Gas Dynamics
Prof. T. M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras
Module - 21
Lecture - 52
2D Method of Characteristics

Hello everyone welcome back we were discussing designing of smooth nozzle such that it does not produce any shocks in the flow, and I will just continue from that point.

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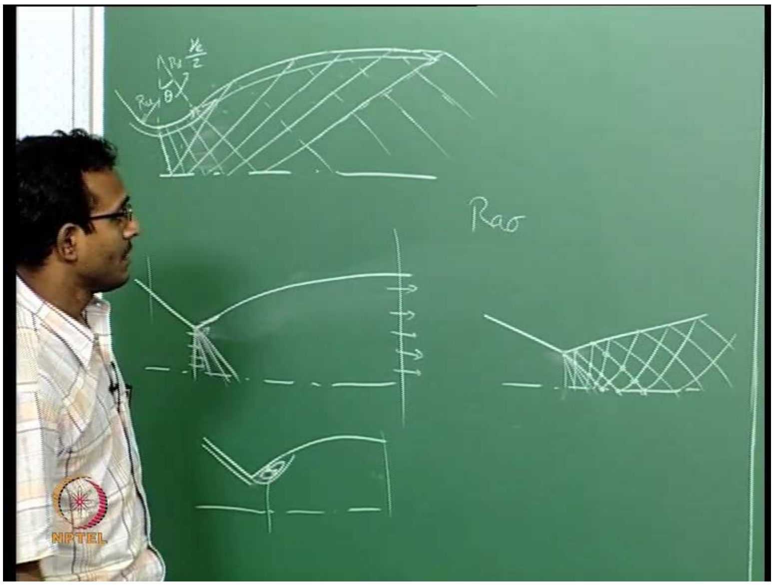

We were talking about from the throat, I will have an axis symmetry line from the throat we will go along a circle up to some theta which is equal to nu exit by two and after that point, we will create new points on the wall such that these points, if you notice what is happening in this region it is expanding the flow is expanding.

Now, if I keep going this way I will get velocity vectors like this as it goes up velocity vectors are out right we wanted parallel flow at the exit. So, now, we have to turn those stream lines back that is the other reason, why the remaining lines will be turning back the remaining wall points are designed such that they will be turning back. What is really happening is this region which is actually sending out expansion fans these results, that are sending out expansion fans this is the smooth corner think about it. Its sending out expansion fans downward those expansions, when they come back they will still be expanding they will still be expanding as, they are going out they will expanding more but after they reach this and come back down, after this point they are all going to be parallel lines; that is the idea of this other part of compression smooth compression wall.

Initially we have a smooth expansion wall and then a smooth compression wall this is a whole sequence of how we will have a nozzle downstream section designed, the diverging portion of a c d nozzle designed. upstream portion, of course we would not worry about too much when we are designing a nozzle they will just typically maximum angle of conversions as forty five degrees, and then it will have a small radius here we talked about this r upstream and this one we called as a r downstream, we can pick r downstream to be anything r upstream decides the initial line, here very first line with double bunch of point on that is given by this r upstream we will move talk a little more about this after we see some pictures.

\What I want you to think about is from the initial point, because of this curve it is going to be expanding the characteristics are going to diverge as they go out and after, it hits that wall and comes back they will all become roughly parallel ideally they will be parallel only after the last characteristic, which is your characteristic coming from this corner point that point of inflection. That is the point where this theta equal to nu exit by two exactly there will be times if you do not have enough resolution in points in here, you may end up with not going and meeting the point at all, in which case you will not have optimum expansion here at the exit you will be slightly less or slightly more or you will have theta slightly varying it will not be all parallel those kind of problems will come up. So, depending on your particular situation you may have to choose the number of points, such that there is one point going and hitting there then it will be easier for you to design your nozzle. If not of course you can have an algorithm , where it will sense that there is the previous point is very close to that point and the next point will be far away.

Then you introduce another point here purposely from there you calculate stuff, it is not easy can be done, you have to use some interpolation in the middle I would not talk about that here it is not. So, important for us I did not use it for my design of nozzle, now once you have this understanding lets go to the screen.

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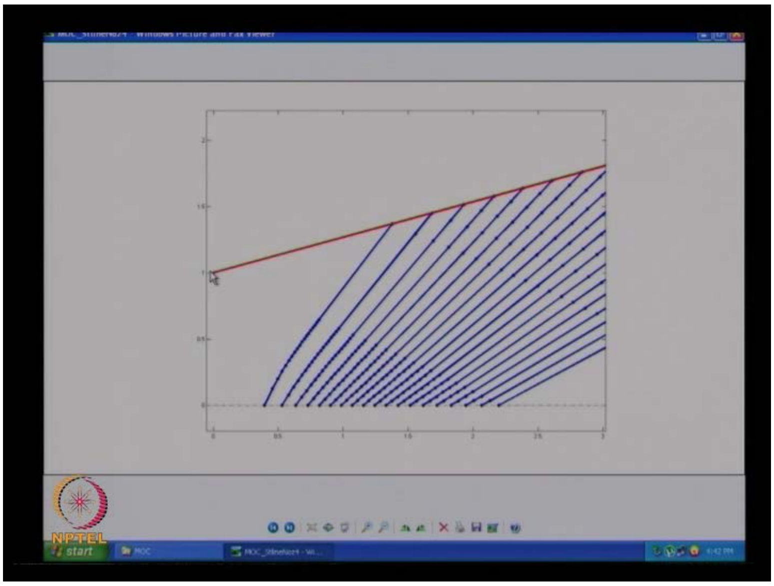

I have designed a nozzle for mark three here for gamma equal to one point four a u can see that this is the throat section this is your a star location ideally, and I have created number of points on this to be ten initial points on my initial line to be ten, how do I get these points what I actually did was we talked about the transonic solution near the throat I took a point and I said that that is n equal to one a was actually it is not one its exactly, I went slightly pass that one point zero five or. So, I saw no ya one point zero five is what I used I think from that point its one point zero five and here actually it will be slightly subsonic we will not pick this point.

