Gas Dynamics
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Module - 3
Lecture - 8
Communication in gases, Stagnation state

Hello everyone, welcome back. We were looking at the animation of how a sound wave generated from a point is carried by the flow. And if the speed of the flow is much more than the speed of sound, then we found that all the waves were restricted inside some wedge like area or we could say if it is 3D, then it is inside a cone; that is where we stopped last time.

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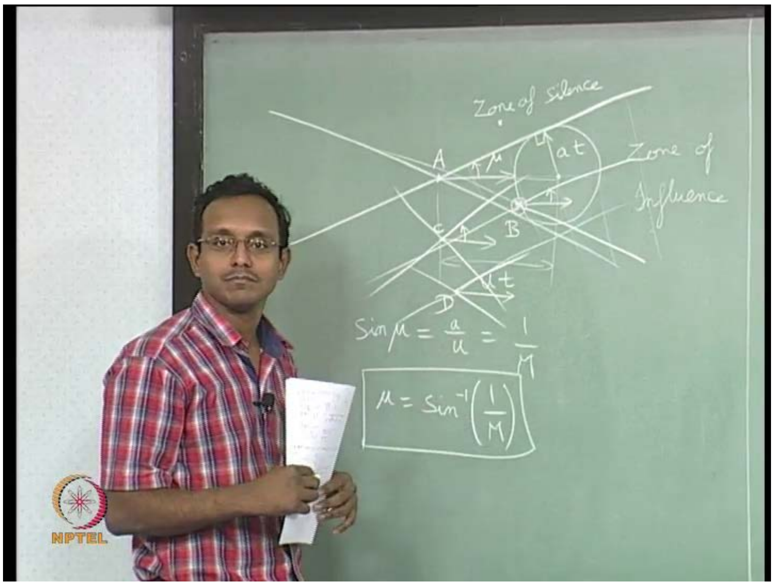

So, we will start looking at that kind of picture from now on. I am going to say I am considering some particular point here currently, and the flow through the point has a velocity vector in this direction. And we already saw that there was some circle which was the wave that was generated by this particular point when it was here. Currently, the point is here and the wave is here; that is what we saw, and this is going to be let us say the point took from here to here sometime t. Then this distance is u times t while this is with respect to this it is just going out circular. So, it is just radius is equal to a times t where a is the speed of that wave specifically.

So, now I have a triangle with this 90 degrees, and this angle we are going to call this specially mu Mach angle, and that will come out to be simply sin mu equal to a by u which is 1 by Mach number for us. So, mu is sin inverse of 1 by m. Now we have this formula. So, now we have to start thinking about what will be the effect due to some point a in the flow. If there is any change that happens when the fluid element crosses this point, then that information is transferred in a circular manner from that point as the point moves down; that is what is happening here, we saw that in the animation last time; that is what is happening out here, if this point travels more than that will be a bigger circle sitting somewhere here.

Now we already said that the information never reaches this particular point outside there; that says that this point if it wants to effect some change in the flow field somewhere that information can never reach this point even if I wait for long enough time, because the fluid is going to be pushing the waves out this way. No wave is going to go reach this point, and because of that now I can tell that this region can be called as zone of silence, while this region what is this region called?

Student: Zone of action.

Zone of action, zone of influence depending on which book you take it is given some set of names, but this region is the region where this point can have an influence on this. No other point outside this whole wedge like area can be influenced by any change in this point here. So, I am going to call this my zone of influence. It could be called the zone of action also by basically saying if I change something only this region acts on it, zone of action if you want to call it. Now this region is something straight opposite; we will come to this after we look at some more points in here. This is just one particular point in the flow field.

What about I pick some other point in the flow field? Let us say I pick this point in the flow field somewhere in here I call this my b. Now if there is any change in this point, then that will be influencing only this region. I am again assuming that the velocity vector is parallel; it need not be parallel. It could be a non-uniform flow, not parallel flow, anything. I just picked up a special case like this. And this mu need not be same as that mu. This could be a mu subscript a, this could be a mu subscript b. Now all I am going to say is any influence any change that I make at point b will be experienced only inside this region now and not in any other regions.

Now if I extend this line this way, b will be affected by point a, because b is sitting inside this wedge of a. So, b is in the zone of influence of a. So, b will be affected by a. Now I pick another case. I pick a point here. Let us again say velocity is going this way. I have put some other angle; it does not matter. Let us draw it with respect to that angle, and these are my mu values for this point c let us say. I am going to draw so many points on this. It is going to get confusing after sometime, but anyways we are looking at point a, it has these zones; point b it has some set of zones.

Now I am taking a point c specifically in this location. Now b is sitting in point c is downstream zone which is your zone of influence, which again means that c can also influence b. If c can also influence b, then a and c are both influencing b, right. There is a whole bunch of points in here which can influence b; that is what we are coming to. Now I can tell that the properties at this point depends on all the properties of a and c also. I am going to say also because maybe I will introduce a boundary condition at b in which case it will depend on these plus the extra boundary condition I imposed at b; that is also possible, we will keep it that way.

