Gas Dynamics
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Module - 03
Lecture - 05
Equations

Hello every one. Welcome back. We have as of now derived equations for conservation, and we already reviewed. There is no more of review. We already entered in gas dynamics. From now on I believe the fourth class we are on time. So, we are going to start with one-dimensional flow. In reality, no flow is really one-dimensional.

So, we have to think about what is one-dimensional flow and then, once we understand what we are talking about as one-dimensional flow, we will go into deriving again equations for one-dimensional flow. What we are going to do is, we will start with the original equations, we derive all this time, simplify that to go to one-dimensional and after that we will mostly use only one-dimensional equations, nothing else. So, what do you mean by one-dimensional flow and why is it useful thing to go for any ideas? Let us answer the first one.

What do you mean by one-dimensional flow? First flow parameters are function of only one direction if a flow velocity has a unique direction. Flow velocity has one and only one direction. Flow properties are function of only one direction and those are like very constraining terms, ok. Let us look at the other thing. When I will use one-dimensional flow approximation, then we will get back to this. When I will use a flow, when will I call a flow one-dimensional flow through a duct as in you are thinking as straight pipe or can the duct be curved like this, it can be curved also and we can still use one-dimensional flow areas should not change. They are same. I do not want to accept that fully also, but now we will go back to my way of looking at it.

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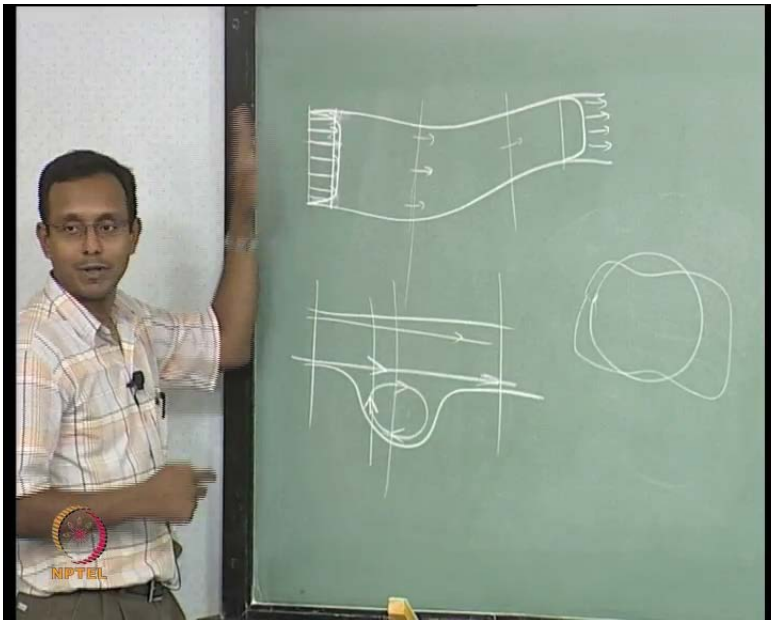

So, I am going to say there is some general flow through a duct. We will start with flow through a duct. Simple half some crazy shaped duct. I do not what shape it has really. Let us say something which is having area changing and there is flow inside this. Let us make it slightly nicer than this, something like this. Now, I am going to say my flow is entering this way and is coming out through this in reality.

What it is doing is going to have a boundary layer which means the velocity profile is not this. Top had profile or uniform profile as we have drawn. It is going to be more like this towards the end. The velocity will go to 0 on both sides, and this will be more like the actual velocity profile there and the same thing will happen here. Let us say I will draw it one section before. It is going to be some kind of velocity profile. Do not worry about the integral not being same. It should be the same, but I have not drawn it correctly that we would not worry about right now any ways.

So, if this is the flow and I am not really worried about the change in this direction, the perpendicular to the main axis of the duct direction locally. I am saying locally is very important word. There I am going to say at this point. This is my flow direction at this point. I am not worried as long as I am going to say when the flow is here. This way flow is everywhere. Here, it is only this way that is I am not having any huge recirculation close inside. If I pick a case more like this, if this is my dump, then most likely the flow will just go straight with the huge recirculation sitting here.

How do I know this? You should know some fluid mechanics. This is not fluid mechanic course I would not go in to, but if this is the case, now if I take this point flow is not always going in one direction, and if I take this point, this velocity is significantly high or it is at least of the order of this velocity here. So, those are things which I cannot neglect. This flow is not really one-dimension, ok.

When I look at this flow, it is not really very bad. If I look at any cross-section flow, it is roughly going perpendicular to that area of cross-section, nothing else and if I am willing to neglect the changes near the wall steps, near the ends, I am going to say it is not really changing. It is changing in say 5 percent of my area or something like that. I will neglect it or I am not interested in that change. Then, I can replace this velocity profile by a flat velocity profile with equivalent must flow at this.