So from here I go along the characteristic I go along the characteristic, some delta y down ward and then I will find the next point there some delta y down ward I said I just divided this a throat height by ten points. So, I will get nine delta y is here from there to here I moved that height down along the characteristic found the exact location from the transonic solution there, I will get my theta and m values there from there I will go along the new characteristic found by this point to here again delta y down find the new characteristic like that I am going along one characteristic line.

I am going along the right running characteristic fully all the way up to here that is what I have done here this is the easiest thing to do, if you think about I have done this first characteristic once, I have done this remaining characteristics are whatever we did before from the second point, here go to the wall the circle goes and ends somewhere here for me till here it is the surface actually circle ends here for me. So, first point goes there inside the expanding circle from there I send out a characteristic that will again have ten points, the third point from here goes there forms the second point of this characteristic and then becomes that last point on my expanding circle till there, from there I will get another characteristic and now you can see that the gap between the characteristic lines are expanding very highly here, and after it reflects on this symmetry line you can now think about this line going back out this way as positive characteristic, and this line going out this way as a another positive characteristic, and you can say that they are still expanding this lines and this lines are still expanding out you can see.

Now after that you can again see this one is expanding a lot, here and then when it goes out there it is still expanding this line is going out there, while the next line the last line is expanding all the way out up to here that is also expanding, but when they go and hit the compression wall here and when they come back they are coming back almost parallel by the time it comes and reaches this point, this expansion going out from this way is counted by this compression this way for each of the characteristics and they finally, become parallel roughly when they come here, when the characteristics are parallel you are going to have uniform flow field when the characteristics are expanding then you are going to have increasing mark number, when the characteristics are converging you will have decreasing mark number. So, here you are going to end up with after this point I will not plot, because that is not end of my nozzle after that if I plot they will just be parallel lines going all the way straight to the axis and from there each of these points will go back out parallel. and all the mark angles after this last line will be the same as my exit mark number mark angle, which happens to be for mark three it will be some angle whatever it is sin inverse one by three, it will just come out to be to see values in here that is what you will get here by the way these regions mark angle will not be sin inverse one by three only, after this last line from there from the expanding circle after that only it will be mark three this is the last line, where it is not mark three downstream of that everywhere it has mark three theta equal to zero, this is what you are discussing before. So, it just comes out to be that.

So, if I now make a nozzle with this particular red line profile, I will get uniform flow with exit mark number three of course, I checked with same surface given back to a method of characteristics code, and I got the value of the exit to be three points zero five mark number for this particular case. So, it is not exact which means I have to have more number of points on this expanding wall to be exact I did not do that in this case. So, we are not exactly accurate it is just three point zero five small error, I am ready to live with that and I would say one more thing typically, if we are designing something by purely invisid analysis this method of characteristic is purely invisid analysis. So, if you redesigning something for three point zero five most likely, I want to get two point nine two point nine five mark number when you are making the nozzle and running it in experiments, because there will be some effect of viscosity boundary layer forming area will be slightly lesser.

And because of that you are a by a star will be lesser and you will get lesser mark number, but you will get almost uniform parallel flow field there with this kind of design there is some error in it. So, I will say slightly over design mark number. So, that you will end up with a uniform required mark number there, I have designed one nozzle for two point two and we go two point zero five finally, and we also designed one nozzle for I believe four point one and we are getting three point nine. So, that is the kind of numbers you will you can expect it will not be exact it will be close it will be slightly lesser typically that is one aspect of it, now I am going to think about another case instead of solving for what is the wall that is required to get uniform flow field, now I am going to say I am prescribing a wall I am prescribing a wall that is straight line nozzle straight line straight line this is the nozzle.

I am giving if this is the case I have if this is the case I have of course, I have to give a point expansion here that is the only option I have a sharp corner I do not have any theoretical analysis for r , in this case you have a radius upstream here that radius up stream, we will use in our transonic flow analytical expression and I will get our initial line here, but now if I add u equal to zero, I do not have any way of calculating analytically, because it is a sharp corner sharp corner is typically analytically, it will go to infinity things do not work very well curvature is infinite things will go wrong a not curvature radius is zero. So, things will start going wrong because of that we will not use that method and I will have to think about using point expansion point expansion we already talked about right; its I am going to assume the very first line to be a straight vertical characteristic.

And I am going to assume flow here is exactly parallel to my symmetry line and my mark number is one it is not a very good assumption, but it is the only thing I can do right now. So, I will assume it that way and from there I know the final angle of the wall locally it can be changing far away, but locally that is the angle and I should be knowing that and once, I know that now I can think about from mark one to change to that angle how much should I turn. So, you just give that kind of that much of expansion fans here and how many ever points you want on this you can give typically I will give ten points, but I have done an example here with twenty points anyways I can choose whatever point, I want from here now one more detail I want to talk about in this nozzle design, if I had a situation if I had a situation where I do not have that upstream r the radius upstream here.