Now I am going to look at this specially. Let us pick a point d. If I have one more point here, then this is going to have let us say some velocity here and some angle here. Now I pick this d such that b stays outside of my zone of influence; it is in the zone of silence. This can happen. If this happens d cannot influence b. Now what is special about d that is not so for a and c? It so happen that d is outside of this other wedge formed by b not exactly; that need not be the only answer, but there is a simpler enough answer I can think about.

All this explanation will work only if my Mach angles are the same and the velocity vectors are the same. We will pick a simple case; that is what I am saying. Basically, uniform flow constant Mach number flow parallel flow; that is the only case where I can explain these zones very clearly. I did not pick such an example here.

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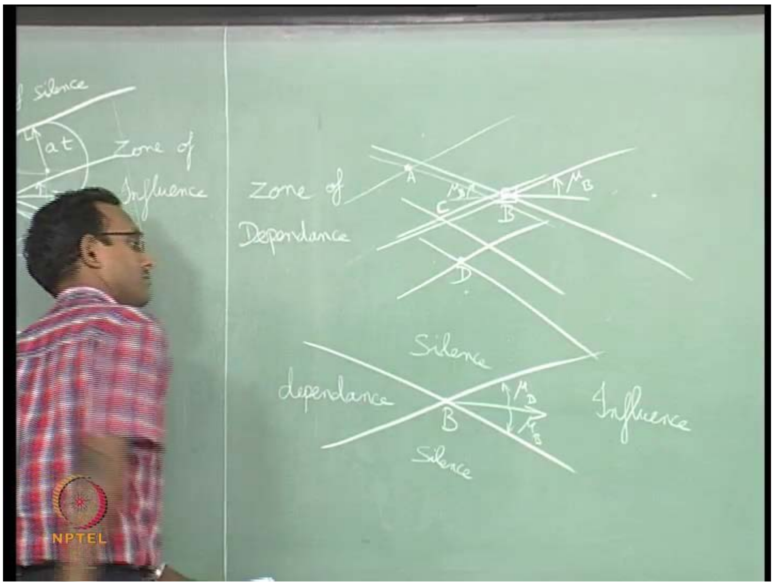

Let us go pick such an example again. I am going to say I am having that point b here and I had another point a which will also be same Mach number which means it will be the same cone angle. And i pick another point c, and I pick another point d. Now we are interested in the point b. This is the only point we are interested in really. What am I seeing? Basically if the points are inside a and c inside this cone formed upstream of b with the same sign inverse 1 by mu b. It is specific to that particular Mach angle mu subscript b. This side also it will be the same mu b; if say that my points are inside this, then it has an effect on b; if it is outside it does not have effect on b.

So, I am going to say that the b value here depends on all the points inside this wedge like area; in 3D it is going to be a cone, inside this cone upstream cone which is called zone of dependence. That is these are the points that this point b depends on. Now I have too many points in here. I will just simplify all this and just draw one picture. For now I will label it b; it does not matter what the label is. And this is my flow velocity direction; flow velocity is going this way and with respect to that there is a mu b that way and this way; that is what I have done. This is also mu b by the way.

Now I am going to label all the parts. This is my zone of silence; this is also silence. This is my zone of influence, and this is my zone of dependence. These are the different zones for one particular point b; this is what you have to get used to. Now in real life flows it is not simple flow like this; it is not always parallel flow we considered in this here. In real life it is going to be very different, may be the next point the flow velocity vector I do want to disturb this picture, I will go here.

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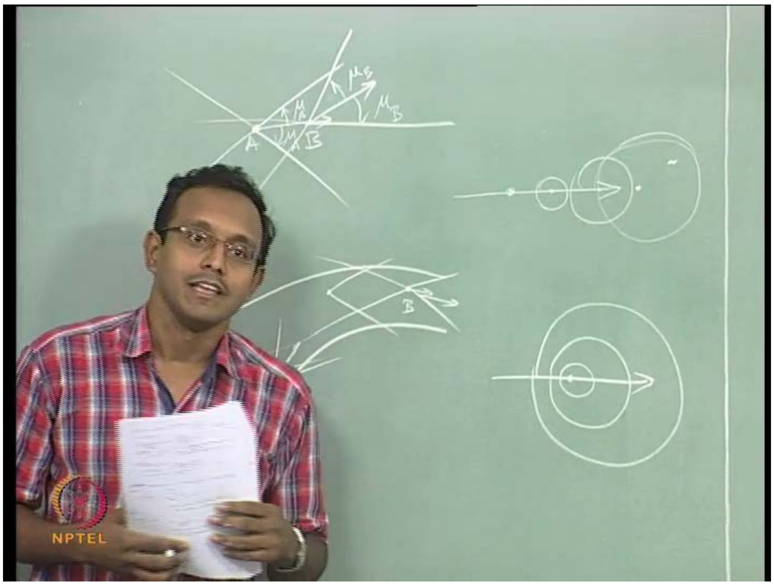

But the next point here the flow velocity vector maybe this which means the flow is going like this and turning. If that is the case let us say this is my a and this is my b, for a based on the Mach number here it is going to have something like this. And of course, it is going to go this way, but for b it is with respect to that flow velocity vector. So, it is going to go like this and this. So, I am saying with respect to local velocity vector I have to choose this particular thing. Now can I apply this zone of influence zone of silence here? Yes, I can. In fact, I did not assume that mu a and mu b are same. In fact, I tried drawing mu b to be a little bigger.

