Gas Dynamics
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Module - 3
Lecture - 6
Energy versus momentum equations,
Pressure waves in gases

Hello everyone, welcome back. We stopped just short of completing momentum equation derivation and full; we went up to a point where we derived the momentum equation up to the differential form and the integral form. I just have to give some inferences from those.

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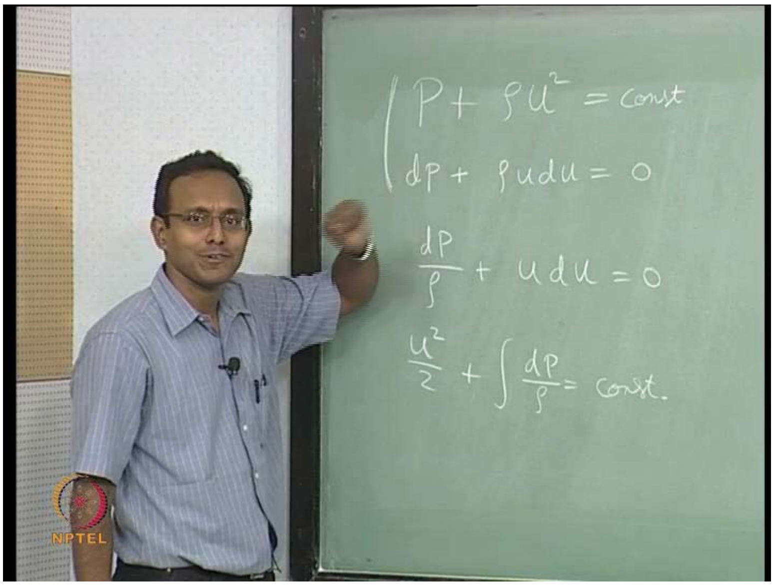

From the large control volume we had this relation and from small very thin control volume, we have this relation. Now we have to find the link between these two, but before that I will just give you one more aside so that you get comfortable with compressible flows. So, I will go back and integrate this expression. When I integrate I will rearrange this row below this d p. I will write it like this. We know density is never zero; I can divide by density, no problem.

So, now if I integrate this I am going to get constant which is your integration constant. So, if I say now that my density is constant irrespective of what pressure it is, when I am in incompressible world; if I am in incompressible flow condition, then I can take this 1 by rho outside, then I will get integral 0 to p of d p will be just p. So, I am going to have p by rho here. Then that looks like your Bernoulli's equation; that is if I assume density to be constant, then I will go to a point where it becomes Bernoulli's equation, but in reality we do not know that whether it is constant or not in our case. It may be constant in some special situations, but mostly it is a variable, because we are in compressible flow.

So, because we are in compressible flows we cannot take this out of the integral just like that. And so, now I will have some value for this, and that is going to cause some other extra terms in your Bernoulli's equation. We will look at Bernoulli's equation derived from some other point of view may be in one or two classes later. I want to derive it in that Bernoulli's equations only for incompressible flows. As one extra thing i will do. Now that is one part. Now the next thing I want to talk about is how are these two equations related. They are both the same momentum equation. One is p plus rho u square is constant; other is d p plus rho u d u is constant. If I just integrate this in a simple world I should get a u square by 2, but that is not there.

What is going wrong? It so happens that the same answer as this, density is sitting here. If I keep density is constant it will become u square by 2, but this not derived for density constant. This is derived for full compressible flow equation, and so this will be different from this. But I will tell you a quick way to derive this. By the way one more assumption we have in this top one was you wanted to eliminate this f x variable; let the force on the side walls, we wanted to eliminate that. So, we said a 1 equal to a 2. This is a very special case of constant area duct.

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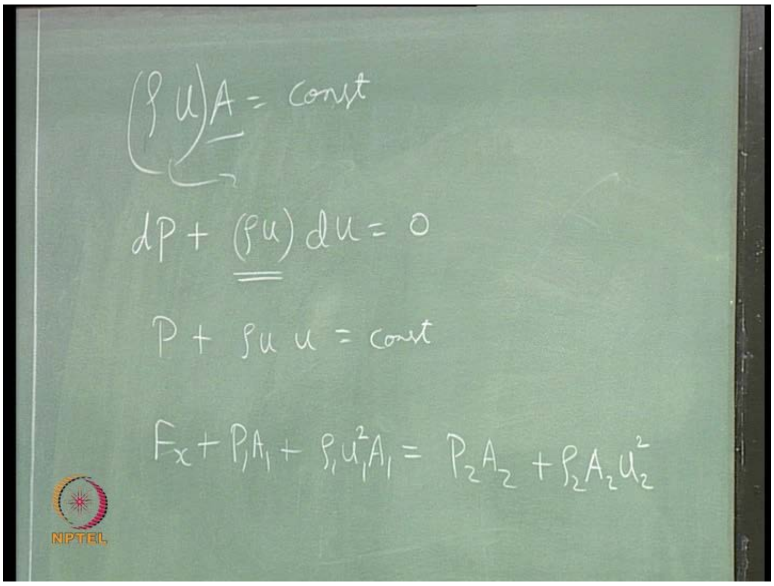

If I say it is constant area duct, then from mass equation this is a constant which means now this is a constant. If this is a constant now I will go and write my differential form where this is a constant. So, now I will suddenly if I integrate this, I will get to my form which is the expression which we have up here. That is the link between this and that. Typically no book gives you this kind of explanation. Just you are expected to do these by yourself and figure out what is going on. You should be trying to link everything you have learnt already; try and link everything one way or the other.