So, I cannot use my transonic solution then I will assume my initial curvature of my initial angle of my wall to be exactly equal to theta equal to nu exit by two as from the other way the other calculations I will directly do this. So, what; that means, is this r downstream that radius is also now shrunk to zero, which means all these expansion fans are now coming from this one corner instead of it being smooth wall smooth expanding wall it will be a point expansion that the only change point expansion we already had a routine for it. So, you just have to go and put. So, many points exactly at the same location with that kind of expansion now if you look at your nozzle, it will just be a compression wall this is all you will have it will just be a compression wall its already expanded with point expansion and then compressing wall smooth compression wall this is all it will be this is what is being given in all these standard text books as minimum length nozzle this will be the minimum length nozzle, that you can design with method of characteristics k that is assuming, I still want uniform mark number and uniform theta being equal to zero everywhere, if that is the condition if I have r downstream r upstream equal to zero then I will get only compression no expansion part that is.

The minimum length you can ever get in your nozzle this is a minimum length nozzle typically I will never use a minimum length nozzle as per this design, because if I have a nozzle like this and I make a nozzle like this and go and put it in my flow, if I go and put it in my flow the flow is going to come here; it will never want suddenly turn because viscosity never likes any sharp turns. So, it will eventually end up with a big separation bubble because of this your nozzle is not having the real throat here, but its having some other throat which is a moving throat it is not a constant throat, because it is going to be oscillating re circulation zone it will be changing shape once in a while.

So, you’re a by a star is not fixed ever. So, you will not going to get correct mark number at the exit and it is not a steady flow you will never make a nozzle like this ever if you want to use a this in a supersonic tunnel should never make this, I would say it is better to use a straight line nozzle with a small curvature, than the this nozzle this is the worst nozzle you can ever make, but a better want to make will be with this radius here and then another radius , now you just it will just be expanding smoothly slowly that the flow seeing only small turns at any time it is got to turn slowly, and it will follow the wall very nicely and finally, come to exit value there this is what you will get finally,. So, this is the good thing.

If I think about it this way it is a better way of designing a nozzle there are other conditions you can impose on nozzles which I will just give it as a exercise or whatever I do not even want to give it as a exercise you want you can go read up more on rao nozzles have different conditions that is not same as this m equal to m is constant, and theta equal to zero that is not the same condition they obey some other rules you can come up with different rules for different cases, that you can think about that also if you want, but I will not go into that in this coarse. So, as of now we will go and study a case where I have straight line nozzle like this of course, you should know that if I make a nozzle like this in real life it is going to have a small bump here and a recirculation there.

So, it will not give me correct mark number and steady mark number it will never give that. So, I will just assume that those things do not exist viscosity never exists in real life, I will assume which is really observed any way I will assume that and I will go about doing calculations with point expansion here, and how many ever points I put here say I put five points there I will have five characteristics coming from their first, and the first characteristic is a useless one it is just a straight line thing only remaining four will be useful and I will finally, see that I have one two three four five points if I started with five characteristics the first characteristic going vertically down that is m equal to 1. I will end up with five characteristics five points on my final characteristic after, that if I keep advancing along right running characteristic I will always be having five points, I will always be having five points in here depending on how many points; I start with that is the number of points you will always have after I reach the steady condition.

I can keep on drawing this mesh like this we will go and look at on the screen some example of that this is that straight line nozzle that red line is my nozzle wall which I am forcing and the flow is going starting from this point there is a vertical characteristic. which I have not drawn and there are other characteristics twenty of them the second first one goes straight vertical second one goes to this point third one goes through this point, and this point like that there are. So, many of them the last one goes through this set of points here on the end till here that will be your last characteristic. So, after that whole thing you will now find that I have started with twenty points. So, you will find that from here you will be having twenty points along this overall now the next thing I need to do is just this point is now useless that s its job is already down next point here now I can send it to the wall I do a wall point calculation here from there that point and this point together.

I will do an interior point I get this point and the next characteristic line point either interior point like this, and I just keep on going along this line that is this picture, now I also want you to look at one more detail the very first characteristic our vertical the next second characteristic was here, that is having the biggest gap between these two right and that gap is propagating, if you note this that gap the first line just comes and goes back of vertical this line comes here and goes like this. So, the gap between those two characteristics was this big and that thing is continuing to propagate down here.

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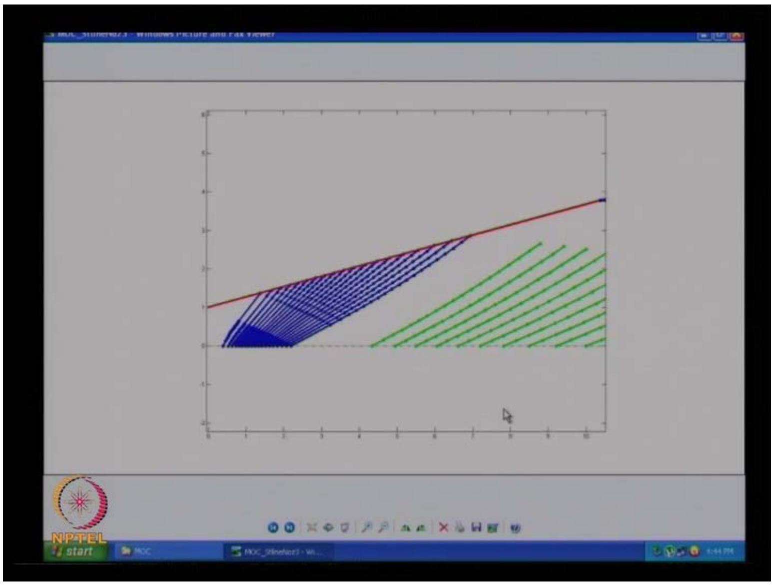

it will be more obvious in the next picture I just zoom out a little more if the same set of points which we are looking till now we were zooming in only this region, now I am seeing a little bigger picture when I see here, that gap if you note this its actually slightly decreasing, why this wall is not a nice wall its not a well-designed wall this wall is such that it is going to give you compression after the initial expansion. So, its slightly decreasing, but not very much they are almost going parallel in here, if you think about and this gap will always exist it is going to propagate through all through your flow field it is going to keep on propagating. So, as of now I have been solving left running characteristics all the way till here and suddenly , if you want to switch to a right running characteristic way of solution I can still do it and that is what I wanted.

