Gas Dynamics
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Module - 2
Lecture - 4
Momentum and Energy Equations

Hello everyone, welcome back to our class, we stopped at just writing the outline of conservation of Momentum, the Equation for conservation of momentum last time. So, we will start from there hopefully we will derive conservation of momentum and conservation of energy before end of today's class and probably go a little more stay on that.

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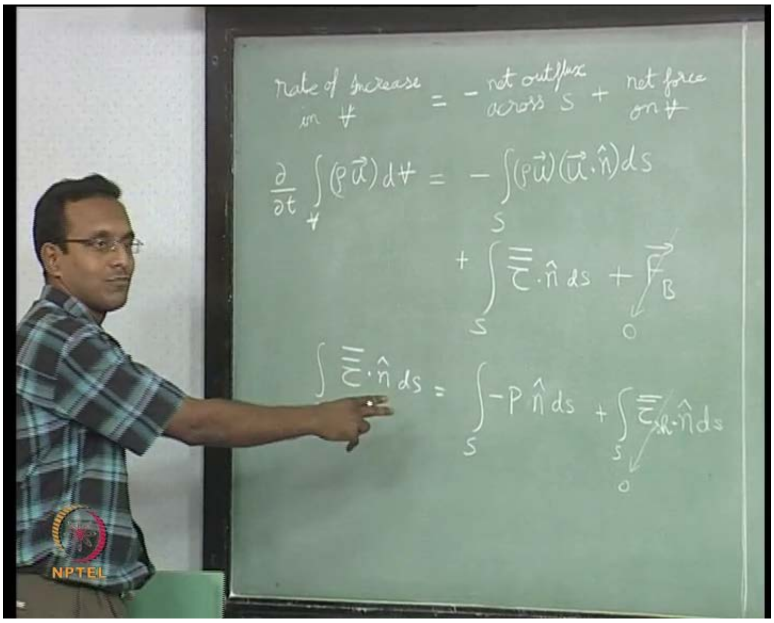

So, when we said conservation of momentum, we said time rate of momentum, rate of increase in volume V is equal to minus of net out flux across the surface S plus the net force acting on volume V. If I say that there is no out flux, then I will say that the net force is going to be directly increasing the net momentum at some particular rate that is your Newton's second law, which is directly, gets you that. This is the extra term, which is the last term from the volume, we keep it this way from in a simple expression, we will write for rate of change plus all the remaining forces.

Now, we could have so many forces possible, so we will just write it as the stress tensor is the full matrix of a 3 by 3 form of it, and that is going to include along the diagonal, it will be the normal force components and the half diagonal terms will be your sheer stress terms. Right you would know tau x y the most common one used in Newton's law of this like that. So, this is the full equation, I am going to say that this f b is here body force term, that is if there is any say magnetic field across and that is going to cause some bulk force or gravitational force. This is all force per unit volume the way, I have written it, no it is actually absolute force, it is no per unit volume, it is just absolute force no per volume as if no.

Now, we will say we will start simplifying this and I will say this is going to be equal to 0 in most of my problems in gas dynamics, we will mostly say there is no gravitational field or any other body forces electric or magnetic field. We do not have any of that and this, we will simplify a little further. Now, I will write this whole term by splitting this into diagonal term and the half diagonal term, I will keep the that only the diagonal term part.

I am writing it as tau sheer it is means I am having all the diagonal components of this 0 and then I am taking, I have to take a dot product with this, dot product with the perpendicular vector from there surface vector. And this is the force is acting along the n direction normal force, if there is a surface and the pressure is acting from outside on to it, the normal is going to be out of the box and pressure is going to be inward.

So, that is giving you this minus sign in there and then in sheer force, it will automatically take care of this sign based on this dot, It will take care of it right side. Now, we substitute these 2 inside here and we are going to say in our simple gas dynamics world, we will neglect this. We will say there is no sheer force as in we are going to consider inviscid flow problem as of now. Of course, we will come back and say friction can be taken into account at a later stage as of now, we will say there is no friction there is no sheer forces inside my flow. So, we will neglect it for now, only normal force component will be there. So, now we just have to substitute all those inside.

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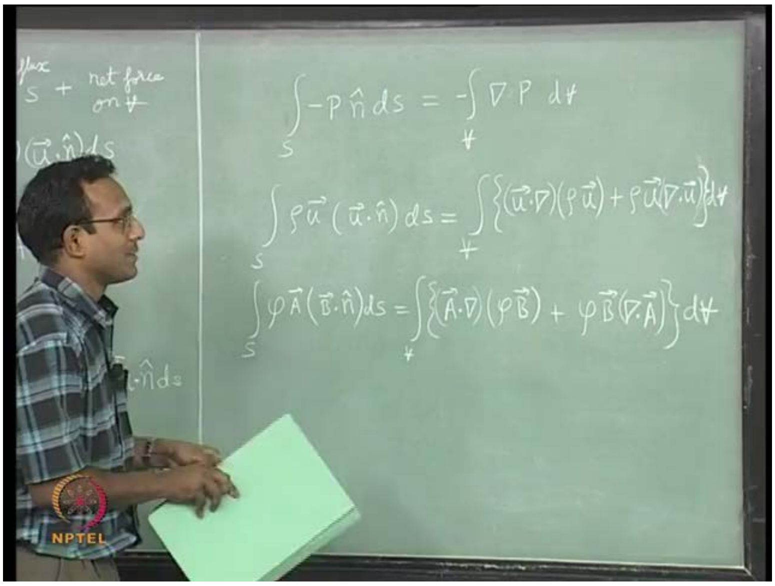

Of course, I can just take this, I want to convert all of them to volume term, volume integral term. So, this is going to be del P not, del dot P that is just gradient of scalar pressure is a scalar field. So, the gradient of that is a vector and that is your force direction, we just got to this form that is just 1 term is still have other terms. We want to look at the next term, the advection, term this term when, we want to convert to the volume integrals, it is not very easy. So, we will write a vector identity below this and go by parallel that is easier for us.