I am removing that central velocity, a little bit adding it to the corner, so that my masses are same. The mass that is actually flowing here is imagined by me to be flowing in this corner, so that the velocity profile is uniform. That is the way I am going to look at this change. I have drawn and exaggerated. It will not be really this big. It will typically be very small. If it is just flow through a close to one-dimensional duct, I did not tell what the cross-section shape is going to look like. If I take this cross-section and look at the shape in front of me, it may be looking however crazy.

It feels like this may be the duct cross-section. I do not care as long as the flow is going only perpendicular to this duct. This area is nothing else, no other flow that is my assumption currently. Typically, if there is such a flow, nobody will want to use 1-D assumption. I will also tell you that typically they will use it only for something less crazy, more like this. They can use only assumption for this. This is a lot more complex. There will be other effect which I am neglecting here, that is for fluid mechanics people to teach you, but if a flow is going to have such things here, the velocity will be pretty high and I cannot neglect this velocity with respect to this one.

So, in this particular situation, I cannot use 1-D at least in this area, but if I do not care, if I just say I am interested in this section and interested in this section, I can probably use it depending on what my interests are. I have to think about using this as just simple. Example of 1-D. Now, I also told I can think about it as stream tube. Somebody said I do not know, but I like stream tube way of looking at things. Why? What is the stream tube? If I form duct with the whole bunch of stream lines together, how will I think of it easily.

Let us say the flow is currently through the board and I am going to draw close loop on this board. Now, there is whole bunch of stream lines going from every point into the board. If I go on the other side of the board and see all those lines coming, those stream lines together will form a tube and there is no mass that is going to cross this line, right. That is the way I have to think about. I will think about it in a better way. Flow is coming from inside the board towards this side. It is easier to think about.

So, I am going to say there is a whole bunch of stream lines coming out from here, coming out from here, where ever which ever direction they are going to take, but overall I will keep all of them connected at the same way. They cannot cross, right. So, this circle may change shape to something else. Yes it may become something else like this, somewhere else. When it comes to this section, it may have a different shape, but the stream lines would have never crossed each other. That is the basic rule in fluid mechanic.

It cannot. So, I do not need to worry about what happens. As a tube flow inside will remain inside, the mass that entered inside this circle will stay inside. Whatever new shape we are having at some other distance, but it will not go out. No other extra mass form outside has also come inside, no exchange across. That is why it can be imagined as a tube. It can be imagined as physical tubes sitting it the advantage of using this. I do not need to worry about walls and having boundary layers and other effects that does not exist here, ok.

So, I can imagine a flow fully through this whole room. No boundaries, no walls nearby. Still I can imagine a circle here and say all the stream lines going through this. All of them will form a tube. That tube can also be used and that can also be a stream tube. Now, I will imagine this as my stream tube. Now, I will go back and use this as a stream tube and if I can say that where ever I am interested in that particular cross-section, there is significantly only velocity perpendicular to this area and not along the plane of the area. Any other direction if that is not there or I am willing to neglect that particular flow, neglect should be like reasonable neglect. It cannot be like it is 100 meter per second. The main flow has 5 meter per second and I am neglecting 100 meter per second. That should not be the case. That is not really neglecting. It is ignoring. We do not want to ignore.

We just want to be able to neglect if there is such a flow which we can be neglected, then neglect the perpendicular components along the area plane component. I can neglect, then I will consider that kind of flow and that is to be valid everywhere in this whole duct or your stream tube. If it is valid everywhere in your stream tube, then I consider that kind of flow to be one-dimensional flow. Now, the next thing is there that any practical flow that will have such a thing, 1-D flow jet exhaust think about. Read this. No flow will be really that. The reason is I will take that particular example jet exhaust. I am going to say there is some nozzle.