to show here now that I have given different color I saw start solving from here I will give a triangular patch here this triangular patch, where I will from here go to the axis from the axis I will now start calculating interior points up to here and then the next wall point will be here, that I would not worry about right now similarly I will just take every characteristic to the axis from there, I will just go this way all left running characteristics. if I want to switch from right running to left running I will be having to do some patch like this if you already had a right running characteristic pattern I did not draw the full fabric I just drew one set of threads of the fabric other set of threads will be this way which I did not draw I just put points there I had drawn that while you will l find that this set of characteristics are right running characteristics. if I wanted to continue with right running characteristic they will all be just running like this parallel like this all the way up to their that is what it will be and if I wanted to turn it to left running characteristic say I had right running characteristic all the way here and I wanted to turn it to all left running characteristic I had to create this triangular patch here at the end of it.

I will have all lines this way after that I can continue along this way any case if I want to go from one one family of characteristic to the other family of characteristic I need to have this triangular patch.

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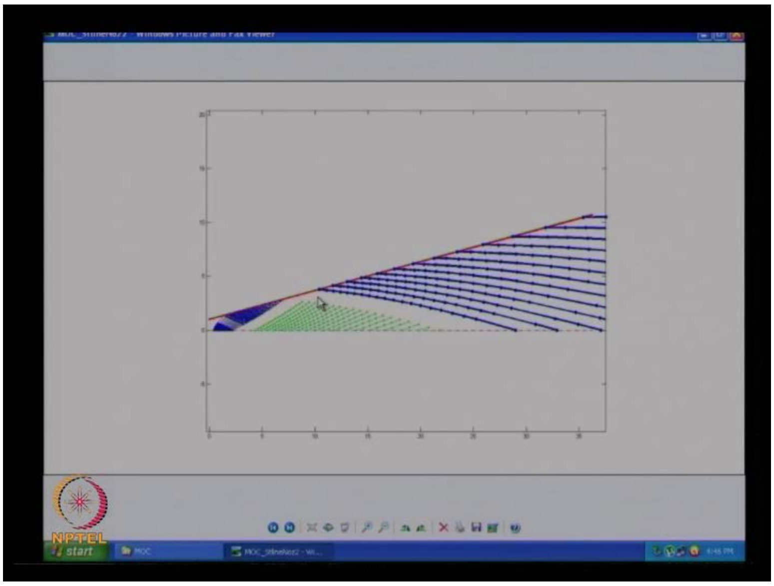

This is how I can show it to once more clearly here till now I was solving left running characteristic from here and suddenly I switched to right running characteristic the reason I will give more obvious in the next picture I wanted to study the flow around the center of the ba nozzle not along what is happening in the boundary that is not what I was interested in I was interested in what was happening in the center line. So, I wanted to go for right running characteristics a I talked about this already. So, this is the triangular patch and you can see that that initial expansion gap is still propagating through all the way up to here. In fact, if you see that its going through this region also between this point and this point between this point and this point just its just going through this way, but the gap will always be there.

it is the only way it will be now if you look at it I want to this point and our nozzle wall ends somewhere there these kind of problems will always come up and solving these kind of problems you find that the nozzle wall exists up to here, but I can calculate only this point data point I want to be much closer I should have started with different set of number of points here then I can probably get much closer result the next characteristic that goes from here let us say this line if it goes up it will go through this and go and have a point out there which is outside of this nozzle that is the best test I can do. So, what I will do is I will live with that point or I will live with this point as the exit I chose the next point as the exit.

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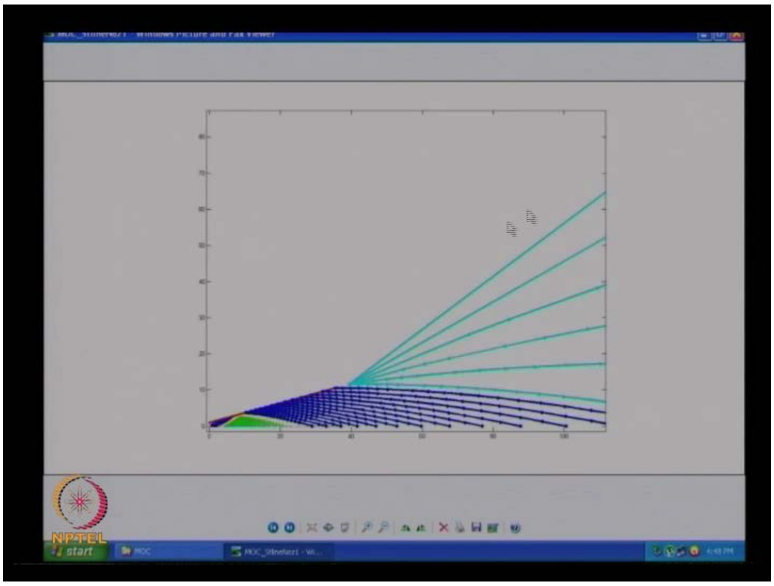

i chose this point as exist exit even though my nozzle has stopped somewhere here as just cheat calculations if you want more accurate you have to do much more number of points here and get it as close as possible to this value and live with that value I just did not bother and after that I gave a exit pressure boundary condition where the mark number should have been eight while this exit for this nozzle was supposed to be four and by the way since straight line nozzle at the exit at the top most point theta the is not going to be zero, but it will be whatever is the wall angle a it is in forced by the wall. So, it. So, happens that it is starting from fifteen degree angle there.

