Gas Dynamics
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Module - 16
Lecture - 36
Non-Isentropic flows-Crocco's theorem,
Fanno flow

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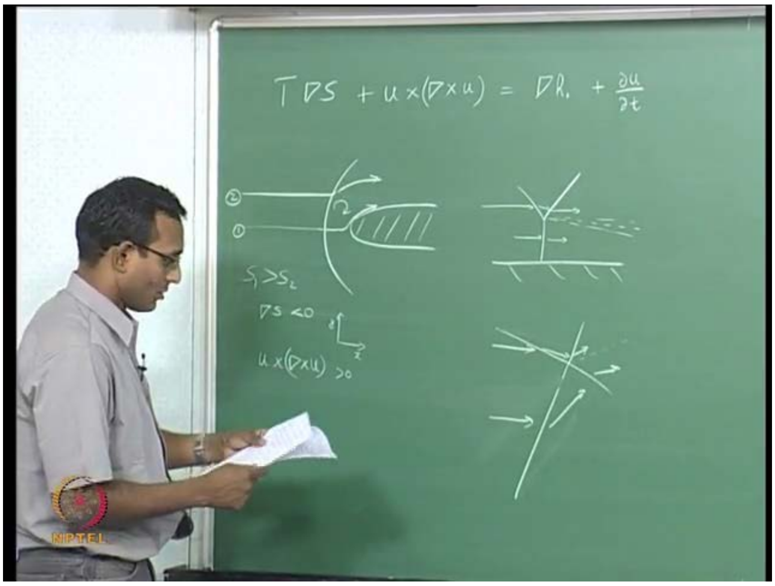

Hello everyone welcome back we were dealing with Crocco's theorem and I said that I will get back to you on examples in this class we defined the full garr equation or full equation for Crocco,s theorem as this I have such a case now if I look at different things even if I have isentropic flow and isenthalpic flow if I my velocity is changing in time I may produce vorticity due to that that is one more thing I can see from there even if I produce if I increase velocity even isentropically I am going to have a case where I will have vorticity produced that is one more statement from Crocco's theorem.

Now, we will see some examples say I have supersonic flow over a blood body what I am starting with I am starting with let us say 2 stream lines like this one stream line very close to the centre line will go like this and the other one is going to have immediate deflection and then it is going to go like this is the flow field we have if I look enable them as 1 and 2 I am of course, starting with the same enthalpy in both of them same stagnation enthalpy in both of them pre-steam condition is the same for both.

So, I am having no gradient difference and since that particular stream tube with this stream line in it is going to be having no change in enthalpy stagnation enthalpy because it is adiabatic flow of this flow through a shock is adiabatic anyway that is the only difference in this yes just going through that particular process here this particular stream tube around this stream line also will be h naught does not change along the stream line.

So, I am going to say h naught here same as h naught here that means, in this region if I look at it I am going to have del h naught 0 gradient and h naught here is 0. So, all same h naught assuming I am having adiabatic wall here I am assuming my model whatever I am putting here is adiabatic it may not be the case in real life if there is a aircraft going supersonic it may have some heat transfer in also we will neglect that for now.

If I think of such a case then I am having a condition and I say it is a steady problem I will neglect this dou u by dou t term also. So, whatever I have the right hand is 0, but now, I can say that entropy on the top layer is different from entropy on this layer which one is higher 1 is higher or 2 is higher center line 1 is higher if 1 is higher why is 1 higher this is the normal shock very strong shock while this 1 is very less strong shock it is more oblique as I go away from center entropy will be decreasing lesser and lesser and when I go far out somewhere out there it will become mark wave where entropy does not change at all that can also happen.

So, entropy change decreases as I go there. So, which entropy is higher after this shock condition is it 1 or 2 which one is higher we talked about entropy jump is higher which one is higher at the end when I look here after shock condition I want to apply this equation. So, what will be the condition let us say entropy initially is whatever 1 joule per kelvin or whatever some unit here it is jumping higher value here it is jumping lesser value what will be the final thing which one will be higher simply enough right nobody is thinking great I will just tell you 1 will be higher why we start with the same value this one increased more than that one. So, this will higher value compared to that value that is all.

