I plotted and the value of the residue that you get, the error that I plotted, that seems to depend on the grid size. Actually the way we calculated it, it could depend on the grid size. Whereas if you do it this way, it is not going to depend on the grid size, okay.

This is actually an integral. So if you were to, right, so you please, you want, you want evaluate this or you want me to write the expression to evaluate this? This double integral, this is an integral over an area, okay. So you can, you can try to evaluate it and see what you get, okay. So when I say the norm of R, when I say the norm of R, I mean the square root of this, I mean the square root of that and in a sense it will turn out to be the square root of the sums of Rij and so on, right, okay.

But you have to be a bit careful because when you evaluate these integrals, there is, you can use, when you evaluate the integral for instance you can take rij times this area to find the integral over that area, something of that sort, okay. Is that fine? Okay. Only thing it is R squared here, it is not rij, that is the thing to remember, okay. Is that fine? Everybody is with me? Okay.

(Refer Slide Time: 06:39)



The second thing is, before we go on and since I have talked about the residue, an apt point here is, we have so far taken the actual solution to be x squared-y squared, okay. You have, you have solved a problem where you have taken boundary condition so that the actual solution is x squared-y squared. So it is possible for you to verify that your code works. I chose this because it makes life easy for you.

You are writing this program possibly for the first time, right. I wanted to keep everything simple. What was the truncation error for the second derivative term, do you remember? What truncation error was for the second derivative term? What was the derivative involved? Fourth derivative. The Truncation error is like some delta x squared but the derivative is fourth derivative assuming that we are representing the derivative, I am sorry, derivative is fourth derivative, right, or if you say phi instead of u.

Derivative is fourth derivative. Am I making sense? So the truncation error by this is basically 0, right. So at this point, I would suggest that you chose an analytic function where the truncation error, is not 0. This derivative is not 0. So either you, either chose something that has a fifth degree polynomial z power 5, z power 4, something of that sort. Z is a complex number. Either chose the real of z power 5.

Do something like this. That is one possibility, right or pick something like sine z, okay. Pick a transcendental function, okay, exponential of z, hyperbolic tangent of z, something of that sort, right, okay, fine and see how well your code works, right. So far the problem that I have given you, you need to square everything is well-behaved. It was deliberately chosen. Now you have a situation where you are going to pick a function which is, where this truncation error will not automatically be 0, okay.

So see how well it believes, right, how well does the code behave as well, when you, when the truncation error, is there any change in behaviour. So just try it out, okay, right. So this is important. So the process, the process that we are going through right now is also important. So anytime you encounter equation for the first time, right, if you, if you or a class of equations, you should see whether there is a sub-problem for which you have an analytic solution.

Pick a simple analytic solution and test your code against that analytic solution, okay. Then you can make the known solution as more complicated. Where you do not try to do it all at one shot, that is not a good idea, okay. Do not try to write the program all at one shot. Is that fine? The second thing is where to have an equation for which you do not have a solution, you do not have an analytics solution, right, where here it is supposed to be computation of fluid dynamics, though we are not going to look at Navier-Stokes equations in great detail, right. Right?

It is a, it is an introduction to computation of fluid dynamics, so we are really not going to get to Navier-Stokes equations directly but there are not that many analytic solutions to Navier-Stokes equations. So what you do if you do not have an analytic solution? How do you test your code? How do you test your program? So one way to do it would be, let us take this, let us take Laplacian or phi=0 and say that we do not know our way to generate a solution for this on the unit square, okay.

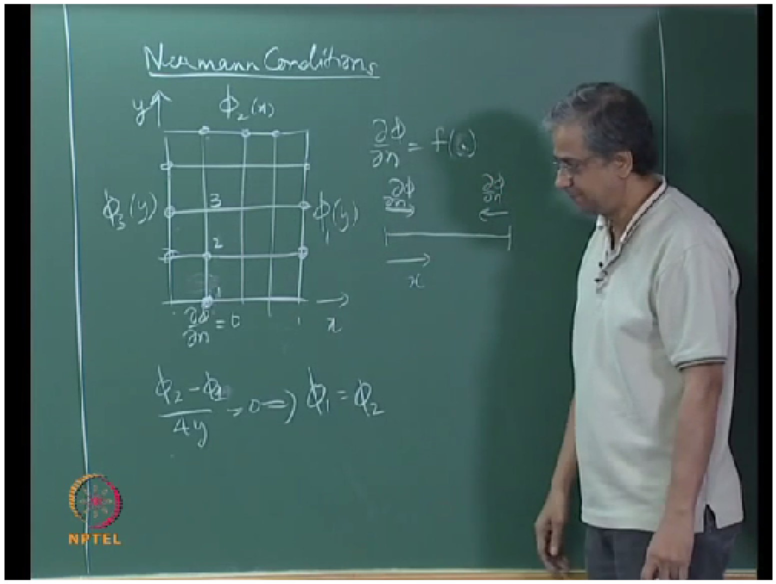
So then you just guess something. So just say instead of saying x squared-y squared, we guess x squared+y squared. We guess x squared+y squared. So anyway you know it is not a solution already, right. So if you substitute it, it will give you a residue. What is the residue that it gives you? 2+2, right. Residue that it gives you is 2+2. So this is a solution to nabla squared phi=4, right.