So, this is a vector identity, we just take it from mathematicians, we will just use it, if you look at it really. It is just differentiation by parts that is all we are doing inside this is nothing rate differentiation by chain rule, I guess that is what it is call, we just doing that inside here, it just look very complex. Of course, we still do not know what to do with this term, we will go back to that and write another identity for that. If we using tensor notation as in the Einstein notation, index notation then this life will become a little easier, I will just do not want to teach you this in this course.

So, we will just live with vector notation and just get over with it, we will just use it only once in this whole course and that is this point, we want to worry about it too much. So, now, I have to just see the parallel between this and this my B vector is a velocity vector, A vector is another velocity vector. This phi is a scalar and that is my density in this case, now you just have to go and find the parallel and write the equation that will become. So, I use this to get to this form.

So, now, we can simplify it a little bit just per unit to change this to flower bracket, now it is consistent, now this looks simple enough to work with, I still want to think about this term, it is not very easy to work with. So, we will go and simplify that a little more again, we will go and ask mathematicians, they will tell me another identity for it. So, we will go to this point and start writing from the beginning.

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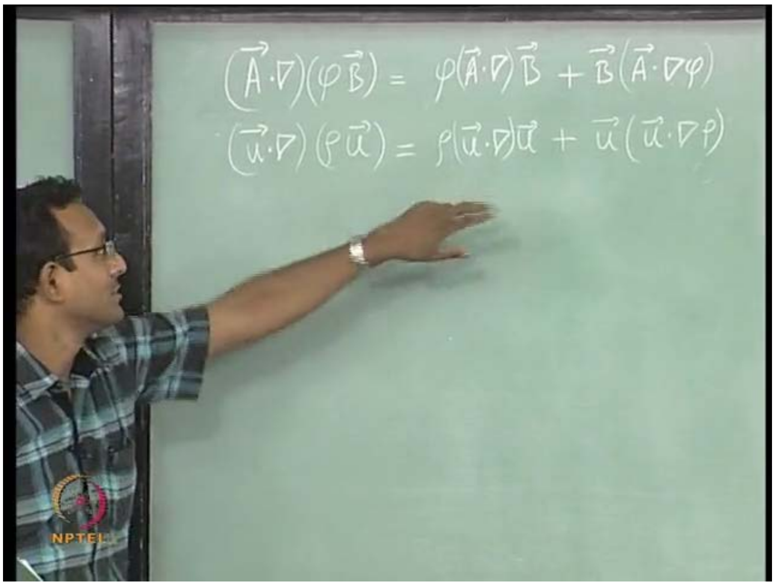

I will pick the identity write now, this is an identity of just the operator operating on a scalar multiplied by a vector, that is what this is it is A dot del as an operator differentiation operator acting on the product of a scalar and a vector that comes out to be this. So, now, we will just have to look at it in terms of our rho’s and u’s, we had an expression that was this, now this can be written as this form remember that here. This is not integral over surface to integral surface volume, that is not what we did here, this is just expressing this in terms of this whole expression.

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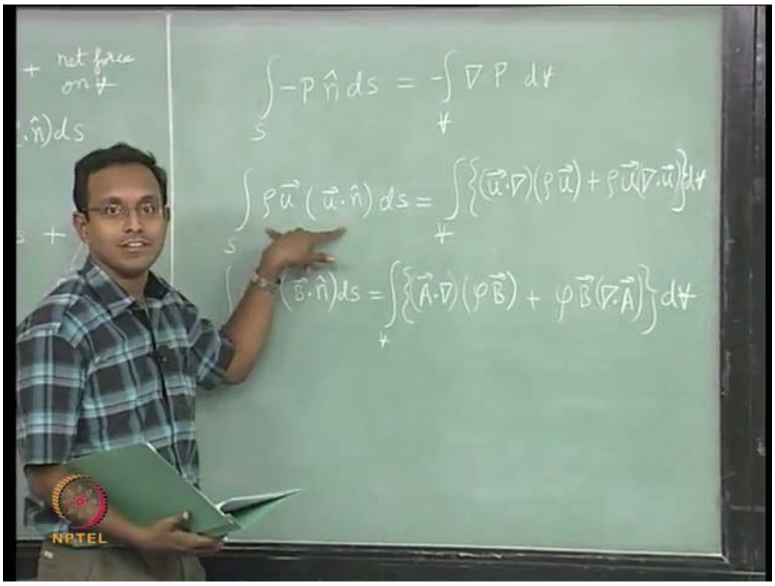

If you go back and look at where, it was sitting here inside this volume integral, that is still here, inside the volume integral. So, now, I have to take that expression substitute inside here along with this expression that will be 3 times inside this, all the 3 terms are going to be representing this 1 surface integral, now I will go back 1 more level.

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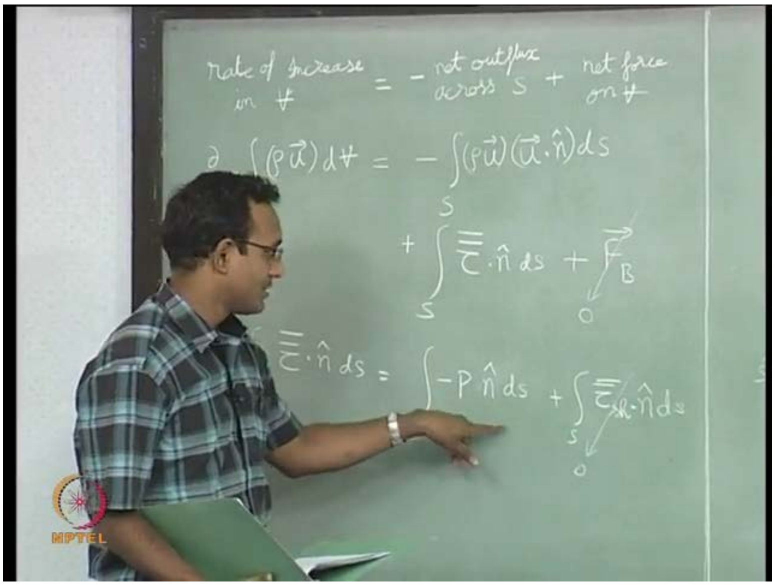

And I will say in my original equation I have this surface integral is going to be replaced by those 3 terms, which we just talked about plus this term, which became these two terms of which, this term we wrote as one simple expression in here.