Now, if this is the case how does my del s what will be my del s will it be positive negative. So, it is going to have dou y by dou x term and the dou y by dou y term let us assume that the change in x is very negligible we will just look at y term or that will be dou by dou y term what will it be dou s by dou y as y increases s decreases.

So, it is going to be less than 0 right hand side of my equation has both these terms as 0 I told it is a steady problem and isenthalpic problem. So, I am going to have only this particular part and in here I found that this is negative which means this term should be positive what is the kind of vorticity I am developing I have to match all these together that is all I am thinking what is the kind of vorticity I am developing this is higher velocity and its going there this is very low velocity overall I am developing vorticity like this into the board.

While my x and y coordinates are like this x y like this. So, I am developing z will be out like this right handed coordinate system this is my positive vorticity del cross v, but I am producing this. So, I am going to have del cross u negative del cross u is negative and I want to do u cross that u cross that will be like this. So, I am finally, getting that term to be positive. So, I am finally, having u cross del cross u greater than 0.

So, now I am finding everything is matching. So, now, I am going to say this is the reason why I am getting a vorticity that is like this in this flow that is what is finally, happening there this is one example I can go and go through this whole example for a different cases I would not go on to details of the other ones I will just tell you I can have a mark reflection at a wall of an oblique shock what is going to happen here there will be a slip line like this what do you mean by slip line we said there is no viscosity.

So, there is slip line, but now if we add viscosity then there is going to be a small shear layer sitting here what is happening inside their it is having vorticity if there is a slip that means, there is a gradient in velocity right which one it will go higher which one will have higher velocity upper one will have higher velocity.

So, I am again going to have negative vorticity I can go through same explanation it will give you the exact same answer finally, you can go through this problem also and get you get yourself convinced I can now think about the more crazy example one weak oblique shock and a very high beta oblique shock I do not want to call it strong oblique shock both are weak oblique shock if you think about strong verses weak, but one is a weak angle one is the high angle oblique shock they cross what does this mean this was probably produced by a wedge that is like this.

This was produced by a wedge that is more like this right what will be the final direction on an average I will draw a symmetry line somewhere like this right. So, the final direction will be something like this of course, I do not have any wedge like this very close to my flow field finally, it is going to look something like this is my flow field finally, I have incoming flow it is turned and it is going here this is my same incoming flow same incoming flow it turned more and finally, it turned like this.

If I think of this particular case also I can now show that this line will have vorticity across why I am having different entropies across these stream lines across these two lines everywhere else it is processed by the same shock if I pick you know un-stream here and un-stream line here exactly same entropy del s will be 0 there del h is also 0 it is a steady. So, there is no vorticity there every term in this will go to 0 if I go across this line they are processed by different set of shocks.

So, only across this line there will be vorticity only on the line there will be vorticity finally, because across this line there is del h naught 0 dou u by dou t 0, but del s is not 0 because of that there will be vorticity produced I have picked a problem again it is going to give the same kind of vorticity if I picked the top one as very strong it will be the other way vorticity, but I did not pick that anyways.

So, that ends the discussion on our Crocoo theorem is just to introduce that there is a mechanism by which we can prove that there is vorticity in a slip line that is all I just gave now we would not worry about this anymore we will just move on to adiabatic flow in a duct with friction present I am saying adiabatic flow in a duct because I can solve the problem for isothermal flow in a duct where the temperature of the wall is fixed or the temperature of gas is fixed that I can work on any particular way where there is heat transfer alert we will currently work with the situation where there is no heat transfer into or out of the gas.

Whatever enthalpy the gas has stagnation enthalpy it stays the same that is one thing and this is one problem where we are going to say viscosity is present the only other problem as we discussed viscosity has present was shear layer after that it is this one in the middle we discuss the energy equation and we know that because there is viscosity present we may have entropy production and because there is heat transfer there may be entropy production now we said adiabatic.

So, there is no heat transfer into the gas. So, there is only one way of producing entropy that is viscose forces that is all here we are going to have current that is coming from the discussion on the energy equation and entropy equation that was already done last class.