So, ideally the full momentum equation for any area condition is going to be this. Inside this we have assumed that it is constant area; affix does not matter and a's are all the same. So, you will finally end up with this relation; that is what you got to this. I put a 1 square instead of u 1 square. It should be u 1 square; here I have done it right. I just switched the terms you know multiplication is coming right here. So, this is about momentum equation. Now we will start looking at energy equation; we will go to a fresh section. Again we are going to start doing the same exercise starting with the full integral from the original first time we wrote the conservation form.

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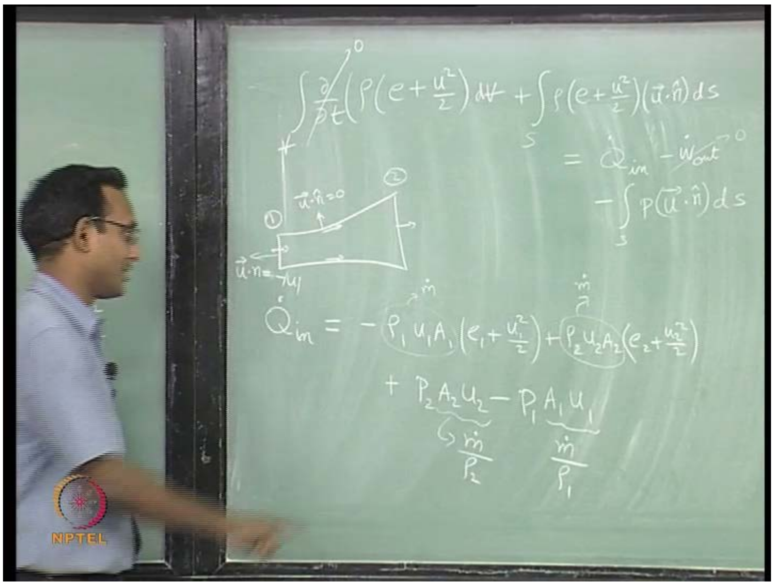

We will start from the conservation form of energy equation and that looks like this. And this is going to be equal to q dot in minus w dot out, and there is a pressure work which I did not take into account minus integral over surface p u dot n d s. This was one of the forms of energy equation which we wrote at the beginning. Now we are going to use this expression, and I am going to take some arbitrary control volume. This is my section one, this is my section two, and I am going to say flow is going along my control volume here and here it is perpendicular. So, here u dot n is 0. U dot n is zero there which means there are no energy flux terms through these sections. Energy is just converted along the surface but not across the surface.

Now the next thing is q n; we will keep q n as such. Let us assume there is some heat transfer somewhere inside. It does not matter, same thing will work out. We can keep it or remove it depending on my convenience; we will keep it for now, but we will remove it after we write the integrated form. We already said that we are looking at only steady problems. So, this term goes to zero, and I am going to say u dot n is zero for those two surfaces, but the other two surfaces here n is this way. So, u dot n is minus u minus magnitude of u. You get to that; there it will be plus of magnitude of u. So, if I use this and write it all together, I will rearrange it left hand side to right hand side. Q dot in will be equal to I am neglecting this w out.

Currently let us say there is no turbine work or shaft work or anything like that in our problem. So, I am going to have just terms from this integral which will be rho u a times this quantity, and that should be at section one which will have a minus sign. So, rho 1 u 1 a 1, this is one term. Then the other term will have a plus sign, because they are vectors are same direction. It should be rho 2 u 2 a 2 times c 2 plus u 2 square by 2; that is that term. Only other term remaining is this; I have taken it across the other side of q. So, it will be plus. Now I have to look at this term again. U dot n it is zeroing here, is it logical? Pressure is acting this way; is it not having any work done on the system? It looks like there is no work done on the system due to pressure, is it logical from this section.

Of course, it is going to give some value here and here. It so happens that force is applied perpendicular to the velocity and f dot u is power. So, because of that it is going to be eliminated. Actually the force is p times n vector times d s that is your force pressure times area and area has a perpendicular direction to it. So, pressure is acting perpendicular to that area dotted with the velocity is your work done, rate of work done; that is what it is supposed to be. So, it so happens that force is acting there, but it is useless for us. Remember if there is sheer force along this line, it will add or remove energy from the system.

Currently, we are assuming that there is no viscous force here. We are saying no sheer forces possible. We already removed the other terms; we said tau x y terms we removed from here when we derived this whole integral equation. So, if we have that we will have some other term extra here nonzero terms. Now we will have only contribution on this surface and the final output surface, only these two surfaces. So, again I am going to say it is minus u here, yes?

Student: Sir, in real case of course we can.

In real case there is sheer force. We will not deal with it right now in our simple compressible flows course. When we are thinking about sheer forces on the wall, it will be a small factor compared to the huge amount of energy that is passing through. There will be a little bit of power wasted against friction on there, and that will be relatively small, unless you have friction very very high. We will deal with the flows with friction after sometime. As of now we will assume frictional flows not much effect on my energy; that is my assumption.