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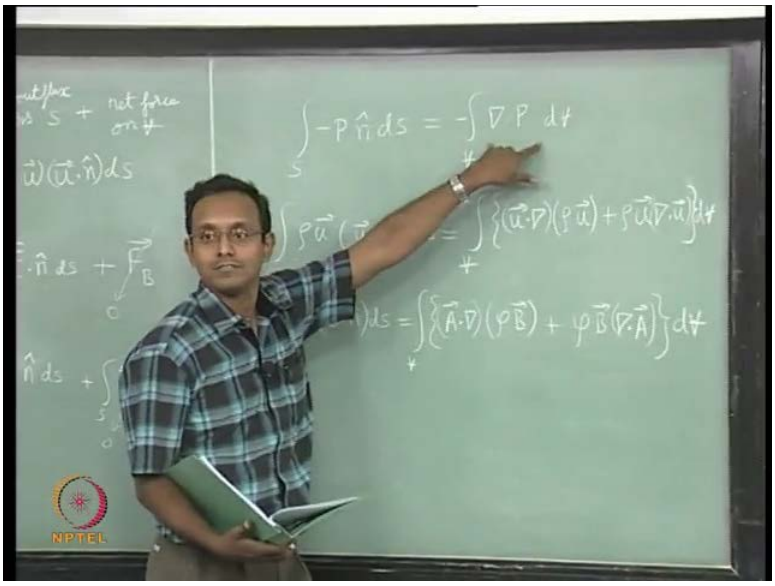

So, there is whole set of equations, we will put all of them together and write it once, the full equation in volume integral form.

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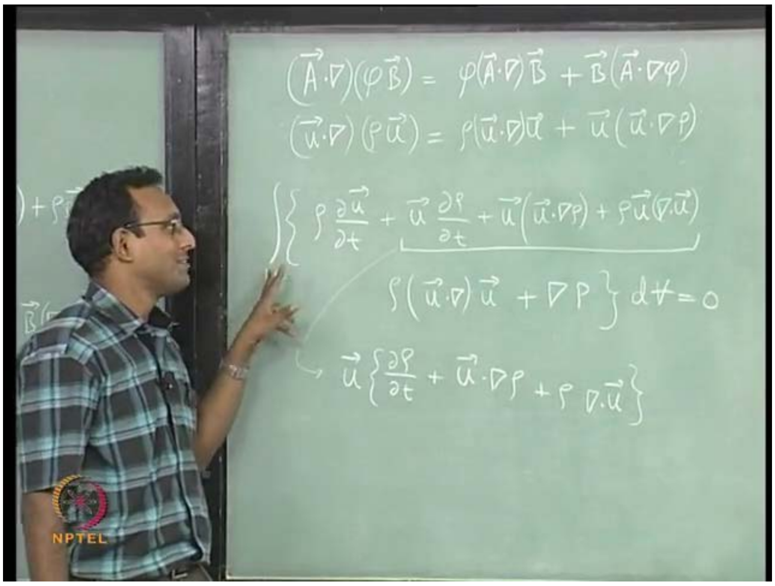

I will fold and write here, I want to give some gap a little bit here, it helps after some time. This whole thing is 1 equation of which, we know that there is your del P term, that coming from tau tensor dot n d s integral converted to volume integral. These are the three terms, which we just talked about of which 2 of them are just sitting here directly above. This is the first un study term, now the time derivative is operating on individual terms instead of what, we had before as a group dou by dou t of rho u together we have, it is operating on individual, that is the whole thing we have.

Now, we want to group it in a nice fashion, we want to say, I will group these 3 times, that is why I wanted that gap here. I want to group these terms all of them have a u vector multiplied by something, now we will group them that way just take that bracket alone, it will look like u vector multiplied by it will look like this. Now this is just your mass equation, which we just derived last class, this is u vector multiplied by mass equation and if I say my flow is obeying mass equation, this will automatically be 0. So, my expression can be simplified to just this term plus 0 plus these 2 terms, that is what we have going to use from now on. And now, we will say again what are logics, we used last time we said the integral should be valid for any small volume d V. So, ideally my flower bracket inside the integral that must be equal to 0. So, we get to the differential form of the expression remember the integral form which, we started with and the differential form, we will use all of them. So, what is the integral form, we will go back here.

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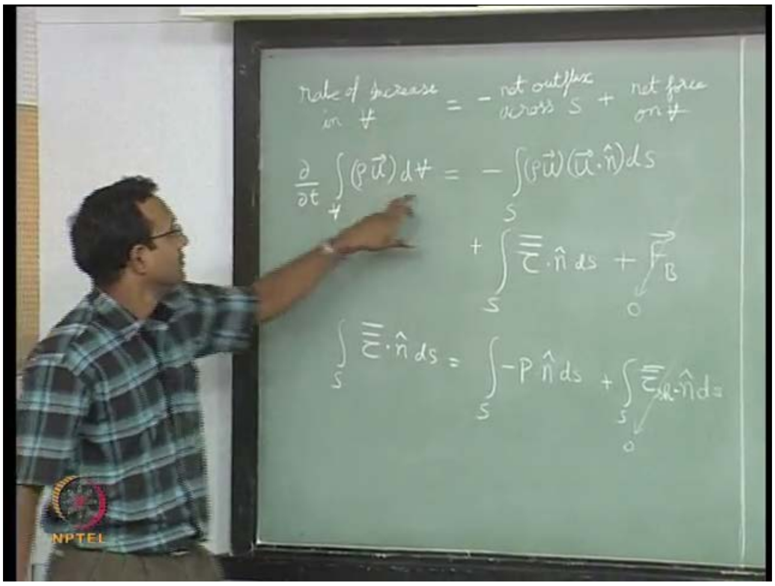

This original thing is my integral form is this whole expression of which you should remember this form along with the change to derivative form, which will be in here, this whole expressions.