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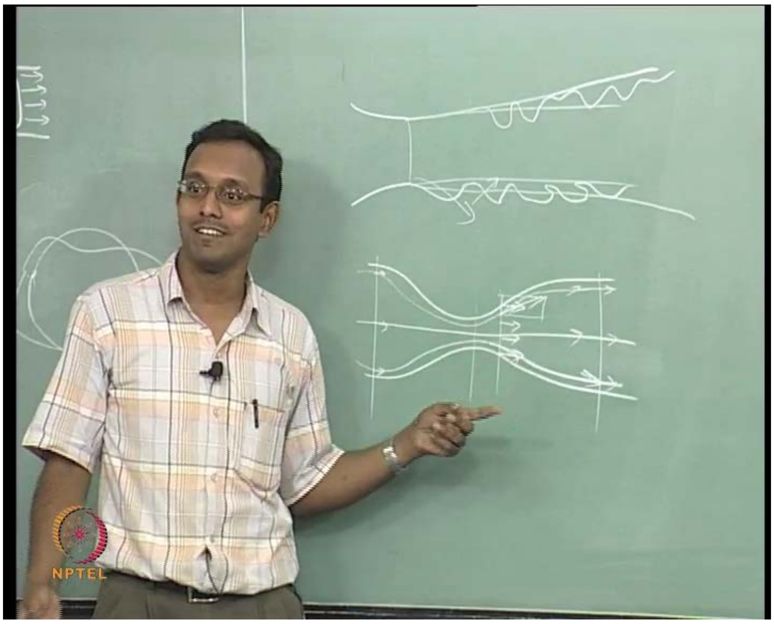

This the exit plane of the nozzle and there is jet coming out. It will not be just going straight. What will it do? It will start mixing with the outside and this region is going to have sheer layers adding forming all kinds of vertex rings formed whatever depending on the Reynolds number. So, the flow is going to be doing all kinds of things like this. You would have seen pretty pictures like this on the web or in fluid mechanics books. We do not want to go in to details of it. If I think about a particular stream line, this is not really stream line I have drawn. I have not drawn a stream line, really. It is probably not even a clean straight line, something else. If I draw stream lines, it will be looking sometimes going this way, sometimes going this way and all.

So, this flow is really unsteady, but can I neglect is the question. I am going to ask again. There is some deviation. I will consider this as a stream tube. Now, the straight thing if the flow is just going straight like water from a tap water, from tap forms that nice stream tube by the way flow inside. That is all inside that stream tube found by edge between water and air nice example. So, I am thinking about that kind of stream tube, but the edges are shaking, oscillating. You would have this again in water if you increase the flow rate a little more, ok.

If I am willing to neglect those changes, I will say in real life that it is not exactly forming that straight thing. It is oscillating in time, but it does not affect my understanding or it does not affect my particular study. It does not affect what I am interested in and then, I can still say I will just draw something along this line. This is what typically all the sheer layer people will do. They will just draw some average line that way and this way, and say my jet grows at this rate and then, they will just talk about growth rate of a jet and just worry about such things. We can imagine that kind of thing as an average line.

Now, we will just think about time average, its quantities. I am not interested in instantaneous values. If it is instantaneous, may be my stream tube would have been curved a crazy something like this. We are willing to neglect. That becomes here close to 1-D, not really 1-D, but we will assume it to be 1-D. So, we will call that the quasi 1-D problem. Actually, it is quasi static 1-D problems, quasi stationary 1-D problem whatever. Now, there are other simpler examples which are more common to gas dynamics flow through a nozzle. I have a flow that is going like this, coming back out like this. Of course, I have there is a wall that is going like this and inside that there is flow. Now, I am going to assume that the flow tracks the wall.

By the way if there is separation, the flow does not track the wall, then there will be recirculation zones. That flow cannot be really considered as 1-D. Why? It will have the same problem as this recirculation zone. It will have the same problem. This velocity will be comparable to this. I cannot neglect this. We will go back here. Let us say we would not have separation. We will have nice flow and the center line; stream line will just go straight. If I have flow like this at any cross-section, at this cross-section, it is really nice. All the stream lines are perpendicular to the area.

My assumption is perfectly valid. No problems. This again is valid. Everywhere it flow perpendicular to that cross-section area at the center. So, it is valid. It is it really valid here. The flow is having some other component that way and other component this way. Let us have these components. Am I willing to neglect, that is the question. I am going to ask. I am going to say the actual velocity this way is very small compared to the velocity component. This way I am going to split this actual velocity vectors. It will be more like this. I am saying this velocity is very high compared to this velocity. I am going to neglect this component and I will assume that my flow is going straight parallel to this. Any real flow does this.

Now, it will always follow the wall, but we are assuming that it is going straight and still follow the wall. Somehow we would not talk about it. It is still following the wall. We would not draw this kind of stream lines in 1-D flow. Why it is 1-D? There is no stream line concept in 1-D. In 1-D, all stream lines are parallel always, ok. Only when there is 2-D, I can think about a slope. Really 1-D there is no slope, right. Hope fully you are comfortable with that d y by d x gives you a slope y and x. It should be there. There is only x. There is no slope really.

I can just talk about velocity gradients along this. That does not give you a slope and so, run get a stream line. Stream line needs slope of velocity vector to draw. This r slope of the stream line gives you the velocity vector direction. Hope fully you know this. So, we cannot define a stream line in 1-D flow, and we are going to assume such things here. We are going to say this is clean, very easy to assume here at this cross junction throughout when I go out to this diverging and converging area cross-sections, there things may not be very nice, but will assume it is close to 1-D. Again, we have gone to call this as quasi 1-D flow, ok.

