Gas Dynamics
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Module - 3
Lecture - 7
Pressure Waves in Gases

Hello everyone, welcome back, we stopped last class with a little confusion about one of the derivations, there was actually nothing going wrong there, we just have to blindly believe math, it just works out right after some time. So, anyway we will start from a point where, we will go back two steps and come back again. We were interested in deriving speed of some pressure wave in a gas and we pick the gas velocity to be 0, that is what we did.

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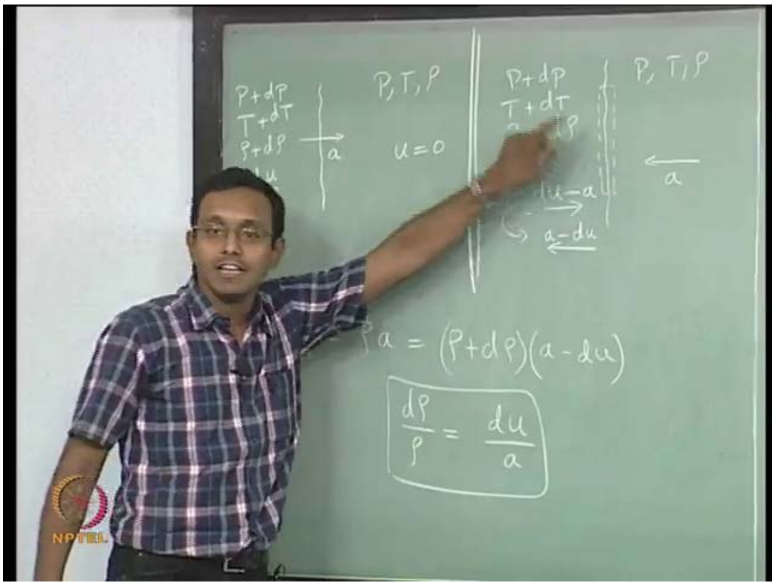

And where, we were saying this is my wave, it is actually a planar wave, I am just drawing it as a wavy thing, just so we know that it is a wave. And I am saying, it is travelling with a velocity a and into a medium which has 0 velocity, it has a particular pressure, temperature, density. On this side, I am going to say the effect of the wave is such that, there is a small change in pressure, temperature, density, etcetera and velocity is also changing.

I am going to say because of this, this is going to have a d u so my velocity will be, I am going to say 0 plus d u but we should just know, that it is increased by a velocity d u, that is all matters. We can later do the whole derivation for our u naught equal to 0 also and you will get the same results, results will not change because of this. From here, we transformed to the other coordinate system, where the reference frame is fixed on the wave itself which means, from here to here, I am adding negative wave velocity component everywhere that is what I do.

So, I am going to have velocity a here and this will be I said, it is a positive d u this way, minus a the other way. So, it is going to have d u minus a velocity this way or equivalently, a minus d u velocity this way, both are equivalent anyway, I will keep it this way this is simple enough to explain things. Now, remaining terms pressure, temperature, density, they go to P plus d P, T plus d T, rho plus d rho this is what is happening, we have picked such a case.

Now, we picked a control volume like this, simple control volume no change in area and very thin control volume that is what we picked, across that particular wave that is what we did. And then we showed that simply m dot by A equal to rho a equal to rho plus d rho times a minus d u, this is just directly from your mass conservation. This can be simplified to a form, where it looks like d rho by rho equal to d u by a, this is your mass conservation across that wave in this reference frame. We are having reference frame fixed to the wave with respect to the wave, the fluid is coming in and going with some other velocity, some other pressure, temperature and density, that is what is happening.

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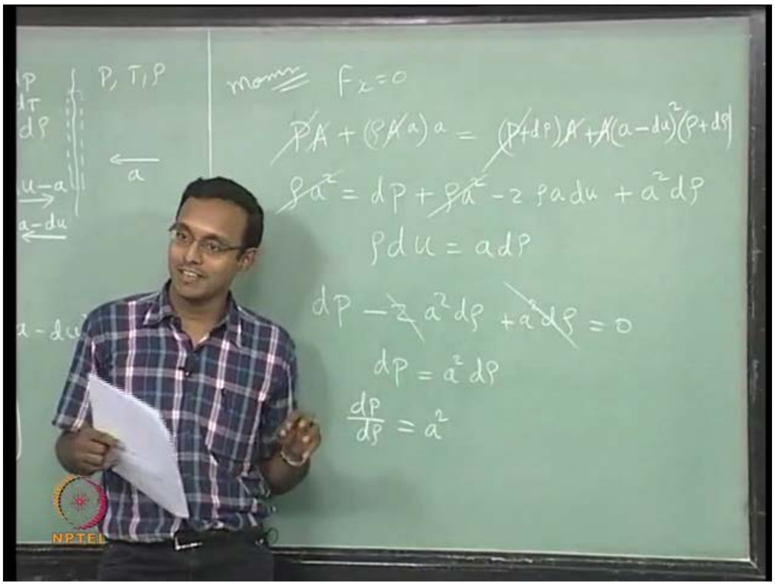

Now, we have to go for momentum conservation where, we have picked a thin control volume with the constant area. So, I am going to say, my F x is equal to 0 from your momentum equation for 1D flows, we already derived that and then I will write the remaining terms, I will go by what we did last class. So, it is going to be, I wrote it slightly differently this time that is all, it is not any different actually, it is just rho a times a square the velocity square that is all.

It is going to be equal to P plus d P times the same area there is no difference in area across, I have thistimes area I have left area out. Of course, you can now immediately cancel the areas out, it is all the same area, I can remove all the areas. Now, the next thing I can cancel will be the pressure directly, the pressure term is sitting there, pressures are getting cancelled.

Now, if we look at remaining terms, we will write one more stepso that, it is easier, rho a square on this side, other side it is going to be a whole bunch of terms, first term is just d P plus all the higher order terms, which we can neglect. Other higher order terms will be d u times d rho timesa that is one particular term, like that I can have different possibilities. I am neglecting all that because I am going to say,it is a very thin control volume in here but the change will be really, really small across here.