So I started off with an equation for which I did not know a solution. Now I have an equation for which I know the solution. Am I making sense? Of course, there may be difficulties associated. Sometimes there are difficulties associated with adding a right-hand side, okay, right but if you are trying to figure out how well am I representing this nabla squared phi, how well am I representing these derivatives, how well is it working, then it is possible for me to actually turn around, then it is possible for me to actually turn around, right, substitute some guess solution in to the equation.

It leaves the residue and just say well that that does it for me, okay. That is an equation for which I have a solution. Is that clear? Okay? Because very often you will hear people saying, oh I am trying to compare it to experiment because I do not have a solution. I am going to compare it to a, you know, the solution of another code because I do not have a solution. Not necessary, right. You can generate an equation that is very close, for which you have a solution.

What it will do is it will add typically the source term that has its own consequences, that has its own consequences, as long as you are aware of that but basically you can turn around and generate an equation for which you have a solution. Is that fine? Okay, right. So this is really as far as the residue goes. So the residue is not only useful to check whether you have a solution. The residue is also very useful for you to generate a solution, okay. Is that fine? Everyone? Right. Let us do the last part. I was talking about the Neumann conditions.

(Refer Slide Time: 12:19)



So, conditions, so effectively what you can have is, you can have, we will still stick with our unit square. Is it okay? There is a very simple reason why I am sticking to unit square or you can change to a rectangle or whatever. If the geometry change, then we cannot use the Cartesian mesh, right. Then we will have to do something special. Either we will have to use unequal meshes or we will have to get in to some, the game of grid generation which is a completely different course, okay.

So I am restricting myself to squares and rectangles simply because it illustrates what we need for the introduction to CFD part. So on one of the sides, right, it is possible that what you have is a condition that is like do phi do n=either the function or 0, okay, in particular 0. We say 0 because this is a condition that we are used to in fluid mechanics, right. Do phi do n=0 and on the other, other sides, you may actually be given a function of, you may actually be given functions of x and y.

Am I making sense? Okay. So if you were to discretize this, if you were to break this up using gridlines, okay. On these nodal values, it is possible for you to find the phi value directly from the function specified. So what do we do here? What do we do at the bottom? I need a proposal. So I can use a finite difference method to represent. So I can use the finite difference method to represent do phi do n at this point, okay and there are many ways by which we can do it.

One possibility is that the simplest thing, so if this point is 1, this point is 1 that point is 2, then you can just basically say phi 2-phi 1/delta y is 0 telling us, phi 1… That is easy enough to do, okay. So that is the boundary condition. So you would actually evaluate this point on the boundary. You would actually calculate this point at the boundary. Where values of the function are given on the boundary, they remain a constant, they do not change.

Where the value is not given on the boundary, it becomes part of your iteration. Whatever the value is at 2, the same value is given at 1, that is one possibility, okay. Is that fine? If you are not satisfied with the truncation error of this representation, you can actually use 3 points, right. You can use higher-order 3-point representation to find the first derivative at this point, okay. Everybody, right?

And at this point, at this juncture, I want to point out something. It is okay that we had do phi do n=0. You could have do phi do n=some function. Either function of n or function of y or whatever it is. You could have do phi, do phi do n not function of n but do phi do n is a function, some value, okay. Do phi do n something that, that changes, okay. That changes, one it could change along this, along this length, right. So in this case, I do not want to say t or whatever.

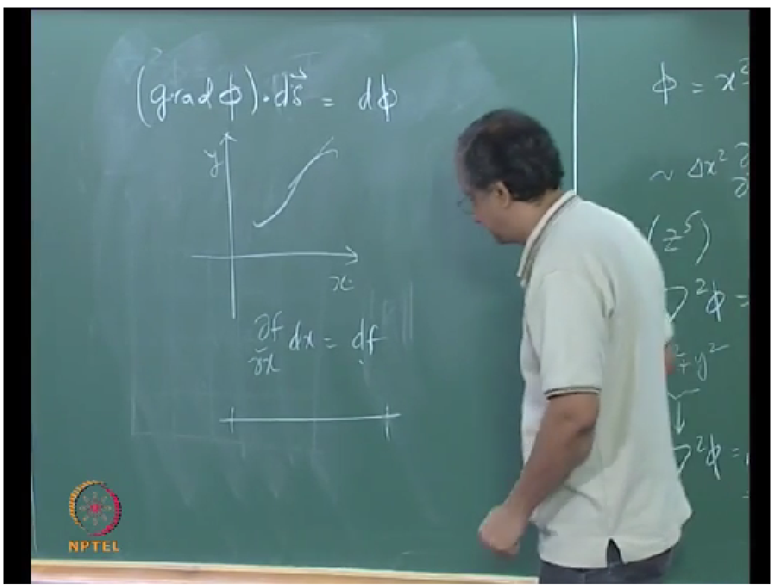
So do phi do n which is a function. I will just put a dot there and your, the problem may be that if you have to integrate this, right, consider a situation where for example that this is heat flux or something of that sort. So somebody gives you a boundary condition where this is heat flux. They are telling you how much energy is flowing into the system, okay. So this is the rate at which the energy is flowing in to the system, right.