And if I go down straight it will become zero degree angle at the a symmetry line that is how the flow will be. In fact, you can still see that expanding region it is just bouncing up and down across and that is your initial expansion fan from the throat this is what will be the problem of not designing a nozzle very well if I do not design a nozzle very well we will start having either an expansion fan coming into your flow field or a shock coming into your flow field it will just be bouncing up and down up and down up and down and finally, it will reach your flow field region which is of interest for us if you design a nozzle really well then all the characteristics are going to be exactly parallel and there will be nothing expansion fan or compression fan thats coming in into your region that is the reason why we want to go for method of characteristics to design nozzles up to here we did not consider any cases where it will be shocks produced I could have had a o I did not solve for after this the next characteristic point I have to solve for will be from here send out pressure boundary point from there it will just continue pressure boundary this way and you can realize that the plume of this nozzle jet is coming out to be this big it is going to be something really big along this line it will be this big the flow is going to go all the way around up to there that is what you should be seen I did not show you that case probably I will try and show one picture of that next case next class.

Now, what if I chose a pressure outside the back pressure to be such that it is a over expanded nozzle condition then what will happen there will be a shock at the exit if that ever happens then you will be in trouble typically what happens is you will have characteristics crossing each other that is the thing I want to talk about right now we talked about little bit in terms of expansion.

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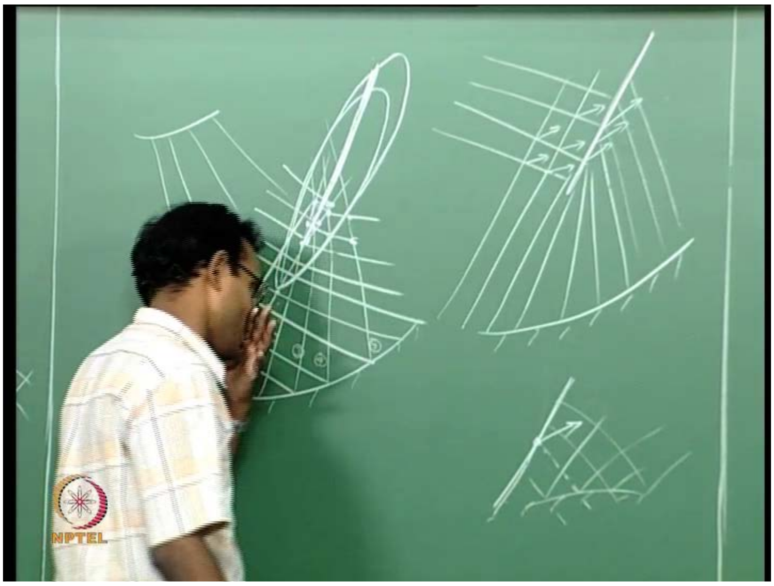

We said if there is a wall like this my characteristics will be going away from each other, but if I had a wall like this I said my characteristics will come together towards each other now if I do not bother about their crossing each other and I just keep calculating using the interior point codes what I will have is a funny situation where I will just draw that case here say I had a case like this if I keep solving this you will get to a situation where the first line is crossing the second line and then the second line is crossing a third line third line will cross the fourth line like that. So, if you think about it when I start solving I will solve in a logical order right we will just say I will go meet with the first point and then use that and this point to get the second and from this and this I will get this from this and this I will get this like that that is the we will go, but when things are switching around like this you will start having problems till this line there is no trouble the next line what will we do

if I do not worry about anything I will calculate this point first for line one I will label these lines for this particular characteristic I will solve with line one first and then interior point with line two will give me a point somewhere here will give me a point on the back side of the point and then from there everything else is nice. So, it will just go like this if I have another point first line is here second line is here third line here fourth line here fifth line here this is the way it is going to be solving if I think about one more I will draw one more and then I will stop first line it goes here second line will be here actually I will draw it much worse than what it is now my third line is more upstream. So, it will go here one two three and then now comes my fourth line which will be out there and then half of the line and it goes if you think about what is really happening here if I imagine this to be a m o c fabric I imagine this to be a fabric this threads inter leaving across each other now I think about a situation where I have this piece of paper I take this one corner of the imagine this to be a fabric I am going to take this and bend it it is not actually nice to bend this I will actually bring a cloth there is a cloth here. So, I have a cloth and I am going to imagine this a I will keep it here I will imagine this to be wrinkled like this.

if it is wrinkled like this, but half way through it is not wrinkled any more this side it is not wrinkled suddenly its wrinkled from here alone there is some triangular overlapping region if I if I have something like this and if you imagine the each of those threads are going in and coming out like this and going back like this that is exactly what is happening with your method of characteristics a here characteristic mesh if you think about these lines are just going these are just cutting through with this if such a situation ever happens on your characteristic mesh then you are having a situation where there must be a shock replacing the this whole set of regions if you think about this what is happening here this region has gone on top of this region this is gone on this region of the cloth has gone on top of the other region when that ever happens then I have to think about replacing this whole region here with a single shock what is our common logic from our gas dynamics when compression waves come together they form a shock right, but here we did not do anything about that we just let the compression waves cross each.

