Gas Dynamics
Prof. T.M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 16
Lecture - 37
Fanno Flow-Relation, Plots and Discussion about Chocking

Hello everyone welcome back, we were writing relationship between d u by u and every other property derivative by it is value I will write down those values again once.

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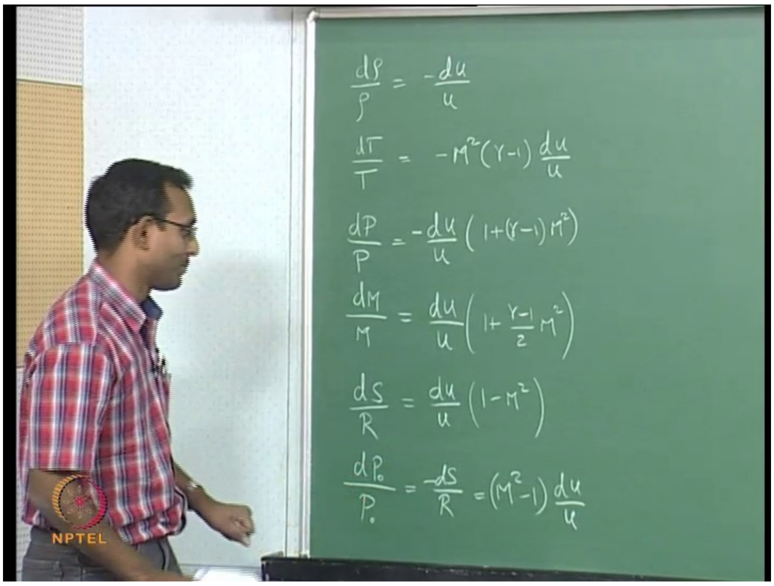

So, that we can go for easy reference this was one of them we had, and then we went to energy equation here we wrote d T by T we had all these relations plus d S which was I will write it as d S by R. So, that its non dimensional again these were the various relations we had and we already discussed the effect of changing velocity based on this particular expression. And we found that if I had mach number less than one it will go one particular direction based on these two expressions together.

If I had mach number less than 1 then d S will be greater than 0 only for one particular situation that is d u will be positive. And if mach number is more than 1 d u will be negative that is mach number will decrease, d u negative means mach number will decrease d M will also be negative. So, we said that the flow has a tendency go towards

M equal to 1 that is where we stopped, and then we said we still have to use the momentum equation, which we did not use in any of these we wrote it like this.

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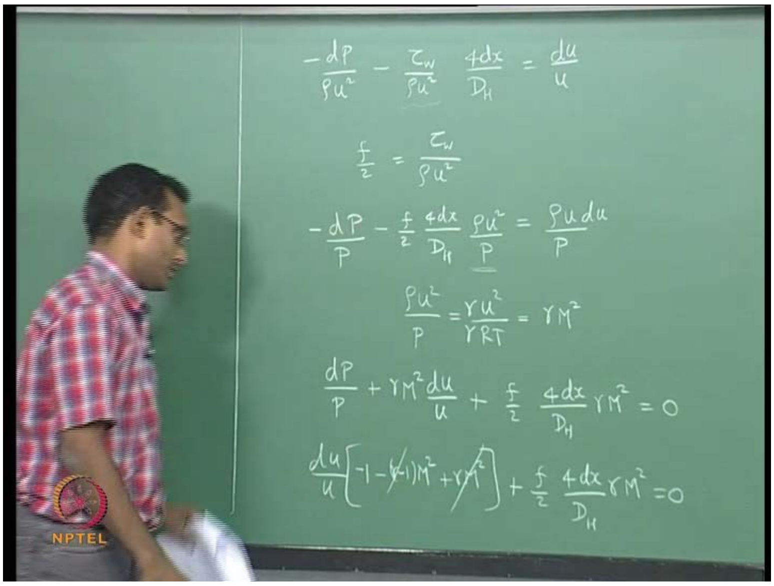

And then we took a deviation and trying to get to one particular form in terms of fanning friction factor f, this can be written as f by 2 from last class definition, we getting to this particular form you can just rewrite it as f by 2. Once we have this as f by 2 I can rewrite this and I am going to use fanning friction factor here or I can use the doozies friction factor, and put 4 f instead of that whichever way you think about it.

You can use of one those two numbers here we have a relation between these 2 f dash is equal to 4 times f fanning this is f fanning. So, there will be books which will say 4 f instead of this also depending on which one they used they change the numbers, now will start using this relation inside there, and rewrite this I want it as d P by P instead of rho u square I previously wanted it to be d u by u. So, I just did this rho u square dividing now I wanted to be d P by P because, I have expression for d P by P already.

So, I will write it as d P first and then I divide whole expression by P with substation of this for f by 2 minus f by 2 into 4 d x by D H multiplied by my rho u square it is going to be equal to rho u d u, this is the expression I have. Now, I will divide this whole expression by p, so that I get a d P by P here, now I had this term rho u square by P can be written as u square by R T P equal to rho R T. So, I can now multiply divide by gamma, so I am going to get gamma M square here, this is age old derivation you should done this, so many times in gas dynamics anyway.