So, now when we look at ordinary flow fields typically we did this flow through a nozzle or flow of a jet in all those problems we do not consider viscosity as much and I told you that is the special case in a jet flow where there is the boundary will slowly have shear layer developing and it will slowly hit into the main flow other than that we did not use viscosity anywhere .

So, is that really valid if it is not valid it will not be in books . So, happens at for most common applications it works very well I am just going to say yes I can leave with not using viscosity all the time because simply because the inertial momentum or the forces inertial forces involved are very high compared to the my viscous forces that I am having in my flow system viscous force or the friction force is really small and it happens only at some corner of my flow where the fluid interacts with the wall typically .

That I will start neglecting this in my overall bulk assuming that my bulk flow cross section area is much more than the viscous interaction area which will typically be very small if my flow duct is big in size when my flow duct becomes smaller and smaller in size they become comparable you will see that immediately in after we derive the whole thing there is a particular called hydraulic diameter that will take into effect all this parameters naturally where we justified and not using friction in a nozzle flow in a way yes, but if you want to do more accurate results you still have to use that also in that friction with area change is the actual problem I have to study if there is a flow through a nozzle and if it is a heated wall then I have to consider heat transfer also along with it we will ignore all that currently we will just say in a nozzle flow we will ignore heat transfer or friction that is a simplest way to solve it which we did already.

But if I consider any duct even if it is a straight line duct just constant area duct if I have flow for long enough distances the net integral force I am generating through this whole duct against the flow will be high if the length of the duct is high for any small distance if I consider the net force produced by friction will be very small, but if my length of the duct is very long then slowly the integral of all the forces on the fluid totally over all the walls surfaces due to friction is eventually going to win over my other forces pressure force or inertial force or everything else.

So, typically we will start using friction only when the duct lengths are somewhat long how long is long it depends on the flow problem specifically we will go to numerical examples later on you will emphasize that fact later as of

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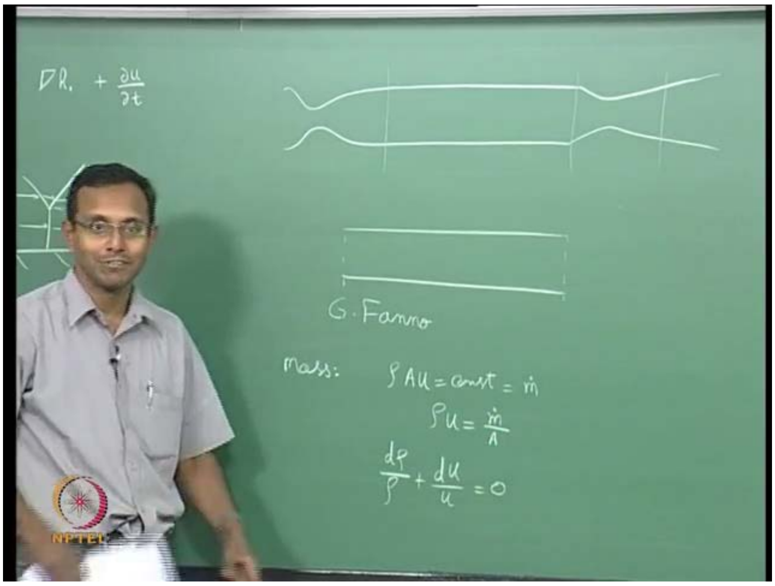

Now, if I am going to say I have a flow field like this very common we will go deal with it later a very common flow field like this. I am going to say for this nozzle like shape up to here I will neglect friction and this nozzle like shape here I will neglect friction and this intermediate region which is very long I will consider friction typically this is the simplest way to include friction very easy to do this one compared to friction along with area variation in this particular case I am more justified and saying friction is neglected here than this one because area variation is slow and it takes long time this length is not very small.

So, you have to think about that also you may not be always very well valid assumption sometimes it may not be very good and we are going to deal with in this particular course only constant area duct I will probably outline how to deal with the whole thing in a generalized notation later, but just a constant area duct we are going to deal with right now this particular problem was first formulated by

I believe his name is Gino Fanno anyways some G Fanno I think it is Gino it is an Italian engineer who did this for the 1st first time he did not get reorganization for his work till he died that is all I know about him he did not know that he is going to become famous. So, anyways this particular flow field is called Fanno flow it is commonly referred to as Fanno flow what are the assumptions I am going to say it is a constant area duct steady one d perfect gas flow in a adiabatic condition these are the assumptions in constant area adiabatic duct steady one d perfect gas flow in constant area adiabatic duct with no body force or mass addition.