Anyways, such cases can happen in real life, and what we are going to do is we are going to think about from here it has moved to the new point, and now I will extrapolate this. And I can tell this point will influence this point, but if this point is very far away then I cannot say that this point directly influences that based on the cone. Then the problem becomes very complex; let us say I will give you a crazy flow situation something like this and flow is going inside this. If I pick a case like this and I am saying it is fully supersonic flow going through this whole thing, then I pick a point here a.

This is having a velocity vector like this which means that the zone of influence will be here, zone of dependence will be here, zone of silence here are here. If I extend this, this is not going to go anywhere out, but I will tell that a is going to influence a point here b. The velocity vector is slightly tilted something like this. Even if this extends out this way, I am going to say that a will have an effect on b, why? Because there will be some other point here which will have zone of influence like this. Now there will be some other point here which will be influenced by this point, and that will have zone of influence like this.

B is sitting in this influence point which is sitting in this influence which is sitting in this influence. So, typically I can tell that every point downstream will be influenced by a point upstream. It will not be the case only for some special situations like my next point I am picking is somewhere here. This is still slightly downstream, but this point may not have an effect on this point, but most of these complicated cases all downstream sections will be influenced by the upstream sections. In all these I did not talk unsteady; I am talking steady. If it is unsteady what happens? If it is unsteady I am going to think about that animation we had yesterday where I just now created a disturbance and fluid element is carrying it.

At this point it is going to be this big; at this point it is going to be this big, the disturbances is just now growing. So, even this point is currently not influenced by this change at this instant when the fluid element is here. When this is going some more distance only it will be getting affected. The picture we have drawn about zone of silence, zone of influence are all for steady state conditions. Remember that when we go to unsteady things may not be the same exactly; it will keep changing. I will keep talking about steady and unsteady several times, so that I am going to lay the foundation for the future; towards the end of the course we want to go to unsteady gas dynamics, where if I do not tell you now that there are other possibilities, it is very difficult for me to put into your mind that there is something else possible.

So, I will keep on telling you that there are other things possible; in here currently this point does not know that there was a change here at this time instant. If I wait long enough, this point when it reaches here this circle would have become bigger. There it is already knowing that there was some change here. Now this point can be considered as within this cone; till that time even though it was within the cone, it was not experiencing it, why? I did not give it enough time for the wave to go and reach it, for the wave to go and tell that the pressure changed here, say, compression or expansion wave so pressure changed there; that information never went through, that is a possibility.

Now we will go back and look at this picture again. This is valid only for supersonic flow. What if it is sonic flow, perfectly sonic n m m equal to 1, what will happen? M equal to 1 mu will become sin inverse 1 by 1. So, it will be sin inverse 1 is 90 degrees. This is going to be a straight vertical thing like this; this one will be a straight vertical. There is no real zone of silence; there is no real zone of silence, zone of silence gets crushed. So, everything before is zone of dependence; everything at site is zone of influence, but of course, remember all these analysis we are making lot of simplifications; m equal to 1 region are equations may not be exactly valid, we should not be talking about this case especially.

Our expressions are very well valid for m greater than 1. So, we will keep it that way. So, what will happen if it is subsonic? If it is subsonic there is no solution for this, sin inverse 1 by m less than 1, right. So, sin inverse of something more than 1 there is no solution for it. So, this particular picture does not make sense for subsonic condition, why? We will go back that side and remember recall the animation. We have this; there is a point that is continuously getting disturbed. And we saw that the waves were doing this, something like this, right. There is some wave going forward; there is some wave going back which is going faster.

Why was that going faster? It is being carried also by the fluid; here it is trying to run this way, and it is carried this way by the fluid. This is where I gave the treadmill explanation; it works well here. There is a belt going this way, and, say, somebody was going to run on top of it this way. Then they will go slower versus on that side they are going faster if they are on that side; that is a good enough example for this. Now if this is the case, then the points upstream will also know that there was a disturbance here. Points downstream will also know. Every point in fact will know, provided I wait for long enough time.

If I wait for infinite time, then definitely this will be reaching all the way to infinity in all directions. Even I think about a steady problem in subsonic flow, I am assuming I will wait for infinite time and I have reached the steady state. I waited for infinite time nothing changes. I now take a picture and show you the flow field; that happens to be your steady state flow field; that is the way we want to think about it. Even in subsonic flow if there is a point here and the very first disturbance I created is just sitting here, then this point does not know even if it is upstream. It does not know even if it is downstream for that matter, some other point here.