Now, the only difference in this particular term is rho u by P which is equivalent to gamma M square by u. So, I can rewrite the whole thing and I will take all the terms to one side of the equation, so that it will become positive term d P by P plus that rho u d u by P I will rewrite it as gamma M square d u by u. And then this one term I do not want to cancel this 4 by 2 as to will just keep it as is, so it is going to be f by 2 4 d x by D H, will get it to this particular form wait I missed one gamma M square equal to 0.

I just I took these two terms to the other side of the equation, and then I substituted this gamma M square in terms of where ever there is rho u square by P I have done all that, and am having this particular form here I have d P by P d u by u and d x. Now, I want to rearrange things such that as I go along my tube I want to find only mach number variation nothing else.

So, I want write d P by P d u by u in terms of d M by M do I have that relation I go back here, I have d P by P in terms of d u by u and that will make my expression having here d P by P in terms of d u by u will mean two terms will have d u by u. Now, d u by u I have in terms of d M by M that is here, I just have write all of them together inside that form that is the next thing we going to do. So, I will write just d P by P in terms of d u by u first d P by P term happens to be minus 1 minus gamma minus 1 M square plus gamma M square, these two terms together becomes this.

And the other term I will leave it as is, will look at here minus gamma M square and plus gamma M square will get cancelled. So, I just have M square minus 1 here finally, d u by u times M square minus 1 is the only term left, now I will go look at here d u by u can be written in terms of d M by M if I take this bracket to the other side. So, I will put that expression inside here along with this M square minus 1 write the whole thing here.

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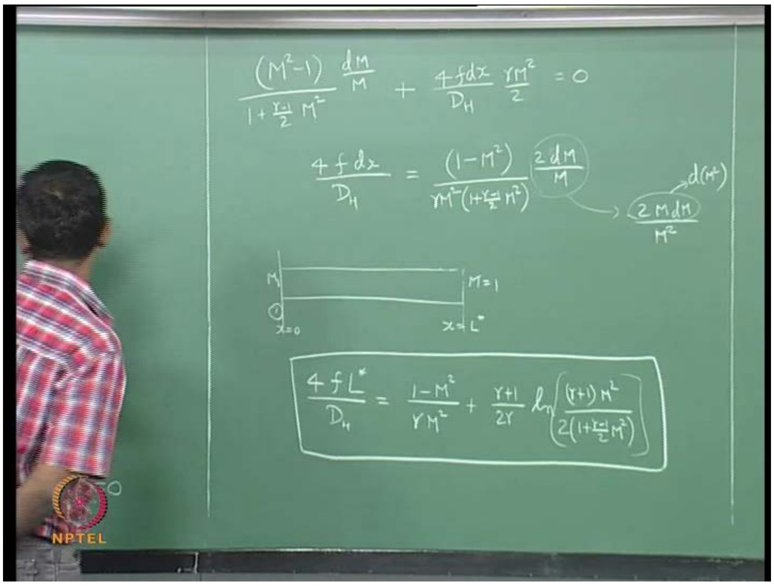

So, it comes out to be M square minus 1 times d M by M divided by that term in the d M by M expression plus actually I will write it the same form we did 4 f d x by D H gamma M square by 2 equal to 0 this is the expression I have. Now, I want to take all the mach number related terms to one side of the equation, so I will get it to 4 f d x by D H equal to there is a minus which I will switch over with this M square minus 1. So, it becomes 1 minus M square 2 d M by M divided by gamma M square M plus gamma minus 1 by 2 M square this is my expression.

Now, I just want to rearrange this particular 2 d M by M as 2 M d M by M square, now this 2 M d M can be written as d of M square. So, it this whole term becomes d of M square by M square, now if I look at it every term that has mach number is all M square. So, I can might as well substitute this M square is equal to some y or something, and start integrating that expression that is why I have made this thing. I am not going to do the integration here, but will just say that it can be integrated as of now, and what will be my integral limits this is d x.

So, I have to start from somewhere I can start from x 1 and go to x 2 and integrate that, then it will give me M 1 going to M 2, but that is not very nice. So, what will we do will be we know that given long enough distance, my flow reaches M equal to 1 right whatever we discussed already, we know that the flow has a tendency to reach M equal to 1. So, what we will say is I start with some location one which might be x 1 or I will call that as x equal to 0 I will set it can be any value.

So, I set it to be 0 x 1 can be 0 also from here I am going to go to some length x equal to L, and in this time it started with M 1 and ended with M equal to 1 that is what it is doing. So, I am going to give these as my limits to my variables x is going from 0 to L and I am integrating that, while M is going to go from M 1 to 1 I will just do that whole thing. Instead of M 1 I use it as M later I can just replace M 1 with M is just anyway simple variable, what I start with starting mach number is my M 1 that is all I am having.

And now, since it is M equal to 1 I will put a star on top of this L as we are doing this for a long time that says that it is the length that this flow needs to travel along the this tube to become M equal to 1, this just special length not any length. We later find that this is your maximum length that it you can have, if you go beyond that the flow will not be just doing the same thing it will do something different that I am suppose to discuss today's class anyway will get back to that.