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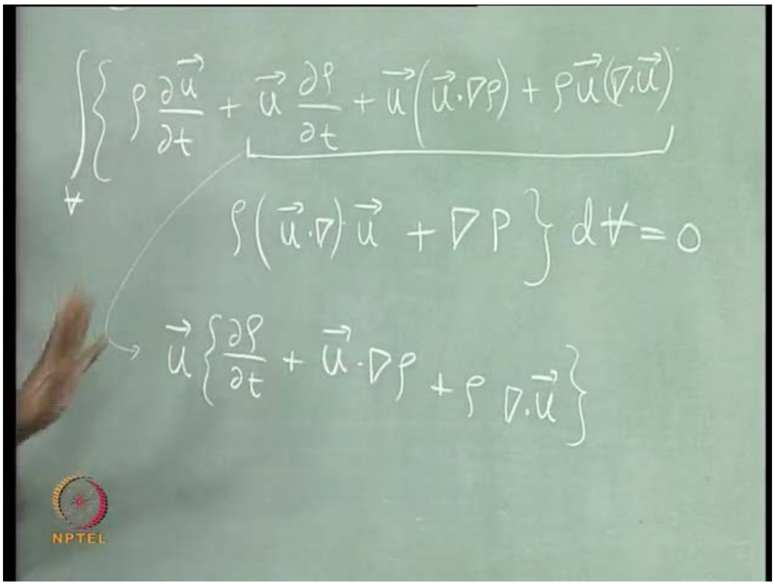

Up to here, you want to keep both these expressions, we will use 1 or the other depending on the situation, whichever is convenient. So, I will get a final expression, I will write it in a next board.

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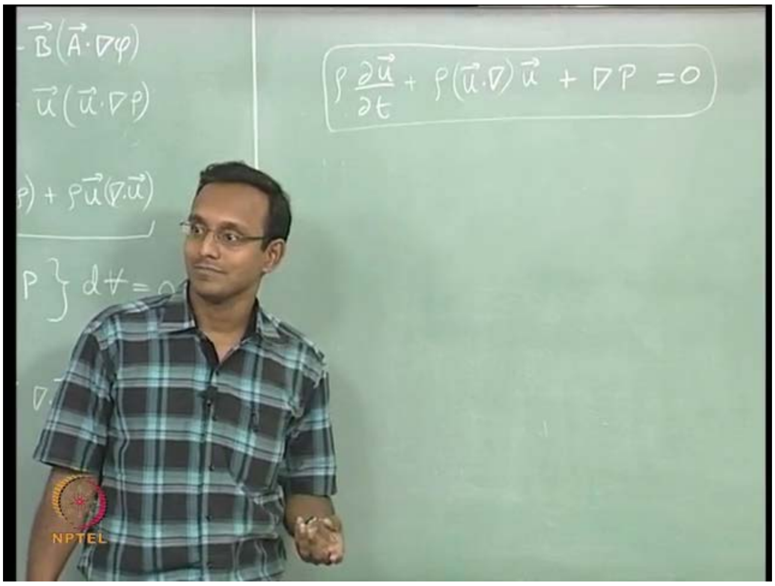

This happens to be your differential form of full momentum equation, of course, it is 3 equations together I have to put this vector symbol here on top, it is actually, 3 equations together. If I look at it there is x component 1 full equation is there, y component that is a full equation, z component is a full equation, this is grad P is going to have 3 components in there.

So, each of them will go to one particular equation specifically, If you want to think about it dou P by dou X is going to be in X momentum equation, dou P by dou Z will be in Z momentum equation or the W momentum equation, which I am using. Now is this expression for compressible or incompressible flow.

Students: Incompressible.

Why is it incompressible flow.

Students: Compressible.

He already changes the answers. So, he say it is compressible flow, why is it valid for compressible flow.

Students: (( ))

Del dot u equal to 0, we used somewhere, where did, we use it.

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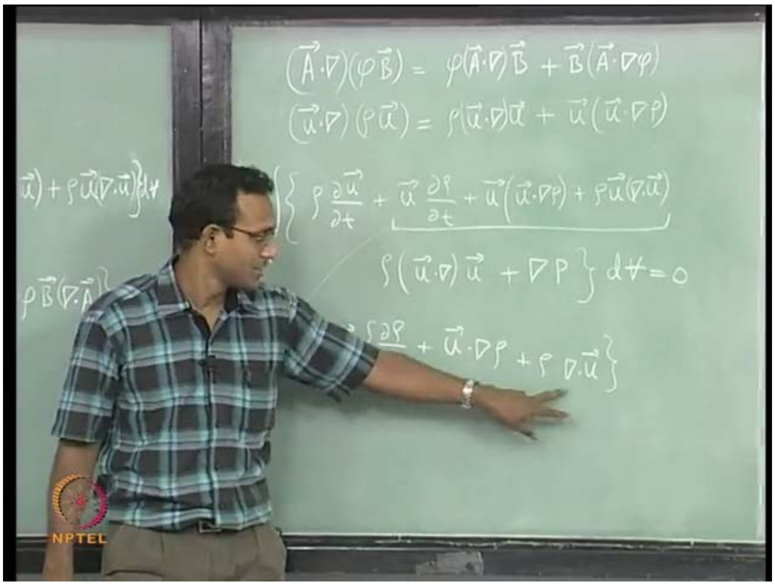

I want to go say it is here, this is a mass equation, we said this whole term is equal to 0, in mass equation, that is we saying right. So, if we look at the original expression for momentum equation, if I do not use this simplification, the actual momentum equation happens to be this, whatever is inside this flower bracket equal to 0. That is your original momentum equation, if I look at it that way, I am having this dou rho by dou t term. So, in theory I have to say the density variation is also taken into talk on to in this excepts for I am simplifying this expression using mass equation. Because, mass equation is also satisfied in my flow problem, mass equation and momentum equations are satisfied. So, I will simplify this, so even though I write my expression in this form.