Even that point does not know that there was a change here; unsteady problem is a lot more complex. Now easy way to think about this, say, I have this and I move it this way, there will be some point out there which still did not get the wave from here. If I wait short enough time, then that point never moves because of this, but if I wait long enough, if I wait for long time say a few hours and this is continuously kept moving like this, then there will be flow field there which will be responding to this. When we think about subsonic flows, typically they will say the fluid upstream always knows about what is happening downstream.

When they make such a statement they are assuming they have waited for infinite time that this wave has already crossed all the points in the flow field which I am looking at; that is an assumption remember that. In unsteady world that would not work; I just now created the disturbance. I did not wait long enough, but I want to see what happened to the fluid. So, maybe some points will be influenced, but some points will not be influenced. So, life is a little more complex when we go unsteady. We will look at more unsteady problems later. We will get back to a little more simpler things to think about; first we have two settle some other analytical stuff which will left halfway.

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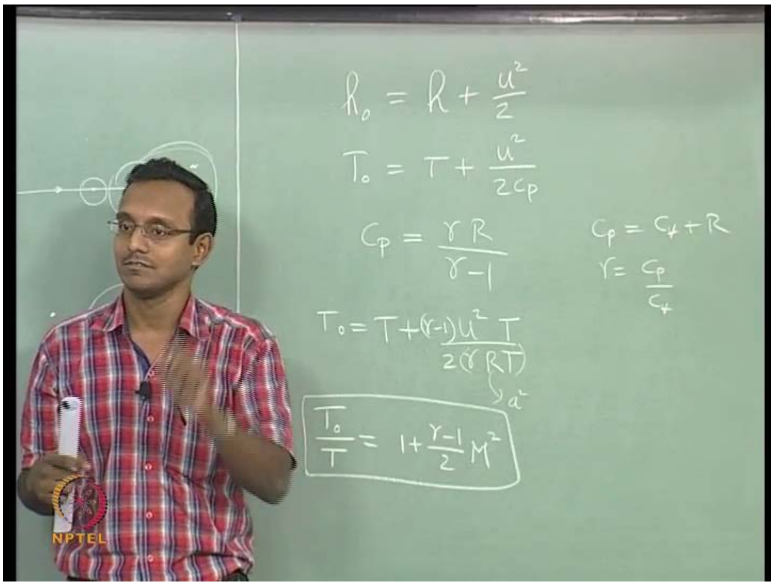

We will go back to equations again. So, somewhere along the line while I was deriving we got this form; oh, by the way anybody has any questions on any of this, these waves and moving, subsonic, supersonic. So, now we will go to this the next section; this is again more analytical stuff. So, from here we defined this to be stagnation enthalpy. And now I am going to say if I have calorically perfect gas, then I can write; what if it is not calorically perfect gas? Then the c p here may not the same as c p here. So, I have to keep the c p naught and c p separately and I will have to calculate them independently.

It is not going to simplify to this form; that is all, but that expression is definitely your energy equation. Assuming it is adiabatic, steady flow, no heat transfer from anywhere else and all those other things. This equation is for one special set of assumptions remember that always. Now I want to rearrange these terms. What is c p in terms of r?

Student: Gamma r by gamma minus 1.

Gamma r by gamma minus 1 where I am taking this to be per mass basis, why, because h is per mass basis by definition for us; our convention is small h is per mass basis. So, this c p will be per mass basis. So, I am going to get to r will be per mass basis, specific gas constant, and you can check this. This comes simply from c p equal to c v plus r and gamma equal to c p by c v. We can just divide this whole expression by c v; substitute this, you will get to this form immediately. It is not very difficult to get, simple enough to get. So, now I want to substitute this c p inside here. What will I get? I will have this form.

Now I want Mach number here; that is the motivation. So, I want gamma r t here. If I want gamma r t I will put t here and t here. I multiply numerator and denominator of this term alone by t. So, I have gamma r t which is replaceable by a square. So, now u square by a square is my Mach number. So, now I can write this expression very commonly used in gas dynamics very very commonly used in gas dynamics. This is the relation between stagnation temperature and static temperature. When I gave these kinds of definitions, now I have to know the exact meaning of each of this. When I say static temperature I mean if I follow the fluid as in I am sitting inside the fluid element imagine that; I am sitting inside the fluid element with the thermocouple on it in my hand temperature measurement device in my hand and I am measuring what is the temperature.

I am being carried away by the flow. If that is the case, then I will get this t static temperature, but if I am sitting at one point making the flow come to rest at my thermometer or thermocouple. Then I am going to measure stagnation temperature; that is I am bringing the flow to rest and then I am measuring the temperature there, is that correct? How will I say that? I am going to say u becomes zero; if u becomes zero, then t is equal to t naught. I am measuring temperature at that point when u becomes zero.