If the change is really, really small then the product of two such changes will be even smaller compared to all the variables like a, P, T and rho, etcetera. So, we can neglect those small changes, so that is why I am neglecting all the d u, d rho terms which will come up here, with respect to my rho a square or with respect to my d P, I am neglecting all the other terms, that is what I am doing.

Now, of course I am going tocut off these two also so this side will be just 0 now, I have to use, this is where we were stuck last time.I was expecting a plus rho a d u finally, it came out to be minus 2 and that is what caused error. Now, if I go back to here, all I have to do is, rho d u is equal to a d rho remember that, we will write it here. I have this conversionso, now I will just substitute this rho d u inside here with this, that will again become a square d rho.

So, I will finally haved P minus 2 a square d rho plus a square d rhoequal to 0, that is what I have, which will just immediately simplify tod P equal to a square d rho. Now, I will use this calculus suddenly and I will tell for infinitesimal changes, d P by d rho is equal to a square. Now, this is just gradient of pressure with respect to density, I did not tell anything about the remaining two variables.Ideally in thermodynamics, we learnt that I will have typically 4 variables,of which two of them are here, other two could be temperature or the entropy, volume, whatever.

I can keep something constant and do this job, what do I keep constant, I can do d P by d rho keeping temperature constant or I can keep entropy constant or I can keep Gibbs free energy constant if you want, I can keep anything constant and do this process. So, we will take a short cut here, we would not go the regular acoustics method, acoustics method is the correct method, we will take a short cut here. And I am going to say, the wave we picked is a very, very weak wave, we just wanted to see some pressure pulse so we picked any weak wave.

If I pick a very weak wave, the pressure change it has is extremely small, the density change it has is extremely small, the temperature change it has is extremely small.If I say, my temperature change is extremely small then the heat transfer across that border is very, very small. So, I am going to say, there is no heat transfer in this problem or I am neglecting the heat transfer in this problem. Then I am going to say, there is not much of work done in any other form in this problem.

And I am going to say, there is not much of friction in this problem, none of those nonlinearity is sitting in this problem. After all this, now I am going to say, that I can approximate this process to be isentropic of course, it is still a hand waving argument, I accepted myself that it is a hand waving argument, it is not a clean argument saying d P by d rho should be done keeping entropy constant. But, we are going to say, the process we have picked is such that, it is almost isentropic.

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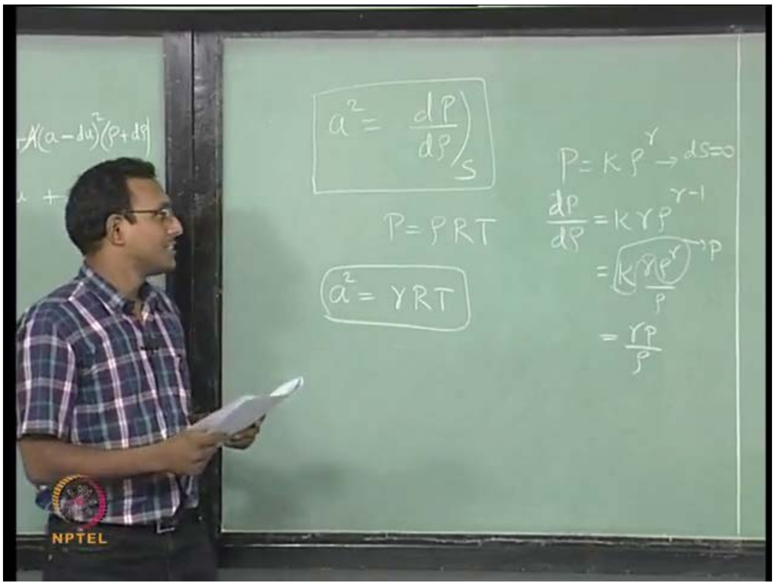

So, we will say, d P by d rho keeping entropy constant will be equal to my speed of sound square. Now, I suddenly started saying speed of sound, all this time I said some pressure wave now, I am saying that pressure wave is a form of sound and I have included or actually sound is the form of pressure wave, that is a better way to put it. And so I am going to get to this form, this is valid only for isentropic waves, it is not valid for non-isentropic waves, non-isentropic waves exist in your flow and they will be called shocks after some time.

So, now we have come to a point where we have some speed of any pressure wave in a gas and of course, you can now link this to, if I have my ideal gas P equal to rho R T and I have is entropic condition. What is the isentropic relation between P and rho, P equal to some constant times rho power gamma, this is the rough way of putting it, that constant will be something or something like P naught by rho naught power gamma, something like that it will be.

So, now, d P by d rho will be what take derivative of this, this is already d S equal to 0 assumed, that is I am assuming entropy is constant already, this is the relation between pressure and density. Now, I take d P by d rho of this as going to be K gamma rho power gamma minus 1, which can be rewritten as K gamma rho power gamma by gamma, which I will write this together as my pressure. So, it becomes gamma P by rho, this is just taking derivative.

Now, I am going to use P by rho as R T so I will get to a nice final form, very commonly used expression in gas dynamics, a square is gamma R T, very commonly used expression where, R is specific gas constant, it is specific per mass basis. So, it is universal gas constant divided by molecular weight in kilograms, that is what this is going to be. And typically, you will see square root of gamma R T in so many places in gas dynamics, it just comes up from here.