So, finally, if I do all this I integrate this and get you final expression it is going to look like 4 f L star by D H is equal to 1 minus M square by gamma M 1 and gamma M square am I could write it as M 1 square M 1 square everywhere, I just I just say I will just replace it with M here plus gamma plus 1 by 2 gamma natural log of gamma plus 1 M square divided by 2 times 1 plus gamma minus 1 by 2 M square. Log is on this whole thing, this is a very important relation for fanno flow in fact, the most important relation for fanno flow.

Now, once I have this relation I can get other variable changes, but I will do the basic simple math way to get the other variables, which will be I will take this d M by M and replace d M by M in terms of d x in all the other equations. Let us say I will pick this equation here, I can replace this d M by M in terms of d x now, then I will get d u by u in terms of d x. Then I can get velocity as a variation as a function of x, if I integrate that particular equation.

Similarly, I can put that d u by u in terms of x into this I will get pressure variation as a function of d x d P by P in terms of d x. Similarly, d T by T in terms of d x I did not write one more here P naught by P d P naught by P I will write that also right now, d P naught by P it is not anything great d P naught by P naught can be shown to be equal to minus d S by R. So, it is just negative of the previous expression there, M square minus 1 times d u by u, it is just going to come out to be this, just the previous expression negative.

How do we know this, we already derive this particular thing long back will just keep it the same. Since it is adiabatic it will come out to be this, if it is not adiabatic it may not be this simple an expression there will be a T naught variation also will ignore that further. Now, if I do all these integrations I will get a whole set of expressions whatever we are interested in I will put here.

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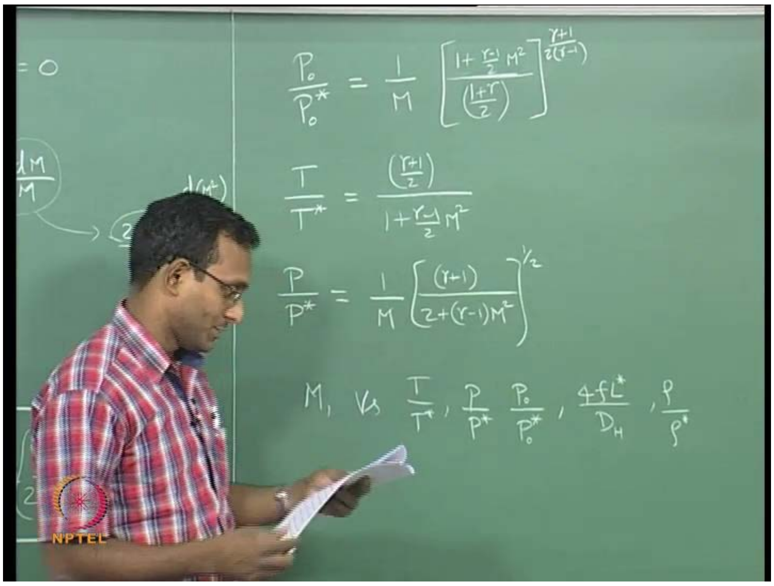

One of the most important things P naught by P naught star, basically I am going to say my flow start with a P naught and ends with a P naught star. So, that is this and during this time it goes from x to 0 to x equal to L star right, and I am expressing that in terms of M square bracket should be for the whole thing T by T star this is nothing right this T by T star is just coming from T naught being constant, simple enough expression this comes from T by T naught T naught by T is equal to 1 plus gamma minus 1 by 2 M square it is coming from that formula itself.

Just directly nothing special for T by T star P by P star by the way this is similar expression as this one, I just multiplied numerator and denominator by 2 to get this form it looks simpler than the previous one. And this can be obtain from this expression, and you are isentropic relation P naught by P, P naught by P and P naught by P star appropriately multiplied with this particular ratio to get this that can also be obtain.

And of course, now you can find delta s from here I do not want to go and do that right now, you can go and do a delta s if you want can be done. Typically all these variables are listed in compressible flows tables, listed as M 1 as the primary variable verses T by T star P by P star P naught by P naught star 4 f L star by D H rho by rho star, this rho by rho star may not be given in all the books. But, some books give this also, you can of course, get rho by rho star from T by T star and P by P star P equal rho RT still valid.

So, now, we have a expression for all this I did not write an expression for rho by rho star I know you can get it from here, I do not need to worry about that. Now, what I want to do is just go show you the plot of how things are, will go over to the screen.

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If we look at this I have plotted all the primary variables whatever we have, I am showing here several flow parameters as a function of mach number here. And the first one the blue is your f l max by D l max is same as l star as of now I want to prove to you that it is l max for now, just take it as l max for now. And what we are seeing is it is something very high, when it is very low mach number, and then that particular value drops drastically to 0 at M equal to 1. And then it goes back up again when the mach number is higher than 1.

And we know that if my mach number is more than 1, the flow wants to go towards M equal 1 when my mach number is less than 1 it wants to go towards M equal 1 always the flow wants to go towards M equal to 1 remember that. So, what we finding is if my mach number is let us say 0.5 it has some particular f l max by D, and if I fix my f and D to be the same for two different comparisons, my l max is some particular value for this.