So, I am not having that extra term which is supposed to be here friction term. We will not have it for now. Now I will go here and I will just substitute these terms in here. There is already a minus u with this coming this side it will stay as minus finally, So, I am going to write that term as p 2 a 2 times u 2 minus p 1 a 1 times u 1. So, I have these many terms here. I want to rearrange this simplify it a little bit. So, I will take this and then look at my mass equation which was rho 2 u 2 a 2 is equal to m dot my mass flow rate. So, I will write this as m dot by rho 2.

Similarly, I can write this as my m dot, the mass flow rate by rho 1. I did not use subscripts m dot 2 and m dot 2, because it is supposed to be same mass flow flowing through; that is my continuity equation. I am saying mass flow rate is constant from section one to section two; that is also taken into account here. So, I am just having m dot, and that is going to simplify it like this. Similarly, if I look here rho 1 u 1 a 1; that is my m dot directly. This whole term will become my m dot, same thing here. This is also the same m dot. Now I will incorporate all these changes and write my equation again there.

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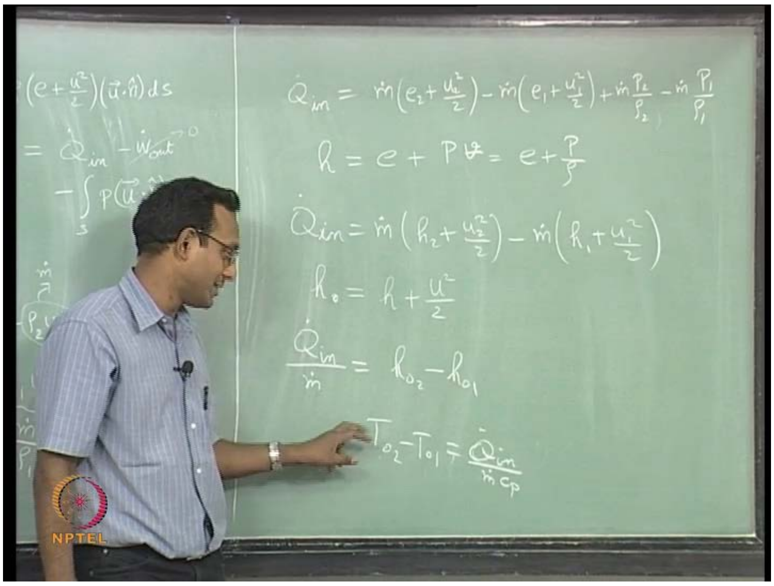

So, I have this form. Now recall that h the enthalpy per unit mass is equal to e plus p v where v is my specific volume which is 1 by rho, right. This is volume per mass which is reciprocal of mass per volume. So, I get to that form. So, now I can rewrite this whole thing clubbed together. I get a simplified form like this. It does not have any a 1 a 2 sitting there directly except through these m dot expressions. It is still valid for any areas; a 1 and a 2 need not be the same. They could be different, and it will still be taken into account inside here automatically. It is still sitting inside your m dot term.

Now we will go and implement that particular assumption you were talking about; we are assuming always that heat transfer is zero. So, now we will go and say at this moment we will go and say this is equal to zero. I came up to this point because I want to show that in case we have heat transfer, my enthalpy may be different from initial to final. If that is not the case enthalpy need not change; that is what I wanted to say. Now for convenience we will start introducing one variable h naught. H naught, it is called the stagnation enthalpy. We will go deal with it later. It is given as h plus u square by 2 directly from here. If I write it like this then my expression becomes extremely simple; that is why I am doing this.

So, if I use this inside here I get to this as my energy equation. It is extremely simple form of energy equation. Especially now if I say my heat transfer into my volume is zero, then I am going to say h naught remains constant; that is my stagnation enthalpy does not change, it is very specific. We will go deal with why this is called stagnation after sometime, but I will just you a quick answer. If I say my stagnation enthalpy does not change I can go to a condition where I will slowly decrease this velocity and go to zero velocity, the energy from kinetic energy. This is actually kinetic energy per unit mass, right, m u square by 2 divided by m, and this is your enthalpy the c p t term. This is again per mass, enthalpy per mass.

So, what you are having is total enthalpy per mass. If I slowly take energy from kinetic energy and give it to this keeping this constant, then I am going through some particular process where I am taking my fluid to rest, some imaginary process by the way. It is not a real process; it is not happening in flow really. I am going through this some imaginary process taking it to rest. And if I go through that particular condition of taking the flow to rest, the final state is called stagnation condition. That is flow is stagnating; it is not moving. So, that is how you got to this name; we will go deal with it more later.

Now I will go back and derive this expression in the other form actually it can be written for calorically perfect gas as I will write it in terms of q n right now like this also if i assume calorically perfect gas, right. I am saying c p is the same for any temperature. And so, I can full out c p out of this, and I will just have t 0 2 minus t 0 1, stagnation temperatures they are called. So, I will keep it like this; we will go for more explanation of this later. We will continue with derivation of energy equation, but now we will derive it from thin slice control volume.