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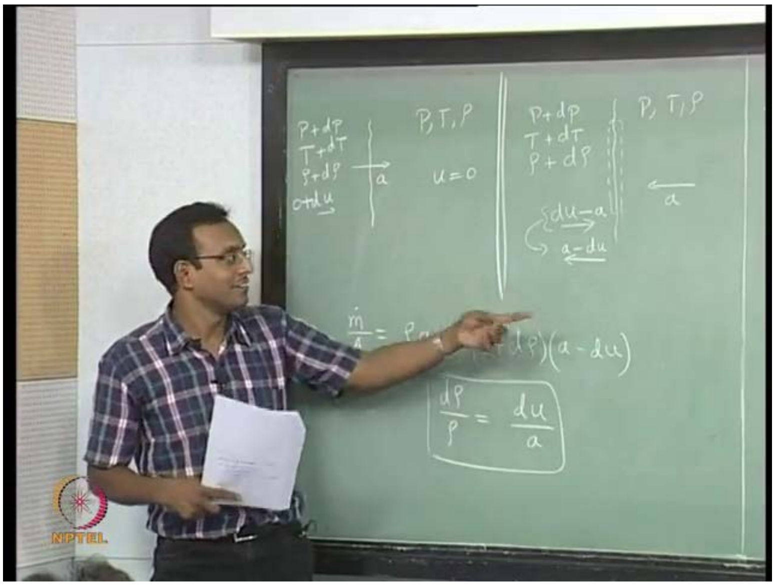

We started at this corner we said that, we are doing all these analysis for u equal to 0 that is, a gas is still in the air and I am having a weak wave moving through this. I am not going to derive the whole thing again for u not equal to 0, but it is equal to some capital V. You can solve this, I will just leave it as an exercise for you people, you can solve this whole thing and you will get to the exact same expression. You will find that, that V will get cancelled in every place like this, you will have a rho a plus V here and that will just get cancelled automatically, it will be a minus V by the way.

I will just get cancelled automatically, that same thing will happen here, it will be a minus V minus d u, it will get cancelled a minus V directly. Like that, every place it will get cancelled, you will end up with the same final form of this differential equation.You will just get exactly the same equation of course, I assumed I did some hand waving argument in the end there.

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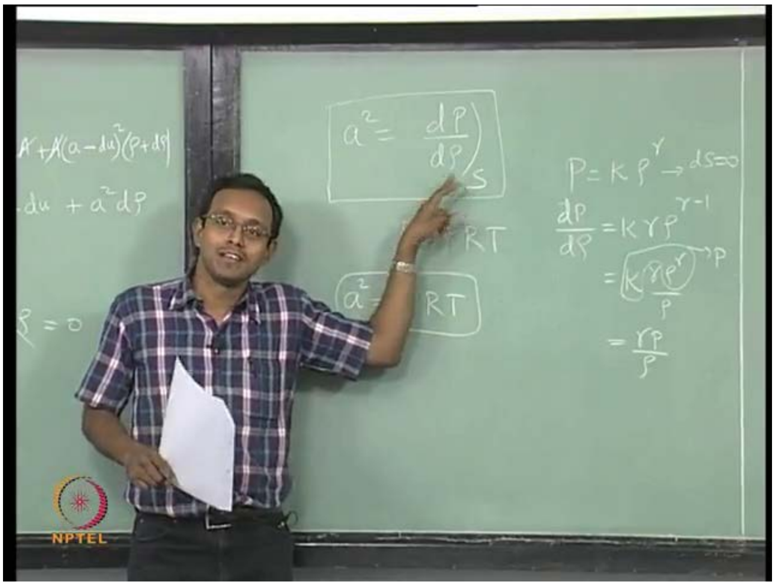

Where I said, I am assuming weak wave and I am saying, there is not much of heat transfer happening anywhere across the fluid elements and so I am going to assume this to be roughly isentropic. In reality, pressure waves can be proven to be exactly close to isentropic condition actually, if you go to acoustics people, they have a much more rigorous proof mathematically to show that, these waves are isentropic. And so my speed of sound square can be linked with d P by d rho, keeping entropy constant.

We would not do that, we will just say we got it from the simple assumption of course, you know, that somebody else is already rigorously proved it, we would not prove it rigorously anymore. So now, after we come to this point, we will keep all these expressions we obtained, we will start thinking some physical field again. Next thing is all about physical field today, we started with initially saying that,fluids or gases communicates from one point to other using what, using pressure pulses, pressure waves.

And now, we have an expression for, what is the speed, at which these pressure waves move, that is a one physical field for things. Now, I want you to think about those childhood days when you had these, when you are awaking up early in the morning, there is sun ray coming through your window on to your floor. And you are seeing these dust in the sun rays, everybody would have seen this in your life some time or the other if not, you can see it even today in your house, it is always there.

So, now, those dust particles are flying around, you try catching one of them, anybody tried this, everybody must have tried this when you are small kid.You can never catch them why, it so happens with the air extremely light and by the time you go close to it, pressure pulses from your hand send information to the immediate previous fluid element that, there is something moving. And that sends information to the one more fluid before, that this is moving and all this is coming closer.

Finally, the dust particle knows that there is something coming towards it, it is moving away from you. So, you try to catch it, it keeps moving away,that is what is happening inside. This is also part of the reason, why you cannot catch mosquitoes all the time but mosquitoes are bigger in size, it has it is own inertia, it has to accelerate to some speed. So, if you go fast enough, you can hit it even though it knows it is going to get hit, it knows that it is going to get hit by the time you go close.

But, they still cannot escape because it is like you are standing in middle of the road, there is a lorry coming, you know it is going to hit you, you still get hit why. You do not have time to react, you do not have time to move away, that can happenor if you had fast enough response, you just jump out of it, that also happens most of the cases, so that is what your mosquitoes also doing. So, if you want to hit a big fly, you do not need to go that fast, it has to take lot more effort to fly. So, if it is a big bug, you do not need to go high speed to catch it, if it is a mosquito you need to go high speed to catch it.

Otherwise, you miss it most of the time, you would have tried itso many times, you know that already, all this is happening from gas dynamics by the way, gas dynamics everyday life. So, if I think about any situation, any flow where there is unsteadiness, that is something is changing in time, there is gas dynamics. In a way, in the sense whatever we dealing with today will be present, how will it be present, I am going to think about it as say, I am going to move my hand towards this, there was a puff of wind going towards this paper somehow, how.

So, I will hold it like this, there is puff of wind going that way why, that pressure pulses this fluid element very close to my hand tells the next fluid element, that this hand is moving this way. Because of that that, next fluid element is telling the previous one, that is telling the next one, that telling the next one. But, it does not tell instantaneously all the places, it tells at a specific rate at what rate, that is related to this. It depends on the temperature of the gas, the particular specific heat ratios of the gas, the specific molecular weight of the gas all that, it depends on all that.