So, the temperature will become equal to t naught. In this whole process I have resumed adiabatic process for the fluid coming to rest at my temperature measurement device; that is the special case. Typically, if I put a thermocouple or a thermometer in my supersonic flow actually any flow subsonic or supersonic, I am going to measure only stagnation temperature. There are special ways of measuring static temperature we would not deal with that in this course.

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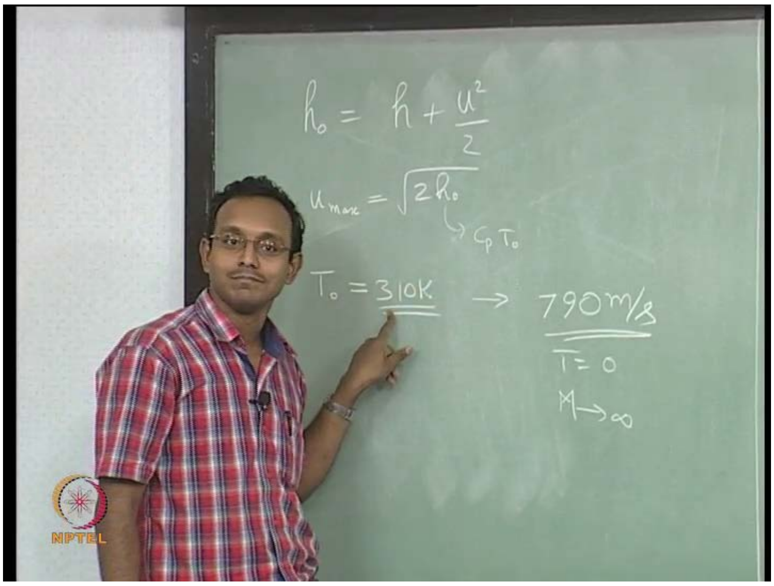

Now there is one more special thing that we can look at based on this expression. I am going to pick this h naught equal to h plus u square by 2, and I want to say what if I take all the energy and put it into kinetic energy; that is the maximum energy ever possible for the fluid to achieve with the given total enthalpy given total energy. So, it is going to come to u max is equal to 2 h naught square root; it is comfortable with that. So, if I find let us say I will replace this with c p t naught; h naught is c p t naught if I write then I will give you numbers. If I assume t naught for my gas to be 310 Kelvin or kind of room temperatures, then this is going to give me roughly 790 meter per second.

If I start with a gas at this temperature and make it accelerate to a maximum velocity possible with the given energy already present in the gas, then it cannot go anything more than around 790 meter per second. It could be 791 point something; I just approximated it to 790. When this happens what is the temperature of the gas? It should be exactly zero, because I am saying h is zero; all the energy is inside kinetic energy only. So, this will go to a point where t equal to zero. If t equal to zero what is my Mach number? Infinity, Mach number goes to infinity, why? Square root of gamma r t is in the denominator; u is having some value square root of gamma r t is in the denominator; it is going to infinity.

Infinite Mach number does not mean velocity is in finite, clean example directly here. Infinite Mach number just could mean 100 meter per second if my t naught was lower, but Mach number could be infinity. It can be that also. What will happen if Mach number is infinity, anything special? If Mach number is infinity, what happens to your mu? The sign inverse 1 by infinity will be what? It will be zero; there is no cone, there is no any zone of influence or whatever. It is all straight line; nothing influences anything else; that is a very special case. We will not deal with than ever in real life.

So, now we will think about the next thing pressure. Now I am going to say instead of just adiabatic process I am going to take the fluid through an isentropic process. We would not go into details of why we want to think isentropic now; adiabatic reversible is what I want to call, because if it is not reversible then I am going to spend some energy into the non-reversibility of the problem.

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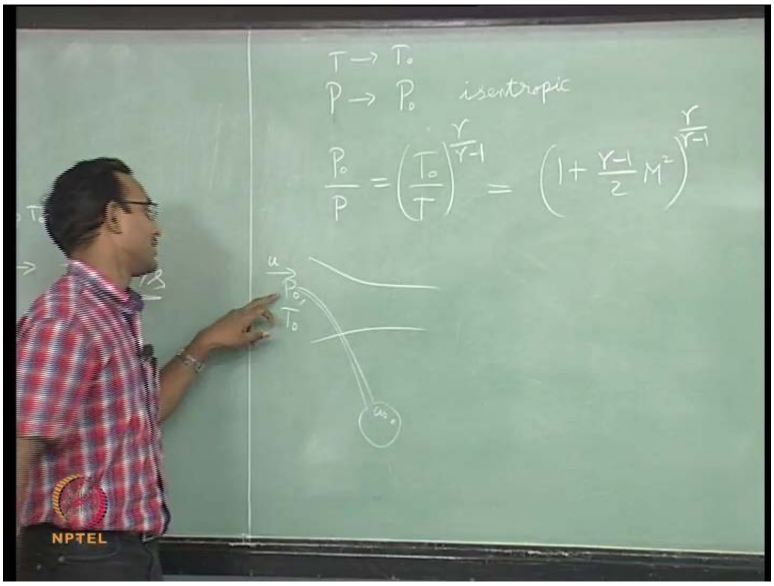