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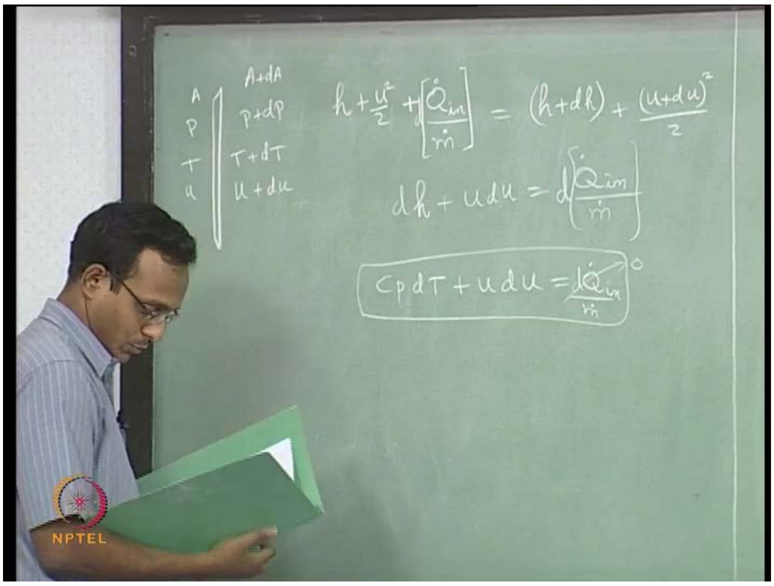

Again we will go back to the small thin control volume, and I am going to say this is A, this is A plus d A, p, p plus d p, etcetera. You can write it for u, u plus d u, rho, rho plus d u, rho d rho, everything. Now we will take the other form which we just derived and then write it here for this control volume. We know that that particular expression we wrote is true for every control volume. So, we will use apply that energy equation for this control volume. So, I am writing it as I have to think about it little and say this way. I have this form; this is equal to h plus d h. How will I get to this? I am going to say it is t plus d t here. I multiply this with c p, multiply this with c p to get to this form.

This is your c p t; this is your c p t times t plus d t; that is all I am having plus u plus d u square by 2. Of course, I am having u, u plus d u also. So, I am using that expression here. Now if I expand these and neglect that d u square term saying it is a very very thin slice, d u square is extremely small compared to all the magnitudes of other variables.

Then we will get to a simplified form, and you will also see that h plus u square by 2 will get cancelled with this particular h plus u square by 2. So, I will have a simpler form. I will go to d h. By the way I have to put, okay I will deal with it after this; I will come back to it, d h plus u d u q dot in by m dot I am having as of now.

We will keep it like this, but now I will say this is a very thin slice control volume which is a very differential control volume. And if I am going to say there is some energy added inside that control volume per unit mass, then this is a differential heat added. So, I will put a d in front of it. I am going to say it is a differential thing, because it is a small control volume. Ideally I should have done it here itself; ideally, I should have put it here itself. It is a very thin slice. I am going to say it is a very small amount of heat added. So, that is what is given here. Of course, I am keeping all this, but I am immediately going to through it away. The next line I am going to say q dot n is zero.

So, I will write another form. Keep this form also; if you want you can keep this as d q n and set it equal to zero. This is also fine. By the way it is d q n by m dot always, q dot n by m dot or you can write it as q is defined as a new variable per unit mass q if you want. Currently we are keeping it as just q actual energy. So, we have derived things in different forms finally. Let us go and do a little more; let us look at it from the point of view of constant entropy condition. People will say that Bernoulli's equation. I am again going back to Bernoulli's equation. People will say that Bernoulli's equation is an energy conservation equation in some books, but we derived Bernoulli's equation just in today's class, and we got it from momentum equation.

So, both are the same. Momentum is not same as energy, but we are getting momentum equation giving you Bernoulli's equation instead of energy equation. But Bernoulli when he derived he just looked it as potential energy, kinetic energy of the fluid he said, and based on that he derived stuff.

Student: What is it for compressible one?

He assumed it to be incompressible flow, okay.

We will go back and see that again; Bernoulli's equation is for incompressible flow, but he derived it from energy equation; that is what I am going to focus here. Of course, we already proved that from momentum equation we can get to Bernoulli's equation if we assumed density is constant; that is we assumed incompressible flow; that is already valid. Now what we have to see next is we derived it from momentum equation.

Bernoulli originally derived it from energy conservation, are they both the same? It so happens that they come out to be the same if you have invicid incompressible isentropic flow; actually invicid isentropic flow is fine, you will get to that point. You will get to that special condition; that energy equation can be derived from momentum equation and mass equation. If you say the flow is isentropic and incompressible; these two if I say I can write it the other way, and you will get to that expression.