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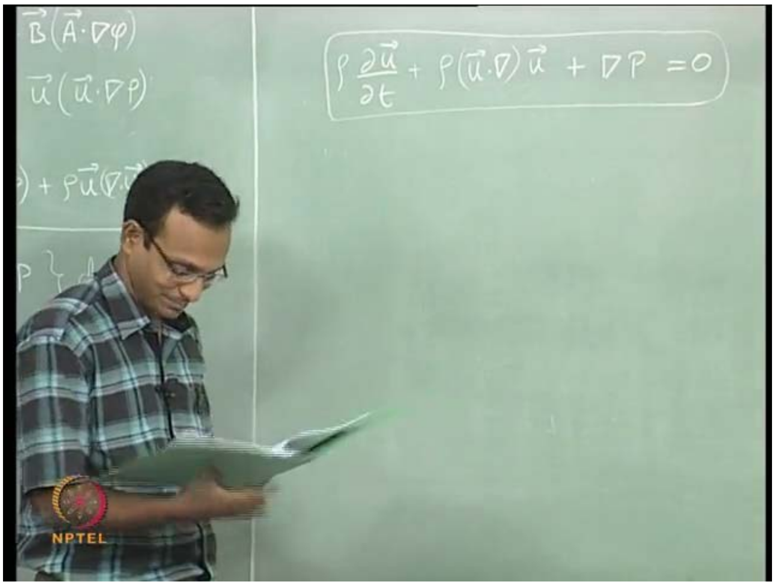

It is valid for compressible flow that is an important point to have to remember, it is valid for compressible flows. Even though they does not seem to be a direct time variation term for density, that is the typical thing people use in interviews they will tell you to write this expression. And then they will ask whether it is for compressible flow or incompressible flow standard drop just tell you that, do not want to fall in that trap anyway. So, now, we will move on to consideration of energy.

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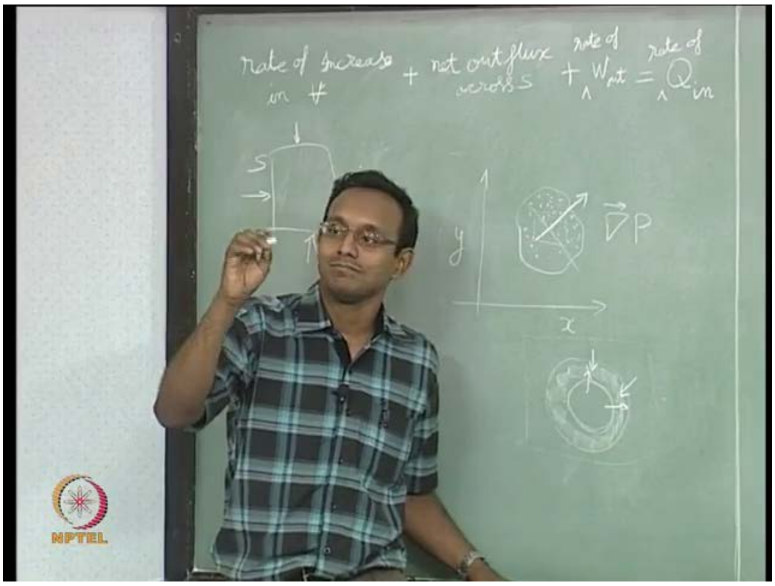

This same thing can be used here rate of increase in volume V, I am going to say how much is being collected inside my packet, we talked about this accumulation term, that is going to be there plus the net out flux. I am writing little less in here ideally, I have to write net out flux is net rate of out flux of energy across surface S all that I have to write here really, I am just writing here, net out flux across s. That is how much am I keeping inside how much, how I am sending outward out of my control surface, those have 2 things.

And then other forms of work done by me on the surroundings, that is a next thing remaining thing. I will just call it W out for simplicity ideally, I have to write net rate of work done on the surroundings, that is what I have to here ideally, all these let us say are going out from the system. Now what is the input, the heat transfer in that is only things or some other source term that is producing energy inside, all these are just rate of increase of energy inside the volume, but what increased it. Everything seems to be using of energy is for something, what increase the energy that is your net in flux, I will just call this your actually. It is not just Q in this is not just W out ideally, I have to write rate of you are thinking about time rate of change of quantities, in this case it is energy equation. So, it is energy quantities energy.

Now, let us look at Q in term, what is a net rate of Q in, what are the various ways by which, I can give energy into my control volume, Q is heat transfer. So, what are all the various things that is pass, some kind of what is a process, we are looking for the process not heat exchanger say as that is a mechanism by which give heat to gas yes convexion not really convexion. It is not convexion, we are looking for conduction basically, heat is being conducted, because the walls are hot and the gas is cold something like that.

So, there is heat transport due to temperature gradient, what log law of conduction named after who, Fourier's law of heat conduction, it should have been sometime in high school you are on be on this for first year at least in engineering. So, now I want to look at how much is the net heat into my volume. So, I have to look at my control volume and say every surface, this is my surface of my control volume as and I am going to say every surface there is some amount of heat coming in.

Which means, temperature here is higher than temperature here, what is a direction of my temperature gradient, gradient has a direction by the way, what will be the direction of my temperature gradient, del t has a direction. It is in the direction of nobody here thought about del let say del P, we look at that just now momentum equation, what is the direction of del P. Normal to iso surface. So, let us use that terminology here, I do not know whether.

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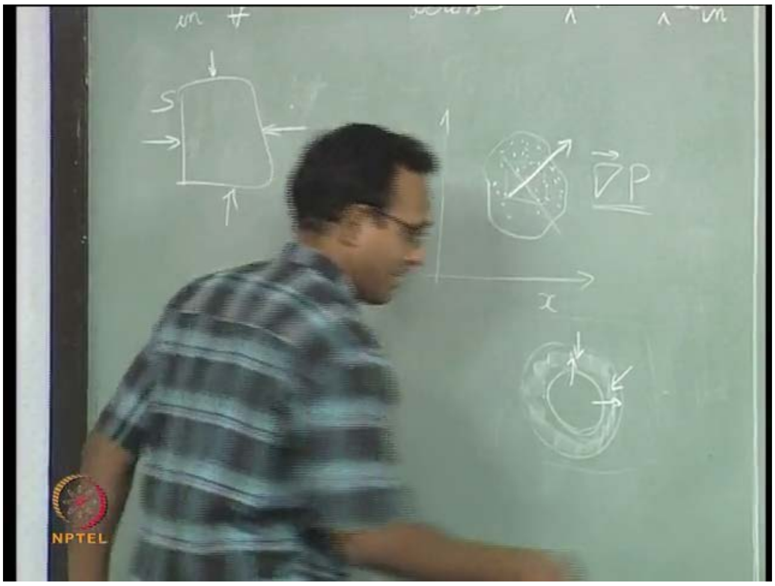

It will work very well, let us say I have my space x y coordinate system and I am looking at some random gas volume. And I have some pressure distribution let us say wherever I put lot of doubts pressure high there something. Pressure is high here, low here, if I take a gradient over this volume here, let say if I take grad P del P. This happens to be a vector, it will have one direction finally, because it is a vector, it has to have a particular direction, what will that direction be it will be towards the highest change.