Other and go which is never the case when compression waves come together they cannot ever cross the second wave cannot cross the first wave they just both club together and become a stronger wave that is all it can do. So, all I have to do is just replace this whole region by one shock which will take care of the flow field before and after this whole situation I will not go into more details of this you have to just have to know that if such a thing happens there is a shock present in your nozzle I do not want to go into details of how will I put a line here what should be the angle what should be the strength at each point of the shock of course, you can do all that with method of characteristic if you going to details of it I will just not go into too much of that in detail I will just say I can put a shock here with varying angles which will decide my shock strength it is all oblique shocks each one will decide my shock strength I will just leave it there as of now that is a possibility in doing this.

Now, once I replace it with a shock what will really happen I will have another case here I am replacing this region with a shock then the characteristics this side you will not change, but my characteristics other side will lose a lot of points coz I am removing the whole bunch of points in my crumpled cloth kind of mesh and putting one line with it. So, whatever points that are coming this side are now supposed to go in a different pattern this is the original pattern that it will see and if you look at it velocity vector after this oblique shock you know the velocity vector here is the angle by sector at any point on the characteristic mesh hopefully you remember this characteristic lines are here two mark angles which means the angle bisector between this will be your velocity vector direction.

velocity vector direction for each of these cases is something like this angle by sector now when there is flow coming like this and there is a oblique shock like this flow will turn more up. So, it will be like this finally, final velocity vectors are like this now think about this being one set of characteristics what characteristic will it be it. So, happens that these are all left running characteristic when I look from this point what happens is when they go there they are still left running characteristic they just vanish because on that side of the shock is where it has to go there is no point if you think about one shock line and a point here the downstream location will have an angle like this and the mark number is lower. So, what happens is characteristics will be like this this characteristic never runs up stream of the shock this is just lost we will just have one set of characteristics going down from downstream of the shock this is all you get finally, we will just have to live with this of course, from inside you can see you have other characteristics going down there and maybe there is a wall here and it will interact and come back out from there.

May be if there is a wall here or an axis point or whatever depending on that it can bounce off the boundary and come back inside and create a mesh field there that is still a possibility, but that is a sep separate possibility I will probably show a picture of this next time I dont have a picture directly with me right now for this case I will show a picture of this and the picture of pressure boundary conditions probably a picture of this crossing of the mesh the cloth crossing kind of thing show all these three next class most likely, but if I am given a flow where a body kind of external conditions then how will I solve the problem.

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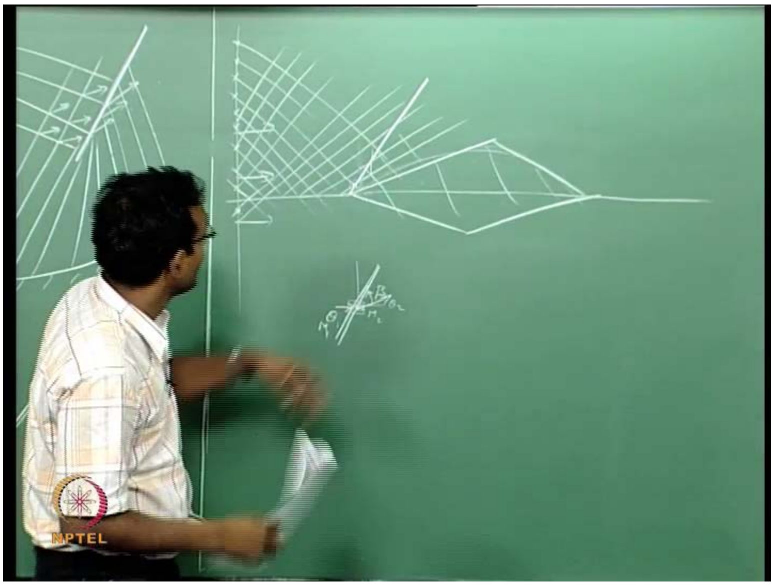

So, I have a situation where I have a some diamond shaped edge fair foil and I have some supersonic flow parallel flow in coming do I need to give you initial conditions for this problem initial line its already given when I said parallel flow uniform mark number. So, I can pick a set of points at some condition and say I know the location of the points because I picked the points and at all those points I am going to tell the mark number and theta right that is what we need for every characteristic point I need x y theta and mark number. So, I will give all those for all these points I just distribute set of points here now all I have to do this start running characteristics from there I just have to start running characteristics from there in positive and negative directions and of course, I know it is not useful.

So, I do not want to draw remaining portion let us say I will solve only the top half of my edge in which case it comes to the symmetry lane and goes up or equivalently I can solve the full flow field then this point could have come from here and it is running through from here that is what will be the case which is what I stimulate by saying symmetry line it bounces off of the symmetry lane ideally it is coming from the other side it is like a mirror line if I think about the next one its going to go like this it is equal to coming from here and going like this that is what is happening ideally that I should not be talking to you guys.

Now, it should have been given to you on some other fluid mechanics course now I will go to this point suddenly I have to give a wall condition there till now it was symmetry line suddenly it is a wall condition if I just go and start solving with wall point condition there you will start having characteristics crossing characteristics will just bounce back and go up stream instead since I know that this is a shock shock definitely exist in this problem I can do a better solving procedure where in I will say I will put a shock first I will put a shock at the beginning point only and then when this characteristic comes and meets here I will find out the point here the upstream of shock that is at the point what is the value as if the flow upstream of it is what I am calculating I will calculate that point and then I will look at that local region.