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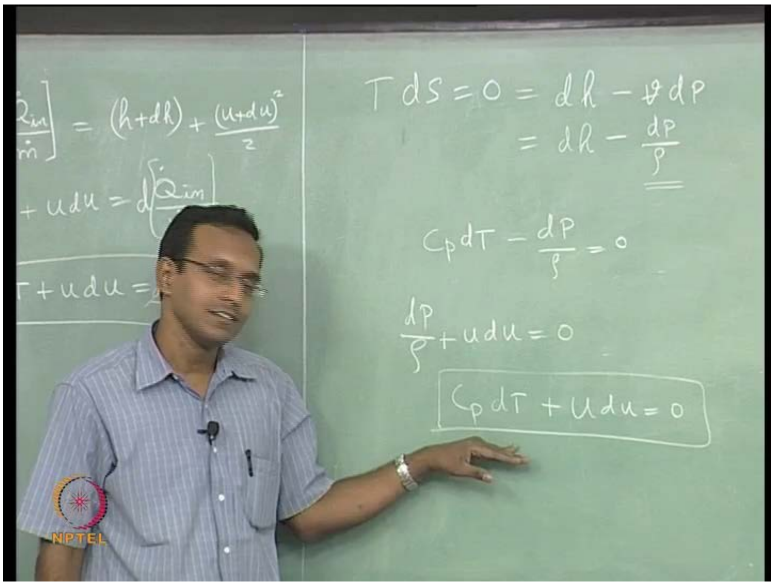

We will look at it from isentropic condition. I am going to say t d s equal to zero. We already started using this thermodynamics remember; this is next time we are going to use it. And of course, we write it in this form in terms of enthalpy from now on. This is a common form, but we do not like it; we write it in terms of density, rewrite it in terms of density. Now of course, you should know that we have already used p equal to rho r t inside here in deriving this, right. In thermodynamics class itself the beginning review class itself, we already substituted d e plus p d v with p v equal to r t to get to this form this specific form; p equal rho r t we used to get to this form. We have already used that; remember that gas equation is already used inside here.

Now the next thing I want to think about is substitute this and simplify this expression. I am going to say this is equal to zero; I am going to say this is c p d t. So, I am going to write it as c p d t minus d p by rho equal to zero; I have this expression. Now I am going to substitute momentum equation into this. I will substitute momentum equation; what was my momentum equation? I erased that already; anyways this was my momentum equation already. So, now I will just substitute this inside there, and see what happens? D p by rho happens to be equal to minus u d u.

So, if I substitute that inside here I am going to get c p d t plus u d u equal to zero making them to this form. Remember I started from isentropic; I said it is isentropic. I said p equal to rho r t I am using; I am saying I am using momentum equation. I did not use energy equation, but I finally ended up with my energy equation. You can compare it with this expression. It so happens that this and this expression are coming out to be the same; this is what I was just now talking about. Energy equation is now not an independent conservation equation. It is a dependent equation; it can be obtained from some other equations, a combination of other equations; that is what we are ending up with.

So, is this right always? It is not right always. It is true for the special case of come back here; if you look at here d q is zero. Only if my d q is zero I can link this and that equation like this; otherwise I cannot it is a special case. So, I am going to say if I have a special flow which is isentropic, no heat transfer, nothing, no friction, nothing. And I can use mass equation and p equal to rho r t, mass equation momentum equation p equal to rho r t and d s equal to zero. Four equations I have used and I get energy equation in here; that is what I am ending up with finally, is this right? So, we will go look at it from math logic point of view next. How many variables do we have in the flow? Think about it while I will erase this. Three variables, I do not think it is right, but anyways let us label them.

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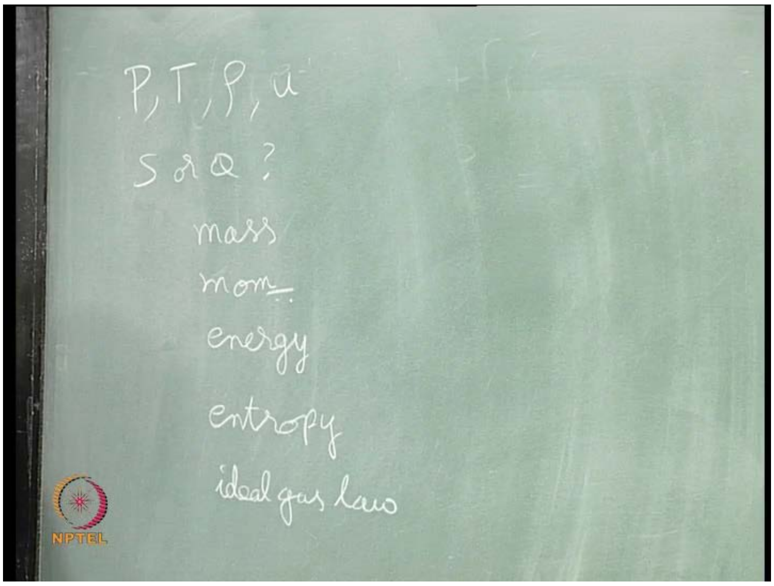

So, give me variables, pressure, temperature, anything else?

Student: Density.

Density, anything else; it is a flow.

Student: Velocity

Velocity, anything else; what is that?

Student: Volume

Volume is actually related to a density in a way; of course, if you want extensive quantity I have to give volume separately, but we are thinking all intensive properties right now. Entropy, that is the question; entropy or heat, okay. I will put a question mark here as of now; we will keep it like this. Now how many equations do we have? Three equations, let us look at them; mass, momentum, energy, anything else? Entropy relation, anything else, ideal gas law p equal to rho r t; I have all these equations. I have five equations. If I say my q is zero or my flow is isentropic, I do not have this variable as a variable really; I know the value of this always. It is not an unknown in my problem.