So, we will pick isentropic process where I am going to think about the pressure of the gas goes from p to p zero. Already we looked at adiabatically taking the gas from some temperature t to t naught. How did I do that? Decreasing the velocity slowly to zero velocity; so, I already did this. So, now I am going to say this one is done isentropically; that is previous one adiabatic was enough. Now I have to say that it is adiabatic reversible. If it is not reversible then there will be entropy loss in what form? Delta s equal to I do not remember exactly r lon p naught 1 by p naught 2. We will get to something like that, r lon p naught 1 by p naught 2.

So, there will be some loss in entropy, some energy lost in creating entropy so that the flow happens that can happen in real life. So, now we will say that there is some such a thing. If this is the case, now I can write a relation for this process in terms of t. Actually it should be c p by r which I am replacing by gamma by gamma minus 1. I am getting this form; how did I get this, where did I get this from?

Student: Some Isentropic relations.

Isentropic relations, something much more fundamental from here; I have to rewrite this as r by t and you are going to get to that, sorry r t by p and then I will get to this form. And I am going to say isentropic this is zero. I will rearrange the terms. I will get to this expression. From there I will get to here; that is where you are getting c p by r. So, remember the thermodynamics heavily dependent on thermodynamics; gas dynamics is completely standing on thermodynamics, remember; that that is why I spent three classes reviewing thermodynamics for you people. So, now once I know this I can write one more expression from the previous relation.

Student: Sir, this t naught by t is also isentropic?

T naught by t need not be isentropic; all it needs is adiabatic. It needs to be isentropic only for pressure; temperature it is enough that energy equation saying there is no more q dot in. If that is the case, then that expression is valid. You just derived it from energy equation just today, right. When we derived it that way we found that it was directly equal to this directly from energy equation; that was only for adiabatic steady flow and I got to this point. Now I have to go to the next point. People will start talking about these parameters, stagnation conditions and static conditions. This is your stagnation pressure; this is your static pressure.

Static pressure is the pressure if you are moving along with the fluid element if you measure a pressure inside that fluid element, you will see that pressure. If you are putting some block and flow is coming and hitting it, then the pressure experienced here will be related to the fluid element coming to rest in front of your hand and then measuring the pressure here; that will be your stagnation pressure. Of course, now I have to think about is the process of taking the fluid from there to here to come to rest, is it isentropic? I have to be careful there; not all conditions will be exactly isentropic, but anyways in real flows you will never have any real stagnation flow ever. So, what they do is they use this as a way of finding out how much energy my fluid is having.

Pressure is a form of energy, right, p v work can be done from it; that is pressure is just energy per unit volume stored inside the gas to do work in the form of p d v, and this is your other energy. C p times t is your internal energy of the gas two forms of energy that is possible. So, people will use these to describe the gas by which I am saying how much energy is present in the gas; that is what is important for us at any time, because if I know how much energy is present now I can go and find how much energy can be useful. Remember G, the Gibbs free energy, how much energy can be useful for us? That can also be done from here.

So, typically people will just say I have a flow field and I am going to give you p naught and t naught here. If I give you this, then I am telling you a lot about this flow. I am telling you what is the initial energy for the fluid. After that you just have to go through all the equations and then you will solve this whole flow field. We would not solve anything now; we will go through all the analysis first, then will just go on and take one problem at a time and just start solving each of those problems. But in every analysis they will give something like this, does it mean the fluid is starting from rest here? Not really, I am giving you a condition about that particular fluid may be it is having a velocity u right now which is nonzero. And at that point am telling you p naught and t naught; it is a possibility.

So, what I am giving is really a hypothetical thing. I am going to imagine having a separate tube through which my fluid element will go without any friction. It is completely isentropic process, imaginary process my fluid element will go to and it is going to become u is equal to zero here. When that happens imaginary process I can do this on every point of the flow. I can put this tube here and collect that fluid element and going to zero and see what is the pressure and temperature there; like that I am imagining this. It can be done in subsonic flow, supersonic flow, anything; only when it goes to Mach angle Mach cone; those are the things that are defined only for supersonic. Everything else I am talking about here is all for subsonic or supersonic.

So, I could define these and the usefulness of this will be seen when we start solving problems. It is very very useful to have something like this, and I will just tell you one peak into the final thing. If I tell you that my flow is isentropic p naught and t naught do not change all through my flow. We will see why later; I already gave you the answer delta s equal to r lon p naught 1 by p naught 2. If my delta s is zero log of p naught 1 by p naught 2 is zero which means p naught 2 by p naught 1; p naught 1 is equal to p naught 2. I just did this analysis all in my words; we will start doing that kind of analysis later, but that is one good thing about isentropic flows.