The previous one we are assuming that it is not very steady, but we will assume it to be close to steady, that is I will call it quasi stationery. Now, we will go to the next thing and I am going to say it is quasi 1-D flow. We are approximately saying it as 1-D flow, not really 1-D. I am neglecting the perpendicular to stream line direction components. Now, when the velocities are very high, there is one more thing we need to worry about in real flow. What? In real flow, velocities are very high, something turbulent. We are going to neglect that also in here. We would not worry about turbulence.

We will just say turbulence may have its effect, but that is not as high as this main component of velocities. There will be some fluctuation of my velocity, up and down in time. We will say that fluctuation magnitude is very small compared to my main velocity magnitude along my 1-D direction. So, we will consider only that and we will ignore those small things. So, it looks as if it is steady, ok.

It is actually quasi 1-D problems. So, it looks like it is very nice and steady, but we would not pick up any unsteady problem as such right now. We will go to unsteady problem towards the end of this course. Last two lectures are something we will do unsteady where I am talking about suddenly I opened a valve through a nozzle. What happens is we can even look at some pictures from one of my student's research. We can look at that when the time comes out, these are all possibilities, but we will start deriving equations for steady one-dimensional flow.

For now, we will stick to steady problems. Of course, we have already neglected the fluctuations and velocity perpendicular to those main line. What is the main line itself is varying a time. We are going to neglect that right now. So, we are going to simplify the expressions and then, start deriving mass equation, momentum equation and energy equation.

Now, I want to go revisit control volume. What is a control volume? We already did this before. Deriving all the equations is some imaginary volume. Some surface we are going to use inside the flow of feel which is going to mark the region of interest for us, and we are going to study what is happening inside that volume. That is what supposed to be and they already told that it is an art in picking what control volume we want for a given problem. It is an art and if you pick the correct control volume or equations, it may get simplified. If not, it may become very complex. We will see that in momentum equation today. So, we will do two kinds of control volume in all the process.

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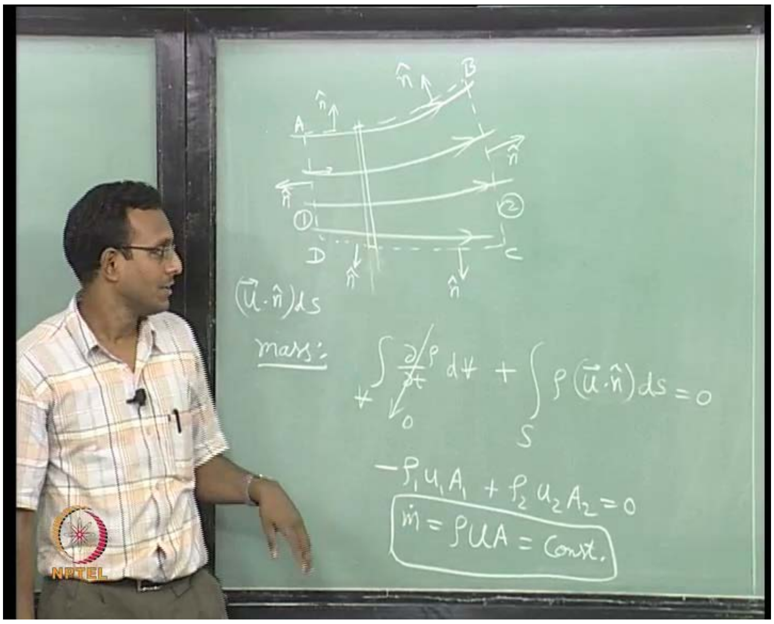

Let us say my flow is like this. Some search flow I can put a control volume something like this. It is roughly a stream tube forming my control volume that is a possibility or I could take some perpendicular plane to all the stream line, and very close to another perpendicular plane, and I will connect them here. This could be my control volume and also, very thin slice of control volume. I can choose either the big control volume or the thin control volume.

They will give different kinds of equations for conservation. If I do this big thing, I am going to talk about what is happening inside this whole volume. When I talk about this very thin slice, I am going to talk about what change that is happening at this point. One is going to talk about integral form; other will talk about differential form. That is what it does. If you already know something about it, look at it.

Now, there are two forms that are possible. So, we want to start with mass equation. We want to derive mass momentum energy equation today itself. We will finish all that are here today. So, I want to go look at little more detail. I will pick the big control volume, right. Now, I will keep the other one, but I want to use it, right. Now, let us mark my control volume edges. The big control volume, my control volume is a b c d. In this case, it is a volume. I am going to assume, it is going one meter inside whatever even it distance inside. So, a normal is this way.