If that is the case, then I have only four variables in my flow situation, but I have five equations possible. So, one of them should be redundant or one of them should become useless equation; in a sense it is not going to give you any new information. I just showed you that energy equation can be derived from all the others. Of course, I can now start proving something else if I want. I can take these three and get to entropy relation, does not matter. They are all interdependent; there are only four independent relations in them; that is the special thing about this. There are five relations possible in theory, but they are interrelated because there is only four variables in my problem, but if I have some other special thing like say there is friction. So, there is entropy change. Now this equation matters; it is no more that simple t d s equal to zero. It will be something more. There will be a friction related term coming up here d h naught will not be zero. So, there will be extra term coming here in which case this becomes a special relation.

Now this and this linked will not give the same expression; they will give something different. So, that is what we need to think about. In a special case where I am having isentropic flow and no heat transfer that kind of simple assumption, when I say isentropic flow it could have some heat removed and friction which produces entropy; that can make entropy zero overall if you want. I can add entropy by creating friction and remove entropy by removing heat. Then my flow may have constant entropy; that is not the case we are looking at here. We are looking at q 0 as 0. If there is such a case and these are the only primary variables, four variables all I need are four equations, four independent equations.

If I have five all of them valid equations, we should know that one of them must be redundant equation, law of nature, unless we have missed a variable here; that is also a possibility. Sometimes you may miss a variable; in this case we have not missed any variable; that is the special case. So, to summarize I have given you a whole set of equations in two different forms we derived. I will erase this and start tabulating that. You have it separately written in your note books, so that you can always come to this page when you need these equations.

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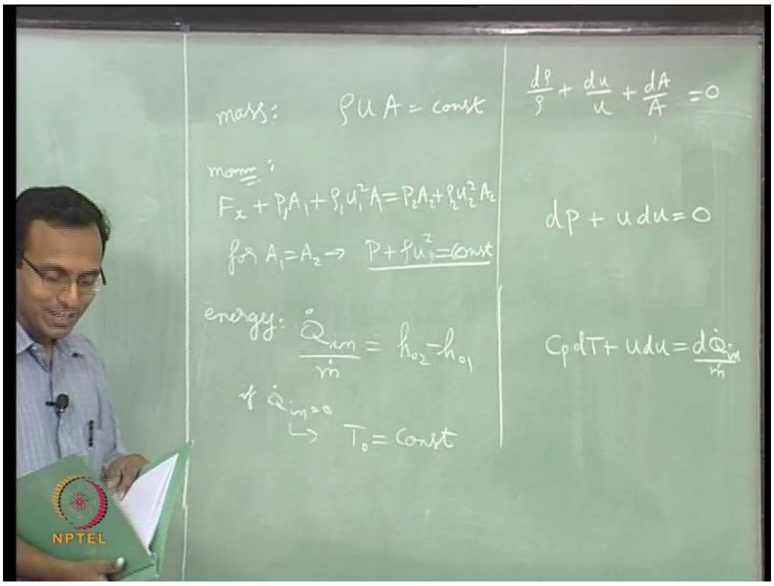

Mass equation we have two forms, rho u a is constant; that is one form, other form is. So, one is the integrated form, one is the differential form. We derived this last class; I am just tabulating it together here one place. Next will be momentum; momentum equation we had several forms. P 1 a 1 plus rho 1 u 1 square a 1 is equal to p 2 a 2 plus rho 2 u 2 square a 2. This was one form for a 1 equal to a 2. We said p plus rho u square equal to constant actually; p plus rho equal to rho u square is equal to constant also we said. And on the other side we had just one simple expression, very easy to remember; that is for momentum.

Now we will write energy. In case of energy we had this expression. We derived this just today just recently, and in case q equal to zero I will say if q equal to zero, then we will get to a form where t naught equal to constant. I am already using calorically perfect gas inside here; c p is constant assumed here. Now the other side we will have c p d t plus u d u. I am writing it like this c p d t plus u d u is d q dot actually, d q dot in by m dot. If I say there is no heat transfer ever inside my any small volume, then this becomes zero and that is the expression we wrote on the other side of the board anyways. So, I wrote this altogether, so that you have one place where you have all these things tabulated.

Now we have derived equations for one dimensional flow, quasi one dimensional flow. We will start looking at specific examples after some time, but we will look at this again one more time; in here we assumed isentropic as of now. There could be a case where the flow need not be isentropic; say, for instance there is a sudden jump in the flow called the shock. If there is a shock that is happening inside the flow, then entropy will jump suddenly in that region. If it jumps, then entropy jump at that particular point is separate, and in that case these equations are no more coupled the same way.

They are becoming different and you have to look at that differently; we have to look at which equations I will use to solve the problem specifically. We will go and do that when we go to shocks; till that time we would not, but I just tell you that that will be one caution we have to keep in mind. Now we are in a position to go and talk about pressure waves travelling in gas. So, we are going to derive speed of sound. What we want to do is we want to imagine a case where there is gas that is very still, nothing is moving.