If I go some mach number closer I find that the l max is become half that is in half that length it is going to be reaching M equal to 1. Mach number was changed by, so much that also says that if let us say the l max was 1 meter from here, this is a arbitrary units remember that it is I am not giving correct units because, f and d values are not given yet. So, if we say it is 1 meters by the time I go from here to here let us say this 0.65 mach number, by then it goes from here to here it would have cross half meter, it is changing mach number very slowly, while travelling long distances for subsonic case.

And especially when I go very low mach numbers, it is changing mach numbers extremely slowly while it is traveling long distances. If I think about 5 meter at say 0.2 0.3 mach number from there it is going to 4 meter, 3 meter, 2 meter, and 1 meter it travelled 4 meters long for that particular f by D. Till it became 0.5 mach number I started at 0.3 and ended at 0.5, while the tube was 4 meter long.

If I go one more meter long it of course, it is going to reach mach 1, when I go closer and closer to mach 1, this particular flow is very sensitive to the friction. Till that time it is not subsonic flow it is not at all sensitive to it, and I think about supersonic flow when I am far away it looks like any small change in length or for a huge change in mach number. Let us think about that way for a huge change in mach number, there is very small change in length.

So, it is very, very sensitive to lengths in supersonic flow very, very sensitive to length in supersonic flow if you think about it. And when I go closer M equal to 1 it finds that when I go from mach 2 to mach 1 it needs only roughly 0.3 meters, if I assume the same f by d it is roughly 0.3 meters it goes from mach 2 to mach 1. While in this case it could have gone from for the same height it would have gone from roughly 0.65 or 0.7 to 1 that is what you are seeing here.

And says that, subsonic flow can handle lot more length of the tube before it reaches mach 1, while supersonic flow cannot important statement there. What this means is if I have a choice between my M 1 in let mach number of my tube to be subsonic or supersonic. If I chose to go for very long lengths it is better off if I go with subsonic flow, than supersonic flow is it logical, if you think about whether it is logical I will simply say that it is a flow through duct with friction.

If the friction is too high, then there is too much resistance I am going to say that if I go subsonic I am going to have lesser wall gradients of velocity. If I have lesser wall gradients of velocity I am going to say, the shear stress at the wall will be very small, so I will have lesser resistance. But, the same thing if it is supersonic flow, then velocities are much higher; that means, the velocity gradient will be very, very, very high, the velocity gradient is very, very high and if my velocity profile will be more like this suddenly instead of as of supersonic velocity profile like this.

Velocities are very high, wall give velocity gradients will be very high this suppose to be from fluid mechanics I am assuming you guys know fluid mechanics. So, and the wall velocity gradient is very, very high friction will be much higher, so friction has a huge effect when the velocity is very high, which is expected right its expected from fluid mechanics any way. So, that is a effect we are seeing here, if it is supersonic flow and seeing that it is very sensitive even for a small distances, there is huge change in mach number.

While if it is a subsonic flow even for long distances there is very little change in mach number. Now, I will go back to the plot and when I look at the next important parameter I want look at is P naught by P naught star, the total pressure of my flow this is the somewhat like the net pressure energy and the kinetic energy together in the flow. We are seeing that the net energy in the flow drops as I go towards M equal 1, I should not look at it going this way though remember that it I have to think about it going this way because, always the flow wants to reach M equal to 1.

So, the flow process is going to be always towards M equal to 1 from both sides, if I pick supersonic condition. I am going to have P naught dropping from it is all arbitrary units right it is lets pick if I put mach number is 2.4 it is 2 times P naught 2.5 times P naught star and by the time it goes to M equal to 1 from 2.4 it is dropping to equal to P naught star. But, in subsonic case of course, I cannot get a exact equivalent here there is less space only between 0 and 1 9 I have values, but what we are seeing is if I think about 0.5 mach number jump I am going from something like 1.3 to 1 P naught star.

And I think about in here it is going to jump for the same 1.3 happening somewhere here, 1.3 P naught to 1 is happening for of the order of 1.7 or 1.75 mach number to 1. The it looks like P naught drop is a little steeper in subsonic condition, than in supersonic condition if I think about the same delta M. But of course, you know that the maximum delta M possible in subsonic is 1, it cannot be just taken as is that is also there.

And one more thing I forgot to talk about is in the 4 f l max by D problem for a given l max I may have two solutions, subsonic and supersonic up to some mach number only beyond that point if I keep on going to vary high mach number it is not going to ever reach f l star by d will never probably go past one. Even if I go to mach 20 it is not going to go passed 1, so after some particular f l star by d there is only one solution like this type.

And P naught again it looks like two solutions, but you have to think about it as P naught is dropping from some value to P naught star that is way you have to look at here. Let us go for the red curve pressure P by P star of course, you have to again remember that the flow is going to go only up to M equal to 1, it is not going to keep on decreasing as we increasing mach numbers that is wrong. If I in subsonic I find that the pressure decreases, if I am supersonic the pressure increases is this logical yes it is because, my mach number is going to go towards M equal 1 in both directions.

And if it is subsonic it is going to accelerate which means my pressure and temperature should decrease, this is coming from your energy equation and momentum equation from basics, we did this in isentropic flow beginning. So, the same reason when the flow is accelerating, pressure temperature will drop and when I will look at it from supersonic flow it is going to decelerate to M equal 1. So, pressure and temperature will increase, they are all going the logical direction.