Each gas will have it is own speed, at which it will talk to each other, each point in the fluid so let us pick some example number so that, we have some feel for things, specifically for air. We need to know about our air, what is the room temperature for us, 298 Kelvin is not really our world, we are in India by the way, you are not in Europe. Anyways 300 or 310Kelvin, for simple calculation I had took 300 Kelvin, 310 is more like our temperature in Madras, you should know that 310 Kelvin most often, I have pick the case 300 Kelvin.

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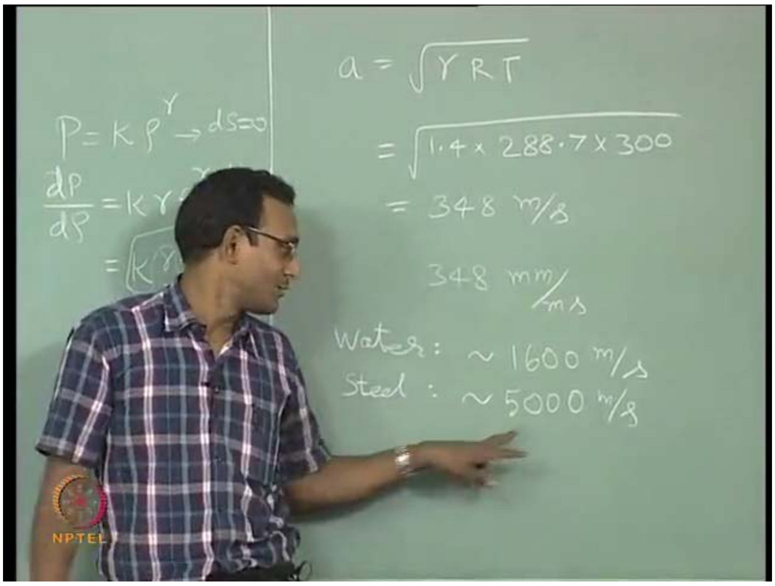

Gamma for air is 1.4, R for air is what, I talked about this already 287 is a wrong number, 288.7 as in 289 if you want, 289 is a better number than 287.If you are using 287, you are assuming molecular weight of air to be 29,while we want to use 28.8 a little better number, that is all I am saying. Atleast in my class use this number, outside in acoustics world use some other number I do not mind. So, I use 288.7 and 300 Kelvin this is the number I used and that number comes out to be roughly 348 meters per second.

Of course, you are so much used to calculating stuff assuming 300 meter per second as the speed of sound, it is not really, it is 348 meter per second as close. But, think about it in a simpler manner to have a feel for, how fast the speed of sound is, I will just rewrite this as 348 millimeters per millisecond to get a better feel for things, 348 millimeters is 34 centimeters, slightly over 1 foot. Let us take it roughly 1 foot, 35 centimeters it is travelling in 1 millisecond that is what it says, sound travels roughly 1 foot or slightly more than 1 foot in 1 millisecond, that is the speed of sound.

That depends on temperature of course, if I use the lower temperature here, this number may be slightly lesser but it is of this order. If you want to have a rough estimate of when will something come close to you, when pressure wave comes close to you, I will give you the ideal example, lightening. It is raining now a days outside, lightning you see lightening, you hear the sound, start counting between these. When you see the light, immediately start counting 1, 2, 3 count the number of seconds, you can exactly tell how far your spark happened up there.

Use this simple calculation, 3 seconds 1 kilometer rough estimate, if you count it 3 seconds, the lightning happened 1 kilometer away from you. If it is happening closer and closer, you know you are getting more danger, it might happen on top of your head next time. But, if it is going far away, first time it was 3 seconds, next time it is 10 seconds, lightening and clouds are going away from you. All that sitting inside this one expression, this use common or everywhere use gas dynamics, it is sitting everywhere anyways.

Now, for other fluids say, for water at room temperature as in 29 degrees Celsius or something, it is going to be approximately 1600 meters per second. In steel of course, I did not give you formulae for these, this will include much more than just simply gamma R T, it will have this bulk modulus term. It will have bulk modulus term and based on that, we did this in the very first class but we said we will stick to gases, this is liquid and solid ideally, we should not be writing formulae for this in this course.

We will stick to gases of the order of 5000 meter per second, it is going extremely fast in 1 second, it goes 5 kilometers in metal, that is the kind of distance. So now, we will go back and start thinking about, what is really happening if I change something so am I again still sitting in unsteady word, I am still talking about I am changing something in time, suddenly I change something and I want to see the effect of it fully. So, how does something change, as in we already did this example of, there is a steel rod and there is gas in a tube.

Now, I am going to take a hammer and hit this steel rod, the wave is going to travel at the speed of roughly 5000 meters per second. If I have 5 meters, it is going in 1 millisecond that is of the order it is going to take now, how did it go through this whole process, those molecules in the top layer are colliding with the next molecules. They are colliding with the next molecules, they are colliding with the next molecules and just keeps on going like this.

If I keep on pressing with the hammer continuously then this will happen but I hit it once and took my hammer away, what does that mean. Now, I am actually sending two waves, one wave is telling, stress here has suddenly increased, that wave is going through. Immediately following that, I am also telling hammer is removed that is, stress has decreased, that is the next information I am sending through this.

So, I am not sending just one wave, I am sending two waves just next, next to each other, one is a compression wave, which is telling the pressure or the stress has to be increased in this medium and the other one is telling stress has decreased. So now, these two are going, if it is not very, very strong pressure pulses then there is no material damage to this steel rod, let us say. I am just wanting to hit it slightly say, I took a wood hammer and hit it with metal thing, nothing happens to the steel most likely.

So, if that is the case then it will have some stress and then it will relieve back to original position so what really happened is, the stress went up some value and came back the same amount. Now, I am sending compression wave, expansion wave, both exactly same intensity and I am sending this through, what is going to happen. The first wave is going and telling, there is some increase in stress, so we have to move away now, the immediate next wave is telling it is all gone, we do not need to move away.

So, what will the molecule see, they are going to start moving and then stop, that is what eventually happens. They will just start moving and then stop but if I let these things go long enough, eventually these two waves will start talking with each other because of viscosity. There is at the pressure difference here and this things will start happening other way, there is a lot of other processes which we neglected in this problem. We said there is no shear effect, there is no other normal forces happening inside the wave, all kinds of assumptions are used in here.