So you say what is big deal. So you just set this derivative equals f, right, that is what it looks like. In this case, you have to be very careful as to whether what is the nature of this. Whether it is a, whether it is a, whether, whether the direction is important, okay. Very often we do not think about it. So we go back to the discussion that we had. So you have do phi do n=0, this is x direction and do phi do n=0, or do phi do n=some function on this boundary, okay.

So I am working up to something. I mean you are wondering what, what is he talking about. See the idea is when you normally talk about a derivative, there are 2 things that are, derivative basically has 2 components to it, right. The way I would like to think of it is basically a linear transformation into direction, okay. This is something that I mentioned earlier in the beginning of this semester.

So the derivative basically consists of a linear transformation into direction. It is very important for me as to where I am going to go now, right and you are used to it from the, you are used to it from the multi-variant calculus point of view. You are not quite what you call it, you do not quite, we do not quite think about it when we are talking about calculus or one variable, okay. Derivatives in one variable.

(Refer Slide Time: 18:23)



So you are used to the directional derivative where I say grad phi dotted with n, right or the grad phi dotted with ds, some differential element, gives me d phi. You are used to this, right. You are all familiar with this. This is it. So this is, this is the linear transformation. This is the direction. This is really in a definition of a derivative, okay. Now you say, so what? How does this, how is this important.

Well it makes a difference, you can say the derivative has something at some point but it makes a difference whether you are walking uphill or walking downhill. The direction is important. It is not enough to say that oh I am on a mountain. It is important to you to know whether you are walking uphill or walking downhill. The direction is important. Do you understand? So if you are applying a boundary condition which is based on a derivative, most of the time when you say derivative.

When you are thinking derivative, when we take a derivative of some function, we are implicitly thinking in terms of positive x direction. Do you understand what I am saying? Most of the times, you are saying do f do x dx is dy, df. It implicitly think, in your mind you are implicitly thinking of dx being the positive quantity. It could be negative. You could be going in the other direction.

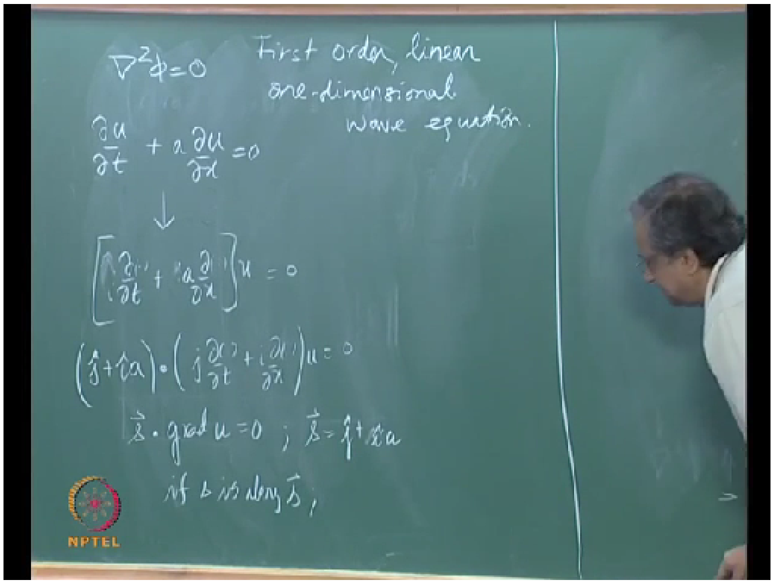
It could be going downhill, okay. So anytime you are applying derivative boundary conditions, you have to be careful, make sure that you are evaluating the derivative properly. If it is, the derivative=0, it is immaterial but if the derivative=something, then you have to pay attention, right. Especially in multiple-dimension, there is a directional involved. Pay attention to what you are doing.

You can get the sign wrong. It is very easy to get the sign wrong, okay. Is that fine? Are there any questions? Right. See normally the derivatives in one dimension, we do not bother but usually, case where students get into trouble is if they are talking about solving the heat equation which we look at a little later in the semester an I give a boundary condition, the derivative boundary condition on the right-hand side because I have a mean right.