P naught does not change; of course, if it is isentropic it is all so adiabatic. So, p naught does not change anyway; all that will happen. Now we want to go to some physical intuition again. This time it is all in terms of plots; this is why I gave you so much of PV diagram and TS diagram at the beginning.

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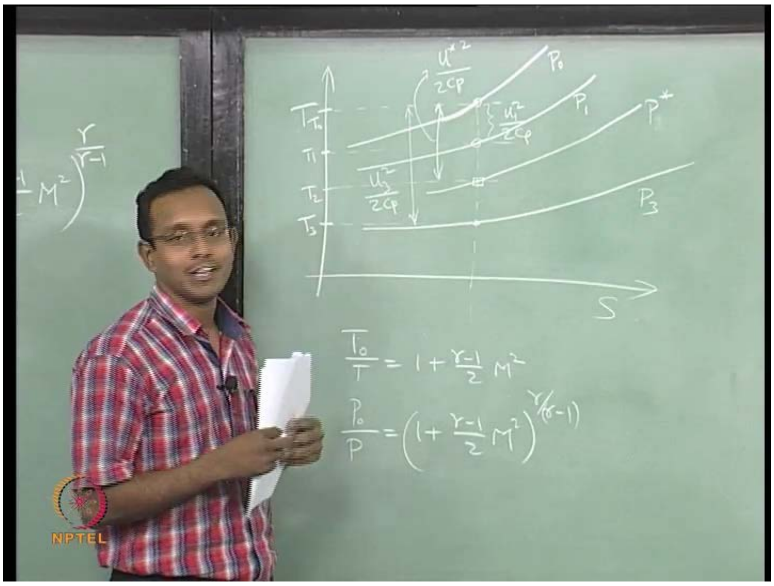

So, you are going to start looking at flow in TS diagram, temperature versus entropy diagram. And I am given that my flow has a particular t naught and p naught. What will a constant pressure line look like on my t s diagram? Horizontal, no; what kind of curve? We solved this exact problem before. It is an exponential curve; it will be an exponential curve going through that particular point I am going to call. It is my p naught equal to constant line or the pressure equal to constant line; that constant value happens to be p zero for my current case. It is an exponential curve; it is not having any breaks, it is continuous. And basically I have given the state of my system p naught and t naught given it is located here.

Of course, now I can find my S; this will be my entropy on this. So, let us say I am going to accelerate this fluid to some subsonic condition, some subsonic velocity. What will happen to my pressure? It will decrease, why, what happens to temperature? Easier to look at its temperature; gamma is always greater than one, you can easily prove it. C p is always greater than C v. C p by C v is gamma. So, it is always greater than one. So, this is the positive quantity; m square I said m is less than one, but still m square is positive. This is positive; the whole thing is more than 1 always, it is never less than 1. So, if I have any velocity even subsonic, then temperature t naught by t will be greater than one which means t is lesser than t naught.

So, I am going to go to some other lower temperature. Now what will be the pressure corresponding to this? Of course, I can do the same thing. I can write something like this and find out; from the plot how will I tell you what the pressure is? At constant pressure line passing through this point; how am I saying that? I said my process is isentropic which means it has to stay on this line; isentropic process is going to be on this vertical line because entropy is the same. So, now I will say this will be my new pressure whatever this value. Let us say we call it p 1, and I can find the value. And what is this value equal to? That will be your u 1 square by 2 c p.

Now I will go to some case where it is fully supersonic. What will happen; where will my point be if it is fully supersonic? I will call that point three. It will be below this. I will draw something like this. This is my t 1; this is my t 3; I will come to t 2 after this. This will be my p 3. Remember each of them is an exponential curve pressures. Now this whole distance; what is this equal to? This gap t naught to t 3; what is that gap equal to? U 3 square by 2 c p. Now I will draw the special case two which corresponds to m equal to 1. This I said is supersonic, three is supersonic. M 3 is greater than 1; m 1 is less than 1; m 0 is 0.

Now we will draw the special case p 2. This is my special case, and typically what they do is they just put a star on top for sonic conditions in gas dynamics, a common notation. They will put a star for sonic conditions; that corresponds to special case m equal to 1. So, now if I want to find this height, what will this be equal to? U star square by 2 c p where u star square can be now written in terms of gamma r t 2, or this t 2 will become t star anyway. We can write it in that way also. What is essentially happening in this whole process of accelerating the fluid; what is really happening here? I have net energy so much. What is happening to it? It is converting temperature energy into kinetic energy; that is what is happening.

Initially it was having all temperature energy or internal energy enthalpy and slowly it is pulling out temperature to be lower value increasing kinetic energy. We are going to keep on doing up to this bottom point. What will this bottom point correspond to? T equal to zero if I put, this will be your u max. This whole height is your u max square by 2 c p; we already did that. So, we know that is what this corresponds to. So, that is the connection you have to get, and if I draw a pressure curve through that what will that pressure be? That will again be zero; it is an exponential curve with zero on the top.