Gradient vector gives you the direction of highest change at that point, which your direction pressure changing the highest, it will be putting that direction for you, which is in a way equivalent, what he was saying. It is perpendicular to the iso surface, perpendicular to that, there will be a line, where I can tell pressure is almost the same. I will get to that, that is your iso surface, but I can have a crazy situation in my flow field where, pressure is very high, in this annulus then pressure gradient will be this way here. This way here, this way here, this way here, like that, but still iso surface is still present.

Depending on the resolution, we will see different things, am I interested in such small differences in distances, if that is the case then we will such effects. If I say I am interested in my minimum delta x happens to be that is big block, there is no pressure variation for me. I will just take 1 average pressure for this, of course engineering problems, we always, we will say I am interested in lengths of the order of something let say 1 centimeter, lengths of the order microns are not useful for me.

So, like that we will set our own limits, depending on the problem it may change, you would is below over an air craft then I am not bothered about lengths of less than 1 centimeter probably. If it is flower my hand then I may be interested in lengths of the order of 1 millimeter, but anything lesser, if it is flow over this chalk probably, I am interested in less than 0.1 millimeter also. Because, the overall size itself is some are small the overall size itself is roughly 10 m m or something.

So, we may be interested in something much more finer that can also happen depending on the problem, you may want to resolve more that is for people, who want to go computational mode other, it does not matter so much. But, in any case grad P, you should know is going to be perpendicular to the iso surface perfect mathematical expression.

But the actual way, we want to we will think about it, it will point towards the maximum change direction, it will go towards change in the for upper direction. We will go from low pressure to high pressure direction that will be the direction that is the way it will be.

Now, same thing for temperature, we wanted temperature gradient, so that, we will have heat conduction, that is what we were looking for.

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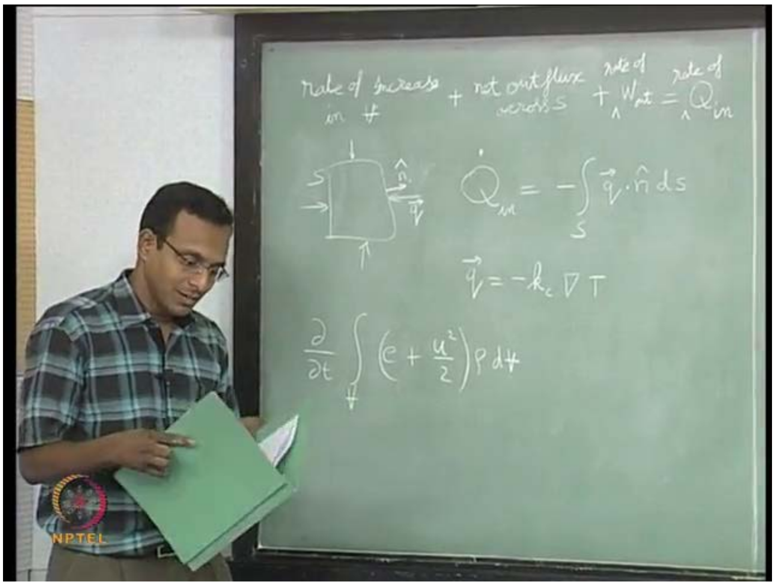

Now, if I say my normal vector is like this and my q vector, the heat flux vector is like this then if I put q dot n that will have a negative sign. Because, n vector is like this q is vector is like this and you will get q dot n will be a negative sign, we want to cancel that because we want to say the q is positive, this q corresponds to positive. So, I have to say my q in, I will put dot saying it is rate of I do not want to go and write every time rate of q in, I will just put a dot on top and say that is time rate of change of q n.

Now write this as minus integral over the surface q dot n d s, now what is this q, that is where your Fourier's law coming conduction coefficient times del T, this is your Fourier's law of heat conduction. Now, I did this gradient of pressure now. Now gradient of temperature, what is this say if I go back to this picture, I am saying heat is going this way in which, means this is higher temperature, this is lower temperature right. So, my grad T will look going outward, but we already defined it to be going inward.

So, there has to be a minus sign, that is one way of looking at it, we just have to be consists, why we put this minus why, we put that minus, they are 2 different minus signs. They are going to cancel each other soon, if I substitute this q inside here, that will cancel that minus sign, we will keep it this way. I will use this form sometimes, I will use this only once, anyways finally will just say q equal to 0 overall, that is also there by the way the q in need not be always through conduction, what are the other possibilities of giving heat to a volume of gas radiation.

Another possibility anything else, convexion will not be working, conduction is the physical process, convexion is the gas molecule let say, I have a hot plate the gas molecule here will come take the heat by conduction. And then it is moves away, some other gas comes here, takes the heat and it moves away that, whole process is what people call convexion engineering wise, it is useful.

They will just define 1 coefficient of heat convexion and problem can be solved faster. So, they have a separate thing call convexion, really it is just conduction with flow that is what is convexion, it is not anything special. So, you would not go into that I am just going to say there is no separate convexion term, that is why I want to introduce this, it is there is only conduction term, there could be radiation term other than that. There could be 1 more thing, chemical reaction can be exothermic producing heat or endothermic.

It can be q in negative, it may be taking away heat, it could be q out from the volume that could also be there, I could have chemical reactions. But, currently in our gas dynamics, we are simply in a flow of system, we will say no chemical reactions, no radiation. But, they are just approximations, I could have a case where it matters, this is just 1 term, we have just written that last term in this 4 term expression. Now, we will go and look at the rate of energy accumulation, that is a easiest term time rate of change of stuff.

This is e is internal energy per unit mass, we already had this convention and this is kinetic energy per unit mass. This is a total energy of the gas per unit mass, this is the gas is energy and this is the flow energy all together. This is all per mass multiplied by density will make it the whole, energy per volume into small volume will mean that small box, how much energy am I having integral over the whole volume. That will be the net energy I will have inside my control volume time rate of change of that, this is your the first term rate of increase in wall inside the volume. Rate of increase of energy content inside the volume, that is this term this is 1 term. Now, the next term is energy out flux that is this net out flux across S, what out flux, energy out flux rate of energy going out across the surface.