I give a derivative boundary condition on the right-hand side which is not a, which has a flux term. If it is a vector, then typically people run in to difficulty because they do not remember that the derivative actually is a, has a, there is direction associated. It is a linear transformation and a direction and in 1-dimension, we do not think about it but in multiple dimensions, it is very clear. It is, in fact in multiple dimensions, very often you call it directional derivative.

It is always directional derivative, okay. It just so happens that in, in 1-dimension, the direction is dx that is what you are thinking of it, okay. Is that fine? Are there any questions? So what you can do is, you can possibly try again Laplace's equation, try to apply Neumann conditions and, right. Said do phi do n=0 on one of the sides and see what happens. What happens to your, is that fine? Okay? Right. So what we will do is, we will now change gears as I had indicated.

(Refer Slide Time: 21:52)



So we have looked at, we have looked at this equation so far. Let us look at a different equation. We will come back to this later in the semester. Let us look at a different equation. I am going to look at the equation do u do t+a do u do x=0, okay. Is that fine? Everyone? So this is the first-order, a simple equation, it has a long name, first-order linear one-dimensional wave equation, okay.

So it says it all. All the derivatives are first derivatives, okay. The equation is a linear equation, you can verify that it is linear and it is 1-dimensional in the sense that it is in 1-space dimension and I have introduced a new term which is time, okay. So actually it is still 2D. As far as we are concerned, it is 2, it is still 2-dimensional because it has x and t instead of x and y. Even I may have, well I have written do u do y+a do u do x=0, right.

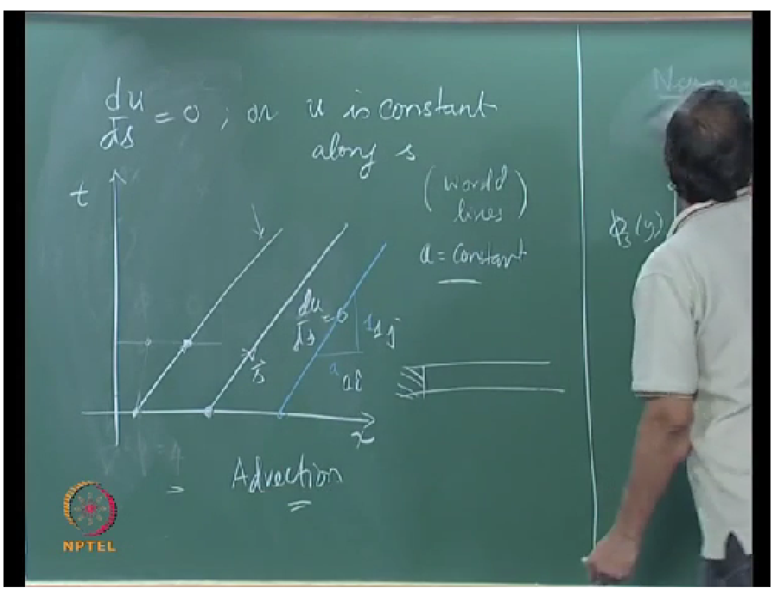
It is just chalk dust. You just interpret that y as t, right but because it is in one space dimension, we refer to it as 1-dimensional problem. Is that fine? Okay? So what is the nature of this equation. What is this equation? What is the behaviour of this equation? Is it possible for us? we had an analytic solution for Laplace's equation. Is it possible for us to get an analytic solution to this equation, okay?

So you may have seen this, you may have seen this for a variant of this in your partial differential equations course. Let me just quickly go through this. It is possible for me to write this as a directional derivative. Actually possible for me to write this as a directional derivative. How do I do that? So do not let it bother you that this is t, right, as I said it. You just chalked us. It could be y, right.

So this is basically looks like j do/do t, that is an operator+ia do/do x, these are unit vectors. Here i is not, it is a unit vector in the standard Cartesian coordinate system, acting on u. Is that fine? =0, what am I doing? And this I can split as j+ia dotted with j do/do t+I do/do x on u=0 which is of course some s.grad of u=0.

Now you will understand why I made such a song and dance about the directional derivative, right. I need a directional derivative. Is that fine? Whereas this j+ai or ia, okay and if the length s, the coordinate s is along s then this basically tells us from the definition of the directional derivative. In fact, it is a derivative.

(Refer Slide Time: 26:09)



This tells us that duds=0 or u is constant along s. Is that fine? Okay? Of course if the differential equation, the right-hand side had not been 0, right-hand side had been something else, then this will be duds=f and then u will not be constant along s, okay because the right-hand side is 0, it happens that u is constant along s. What does this mean? Let us look at this pictorial. So that is x, that is t.

So in your course in physics, maybe you have heard of world lines or something of that sort, right. So this is basically what we are doing. This is an xt plane and we are drawing world lines, right, that is what I, I want you to if you have gone into this, I want you to just think about, right. If you have run into it till your physics, I want you to just remember world lines, okay, that is fine.