What is that local region looking like it is a shock with incoming theta is known in coming mark number is known at that location x y. So, once I known that theta and m and I know this angle local angle beta the oblique short angle once I know these three things I can calculate the downstream of the same shock at the same location I can calculate the downstream conditions. So, it will finally, be going with some other theta and some other mark number m two m two and theta two this is what I will get once I get this as long as m two is supersonic I can still use method of characteristics I can still say I will have characteristics running this way and I can start solving the problem that is what we will be doing except for there will be cases where the characteristic on this side will run up if it runs up this way you cannot solve it you will have this problem when this happens some characteristics will be lost only some characteristics will be useful only the right running characteristics.

Will be useful only the left running characteristics will become useless that also can happen in this case depending on whether the mark number is very high or not if we are very close to one you can manage having characteristics this way most likely you will never have it always shock will come in and block the left running characteristic if this left running shock that will cut the left running characteristic typically if that is the case and I still want to solve the problem and let us say this is not going uniform the edge is some other shape then there should be a feedback from here to that if I want to solve it how will I go through

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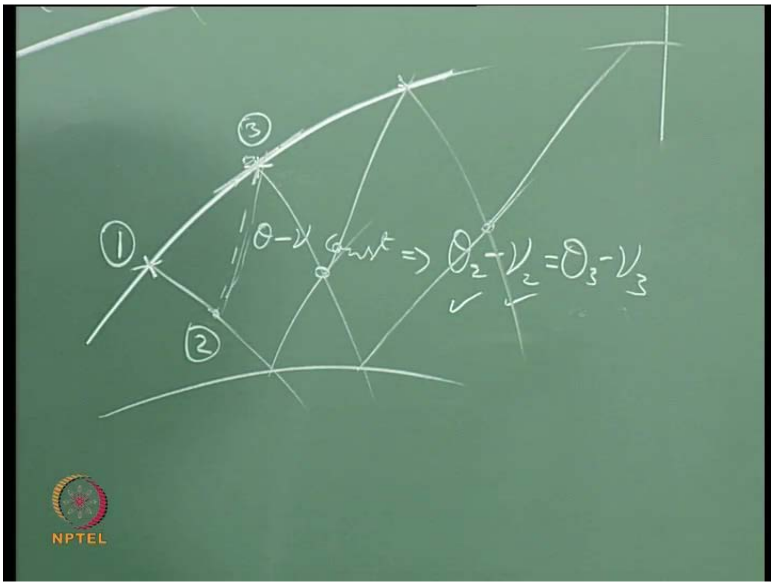

That say I am given some shock point I already know it could be my very first point I know some point already I want to find the next shock point that is when I say that I want to find the shock point I want to find the location of the shock point and the slope of the shock which is my beta of the shock beta is the shock angle want to find those two.

So, my problem now comes down to I am given one interior point and one shock point that is on the shock point now I have to find this point three from here that is what is expected. So, you have to think about how to solve this if I had a interior point and I am sending a positive characteristic it will go and meet there. So, this positive characteristic will have theta minus nu constant which implies that theta two minus nu two is equal to theta three minus nu three now I know everything at this, but I dont know anything at the side say I have uniform flow field coming I am simplifying the problem and I want to complicate the problem.

If I am having uniform flow field coming I know the upstream condition at any point on this curve it is exactly the same m and theta are known upstream uniform conditions. So, if I know the upstream conditions now a I will go back to this picture I am given m one and theta I dont know beta I do not know theta two m two because beta only when I give theta I can get theta two and m two, but I know that I want a relation between theta two and nu two in here it is two in here it is three do not get confused here downstream of the shock is two in this local picture that in here it is considered three I know that theta minus nu should be particular value downstream of the shock that is a given condition. So, now, I will tell you that there is only one beta for which it will satisfied that will only be one beta at which theta minus nu downstream of the shock will match whatever value we are giving theta two minus nu two equal to theta three minus nu three we do not know theta three minus nu three that is the downstream of the point three I do not know the values, but I can find out. So, I will iterate on this beta angle somewhere between zero and theta critical I will not zero mu and theta critical I will solve why the why am I choosing remember they are oblique shock curve.

I want to stay only in the supersonic section a of course, you know that it will be supersonic even a little bit more that side, but I will stay only between mu and theta critical and I will solve for this problem I will just keep on trying whatever beta from here to here till I get to some particular theta value which will match this if I do not get a solution in this I will say this is going to have a detached shock and things will get messy after that the flow is subsonic I cannot use my method of characteristics to solve this problem it will become that let us say we do not choose to solve such problems will choose some good problems which are solvable.

Then I would say that I can definitely find a beta for which I will reach a particular theta and a particular m such that m two such that I will solve that theta minus nu function to be exactly matching this characteristic line that is possible to solve once I solve I am sitting here now I can send a negative characteristic down here which will interact with whatever is on the wall here coming back up and I will get another point which will go up and find the next point like that I can keep points on line I can keep on solving my oblique shock on a some air foil or something I will draw a better picture than this once more.