I gave you I believe 7 different assumptions involved this is the overall way to look at this problem basically if I look at all the terms in my consideration equation you will see all of these terms coming out now, with all these assumptions since I said steady one d I can immediately go and use all my steady one d equations which we derived for any generalized flow before.

So, I will start from there I would not derive everything from scratch I am going to have mass equation which we already have as rho Au equal to constant in this case area is a constant. So, I can pull it out and say rho u is a constant that is the only special thing if I want to think about rho Au is also m dot mass flow rate ok.

So, I can rewrite this as rho u is a constant it is the constant is equal to m dot by a which we know is a constant anyway now from here of course, you can immediately go to the thin control volume we remember we did two types of control volume a big one and a thin control volume where you can just think about small changes it will come out to be differential form and differential form will be d rho by rho plus d u by u equal to 0 you can go and do it or you can we did this long back and just go look at it and say that will have a dA by A also along with it and the area does not change. So, dA is 0 then you will end up with this it is another way of looking at it you can derive it is not very difficult to derive mass equation is the easiest to derive a very now we will go for momentum equation.

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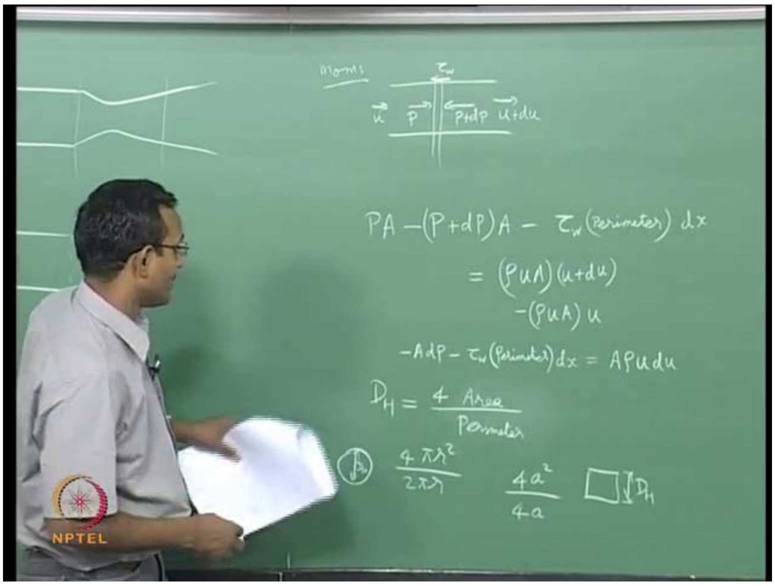

Now, I want to draw this small figure here I will pick this small zone this is my thin control volume way of looking at things I am going to say there is frictional force tau is the wall shear stress of course, I have to multiply this with the area of contact to get the actual force and I am having a pressure P here and a pressure P plus dP here and the whole flow is having a velocity here and this is going to have a u plus d u. So, you want to study this whole thing and of course, it has a density rho here and rho plus d rho there if I think about the whole thing I will finally, I would not go into the detail again I just have to tell that I am adding one more factor here which is PA into A minus P plus dP into A minus tau all into area of contact is the perimeter of the duct times the thickness of the slice I am using.

So, it will be perimeter into I will use as d x currently these are the net forces that are acting on my fluid element inside that small control volume now this going to cause the net acceleration I know my mass flow rate is constant. So, the acceleration is going to be mass flow rate times the velocity I can think of that as one way looking at momentum. So, I can now say this is equal to rho u A into u plus d u of course, I should actually think about rho plus d u and all, but that rho plus d rho into u plus d u is going to be same as rho u that is coming from mass equation.

So, I will just use rho u A here minus rho u A times u this is going to be my conservation form I will just convert this to differential form it will just simplify to I want to use P for perimeter, but P is pressure. So, I will just keep writing perimeter full all the time eventually we would not need it we will remove it into 3 lines we will have this particular form I see that A is present in two of these terms I will divide by A.