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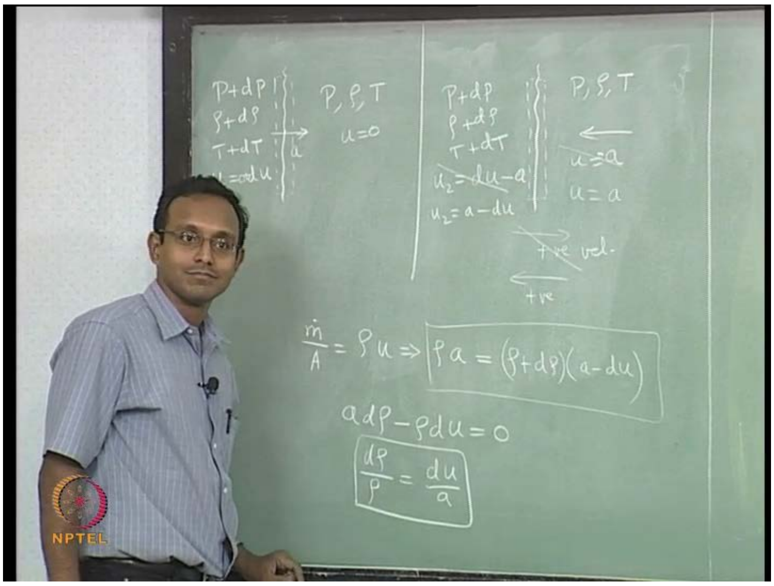

This is just one wave; I just want to draw a curly line, but we are thinking about it is a straight line wave that is going. Just so I call it a wave; I am having curly line here. And it is going at some speed and we want to call that speed a, which is supposed to be our speed of that pressure wave. We will keep it as a, and I am going to say the gas here is having pressure, density, temperature, like this and u is zero. We will keep a special case where gas is still and I am sending a pressure wave into it; that is what I am doing here. And because of this pressure pulse that is going, something must have changed. Let us currently say it became p plus d p; I do not know d p could be positive or negative, I do not know; I am just saying d p.

Similarly, rho plus d rho t plus d t and u will become d u; actually it should be zero plus d u if you want. It is just becoming some velocity value; that is what we are going to have. And now I want to transform coordinates. I want to be in a shifted coordinate system where I am sitting on this wave and seeing what is happening. It is like sitting on a bus and seeing this flow coming in, right, still air outside bus is moving. So, we will feel wind coming towards us that kind of situation. So, I am going to see a different story here; I will draw the other transformed picture here. Wave is stationary with respect to me. Now there is flow incoming which is having a velocity a, and now I will write u equal to a here; u equal to a and pressure is still p density is still rho t is temperature is still t; that does not change. Properties of the gas are still properties of the gas.

Now when I go to the other side of course, things have changed p plus d p rho plus d rho t plus d t. What happens to velocity? What will this be? Now I will go back to this; from here how did I transform to that? I added a velocity of a in this direction. Now I will add a velocity of a to this, this direction. What will that look like? If I want to say that that direction is positive, then this is minus a; there is already d u. So, it will become d u minus a, and this should be actually minus a sitting there if I want to put specific direction to it. Velocity this way is positive for mine. If I say that direction is positive velocity, it is going in negative direction; that is what I am having or if I want to think about simplifying the problem, every velocity is always going to be negative in my case.

So, if I want I will just say velocity in this direction is positive. Now I will redefine my velocity axis; I am going to redefine my velocity axis. So, all my velocities will become negative. If it had a velocity that way 100 meter per second, now it will be named as minus 100 meter per second; that is all will happen. So, I will rewrite this as u equal to a, and I will rewrite this as u 2 equal to a minus d u. It does not matter; I could have solved the problem that way also. It does not affect me at all; it should be solving the same exact way, no difference. So, now you want to go tell that even for this very small change in flow, there is a small pressure wave, because of that there is a small flow induced by it. And there is a small change in pressure, temperature, density all that.

Even then it should obey my mass conservation, momentum conservation, energy conservation, entropy wave, everything; all the equations we derived till now. So, because of that I will just say my mass equation must be conserved, and I am considering my control volume to be a small volume like oh, I wanted to shift in coordinate system. I will be in this coordinate system. I am taking a control volume like this; constant area control volume I am taking. This is very thin on this side, no forces on this side or that side; that is my assumption currently, or it is very thin that we will neglect it; that is the idea.

So, now we want to go and write my mass equation. It is going to be, sorry I should put m dot by a. Now I have to write it specifically; for this case rho times a is equal to in this case it is rho plus d rho times a minus d rho; I have this expression. Now we will look at only this, not the remaining part. This was my original mass equation. I just went to this form from there; we know its constant area. So, I just took all these are constants; rho u must be constant conserved, density times velocity in each section must be conserved. So, I am writing here density velocity, density velocity; I get to this form. Now I have to simplify this. I have a rho a directly from here which will cancel with his; remaining term will be a d rho, rho d u and there will be a d rho d u term which we will neglect; d rho d u is like small quantity multiplied by another small quantity. It is extremely small quantity, we will neglect that.

So, we will have a simplified expression which will be a d rho minus rho d u equal to zero; that is one form I have which can be rewritten as d rho by rho equal to d u by a keep one relation. This is then coming from my mass equation; that is one form. Of course, you should know that I could have derived this from my differential form of equation, that last page there. You can see that the top right corner d rho by rho plus d u by u plus d a by a equal to zero. If I said my control volume will come back here, if I look at my control volume d a is zero. I can directly write to this form. I can do that also if needed. I just derived it from the integral form, and then made it differential and then come to this point. I could have used the other form also; depending on your convenience you can choose one or the other.