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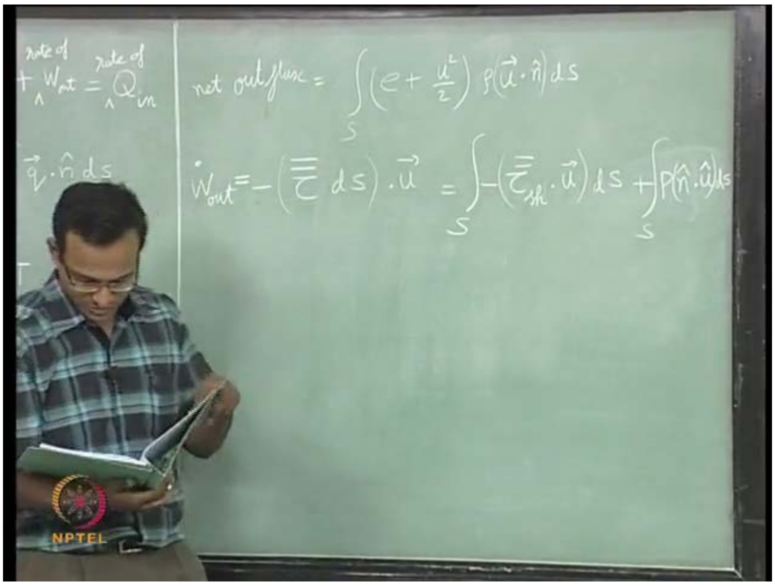

I have to say net out flux. Similar to all the other terms, we did for out flux, it will be u dot n times d s multiplied by that quantity, that quantity happens to be energy per unit volume rho times e plus u square by 2. That is the whole quantity, we are talking about, we get to this form. Now the only term left in this equation, we have an expression for this that is here, we have an expression for this that is out here, then the next thing is this right hand side, which is what is the first thing we did, that is this expression.

Now only thing left is this is rate of work out. Now rate of work out what are all the various mechanism by which my fluid can do work on something around, something surroundings, turbine work yes that is a possibility. That is called the shaft work typically, yes that is a possibility anything else, pressures doing work p d v related work. But, it is flow, we want to find rate of work done, how do you find rate of work done, I want to what is has the units high school physics. I am pushing a block of metal, what is a net work done, what is the rate of work done?

Force into displacement gives you the work not the rate of work for center velocity, were both, there both vectors. So, what should I do that was cannot be generally multiplied should be dot product, you have to know which 1, because we are looking for energy, which is scalar not a vector. That is 1 simple way of looking at and continue mechanics if you give this answer, they would not accept, you have to give more logical answer for that.

But, in here we can give this answer that is you will say, you are looking for a scalar how to get a scalar from 2 vectors dot product easy answer that is enough for us. So, what is my force really, my force happens to be this stress times d s. Now I have to multiply this with velocity along a particular direction. So, I have to dotted it with my velocity, this is my rate of work done, this is overall term, now I have to be careful about this, why because I want to put a minus sign in front.

So, if I have stress say I am fluid element and I am moving forward, but the next fluid element is not moving, what we will be feel that is a velocity gradient. So, I have to full that person along, the next fluid element, I have to full along it is means, I am applying force this way. And I am moving this way, what is the actual force applied on me by the surroundings, it should be the opposite direction right.

Because, the next fluid element is slowing me down, I am doing work against that force this correct way to look at it, you have to think about it and then come up with you do not explanation for it finally. I will tell you this particular way of looking at it, but you may have some other version of it, you can think about it and which, are way you feel like I will just tell you sheer force. It need not be just sheer force, we will look at normal force also after this at sheer force is what is the example? I gave you right now, I could go for normal force also, we will go for normal force after this.

I am going to have this particular force, which is minus this is my W dot out again, I am going to say net rate of work done on the surroundings this is what, we are looking at we will get to this point. So, now, again I will write this stress tensor in diagonal and half diagonal terms separately then I can write this expression as tau sheer dot u d s, I missed here equal to sign here which, I do not think it is a serious trouble. I missed this equal to sign form and then I have to use a correct sign again here n dot u, here the negative sign is automatically taken and talk on, you may should think about it. Again I would say now I have this everywhere d s.

So, already I have to do is the integral overall the surface to get my full force not just full force full rate of work done on the surroundings that is, what I need to do finally. So, I have to just put integral over this over the whole surface that is what I need to do for both the terms integral over the whole surface. Now, you taken into account all the terms all have to be just look at the whole expression, the whole expression is not very difficult to write to anywhere. I will go to the next section.

We have in converted the surface integral to volume integrals. And we will write it without the convention remember that form and then we will convert it and then the whole volume integral, we will just say all the integrand the inside the integral should be 0. We will tell that also and then we will remember the differential form, we will keep both the forms. So, I will write both.

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The first 1 just the whole term, I am going to fold the equation, in case there is shaft work, I will just write this immediately after this point. I will say this is equal to 0 and my problem, in case there is other forms of work out then I will keep that this, whole thing is the left hand side, now that is going to be equal to. The remaining terms again I wrote this term, If I want I can even write chemical reaction here, I just said it is q in rate of heat n through radiation and we will again say, we will make it equal to 0 in our problem same thing as shaft work.

We will again say it is equal to 0 and we want our consider that, now of course I can write this in terms of gradient in t using Fourier's law. If I use Fourier's law, this will become just this 1 term, I will write separately here, this is integral over surface, which I missed here, integral over the surface there. Now I can write this as integral over the volume, this is the first change.

Now, I will use my heat conduction law here and I should have minus sign here, now that will become this term of course, I just tell you, that this is a possibility, we would not most likely use this. We will use only bulk quantities, we would not go for at a point, what is a temperature, what is the next temperature point nearby and find out the gradient, we will never do that in our simple gas dynamics world. It is needed when we want to think about how much is the heat conduction across a shock which, we will never go to in our course here for now we are in simple world.