Here a normal is here. The normal could be some other place. So, something like this. Now, when I think about fluxes, we always have this term u dot n d s. This is related to flux. So, when I think about flux here, this is actually mass flux related. So, it is not really a mass flux. It is actually volume flux, right. Now, we want to worry about that will get back to this actual mass equation has. There is a momentum equation. It has this energy equation. It has its every equation has. So, we want to start here. We want to see what this quantity looks like first before we go to equations at this point flow velocity is this way n vector is this way.

What will that give me? It is 1-D flow. So, actually flow will be aligned along this normal, right. By definition of 1-D flow, we said perpendicular to the cross section flow goes. So, perpendicular that the cross section is where the normal direction will be. So, you are going to get to that point. What will this value be? U dot n value. It will be minus u magnitude. Minus of u magnitude will be what it will come out to on that side. Only positive of u magnitude at this point, it may be different from here to here. It may be different velocities.

What about the top? Let us pick this point. What it will be here? It should be zero. Why I choose my control volumes, such that my control volume edge goes along a stream tube. I pick my control volume such that the stream line is parallel to the edge which means this normal to the edge will be perpendicular to my stream line which means my u dot n will be cos 90 and that will come out to be zero. So, this u dot n will be zero on this particular control volume. I was smart enough to choose such a control volume that will be the same answer, here same answer, here same answer. I always choose along stream lines as a good thing about choosing stream tubes as control volumes, ok.

So, we will keep such control volume. Now, we will look at mass equation. I will start with already derived equation. We had this particular form. We had so many forms. I am going to use one of them. We had such a form before we made it in to volume integral. We had this form. This is the integral equation and then, we said it is a steady flow. We cannot neglect this.

There is no gradient and time. We said we are going to look at only steady flow and then, we going to look at this. I have to integrate this over all the surfaces and it is a 1-D problem. By all the other surfaces, all the side surfaces are all going to give me u dot n as zero including this side surface, and the back of the board surface, they are all going to be zero. Only this perpendicular cross-section thing will remain, only those terms will remain. Now, I will label these sections as 1 and 2. Now, I can write this integral value exactly because it is going to be one value for each section. U dot n will be one value for each section, ok.

So, I can write that value to be minus rho 1 u 1 a 1. How did I get this a 1 as the cross-section area add-section one? How did I get that this is a constant value at that section density? Why is it constant 1-D flow? This whole region, there is no variation here. That is my 1-D flow assumption anywhere. So, I am taking it out of the integral. Sorry, taking it out of the integral. Only thing is integral s d s.

Thus, my total area of my cross-section will be a 1. Why it is I get this minus u is this way n is this way cos of 180 will be minus 1. I get to this form or equivalently I can say take one of them to the other side, and say rho u a is constant from section to section. I can say rho u a will be constant from one section 1 to section 2. Rho u a cannot change. What is this rho u a? What will it come out to be? Mass flow rate.

How did you get that area times length per time? Area times length per time is volume per time into mass by volume. It will be mass per time. You can actually prove that it is exactly mass flow rate. This is just dimensionally I am showing it to be mass flow rate. Whatever I told can be thought of as from here to here if I go with same cross-section, it will sweep that much volume with that density, and that will give me how much mass went pass that area. I will keep that as an exercise.

You guess and solve by yourself. It can be shown to be mass flow rate exactly. So, I will write this as m dot equal to rho u a equal to constant. Very important relation for gas units will keep this, your mass equation if I think about using a big control volume along stream tubes. This is one expression. Now, we will go and use this small control volume. I will draw this control again.

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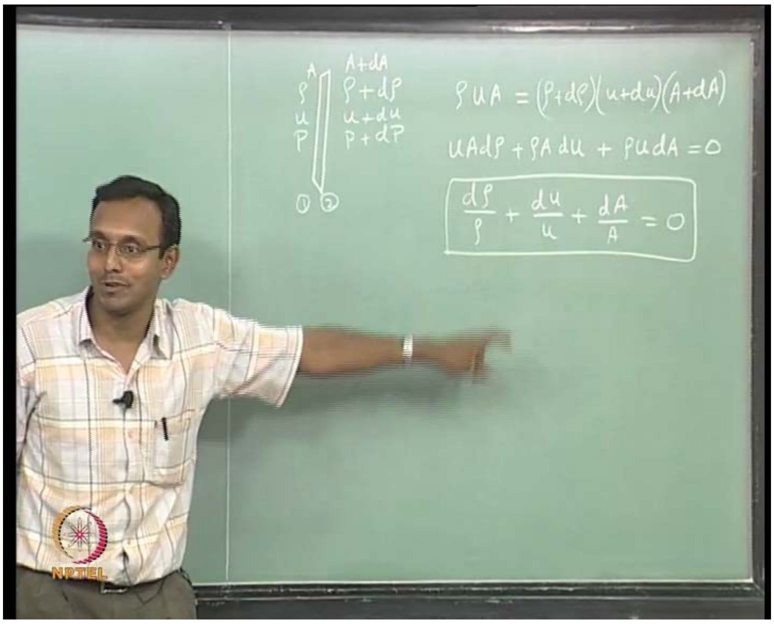