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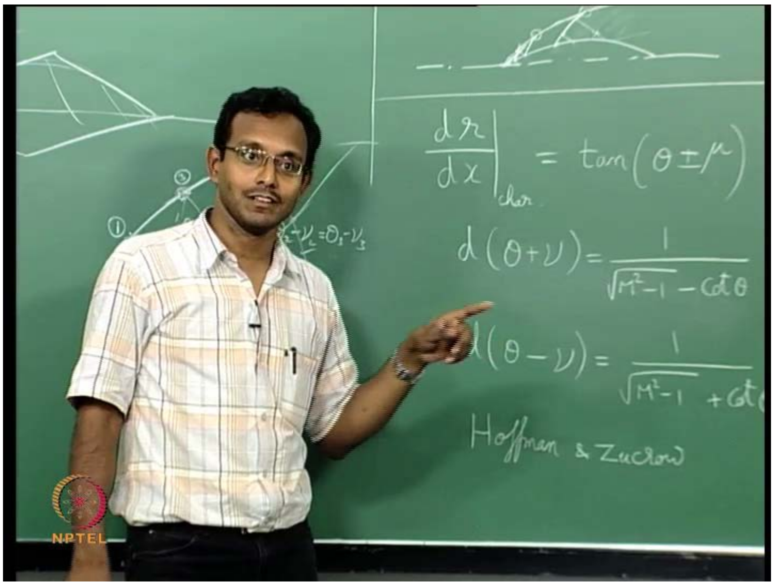

If I have a case like this what I will do typically is use the initial angle here and solve my initial oblique shock by simple oblique shock techniques whatever we did in simple gas dynamics initial angle I will give after that I will tell one point on the wall.

But this is the flow field there I will extend whatever is the downstream of here same value here after that from there I will send out a positive characteristic that will dictate the flow field from here like that I will get this point similarly I can start now solving this characteristic this way which will send out a characteristic and then I will now get the next point there like that I can start getting and eventually I will start a having full shock which will be curving based on the local wall angles here this local wall angle decides y theta minus nu on this line this characteristic line which will decide the theta minus nu at that local point on the shock similarly here theta minus nu will be theta will be lower values. So, theta minus nu will be different from here and. So, this will decide the shock point there this is also telling you some gas dynamics physics if you think about it the point that has a change here will affect only points downstream of that characteristic which is our original anyway thinking the zone of influence kind of thinking.

If I have a point here which is causing a change a change here is my wall decreased its angle if that is happening here when only downstream of this region this is only this region will actually feel the effect of this that is what is decided by this particular characteristic line why because that is the characteristic which is taking the information that there is a change had happened here all the previous points did not know that there was a change here once there is a change here now every point on characteristic knows that there was a change here if I send a positive characteristic this way negative characteristic this way that also will tell the information that there was a change here to all the points down stream here that is the reason this point knows that there is a change here how that point came and told this point that is the way you continue in all this directions this is what we call a shock point routine of course, I did not tell you how to find the location here once you know that beta angle remaining thing is similar to your jet boundary calculations we know this is the angle of the shock and I know the final angle of the shock is this and it has to be a smooth curve from here to here.

I will draw a line from one to three which is average of this beta and this beta tan of this beta and this beta average. So, that will give me one line from here and the other line is coming from average of theta plus mu from here and theta plus mu from here that will be one line from here these two intersection will give me that particular point that is the geometrical way of looking at where the shock will be present with all this you can definitely solve the this problem. So, I think I have talked almost about everything in two d method of characteristics I will just give you a little bit of axi-symmetric method of characteristics that is I will just give the equation and say it is solvable, but I will not solve it if you go and do the same kind of equations and finally, come to a point where d theta plus d nu equal to zero kind of form then I will write that particular final expression only for axi-symmetric m o c you are going to have instead of y I am using r coz now a day's radial versus axial now only two variables that matter now d r by d x.

For characteristic lines characteristic directions instead of d y by d x before now it is d r by d x d radius by d x is going to be equal to same thing as before theta plus minus nu this is not going to change this just deciding your mark one direction, but along each characteristic now I have to specifically say previously we did not have axi-symmetric terms now because of that one by r terms coming here you have extra terms if I go back to two d situation this one by r term will go to infinity sorry zero one by r goes to zero and because these these terms will get cancel and I will just get our original expression for negative characteristic that is negative characteristic is theta minus nu condition.

I will have d d of theta plus nu is zero which means theta plus nu is a constant which is what we had before now it is no more a constant it varies with r that s the special thing here it varies with r and you have to slowly calculate for each point from point to point you have to now take this as your equation and solve for your nu values of theta and nu no it is little more complicated problem to solve I will not solve I will just tell you that it is possible to solve this and if it is a positive characteristic when previously we used to have theta minus nu is a constant now I have theta minus nu equal to this whole expression what m will I use here some average m between these initial point and the final point that is typically what I will do here similar to the previous techniques again I will use some r in between these two values like that I can calculate this, but I will just avoid more details than this by the way if you go and use some serious books on method of characteristics like I will give you the best book I have ever found till now it is authored by hoffman and zukroat I believe the book has two volumes and a second volume deals with fully method of characteristic solutions first volume deals with ordinary gas dynamics they are really.

Thick books that if you ever get to see a book by these people is a really good book on lots of gas dynamics problems I have used this as a reference also for teaching this coarse a this gives you a different approach to method of characteristics it does not use theta and nu as variables along the characteristic line they use pressure u and v as the actual values which actually means that they are not solving this way they are not using the v based method the velocity potential based method they are solving to the actual equations mass momentum energy equations that will be a slightly different approach than this there you will need three lines to be solved left characteristic right characteristic and stream line it use three conditions to be solving at a every point the interior point axis point wall point all the routines will be slightly different than what we have used whatever I have given is really really simple situation where it is easy to solve and by the way this can be extended to three d also people have done three d m o c also I will avoid all that and I want to give three d m o c etcetera.

So as of now I have finished my discussion on method of characteristics except for I will show you some two pictures one will be with the meth characteristic mesh crossing and crossing this crumpling of cloth kind of situation and then a case with the pressure boundary conditions solved and a case with external flow solved that we will do next class and then we will go to some extra portions other than what is needed for the curriculum that will be from some unsteady flow discussions any questions see you people in next class.