Now, we just want to go write this in terms of everything in terms of volume integral, which is not difficult to do like, we did this before. If this can be rewritten in terms of del P dot u kind of term like that, you can write this term also there will be a grad of this whole term, it will come out. We will just write the final form I want worry about how, we got to this, we have the same method used in mass equation, it will look the same. So, I will just write the final form here.

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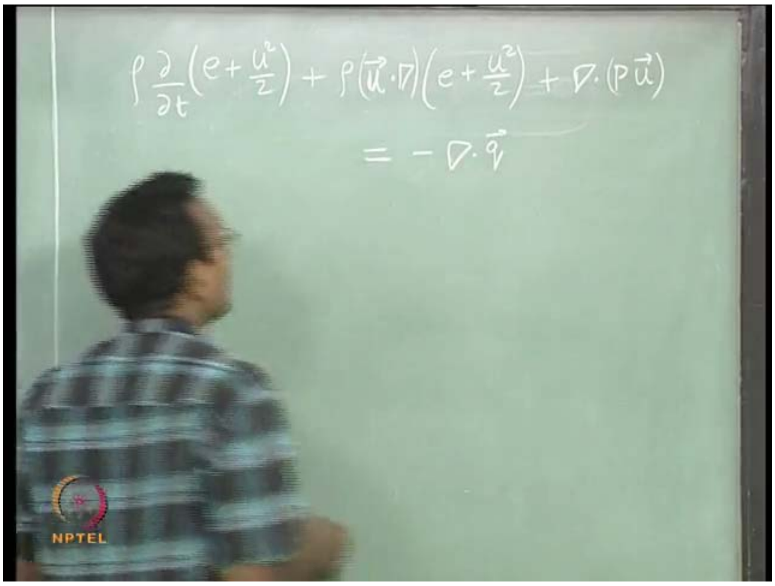

It will be and I am removing the integral and just integral over the volume is for whole thing and I will just say there, whatever is inside the integral mass be 0. I will just write that form. We will leave it in this form actually I just told that it is a grad somewhere.

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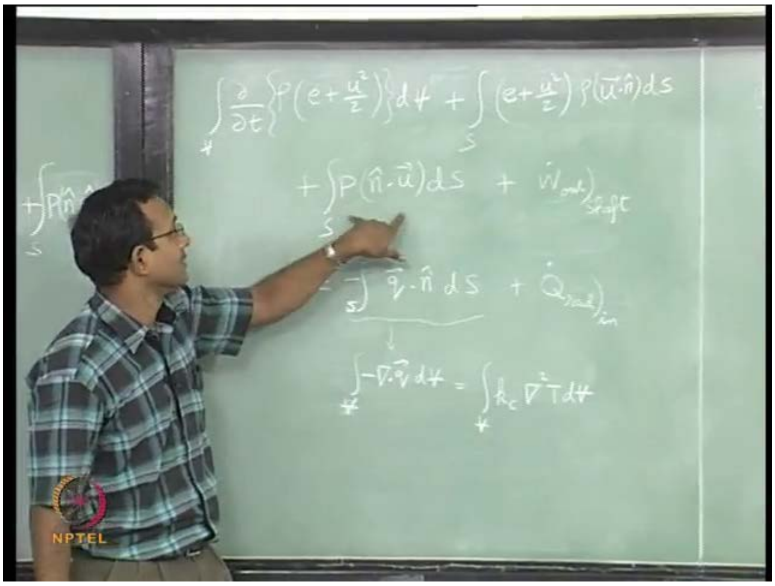

Here in this term I just told when, we make it a volume integral just a few minutes back, I told it is going to become grad P, it would not grad P, because this dot u here.

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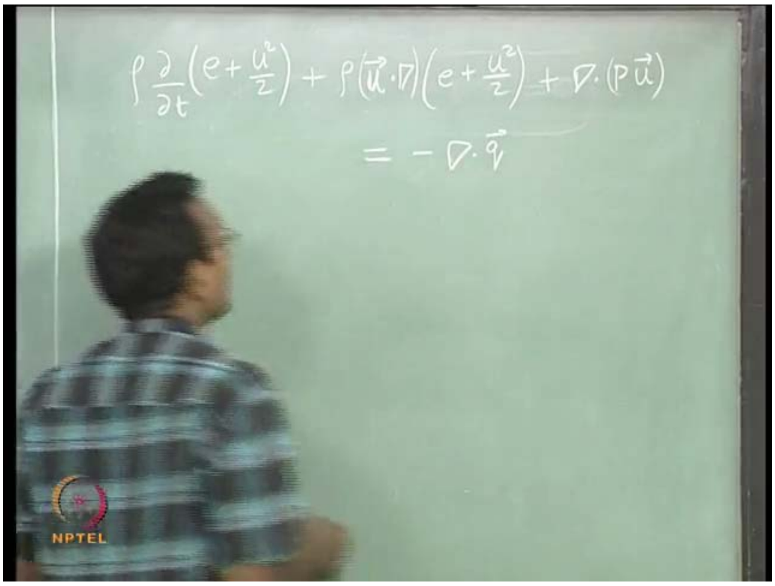

It will just become del dot P u, it will become this term all the other terms was similar to what you did before in mass equation, this 1 term alone and then we get to this form, this is your energy equation, energy conservation equation in differential form. We will keep all these forms. Now we, are at a point where, we have all the govern equations for the flow along with this, we should keep the equation for entropy that is t d s equal to d h minus V d p, we are keep that form or d u plus p d v are not d u for you. It will be d e internal energy change in internal energy plus p d v, that form and P equal to rho r t gas e state equation. So, ideally we have five equations.

We will look at what happens when there are 5 equations, how many variables are, there are 5 equations, how many variables are there, what are the 5 variables we have. Velocity volume not relay, because it is all differential equation is at a point there is no volume for your fluid. It is point pressure, temperature, density, velocity, internal energy is just temperature enthalpy still temperature c p times temperature, we will think about this. We will get back to this next time, there is still one more variable, which I we just missed anyways.

So, next class onwards, we will get in to actual gas dynamics, we just laid the foundation out, we know thermodynamics very well. Now we know, we have used mechanics next class onwards, we will just starts solving problems of different types in gas dynamics, see you guys next time.