I am going to say control volume changes area differentially a small thin slice may have a new area. After that a plus d a d a can be positive or negative. If it is negative, it will be shrinking. Currently I have drawn it as a width expanding does not matter. Now, I am going to say it has some particular density. Density would change by d rho. Any of these d quantities could be positive or negative.

We do not need all of them. I can still write temperature goes to plus d t. We do not need all that actually. We need only a rho and u currently. Now, all I have to go start with this point. I know this is true. 1-D flow I start from here and apply this to this particular control volume as the easy way of going on section 1 for me. Section 2, the other side of my control volume what I may be going to get rho u a equal to rho plus d rho multiplied by u plus d u multiplied by a plus d a. I will get this form. Now, if I multiply it out, it will rho u a here that will cancel with this rho u a. I will also get a lot of differential terms multiplied by each other. D rho d u d a is one term, d rho d u a u is another term. Like that there are so many terms which have multiple differential terms multiplied by each other.

Now, we will neglect those by saying d u d rho is very small compared to rho u. So, I will neglect that term. I have to always say when it very small, it is compared to something else. So, we think about that. Now, I will keep only terms that have one differential term that will be a times u times zero. That is one term possible. Like that I can have rho times u times d a. Like that I can have another term rho times a times d u. By the way I have only three systems. I just named all the three.

So, we will write those three terms and I am going to say it is equal to zero. Really I have neglected some terms. It is roughly equal to zero. If I pick my control volume to be very thin slice, then I more justify will keep it. That way it is nothing wrong. When I make this slice very small, this b rho will be derivative times, this thin slice d rho by d x times delta x that will become your thin slice, very thin slice. That is the limit of calculus mathematicians are with it.

So, I do not think there is any problem. We can say it is close to 0. We will keep it as 0. So, this can be simplified. I divide this whole expression by rho u a, but always keep in mind that when I divide by something that quantity cannot be 0 ever. I am going to say there is always flow in my control volume. Reasonable things we are talking about flow. So, I can never have zero flow. So, reasonable thing I can divide this whole thing by rho u a, this whole expression, every term.

When I do that, I will get to differential form, another very useful expression in gas dynamics. Of course, you do not need t do this thin slice control volume and all that. I can always go back and use this expression and say, I will take derivative of this. I will just take derivative of this with respect to anything does not matter. I will just take d of m dot and that will give you that form directly d of m dot will be equal to 0. I do not need this thin control volume explanation, but it helps in momentum equations. So, we will keep it this way and then, I can come from there to here by just dividing by rho a. That is also a possibility. Calculus can also help you that.

Now, I have a simple observation here. Now, let us look at this expression. This is going to give you some physical intuition. I will keep on adding to these physical intuitions looking at the same expression again and again. Look at this. Now, for in compressible flow, I can say that d rho is 0, then I have this d u plus d a, or d u by u plus d a by a 0. That means, if I increase area, velocity decreases. If I decrease area, velocity increases where I have a seen such flows, a real life water flow through a host end. You pinch the tube water will go faster to a longer distance, right. Simple common example everybody would have at least poured some water through a host, somewhere in your life garden host typical example.

If you pinch the host, it goes faster, right. So, that will work very well. If I say d rho as 0 will go later and I will show you that there is a connection between d u and d rho which will make it go the wrong direction, f m m is for greater than one mark number is supersonic. We will look at that probably in the next class. I do not think I reach there today itself, but will derive momentum equation. As of now we are seeing that simple connection. What you have observed an incompressible flow is valid, but I can just observe that is not as simple a relation anymore because there is d rho in a compressible flow. We will look at more of this as time goes. We will go back and start from that corner again momentum equation, ok.