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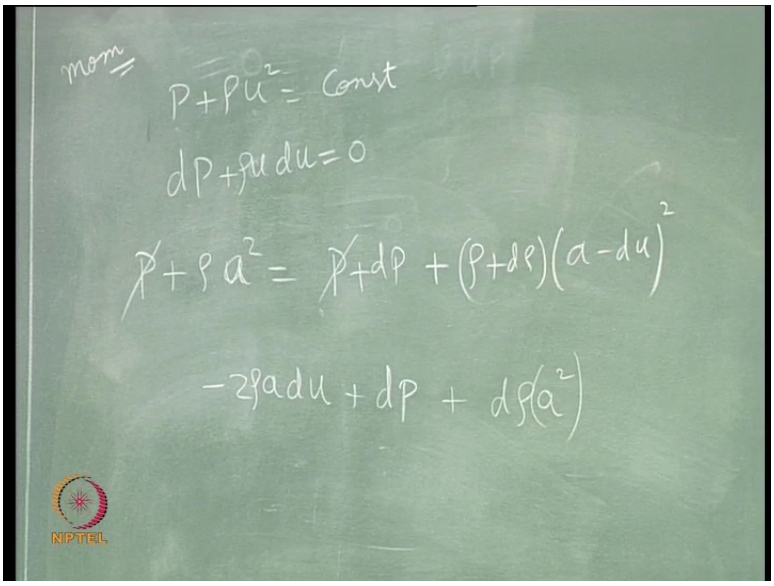

Now the next thing is momentum equation, and of course, we choose a particular control volume here with flat portions and constant areas. So, effects are zero. We are going to take the special case, and of course, if we have a constant area we have a simplified equation, right. I can write it in nicer form. This is a general equation or let us say for a change we will use the other form. This form also I could use. Let us say we will directly use this form; instead of going trough complicated routes I will just directly write d p plus a d u equal to zero. I am taking a shortcut; ideally, you have to go take this one. You will do this equal for both sides; both sides of the control volume we should do this thing and then neglect the small terms same way we did here. Do that for same way p plus d p times a minus d u square times the rho plus d rho; that whole thing you have to multiply and simplify, and you will end up getting this expression.

So, I just wrote it here directly. Now all you want to do is substitute the d u from the previous expression we got. We already had d u equal to a d rho by rho; where did I get this from? From here, this expression in here d rho by rho times a is my d u. I will take this, substitute it inside this expression. So, I will have d p equal to minus a square d rho by rho; that is one expression I have or I missed something somewhere. I missed a rho here sorry. There is a rho missing, and that is why I am having trouble. There should be no rho at the end; this is correct. I was checking dimensions, it did not match. So, I made a mistake here d p plus rho u d u is constant; that is the correct expression.

And I have made the same mistake at the end here also in here also d p plus rho u d u was the actual momentum equation when we derived it; when I wrote it in this table I made a mistake. This is the correct expression. We will keep this; that is the expression I used to get to this point here. So, I have this expression here; from here I am going to substitute this d u times rho as a d rho inside here. And that will get you this form which I will write as d p by d rho. Wait; there is a minus sign which I am missing from here. I took a wrong path; I will go back the right path. I want to take a shortcut from my notes and that is what is causing trouble. Let me just go back to the same old path. I will take this expression and simplify it from there, because I missed a minus sign because I went from here, u will become minus a.

Otherwise, it will not solve the problem. I will just go back to the original form p plus rho u, u will be a square is equal to p plus d p plus rho plus d rho times a minus d u square. This is from p plus rho u square as constant; I am just writing it that way. Now I can simplify this a little bit. Of course, I can remove this, and this can be expanded in one form or the other. Rho a square will get cancelled with rho a square that is forming here. The next term that will come will be d rho times this whole thing where I will remove the d u square term; a 2 a d u with a minus sign will be there, minus 2 a d u times this d rho; that is a term which I am going to neglect again, but i will keep another term which will be rho multiplied by that whole thing.

I do not want to write it. I will just put rho here. This is one term that will be left, and the other term that will be left will be d p. What happened to the all the other terms? There will be a d rho times d u or d rho times d u square; all those terms I am neglecting. Only term that is left will be rho times a square which is getting cancelled with rho times a square. Other term will be rho times d u which is here; rho times d u term is here. D p is directly coming from there. This whole thing is the right hand side I have written here.

Student: Sir d rho times of a square.

I am missing one more term; this is not a nice way to write it then.

In my notes I did not have this path, but anyways let me just think through this once more. In my notes I wrote it in terms of m dot and this problem never occurred. It should be correct even now, yes.

Student: If that in right side is equal to a, then that u 2 should also a plus d u.

This should be a plus d u; no, this is my u 2. Basically, question was when I switched from this positive velocity vector to this is my positive, then I just switched to a minus d u; that is correct. I just made minus sign of this. If this was my minus 1 meter per second, this will become 1 meter per second now from this reference; that is all has happened; that is all I have done till now. So, it looks like I am going fast time; I just wanted to complete this. There is anyway a problem. I will get back to you next time then. I have gone passed my time. I just wanted to finish off with speed of sound today and talk physical stuff next class, but I am stuck here. I will get back to you next class. See you next class.