If we take into account other stuff, after some long distances, this wave will just diminish in magnitude, that is what will really happen. But, that is more of acoustics perspective, we do not need to think about it, what really happens I just want to tell is, this expansion wave will go and cancel the compression wave. They will cancel each other, that can happen just remember this cancel word, I will keep on using this once in a while, after we go to lot more flow over different bodies.

I will start telling this expansion will go and hit this compression wave until this will start damaging, that they will cancelling each other, all kinds of such things will happen. So, it will be helpful to think about it from simple wave perspective later, we will go and complicate it with strong waves as of now we are still thinking weak waves. So now, let us say, I have this air medium, how will I do this hammer business, it is not very easy to think about. So, I suddenly put a piston there and I push the piston, am I continuously pushing the piston or pushing and stopping, two things I could be doing, I could push and stop or I could keep on pushing.

If I keep on pushing at constant velocity, that is just one change, it was 0velocity, it became some other velocity let us say, 1 centimeter per second, that is just constant one change. If I push and stop after some time then I am making two changes similar to your hammer hitting, stress increased and then I removed the hammer, stress decreased. Same thing is happening out there so I could have a piston and I push, what is going to happen to molecules nearby.

They do not have space to sit there, that space is occupied by piston, now so they have to run away from this point. How will they run away, imagine this kind of situation I think about a huge crowded area, lot of crowd say, in villages there will be like huge festivals where, there will be a like a small area, where there will be, so many people who want to come in, that kind of place. Imagine that kind of places where, somebody say suddenly, there is a snake, everybody will start running from there.

Information propagates as people move, they are going to say a snake and these guys are running, next guys will hear it now and they will start running. Of course, there will be people who will want to go see the snake or go and beat up the snake also, we will ignore those possibilities here. If it is a snake it is easy, if I tell something else fire probably, better example than people will run away so something like that. Start imagining such things, somebody created a disturbance, and that is propagating from there ideally, in all directions.

We are in 1D world as of now, one dimension only ideally, it could be any direction, it will go from that point in all directions, pressure does not, gas does not know which direction to go, any molecule individual air molecule does not know which direction to go so it will go all directions, it will try everything possible. So, if there is any change, that change is transmitted to every direction, all three dimensions we have, that is what is supposed to happen.

That being said, the first thing now there could be two things that can happen, snake example is nice because it can be beaten up and killed. So, somebody say a snake, everybody starts running and then after sometime somebody says, snake is killed let us go back. Who will hear it first, only the last set of people, all the others are still running, last set of people know that snake is killed so they will stop, slow down, come back to that place.

Then, the next people will see nobody is running behind me why am I running then they will go back then the next set of people will go back, everybody will go back to the place, something like that, how is the information travelling, there information travelling by peoples response time now. People are looking around and seeing who is running or somebody is shouting, does this make they run and so you run, it is all related to your brain response time there. That is related to your speed of sound from this case, that is the connection between crowd dynamics and gas dynamics anyways.

So, that connection being given now, we want to think about another example where, let us a stadium full of people and there is only one gate and there is so much crowd, that nobody even has space to move, it is very tightly packed. And I suddenly open up the gate, outside there is nobody what will happen, people will start running out of that gate. when people start running out of the gate, there is empty space at that spot, people will start moving into that spot.

Because, there is more space there less space on this side, they will start adjusting and then they find even more space there, they start going out more. And eventually, there will be a point where, the outside room or the outside world and the stadium, everything has exactly same number density of people. Pick this example now, transform it to it is not people but it is molecules, molecules are pressurized inside a gas. Now, I put a hole, outside is very low pressure inside is very high pressure, what should happen, molecules go from here to outside.

When molecules go out, that space becomes empty so molecules find that this place is less pressure than here, they will go and occupy that space. So, molecules are moving towards the low pressure section, what is really happening the other way to look at it, the information that there is the gate open in the stadium passes slowly, how. They suddenly find that nobody is pushing, that is the way information is transferred, response time again, it is just slowly being transferred.

So, as that information travels, people go in the opposite direction, you filled up there is a information that there is some extra space. So, everybody wants to go, that way molecules do the exact same thing, that is given other complicated example. Now, suddenly I want to say, there is already so many people in the stadium, 200 more people want to enter through that gate, they open, 200 more people put in, closed.

How will this information propagate, more crowd this happens in temples typically, typically in Tirupati happens a lot. Every 1 hour they will open it, there will be a wave of people entering it and then they will close the gate, happens a lot in Tirupati so anyways something like this. So now, we have pushed in so many people inside now, what will happen, they have closed the gate, pressure has to be adjust or number of people have to adjust themselves, so that they can manage inside the space.

These set of people are too close to each other when they entered, they are very close to the gate but there are 200 of them, there they will start pushing one by one. When somebody pushes, you will feel the push and you will move away, the same thing is done in molecules. There is a compression wave now, somebody pushing and so you want to move in the direction of the wave propagation, somebody pushing from right I will move to the left.

In the other case, expansion somebody tells you there is more space there, that information is travelling this way, while I go this way. What you have to get, there is the feel for these two waves, that is why I am spending some around 20, 25 minutes on this, it is great to have a feel for this. Once you have the feel for this, unsteady gas dynamics is in your hands already, I want you to take go all the way up to unsteady gas dynamics. So now, if I think about any flow situation, if there is any change happening then these kind of waves will be generated.

When the waves are generated, we are going to go through different processes as in the wave could be a compression wave. If it is a compression wave, gas will move in the direction of wave propagation, if it is an expansion wave it will go in the opposite direction of wave propagation. We got this idea from physical feel room crowd dynamics and I said that, molecules do the exact same thing, you just believe me as of now.

But, we can even look at it from equations point of view, how will I look at it, we already have most of the expressions on the board. But, we will anyway write some set of expressions, it is not very difficult to do. I have to erase something on the board at least let us say, I will keep this picture that is useful.