So this is x, that is t. This is what we are saying. So let us say that we are solving, we are given that differential equation and we are given a condition along t=0, at time t=0. We are given, what is the state of u. What this basically says is that along the direction s, along the direction, this is s, this is along the direction s, this is a vector s, right. Starting at this point, I am going to measure the length s and this basically says along this line duds=0. Is that okay? Everyone?

That means u is constant along this line. So if you take another line and since a is constant, okay. Did I mention a is constant? No. But I mentioned that it was a linear and if you have checked out, if you go back and check to see when, when is it linear, you would need, right, a to be constant or a function of xy. It could be a function of xy, even then it would be linear, okay. So a is a constant, a is a constant, right. a is the function of xy, it would still be linear.

I am getting a few shakes, fine, okay, right. So then you would have a different s and even along this line, u would be a constant, okay. So we have a differential equation. Is there a physical problem for which this differential equation works? So it is very simple. Just let, let us just say we have a stream of water, right. So this is the standard example that I give you. Have a stream of water that is flowing along, along x coordinate direction, okay.

So I am here and in that stream of water, I am going to add chalk dust, okay. So the chalk dust that was added here at sometime t=0, travels along the x coordinate direction in time, right. So when xt, so if the stream is moving at a constant speed a, right, after sometime, after sometime, the chalk dust that I added here, would have travelled to that point. You understand what I am saying? Okay? No?

So basically all it is doing is, so here I have a physical problem that is actually represented by this equation. So I have a stream that is moving at a constant speed. I add some marker, some tracer, right. I add some ink dye or I add some chalk dust, I add something and that propagates. The property u that you are talking about is the amount of chalk dust that you have added, right. That the type of chalk dust that you add.

So I have added blue chalk dust in one spot, I add white chalk dust here and I add blue chalk dust somewhere else and that blue chalk dust travels along that line, okay. At the speed, what is the speed? It travels a distance a in unit time, right. So that is 1j and this is ai. It travels a distance a in unit time. Is that okay? Everyone? Okay, so it is very clear. So what that basically does is, what this basically does is, right, it propagates whatever that is there at this given point, at this initial point, it propagates it at the speed a, right.

It propagates at the speed a and as a consequence because duds=0, because duds=0 which means that it is not as though white chalk dust is being added everywhere. Since I am being added only at one point. If I were to along the length of the, along the length if I were to add white chalk dust, right, then this right-hand side would be an f, the rate at which I am adding that white chalk dust, okay and as you go along, you would be accumulating chalk dust as you go along, fine.

Another possible example something that maybe you are familiar with is if you turn on the water heater in the morning, you want to take a shower and very soon you learn that you should not get under the shower and turn on the, right, tap or faucet or whatever it is. Because what you will get initially is cold water. It takes time for the water, hot water to travel from the water heater to where you are, right.

So if you open the tap to a certain extent, if the water is travelling, so there is a front. So you have a pipe, there is hot water here on the left-hand side and this cold water. So you open the tap, this surface calls it a contact surface if you want. This surface propagates at a certain speed. So if it travels at the speed of wave, then you will know that your cold water and hot water, that interface, what is the, what is the, along the length of the pipe at each time, at each time instant along the length of the pipe where it is. Is that fine?

So it is a very simple equation. It is a very simple equation, okay. But what it does for us is it accepts this property called advection. We use the term advection because for historical reasons, convection is already used up by the convective heat transfer people. So it is convection but just so that there is no confusion in at future times, we will introduce the term advection, okay. So something is being carried and in this case, because duds=0, something is being carried without change in identity.

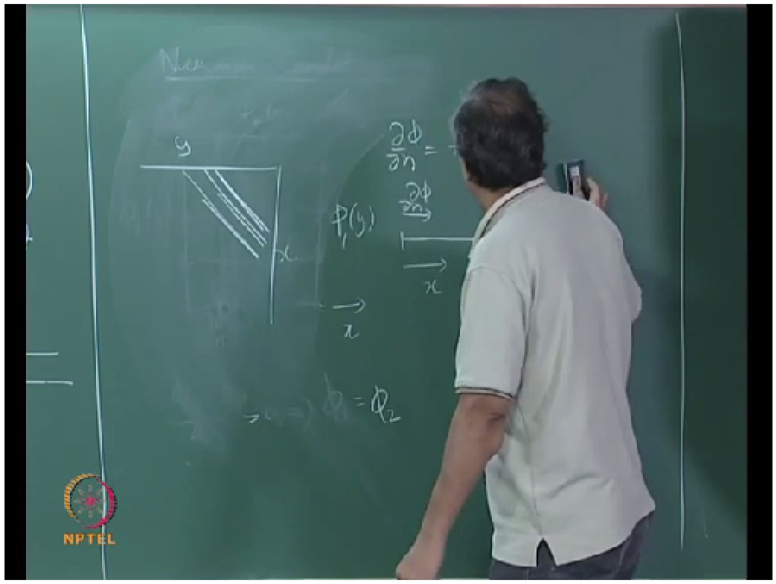
Is that okay? Right? Some property is being; some property u is being propagated without change in identity. This is very different from Laplace's equation. Laplace's equation was averaging, right. Laplace's equation was averaging. Whereas what this is basically doing is, this is carrying, carrying, propagating, whatever that you have, whatever you have, is being propagated in a certain direction, okay.