And the density is going to be something in between this and you know the pressure sensitivity is much more than temperature sensitivity. So, density is going to do the remaining thing which compensates for P equal to rho R T that is how it works, this is the case for gamma equal 1.4 I have plotted, if I show gamma equal to 1.3 oops we have to press this.

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It is just all most the same curves there is a small difference, if you notice this was jumping slightly the green curve, and this blue curve were jumping slightly I will go to M gamma equal to 1.4, just a slight jump not much variation it is not very nice to like look at these plots in this particular way.

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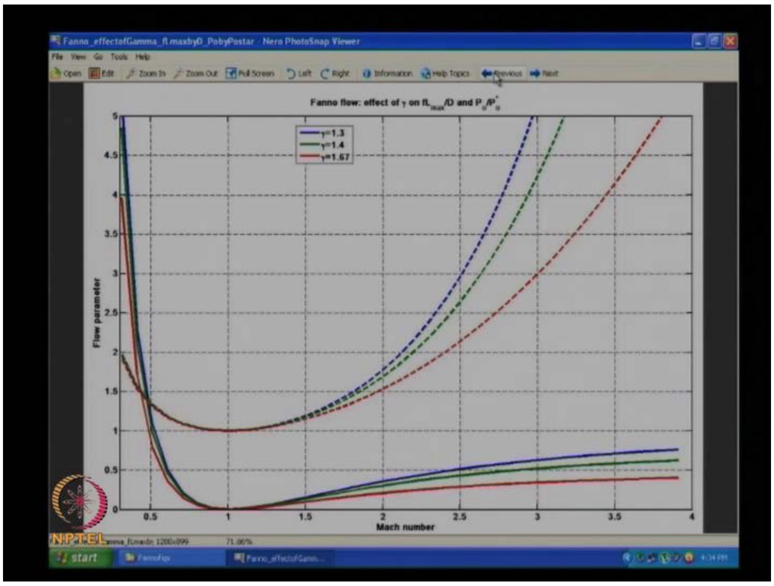

So, I have plotted them together for various gamma values, I have plotted that 4 f l star by d S one set of curves, and P naught by P naught star as another set of curves. Blue is the highest compressibility that is lowest gamma, and red is the lowest compressibility highest gamma of course, I told you gamma is not directly related to compressibility it is only a factor. Always remember that gamma is not the only thing that decides compressibility.

But, it is one of the serious parameters which describes compressibility, so if I pick my f l star by d this particular curve. If finding that there is very little change if I think about, L star variations for the same f and d, so, but there is some change if you think about a little more compressible gas, it looks like it can manage the manage to go a little longer f l star by d is a little higher for the same mach number. If it is more compressible is that logical yes it is because, it is more compressible it can squeeze through a smaller space.

So, even if somebody is resisting they will manage to go through it, when I go supersonic I am finding same thing happening here. But, it is much more drastic change here, I am going to say percentage change in L it is going to be higher as I change from gamma equal to 1.67 to 1.4 to 1.3. So, what is happening here, when my compressibility increases again my L increases, if my gas is more compressible it can manage to go a longer distance it is just managing to go a longer distance.

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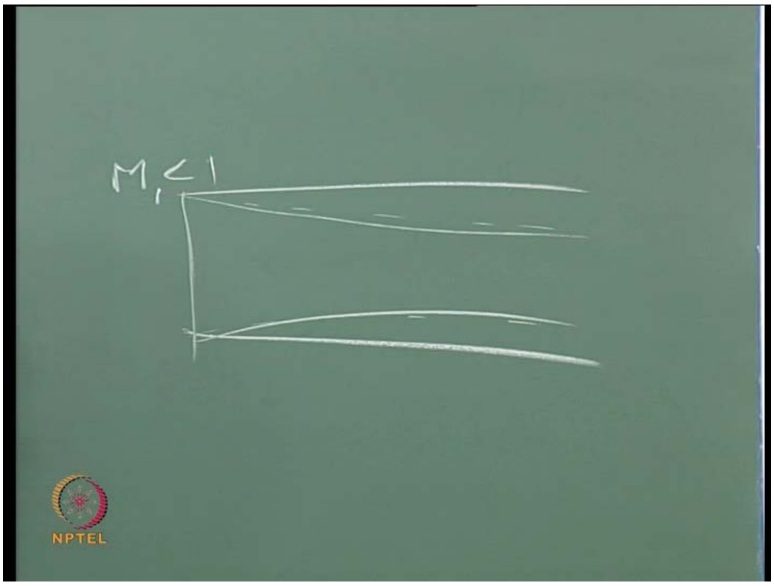

If I let me go to the board little bit, if I think about this from fluid mechanics point of view, I am going to say I have some M 1 let us say less than 1 condition. And I am going to say there is a boundary layer growing I do not want to teach boundary layer in this course, but I will just tell you that the boundary layer is like some block edge in the flow.

It is equivalent to having a wall that is crushing inward it is equivalent to having a wall that is crushing inward it is giving lesser and less space available for the flow is equivalent to that of course, there is fluid moving inside the boundary layer also, I am just giving you a rough estimate it is equivalent to something like this.