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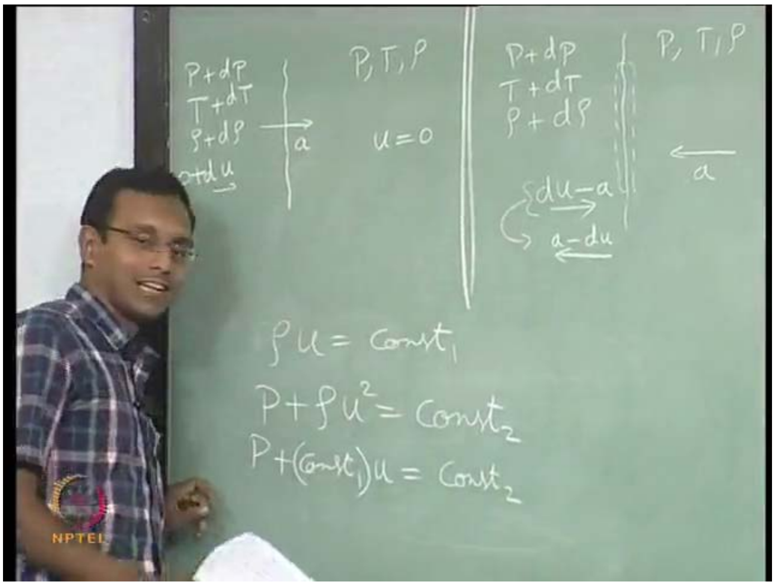

Now, mass equation, simply put this rho u is constant and we already derived that P plus rho u square is constant, for what case. This is a special equation only for one particular situation, area is constant across my control volume, constant area control volume I have picked, it is special control volume a 1 and a 2 are same, that is what we have assumed. So now, I will write this as, I put constant 1 and constant 2 something like this, this is also true from mass and momentum equation together I can tell that, if my pressure increased due to the wave then my velocity has to decrease.

I am directly saying this, how of course, I have to know very carefully that, constant 1 is positive. If constant 1 is negative, it could be the opposite, anyways we will leave that out, we know that mass flow rate is always positive. So, I am going to think about this as, when pressure increases velocity decreases because it has to maintain the constant, this term increase this term has to decrease, that is what I have to think about. So, I am already giving you a proof saying, if it is a compression wave, velocity decreases, is that what we saw.

Let us go look at it from this picture, that is not what I am saying here, let us say we will go look at this picture. In this picture what is happening, fluid is coming with velocity a into my control volume and this is a compression wave, that is going this way. Actually, I am sitting on the compression wave so it is not moving, velocity going this way is a, when it goes that side it is a minus d u so velocity decreased, that is what I have to look at.

Now, what happens in this case this is a special case, here the reference frame is sitting on the wave. When I move the reference frame to outside or with respect to some other gas or with respect to a common reference outside then I am talking about unsteady wave motion, all these equations were derived for steady case. The wave is not standing in one place here, wave is moving, changing position from one point to other, these equations we neglected dou rho by dou t terms, we made it equal to 0 when deriving this.

So, these equations cannot explain this phenomena, as of now we will leave it that way, we will get to this towards the end of the course, last 3, 4 lectures or something. As of now, it will explain this, very easy to explain basically, there is some set of people running this way and there is something saying that, it is too hot there or there is fire there. People will slow down, they will not keep going in the same speed, they will slow down or you are telling, there is z of people running this way and there is too much crowd here.

And you are seeing too much crowd what will happen, you are going to slow down, that is the way you want to think about. The wave is going this way, information that there is too much crowd here that is, pressure is higher here is transferred this way with respect to the fluid. But, with respect to the wave, the wave is just sitting here, we have reference frame on the wave now, just so I can use steady equations, that is what I have done till now.

And I can now use this expression to tell that compression wave, velocity decreases and if I think about expansion wave then velocity will increase why, pressure dropped velocity should increase. Different ways of looking at things now, I will just give you one more extra thing to think about.

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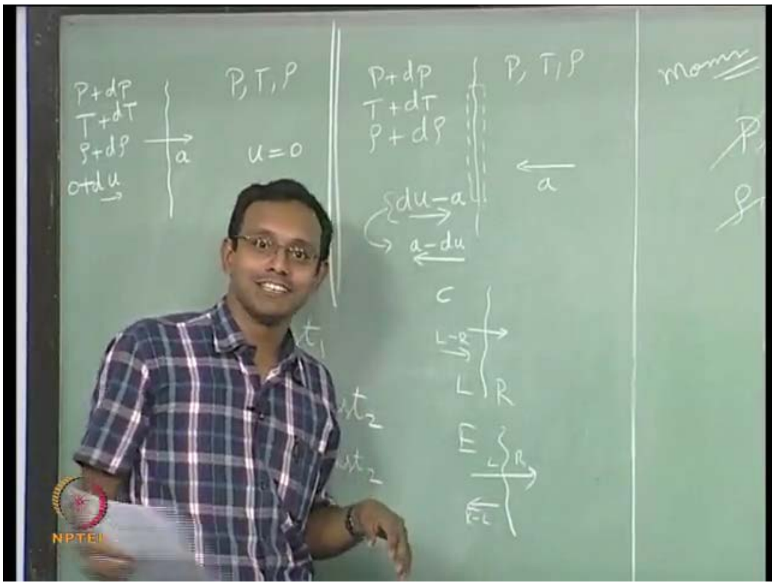

That is, if I have a compression wave let us say, I have a wave here and I am saying, this is a compression wave and it is moving this way, this is unsteady, wave is moving. If I think about wave is moving, I will remember crowd dynamics, it is a whole bunch of people telling, there is too much crowd there run away. Then people will run along with this wave, that is what is supposed to happen, if I go left to right, velocity induced is left to right.

If I have expansion and this wave is going like this it is telling, there is more space that side or there is less pressure here, that information is transferred by this wave. That is an expansion wave, there is less pressure there go there, is what that wave is telling as it goes this way. So, in that case, velocity induced will be right to left, while the wave is travelling left to right, just have this feel for these waves, it will help you when you go and start looking at flow over body.

Suddenly, you will start seeing one wall is pushing the gas up and down and all that, it will all explain things very nicely. As time goes, we will get used to this form of stuff now, I have to extend this, whatever we have talked about till now was about one wave generated from one point or actually one plane a piston generating a wave like this. In reality, this was not just one wave created by this piston, it is actually a whole bunch of points here at that front of the piston, every point is trying to tell that, there is higher pressure there when the piston moves.