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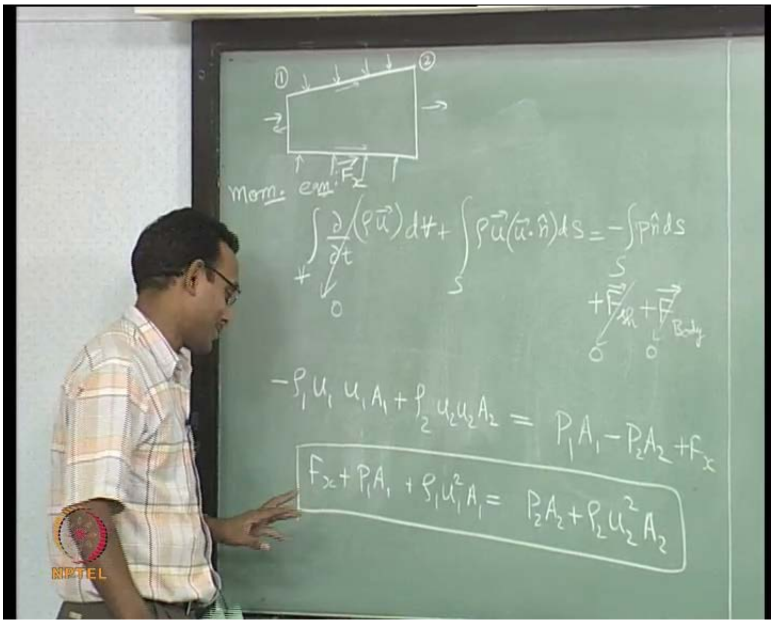

Now, I will start with control volume that nicer to look at I have this control volume. Now, I think about momentum exchange across this control volume and we already have this expression. I will just start writing momentum equation. I will write the full expression as we derived before and then, we will start simplifying things. Now, I will put here plus sheer force plus body force and immediately make a zero. That is what I did before also. I will do the same thing here. Make them zero. Now, I have a main expression, these three terms.

Now, I am going to say I am having only steady flow derivative with respect to time is 0 and make that 0. So, now I have only remaining terms here. If I look at this term, we already said that u dot n is 0 on sides because they are stream tubes and in this surface, it will be minus u magnitude, here it will be plus u magnitude. I am going to have flow in these ways and flow out this way, and I am going to say flows parallel to those surfaces, there that is the idea. So, I can write this expression without the integral anymore because integral will just become integral of d s which will give me area a 1 and a 2. It will give respectively. So, that expression will look like I am writing it separately rho and u 1 is this multiplied by u 1 with minus sign. That is here and then, a 1 is just integral of s d s there.

This term p n vector d s we looked at here this way, but there is no dot product. I just have to keep at all. That is all, but n is going the negative direction. I am going to think about momentum going that direction. So, it is going to give me a negative force. That is all I need to think about. So, that negative will cancel of this negative. It will just say as positive for only this surface because this surface as n on the negative x direction. Let say my 1-D direction and integral s d s will just be this, and this other direction n as pointing along my one-dimensional direction.

So, that will give me just positive single one value. So, that will be a minus p 2. A 2 is this. All that is very typically will make mistake because in here we are neglected u dot n in these surfaces, but pressure can act across a stream line. There is pressure here, there is pressure here and the way I have drawn my stream lines, if this acting pressure is always perpendicular to the surface and n vector is perpendicular to the surface, they are going to give you good number. It will never be 0, right.

So, pressure will always exist. Now, the way I have drawn it, it looks as if it producing a force in positive x direction. So, let us assume there is some force. I do not value why I need to know the exact pressure value here. This is the problem with choosing big control volumes. Let us say we would not worry about that part, right. Now, I will just say the net effect of this whole side surface area everywhere will be giving me a force vector effects along this direction. Of course, my vector does not have meaning. It is just one direction vector, it is just one component vector and it is just a number for me. So, now, I have that force going this way, I have to keep that force also and that will be plus f x. By the way every one of these terms is a one-dimensional vector, ok.

Think about it. I am just looking at x component of the whole force created by everything. Momentum entering this way will give you opposite force like that. Every term I am having this whole thing. Now, I if I just rearrange this term, it looks like this. This is my integrated form of momentum equation in 1-D flow. Of course, remember this effects, this is what is giving thrust in your nozzles. Actually it is not really giving a thrust. It is going to be negative thrust, but this is important term in that. If I use this to find your thrust in your nozzle from throat to the exits, c d nozzle will get negative force, but we would not worry about that. That is not gas dynamics that propositional you need to worry about it, but this is your momentum equation. So, if I say I have a special case where I am going to say a 1 equal to a 2 is a very special case.