So, it will just be a flat line. Yeah, it will be going along the axis. So, we would not talk about that kind of special cases right now. We will never have Mach number going to infinity happening in our gas dynamics. We will give it high but not going to infinity. Now we will go look at this whole thing in the other plot PV diagram anything I do in TS diagram you should be expecting to go look at in PV diagram.

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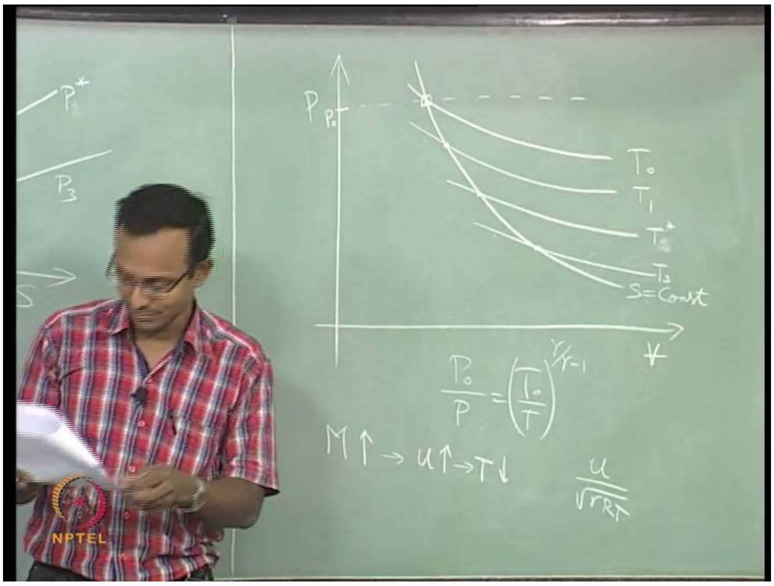

I am not worried whether it is intensive or extensive; we will just keep it something, this particular problem it does not matter. So, I want to do the same process. I am starting with some t naught and p naught and I want to go and find all the other processes as I accelerate the flow to higher and higher Mach numbers. So, I will mark this as my p naught, were will my t naught be? So, I have to basically draw some constant temperature line; what will that look like? Rectangular hyperbola, right, p v equal to r t you can get it to a rectangular hyperbola. So, we will draw something like this.

Let us say this is my t naught. So, this is my initial state; this is where I am. Now I am going isentropic process. What will that process look like in PV diagram? Not a straight line; it will be a curve like this but a little more steep. Remember we did all this at the beginning; it is going to look like this. This is my s equal to constant, isentropic curve. So, because of this now my whole process will have to be only these set of states; it cannot be anything outside. So, now I am going to say if I have a subsonic flow, then it is going to look like this t 1. So, it is sitting here.

What is happening to volume? Seems like it is increasing; now I will again draw the next set of curves t 2 r star I will remove the two, then I will have t 3; this is what is happening here. Now what we are seeing is pressure is dropping which is expected because we already know that p naught by p is t naught by t to the power gamma by gamma minus 1. And we know that gamma by gamma minus 1 is always greater than 1; do you know that? For gamma equal to 1.4 this is 3.5; for air this number is 3.5. So, we are thinking about this is always more than 1 to the power 3.5 will again be more than 1 which means pressure has to keep dropping if t drops; they are related directly, it is not inverse relation . So, if temperature drops pressure drops.

It so happens that pressure drops more than temperature drop. If temperature drops to half pressure drops to much lower than half value anyways. One more thing we have to look at in here is the volume is increasing; it could be specific volume if you want one by density if you want. If you pick out to be 1 by density 1 by density increases means what happens to density? Density drops; that is when I am expanding the gas, actually I am giving the word expanding already. When I am accelerating the gas to higher Mach numbers I am always expanding the gas and its temperature drops, its pressure drops and its density drops. Velocity is the only thing that is increasing everything else is dropping. It is the whole set of things that is happening in there.

Now there is one more special interesting thing. I said that if I increase Mach number my velocity increases. If velocity increases I said that temperature decreases. So, what happens to Mach number? It increases; so, it is like a circular thing, but actually the way to look at it is if I accelerate the flow temperature drops and Mach number increases more than what I do with just velocity. If I double the velocity it does not mean Mach number doubles; Mach number will go much higher than double if I double the velocity; that is the way you have to look at it.

Mach number has more complex relation than just simply proportional to u. It has this inverse thing on t also. How do you know this? I am going to have this u by square root of gamma r t for my Mach number. I am saying if my u increases t decreases. So, numerator increases denominator decreases, Mach number goes much more; that is the way we are going to look at it. So, in the next class we will expand this p naught by p expression using some set of binomial expansion series, and we will start seeing the results from there. Again I want to beat of this Bernoulli's equation thing once more and then we will go look at compressibility effect; that is the whole set of things I need to do. I will do that next class and then we will into 1D steady equation what will happen if I change area; that is the next thing I will get into. See you people next class.