This is, this is very important. It is being propagated in a certain direction. So far us this is very important. So you could clearly what we have done in the sense, if you think about it, this is like we are saying if the coordinate system were not this xt coordinate system but were actually aligned along this line, then instead of having a partial differential equation, we will have an ordinary differential equation.

Really that is what this is. This basically says that and somehow if you have managed to rotate the coordinate system, the physics of the problem that you are solving. This would be, this would just be an ordinary differential equation, right. Say in your mind, I do not want you to think. That is why I said this is just chalk dust. If you go, if you go to, if you go to, of course, I mean this example may not help everybody but if you go to a library, right, where the staircase is at an angle with respect to the coordinate system as seen by the streaks.

So there are 2 rods. There are 2 rods, a library, an institute library has 2 rods that are slanted, okay.

(Refer Slide Time: 35:18)



So that could represent your coordinate system but the staircase to the library is that way, okay. So if you have this as x and y coordinates, if you think about it, what is the given stair, what is this given step, along that the height is the constant, that is the property. You understand. So do not think of, do not think of it as t. It has to be at time, right. It just basically says that along this, this property is a constant.

The height is a constant. So you have these stairs and along different, if I know that these stair, 1 step starts somewhere at a certain height, I guarantee that if you travel along that line, that height will be the same. If you travel in some other direction, then you are in for a surprise, because the height will suddenly, abruptly change. Am I making sense? So if you travel along that line, right, the height is a constant.

These are basically characteristic directions. Is that okay? Right? In this xy coordinate system and there is a coordinate system if you perform a rotation, you can find out what that orientation is, along which the differential equation for that is, it is a constant. Height is a constant. Am I making sense? dhds or whatever it is, it is 0. Is that clear? Everybody? Right? So it need not be, when I say propagation, it is propagation.

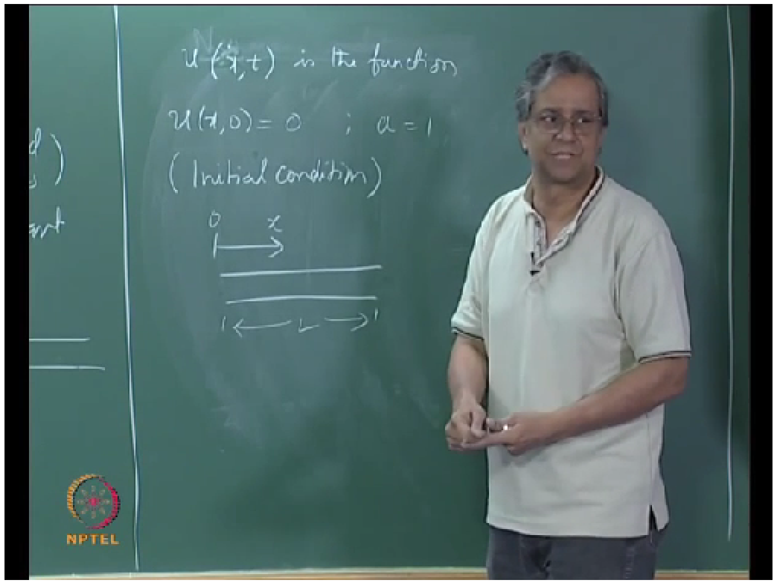
When I say propagation, it looks like propagation, that is because there is time but it need not always be in time. The other coordinate need not be time, okay. The other coordinate need not be time. Is that fine? Everyone? Okay. So this, this equation now results in this kind of a scenario is there are a way for us to somehow solve this? Is there any other way that we can, is there anything else that we can do?

Is there any other way that we can solve this equation? We have lots of examples that had been going back, there are lots of examples that I can give for. Is there any other way that I can solve this equation? You have to go with the notion that, right, remember what I had said earlier. So we are basically trying to integrate this differential equation, okay and all integration is guessing. So you guess, you substitute into the differential equation and you try to see whether you are able to get the solution to, right.

Whether it, whether, whether the guess is a solution to the given differential equation or not. Am I making sense? By substituting into the differential equation and verifying whether it is a solution or not. So normally what you would do, you just guess. So with the information that we have, with the knowledge that we have, is there a way for us to guess? We will see. Maybe we will go along a little further and see whether that… So what could be the, what could be the nature of the function that this propagated?