So, I can now think about it is like a area change problem subsonic flow will accelerate to M equal to 1 if it is supersonic flow it will decelerate M equal to 1 you can think about that exactly same explanation. Now, if it is more compressible gas what will happen, it can squeeze more and growing through for a longer distance that is the kind of physical field you should have.

So, that you can explain all these curves, now will go back to the remaining plots we have, if I think about compressible case for the same mach number it drops a lot more P naught drop is higher in supersonic condition compare to less compressible case. It is more compressible, it is energy drop P naught is much higher to go there, and in subsonic they does not seems to be any difference, there are almost on top of each other.

This remember that if there is compressibility there going to more P naught variations, same things we saw in shock also we would not discuss that right. Now, I told you that is out of scope for this course will just ignore it that way.

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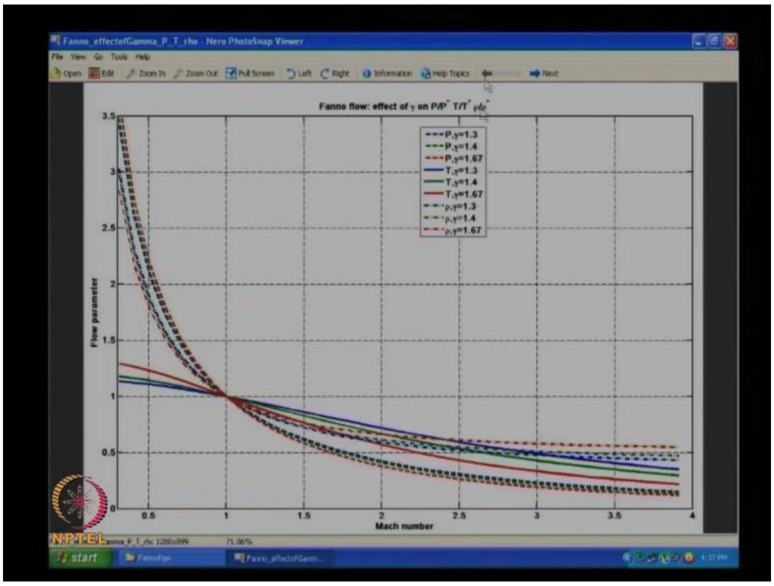

Now, if you look at the remaining curves pressure, temperature and density this I will first show this one, this is the temperature the solid lines are the temperature. And you can see, this is similar to what will expect out of any compressible flow more compressibility than your going to have temperature variation one particular direction. This is for any adiabatic flow, we saw the similar thing even for isentropic conditions and pressure is also doing the same thing, same as what we expected in isentropic condition.

And density will go the opposite because, one is much stronger than the other this same trend we saw even in isentropic flow curves. I would not go into more details on this you can try and explain this by simple arguments it will work very well, now will go back to the board I want to look at this whole set of state changes in a state map.

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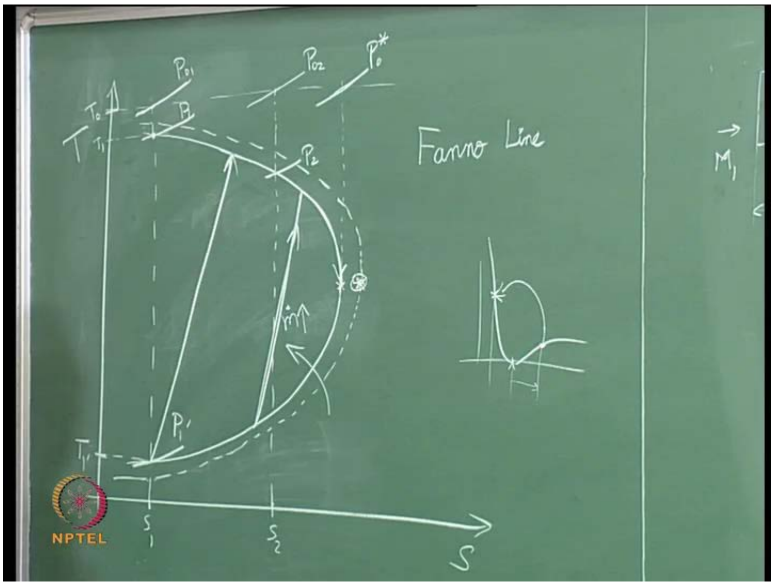

So, I want to plot it in my T s diagram, and we know at the whole flow has a constant stagnation temperature that is my T naught here. And let us say I start subsonic, let us say this is my T 1 and I am going to say it is having this particular entropy when it begins s 1. So, I am going to say this is my stagnation pressure P naught 1 and this will be my static pressure P 1 of course, I am drawing only part of the curve, the curve keeps going this is my actual point P 1 T 1 location is here that is where I am currently the state point.

From here, if I go to some length inside my duct, what should happen my entropy should increase, and my mach number will increase I said it is subsonic. So, mach number will increase which means my temperature will decrease, so I am going to go slightly lower temperature and higher entropy that is where I am suppose to go. So, it is going to go something like this, and I will keep doing this till M equal to 1 and we know that the change is much higher when I go to M equal to 1, you curve to us something like this.