So, then A will go to below the perimeter may be I will introduce right here we will define something called Hydraulic diameter. Hydraulic diameter which is defined as 4 times perimeter divided by area when I say area it is the cross section area available for the flow 4 times perimeter by cross section area that is what I am going to have how do we get to this 4 this is the engineering way of looking at things not all ducts will have circular duct right not all will have circular cross section.

So, engineers used this and finally, found that if I use this formula I will get diameter for my circular duct naturally and if it is a square duct I will get the side of that square you can see that directly if I put 4 into phi r square divided that is my area of circle right phi r square I used area on the top one I did something wrong I am having wrong units right it should be 4 into area by perimeter.

I am having wrong units because of that perimeter in the denominator perimeter here that is why I am going wrong here phi r square is correct divided by perimeter which will be 2 phi r for a circle this is going to give me 2 r which is my diameter of the circle that comes out to be right if I take a square cross section then I am going to have 4 into some area some side is the a let us say 4 into a square divided by 4 a. So, it is going to give me a. So, if I think of a square cross section this is my hydraulic diameter if I think of a circle cross section this is my hydraulic diameter this is how they defined hydraulic diameter that is why you have this 4 just remember that then you will never go wrong in this formula the first mistake I made should never happen because it should have dimensions of meter I had a perimeter by area that will have one by meter assignment that can never happen.

So simple checks always keep in mind you are all basic engineers. So, now, I will use this inside here when I divide this expression by a I am having perimeter by area perimeter by area is 1 by DH just remember that we will start using that actually perimeter by area is 4 by DH not 1 by DH. ok

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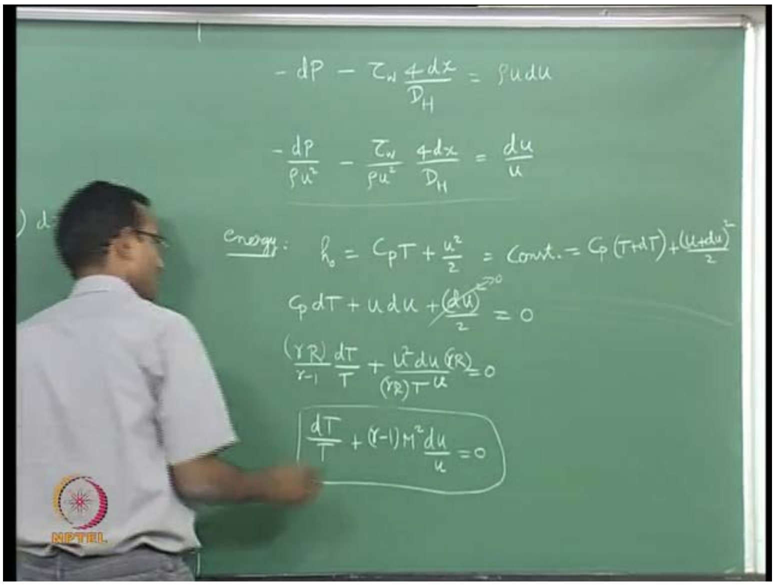

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So, if I substitute that I will end up with minus dP minus tau all 4 d x by DH is equal to rho u d u and end up with this particular form I want to write it similar what we did in mass equation d u by u kind of the form I want to write .

So, I want to write it such that it becomes d u by u. So, what should I do to make it d u by u divide by rho u square divide by rho u square the whole thing I can end up with minus dP by rho u square minus tau all by rho u square into 4 d x by DH equal to d u by u this is the situation I have I divided by rho u square can any of those terms be 0 you keep checking with this if there is flow u cannot be 0 right and density can never go to 0 assuming continuum that is.

So, you keep checking it when you are dividing something for the whole equation always make sure you are not dividing by 0 ever if you divide by 0 map does not hold anymore do not ever do that it can never be 0 any continuing situation that is what you have to think about. We will keep it this particular form we will come back to this later I will start using this after sometime these are momentum equation now I will go for energy equation of course, it is h naught equal to constant as I said it is adiabatic.