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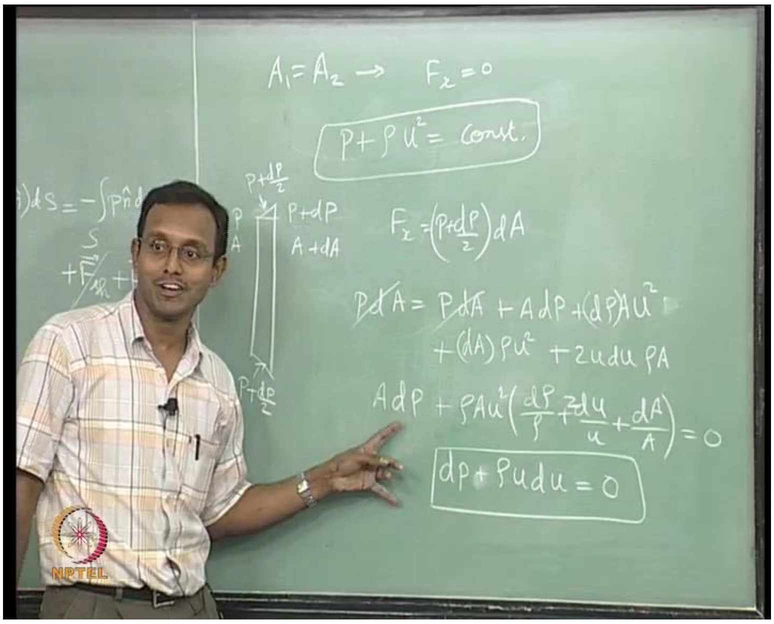

Now, I am not even just 1-D flow, but constant area 1-D, I still can have area cross section shape varying, but the area cannot vary. A square can become a circular as it goes keeping the same constant area. That is still for me a 1 equal to a 2. If I assume that in that equation, then this will immediately tell me that my effects will be 0. Why I will go back here look at the picture this area same this? I am going to say this force is going to be not in this component, but perpendicular to it. That is not my 1-D direction. Anybody wants in stop me there. That statement is true. If I have a single constant area cross section and in that if square becomes circle, things may change a little bit, but if you think about square becoming circle, they will cancel each other. It will add force. Other place it remove force. There is no net force. You look see over all. This is correct.

I am going to say perpendicular force is no more adding to my x direction. My only one direction existing is called x direction.

Let us say that x direction force is not going to be added because it is perpendicular; it is no more in the x direction. So, I can say that is 0. Now, many term becomes very simple. Remember this term also has a momentum equation integral form of momentum equation in 1-D flow. Constant area of flow is a very special thing. You cannot use it for any other case other than constant area of flow, ok.

Remember, this looks very close to your Bernoulli's equation that is where things go around. Let us say, let us go to differential form and then, we will think about Bernoulli. One thing I have time for Bernoulli right now. If I look at that small control volume, I will draw it again and of course, we are becoming a plus b a and every other term like that there will rho becoming rho plus read d rho, everything is there. You would be becoming u plus d u.

Now, I am interested in finding this force here. What will that force be? It will be having an average pressure between this and this. What is the average pressure between this and this? It is p plus d p by 2. So, I will put that specifically here p plus d p by 2 very important same force here, the same pressure here. Sorry, what is the area on which this p plus d p by 2 acting? It will be just the d a.

Why I am thinking about only this component of it, only the parallel component? Say I am imaging you look at a cone a first term of a cone say a conical bucket. I look at it from the bottom small area becoming big area. I look at it and I push along this cone. Only component that matters is along the axis, let say then what matters only that annular region which I see. Some bottom of bucket only that matters.

What is that area going to be? It will be the big circle area minus the small circle area which will be area here minus area here. That will be your d a. That is why I suppose to be. So, my effects has the value p plus d p by 2 times d a. Now, I will go back and substitute that here every other value you know a 1 p and a 1 will just become p a rho u square a. This will have p plus d p a plus d a. All that I want to write that full expression because we have less time. I will just go one term short. I will just simplify it one level and I will write it as I will write below of here d rho.

I have written d a, where this p d a, comes from this f x and I have already cancelled the p a term coming from here. With the p a term coming here, I am neglected all the multiplication of d a times d rho, all those quantities. That is what I have done. So, if I simplify this further, of course this p d a gets cancel. That is what I wanted to show from here. If I simplify this further, I will get to a point which looks like there is a 2 which is very crucial. I miss the 2, there will be a 2 n d u by u. Now, I go back to that corner. Here mass equation, we know that this whole thing with coefficient 1 is 0.

So, I will substitute the mass equation in momentum equation as we did before. Then, I will get to the final form which will give me, I will just write one form smaller. I have cancelled the areas. In fact, I will write both the terms together to one side. I will get to this form, ok and this happens to be your momentum equation in differential form. In 1-D flow, I do not have any assumption about constant area are anything here, while this other term, this one has constant areas assumption. When I differentiate this, it does not give these. We notice that already. There is some difference between this. We will look that next time.