If I want to pick a point P 2 in the middle, it will gone to some other P naught 2 here basically P naught dropped by some amount from P naught 1 it came down to a lower curve P naught 2. And then this will be my s 2, and this will be my P 2 curves, hopefully you understand that there are, so many pressure curves here they are all going to go like exponential increase curves I am not plotting all of them I just plot wherever it just needed. And you can see that this is the kinetic energy component and it has increased right, and I am going to say at this point it becomes star condition that is M equal to 1.

Now, if I start with this same T naught same s value, but supersonic I may be on this particular pressure curve P 1 prime I will call it, supersonic condition for the same entropy I will be setting here, which will correspond to very load T 1 I will call it T 1 prime. If I start here, I know in supersonic condition of course, in my as along my duct my entropy increases, but my temperature will also increase, why my mach number decreases mach number decreases temperature increases.

So, I am going to go higher temperature higher entropy, so I have to go this direction, and again we know that when I go closer M equal to 1 the sensitivity is higher. So, that particular curve looks like this, the curve look something like this, this particular shape is what is called a fanno line, fanno line based on the person who talked about it in his analyses fanno line, the Italian engineer called fanno.

Now, if you think about it I am here this will be my P naught star that particular point will be my P naught star for some particular starting condition, there will be a particular length up to which I can go and go to this particular curve. Till now, we said that there is a point where things will change, what will happen if I have a duct whose length is more than L star for that inlet mach number, I want to start from physical field then will go and give map for it, map is very easy physical field is more important.

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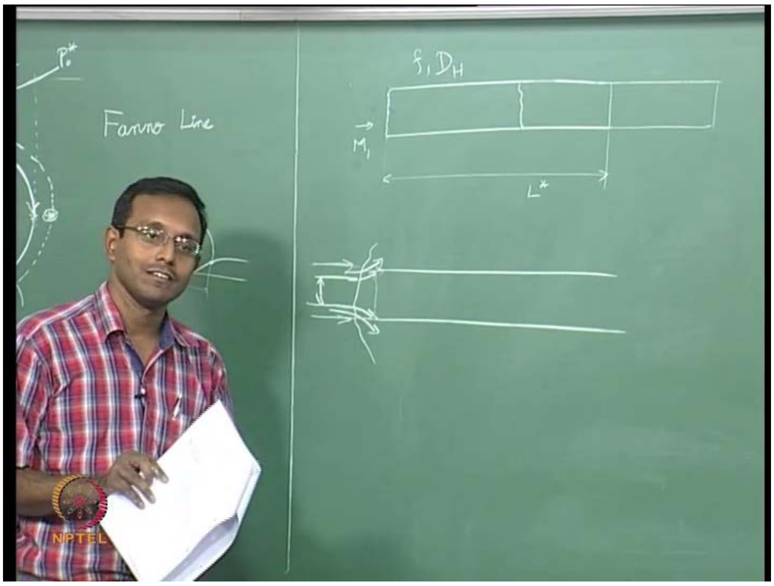

So, I am going to say I have some particular duct with a particular f and a particular D H, so and I have a particular M 1 here I do not care it is subsonic or supersonic, both have a L star. Let us say this is my L star for this particular M 1, let us say I have this if I extend the duct by some more, and try to keep the same M 1 in here, what should happen that is the basic question. What will happen I am having a mach 1 flow here, trying to go through a duct which going to resist the flow by friction on the walls, like there will be a shear stress which is going to oppose the flow.

What does at mean, it is going to oppose the flow means I am trying to return the flow I am going to slow down the flow, what should it do it has to send a compression wave inside. If it is subsonic flow and I am sending compression waves inside, what will happen compression waves always travel at speed of sound. What should happen say I just introduce this length L, all the same it was not there I just added this small tube extra on this.

Now, the flow has to go through this from the time T equal to 0 when I added this, it is going to be a set of compression wave that is trying to go in, what will happen they are going to go faster than this flow. They will keep on going in what will a compression wave do to a subsonic flow, when it is traveling this way it will decrease the velocity, right compression waves are going to tell do not go there come this way along with me right.

So, it is going to decrease the velocity and it will keep on going in all the way to a point where it will go to the front and change the inlet velocity here, it will go and change the inlet mach number. So, what will happen over all my flow velocity at the inlet is going to be lesser, density also will be adjusted accordingly based on the mach number, and finally, I am going to have a mass flow rate that is lesser. The mass flow rate that is lesser will what will be final situation.

If I go back and look at my curve here, this curve comes from all those expressions we have written till now today. Whatever we wrote today as a integrated expressions, will finely give you this particular curve for fanno line, the change in M dot will never be given by this solution. Because, during the solution we assumed that M dot is constant, so this is an unsteady phenomenon, which I cannot express directly from any analyses which I have done till now, unless I go to unsteady analyses I cannot get that we have assumed steady 1 d.

So, when there is steady I cannot go for this changing mach number, changing the length of the duct kind of analyses. All I can say is it jumps to a new M dot, I will just tell you the next M dot solution, where should it be can you guess if my mach number decreases I had subsonic flow and my mach number decreases, what will happen to my T 1, T 1 should go up, but it should look similar curve. So, now I can plot one more curve on top of it something like this I will draw dashed line something like this.