If that is the case and there is whole bunch of points, which are sending out information in all directions, let us assume spherical but it so happens that, the other side is piston so it is hemispherical wave in this special case of piston movement. So, hemispherical waves coming together, they all form one big plane wave and then that moves like this, that is what happens in my piston cylinder arrangement. I will show you animation of this next class, I know that these NPTEL people got it ready and I just did not bring that animation today, we will show you something else today anyways.

Now, imagine there is a case where, there is no walls anywhere, I just have free world everything, no other walls anywhere just free gas, one particular gas present and I disturbed something in one point. What will happen, it will send information in a spherical manner all over, what if I disturb it in three places at the same time, there will be three different waves created, they are all going to go. It so happens that, whether you disturb or not, there is always pressure wave transported from every point in fluid at all times in all directions.

If there is a change, you will see it as a compression wave or an expansion wave, if there is no change, you do not see it anything special. Because, it is just going to tell that single information, no change in pressure so you do not perceive anything because there is no change. But, if there is a change you will say, it is a compression wave travelling, if there is a change it is a expansion wave travelling, you can tell there is a change.

But, always have this in mind, that there is always, always pressure wave is travelling from one point to other inward, why. Gases always have collisions, it is actually consisting of so many molecules inside, in 1 centimeter cube there is 10 power 25 molecules or so actually, 10 power 19 molecules present at 273 Kelvin in 1 atmosphere I think, it is roughly correct 10 power 19 molecules. If there is so many molecules present in 1 centimeter cube 1, CC that small volume and they are all colliding, they are not just sitting ideal.

They have some energy in it because it is having some temperature, they are all colliding in all possible directions. So, the immediate next CC of fluid elements, that particular fluid particles are going to collide with this particular set of particles so there is always information transfer. If the top fluid element is colliding less with compared to this one then we know that, there is less molecules present on the top, which actually means what, there is low pressure there, high pressure here now, there will be wave of information travelling, why.

These molecules find that, it is more difficult to sit in this volume, it is easier to go and sit in the top volume so they will go up there and so this fluid element gets empty. Now, that information transfers to the one below and so there is expansion wave going this way, what about the other side, I have fluid element there is a bunch of particles coming in so these guys have to increase pressure. So now, they have to readjust their space so some of them will see that, this is same pressure as this originally.

So, this is now lower pressure compared to here, that these particles will now move in there so there is a compression wave going this way. Thus, if there is sudden change and I make them come to contact, we will see that expansion wave goes this way compression wave goes this way, assuming bottom is higher pressure, top is lower pressure. Such things keep happening in fluid all times, when I move my hand I have sent expansion waves this way, compression waves that way, why. I pushed the fluid particles this way, here I created empty space where, fluid particles will come in.

So, when I move my hand, the expansion waves go this way, compression waves go this way. When I am talking like this, I am doing so much of change with my lips, that I am producing expansion compression, expansion compression all kinds of variations, all that is happening always. Whether it is subsonic or supersonic, the flow that is happening is a compressible flow, that is the main thing you have to get used to. You cannot always say, that my flow is compressible only if, my mach number is very high. Flow is compressible, if there is d P by d rho non zero, that is what matters finally.

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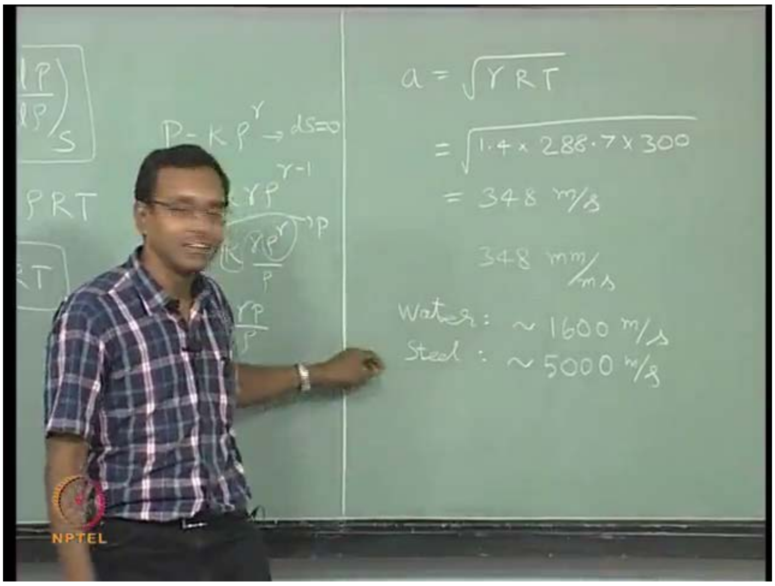

If d P by d rho is non zero, from looking at this point of view, if d P by d 0, d P by d rho is 0, what am I saying there is no pressure variation due to density variation or actually I wanted to say the other way, there is no density variation due to pressure variation which means, it is 1 by 0. This goes to infinity, speed of sound is infinity which means, it is not the molecules that are colliding but they are sitting hand to hand stiff like this, no movement.

So, when you hit one corner, it just goes hits the other corner immediately. That is a very special case, speed of sound is infinity, which is not the case in any fluid in the whole world, even solids speed of sound is not infinity. But, you can say that, steel has very high speed of sound because it is more stiff compared to air, which is you can compress to a reasonable amount, that is the difference compressibility coming in here. If you can compress it easily, speed of sound is lesser, now that being said, we will go to one animation, which is supposed to give you a little more information about.