Let us try a few functions and then see where that takes us. Maybe that will give us an idea as to what is happening.

(Refer Slide Time: 38:41)



So if my initial condition, the typical boundary condition could be u at, u at t=0, right, u of x,t is the function. u of x,0=, let us start with a simple one. Is this enough? Is this enough? Have I specified everything that you need to solve the problem. a=1. Where do I give a boundary condition? Where do I give a boundary condition? Your t=0, this is the initial condition. So you have to be bit careful.

So since we are saying time, now we are going to come up with 2 different, 2 different names. So that is the initial condition. We say initial condition and boundary conditions, that is the initial condition, okay. Why x=0? **"Professor - student conversation starts"** (()) (40:06) Because it is propagating from left to right, because it is propagating from left to right. Does that make sense? Why cannot I prescribe something that x=L.

Let us say that, let us say that, I am actually looking at something propagating through a pipe. The length of the pipe is L. You were saying that I should prescribe the condition at x=0, right. So you are saying that I should prescribe the condition at x=0, right. That is fine? That is correct? Why cannot I prescribe it x=L? It can be. We can or we cannot? We cannot. If a is negative, if a is? If a is negative, No, no. a is 1. That conversation cannot and does not make sense, a is 1.

We will look at it. If it flip sign, why it happens is a different story, right. You are just basically saying flip the coordinate system overall in one way, yes. If a is negative, what is going to happen? Characteristic sign, directions will change. Slope of the characteristics will change. That is the key. **"Professor - student conversation ends"** So you see the thing is, so think about it. I open the tap and I insist that hot water comes but hot water does not come.

You understand I cannot make that hot water. I cannot open the tap and insist that the water at the, at the exit of the tap be hot, right. The water in the water heater is hot. I can turn on the switch and make the water in the water heater hot, right. I cannot turn on the switch, see unless we do something to this contraption but right now, water heater pipe, tap, right or a faucet, whatever.

That is all you have. You cannot insist that the water at this end be hot just because you have turned on the water heater. What you can do by turning on the water heater is eventually the water, hot water will come out. Eventually, because you turned the water heater. You understand what I am saying so you can prescribe the condition here, right. In this case, in this particular case, you can prescribe the condition here.

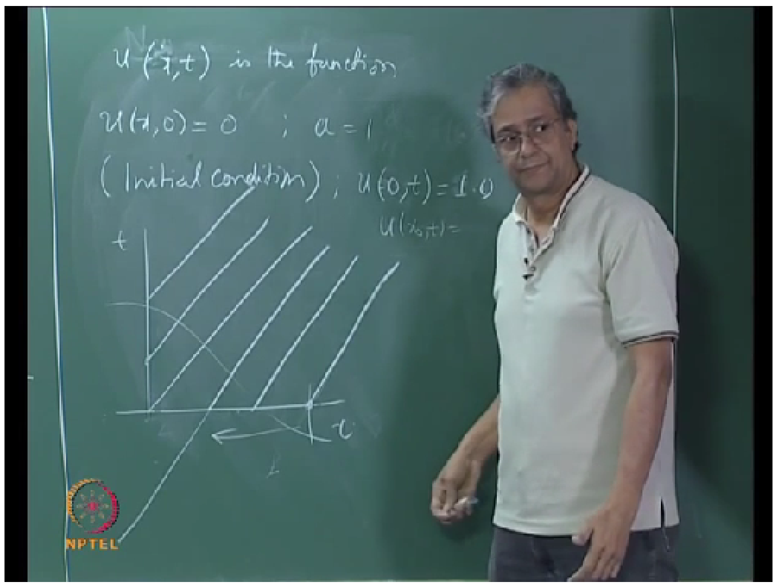
You cannot assert keeping this assert. You cannot assert. So I can put chalk dust here. If I can only put chalk dust at the inlet. I have a stream of water. If I can only put chalk dust here, I cannot put chalk dust here and insist simultaneously that at a given time, right, chalk dust be something else there. It is not possible. And once I have put a certain amount of chalk dust here and the chalk dust has travelled.

The chalk dust has travelled or it has gone along the length of the, hot water has gone the length, along the length of the pipe. I cannot at that point insist, I cannot assert saying that no, no, the hot water has, there should not be hot water. I want cold water, right. You are now committed. You let the hot water in, it is in the pipe, it cannot go anywhere else. You let water out. You are going to get hot water, right.

You cannot assert that it has to go back being cold water, right. Because remember the boundary condition is an assertion. You are going to say this is the value. This is the value of the temperature. This is amount of chalk dust that is there, okay. So because of the nature, because of the direction in which the characteristics are oriented, right which is, which comes back to what Ashok was saying, if you change the sign or a, then the propagation direction changes.