So, I am going to say M dot increasing this way as I go this way M dot increases, going in toward M dot increases this will be my curve. So, now, what happens is if I change my L star to be a little higher, length to be higher than the L star then the solution jumps from this curve to the higher curve. Such that, now it is all matched it and if I start from a lower mach number, what happens to my L max subsonic solution, it will be a higher how do I know that we just saw the plots right plots were like this.

If I decrease my mach number my L star increases further f l star by d we keep f and d constant. So, L star increases, so I go to a lower mach number as starting point, now my L star is higher that is the reason why the flow wants to do this, it has gone there. So, now, it chooses some particular curve, such that will come and make it just L star at that point. So, it will go on choosing till that point, it will not go any further above that is the idea here.

And similarly, if I go to supersonic it can do that, now will go back to that and look at what happens when it is supersonic. If I have a supersonic flow, and it is coming to M equal to 1 let us say here, and I suddenly increased my duct length there said this is my L star M 1 is now greater than 1 for a give f and D H I have some flow. And I increase my length, what will happen again there is going to be retardation of flow.

So, there will be compression waves going in when it is supersonic flow, when there is a bunch of compression waves going in they will come together. And form a shock and as it goes in it cannot keep going in because, at some point mach number will be higher than the mach number of this shock, it is a moving shock it will go and reach a point, where the moving shocks mach number will be equal to the incoming flow mach number and its stays there.

So, then I will go to a point where there will be a shock here, after that it is again 1 D shock right. So, it is going to be subsonic flow behind, now I have subsonic solution from here all the way to the end, if I increase the length further what happens one more compression or set of compression waves that travel in ward because, it is again retarding it. Now, it is going to go all the way up to here because, it is also subsonic flow it will come and each this point, now the set of compression waves is much more strong.

So, they can run against this flow a little more will go and stand where, again the new mach number of the shock will match the incoming flow and it will stay there. Shock would have run a little more ahead, it will go in if I increase my duct length more, and of course, if I keep on increasing the duct length there will be a point where the shock will go set outside this. And then I have only a pure subsonic problem inside this duct that can also happen.

Now, if I look at that in my plot I am going to say I started with this particular solid curve, and I went all the way to reach this point. But, we know that from whatever supersonic mach number L star is not very high right, so what will do it will go this path after some point, and then when I increase my L it will go from some mach number, jump to the same mass flow rate curve across here. So, that it chooses a subsonic solution there, we know that above this line its subsonic solution right it is subsonic solution there.

And when it is subsonic solution, now suddenly my L star is very high for the same f and D H right, I have this curve here right now. So, say I chose this particular mach number and from here it jumped to some other very low mach number right, this is M equal to 1 this is my supersonic thing, this much is the supersonic limit, so it is going to go very subsonic, it is going to be somewhere sitting there. From here it choose to go there what happens, suddenly my L star instead of this being the L star it is, now here.

Now, it can afford to have longer pipes, but of course, it does not want to generally take a very strong shock and go, it will chose only that enough show that much shock strength. Such that it will go to a point where from here it will go and reach this point, at end of the tube it is going to be M equal to 1. It will choose only bear minimum, why I go back and explain things from here, if I extend my length of the duct by some amount.

It will send only that much of compression waves, till it is matching and it will go to a point where M equal to 1 and they exit. More than that there is nothing more need to do right, so it will not do any more extra compression waves unnecessarily. Another way of thinking about it, if I send more compression waves then I am decreasing my P naught further.

Why will flow want to do extra work and it decrease it is own energy more, it will not do it P naught will not drop any more if I make it more strong shock than what is needed to make it M equal to 1, then P naught will drop higher unnecessarily it will not want to do that, so it will do only bear minimum keep it is energy safe that is what the flow will do. So, it will go to the point where this happens, there may be a case where it may have a shock from the beginning, and it will jump here and then from here it is going to have fully subsonic solution.

This shock at the inlet I said this is my inlet condition M equal to 1 M 1 M 1 prime here this is my supersonic mach number. If I increase my duct length any more, then the shock will jump outside the inlet, it will be in front of the inlet where I will still have the same mach number, it will go there. But, after that since the shock is outside I will draw another picture here, I will go till shock comes to the inlet if I increase my length any more, then shock will go sit outside here.

And now, it is completely subsonic problem and I am going to say length is not enough for this, what will it choose now it will go choose a subsonic solution there, it will just jump up and go from that direction after that. What happens? Remaining mass flow, it came in with the same stream lines this much of area should have gone through straight in ward. But, because of this shock sitting outside, there will be a part of the flow that will be turned out in a flow on flow behind the shock is going to be like this.

It is going to turn part of the flow out because, of which the mass flow rate inside will be lesser. So, only the flow between this region will be going in now, so my mass flow rate has been decreased, this called spillage if you go to proportion will you study supersonic index, they will talk about spillage and this is how the spillage happens, this is one of the forms of spillage of course, shock comes out. And if I increase my length more, then there will be more compression waves coming out making this shock even stronger will go sit more outside, which will make it more curved. So, there will be more mass leaking out more spillage, so that the net mass flow inside you see even smaller, this is all just physical field will go and look at how to analyses this problem numerically next class. Any other questions see you people next time.