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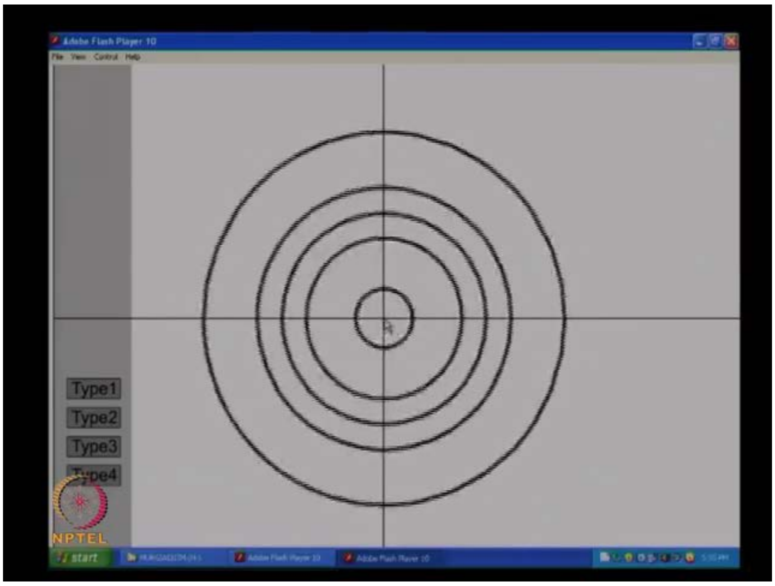

So, we are looking at this where, I have this lines drawn so that, we know the source point source location is just that.

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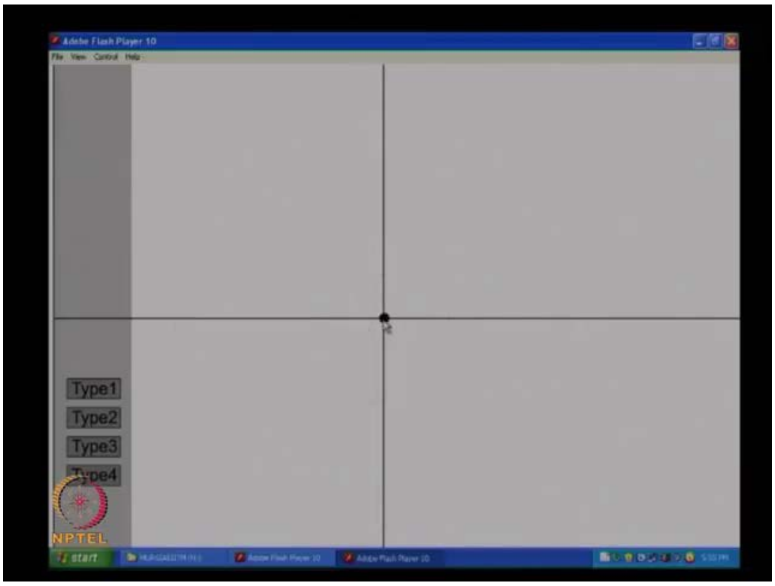

And in this source location, there is just a set of waves created and they are moving out in all directions equally. Now, if I add a flow to this from left to right initially, all the fluid elements are all still, no flow movement, no flow velocity.

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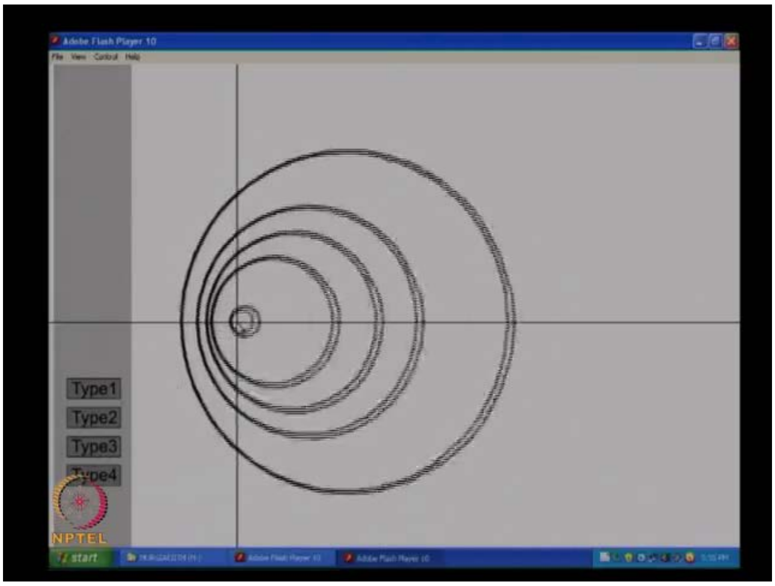

If I add flow velocity from left to right, I would not look at that, it so happens that, waves are travelling against this way and as they go, the fluid is pushing the waves back this way. In this side, it is still the same frequency of waves created by the way but it so looks like, this getting crowded here and it is getting more farther apart in this side, something of that sort is happening here. It so happens, that the velocity is not very high compared to the rate of travel, the way you want to call it, it is subsonic flow.

The velocity here is lesser velocity compared to the speed of these waves in this fluid, that is what is happening here, flow is only along this line. I will go to the next case, this is a case where, the speed of the wave is equal to the speed of the fluid so this is like there is a fluid running this way and the waves are trying to run against it, like this. Imagine a treadmill, you know people run on treadmills in exercise gyms where you will see, the belt is running this way and the wave is running on top of it, so it is just standing there practically.

Both are equal speed, it is just sitting there, that is what you are seeing here but on the other side, if you run the opposite way, you will be thrown out in no time in a treadmill, you should know that. Anyways, that is the idea in here, it is just being added, the velocity of these waves will be it is like running in a train, if you think about it. The train is going this way and you are running inside the train in the direction of train, for a person standing from outside, you will see the velocity of the gas plus the velocity of the wave running on it, both together.

So, you will get u plus a as the speed of the wave here, in this side it will be u minus a actually it is, you can call it u minus a in this direction and you will find that, that is 0 when the special case. Now, I will go to the last case where, my speed is much more than the speed at which the waves can go. If this is the case as it goes, the center of the wave is being pushed out by the fluid, it so happens that, this point the wave never reaches these points.

It is only inside this cone like region, this is not going anywhere outside so that is what we see and this is how, you create your supersonic flow mach angle. You are going to tell now that, if I draw a line along this tangent of all these circles, that angle I get across here, that angle happens to be your mach angle. This is the simple illustration of this, I guess I am going out of time, we will stop here, we will go up next class.

Starting from this point, we would not do this animation again, we will just directly start with the mach cone. We will just start looking at mach cone how it works; see you people in next class.