If a is negative, then you are travelling in the negative x direction, okay. Then the properties being propagated from, if a happens to be negative, the properties being propagated from a positive x, a larger x quantity, to a smaller x quantity, okay. In which case, then you cannot prescribe the condition on the left-hand side. You can only prescribe the condition on the right-hand side. Am I making sense? Okay, right?

(Refer Slide Time: 44:10)



So what we need is we therefore need a condition on the left-hand side which could be u=u of 0, t, so for all time, right, starting at t=0. If it were constant, let us say it is 1 to keep life easy. Then what are you going to get. This is xt, okay. So on the xt plane of course, we have the same characteristics but this time, they are all 45 degree lines because a=1. All the lines are supposed to be parallel independent of how I have drawn them and 45 degree lines and so they also come.

There is something there but we have cut our domain there. So they are 45 degree lines coming from the boundary also. Yeah, please. **"Professor - student conversation starts**" (()) (45:02) whatever boundary conditions that you give? No you are saying that instead of being on this, if I give it on some curve. Is that what you are saying? u(x0, t) = something. No it does not matter. It really does not matter. I am just trying to keep this.

In fact, this is, this problem that I am talking about right now is a rather boring problem, right. Nothing exciting even if it happens. This is a rather boring. This is, this is, this problem that, specific problem that I am talking about right now is a rather boring problem, right. I will admit that. There are, when we come to the numerics, there are interesting things that we are going to see with respect to this equation, right.

But I just want to make sure that, I just want to make sure that there are no issues, right, that we are, we are basically all on the same page and there are certain elements of this physics that important for you. **"Professor - student conversation ends"** For example, so if need, it can be, it can be at x0. What you are basically saying is this particular equation I can actually integrate back in time, right. In a sense, I think, you understand, right.

So this particular equation, you can actually integrate back in time. So in a sense if you say that the value of something, if there is some value here, you can go back in time. But that looks very suspiciously like making a as -a, sort of, right, okay. If you think about it, if you look at the way, it comes back to the same thing. So you can go back in time. This is, this is an equation that you can, you can integrate in time.

So you could say that this is like, you are off, you are, you are, you are hunting for somebody, you are looking for somebody, right and you find a clue and then you are trying or you, you get an order, right. You are, you are, so you then try to figure out, you try to work back to see from where does this come, right. And of course all, all, all, adventure stories that you have read as children, they are always talking about being upwind or downwind or whatever of the query right.

So you do not know, you want to be downwind of the query so that the right whoever that you are hunting cannot smell you but you can smell them, right. That kind of a thing. So you, or you can, so you have to look, so that is, that is basically a matter where was this person, right. Where is this person. So if I give you some value on the characteristic here, it is actually possible that you can trace it back, okay. You can go back in time. Is that fine? Okay? But you could give the value. You could prescribe the value in reality in any point.

“Professor - student conversation starts” (()) (48:01) u of x,0. u of 0, t, yeah. t and the x part, right now whether it is time or not, that is what, what is where we, that is what you have to look at. Whether you are able to integrate back. **“Professor - student conversation ends”** What it finally boils down to is as I said whatever coordinate system that you give, that you can rotate the coordinate system to align and get an ODE along this line, right and get an ODE along and that, that is possible.

It is the nature of that equation, okay. It may not always be possible. Sometimes they maybe some scaling, stretching. There are other, other issues that are, that are involved, okay. Is that fine? So **“Professor - student conversation starts**” (()) (49:03) Along the characteristic, yes, in this case, yes, the solution is propagated. **“Professor - student conversation ends”** No, no if you prescribe the condition here, what I am saying is you can, if you say, when you say, when I say prescribe, in a sense in this case what you say is, I discovered that the value here.

If you insist that this is time, you say I discovered that the value here is 1, then you can integrate back and figure out for along that x at what point in time was it 1. Do you understand? If your domain, if your domain is that, your domain of interest is that, you can locate the exact condition. If you say, if you look at the exit condition, for instance just say your hot water pipe, your pipe in your house is 1-meter long and the water is travelling at 1 meter per second.

So you know that if it is, right, 40 degree or 50 degree or 60 degree Celsius at the exit, right now, then 1/2 a second ago midpoint, it was 50 degree Celsius. 1/2 a second ago midpoint, it was 50 degree Celsius. Is that fine? Okay? So you can integrate back in time or you can integrate forward from that point onwards, okay. Yes. **“Professor - student conversation starts**” Reading t = 0, it is u(0,0) is (()) (50:27) where we take t=0, Yes, there seems to be a problem at 0,0, right. So this is. **“Professor - student conversation ends”**

So we will, we will look at it. We will plot this; we will plot this to see what happens. So there is actually a sort of a step kind of a thing at that point, right. I mean just like you have cold water on one side and hot water on the other side and there is a very sharp interface between them. Is that fine? Okay? So we will get back to this in the next class. Okay. Thank you.