So, it is h naught equal to constant CpT plus u square by 2 equal to constant that is what I have which means this is equal to Cp into T plus dT plus u plus d u square by 2 is equal to that also from this part and the last part I can get some relation for dT by T etcetera. So, we will start taking this and the last part and comparing the terms we will see what is remaining what is remaining will be Cp dT plus u d u plus d u whole square by 2 these are the only terms that is remaining.

So, this will be equal to 0 this is what I have now I will say d u is. So, small that d u square can be neglected compared to all the other terms this is the standard thing we keep doing with the differential form is the standard calculation now I want to write this as dT by T similar to d u by u I will write this as dT by T. So, when I write it as dT by T I divide this whole equation by T and I will write Cp as in terms of R what it will be gamma R by gamma minus 1 I will put a T below now I have divided this term by T plus I have this particular form, but I want this to be d u by u. So, what will I will put u square and the u at the bottom that is same as just u on the down I have this particular form you want to rearrange this further if I think about taking this R to the denominator I will divide this whole thing by gamma R let us say I multiple and divided by gamma R here I am doing the whole thing in one equation I want to make it look good finally, that is all.

So, if I do that I can now say that this gamma R can never be 0 and just cancel this gamma Ris a common factor out of this that cannot be the 0 only the remaining term is 0. So, I will finally, end up with dT by T plus gamma minus 1 times M square how did I get m square gamma R T is a square u square by a square I have times d u by u equal to 0 now I have a relation between dT by T and d u by u this is one thing I have we will come back to d u by u d and d x later.

We already have from mass equation d rho by rho and d u by u those are the thing we have as of now, next I want to look at state equation and get a dP by P. So, as of now I use mass momentum energy.

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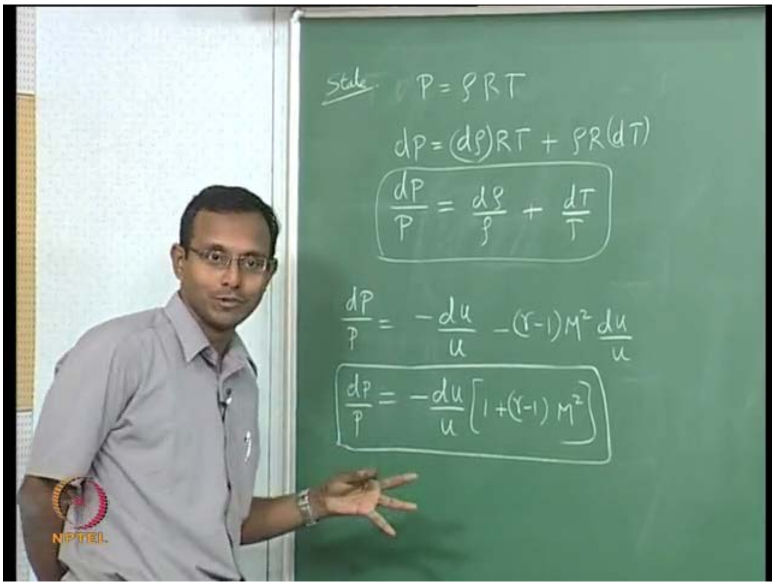

Now, I am going for state equation P equal to rho RT if Pequal to rho RT dP equal to d rho RT plus rho R dT write just differentiation by parts now I will divide this by P because I want dP by P where I use P equal to rho r p here. So, I will divide this by d rho RTt by rho RT I will get d rho by rho again this will become dT by T this is another relation I have.

Now, if I substitute this by d rho by rho mass equation what will I have minus d u by u I will have and I am going to substitute dT by T from energy equation in terms of d u by u. So, I will finally, end up with dP by P equal to minus d u by u and then I will use dT by T from there that will be a minus gamma minus 1 M square d u by u this is the expression I finally, get. So, I will pull out d u by u common minus d u by u common I will have 1 plus gamma minus 1 M square that is the expression I have this is dP byP.

So, now I have dT by T in terms of d u by u d rho by rho in terms of d u by u dP by P in terms of d u by u, but suddenly dP by P has a M inside and dT by T also has a M inside I do not know how m varies with u.

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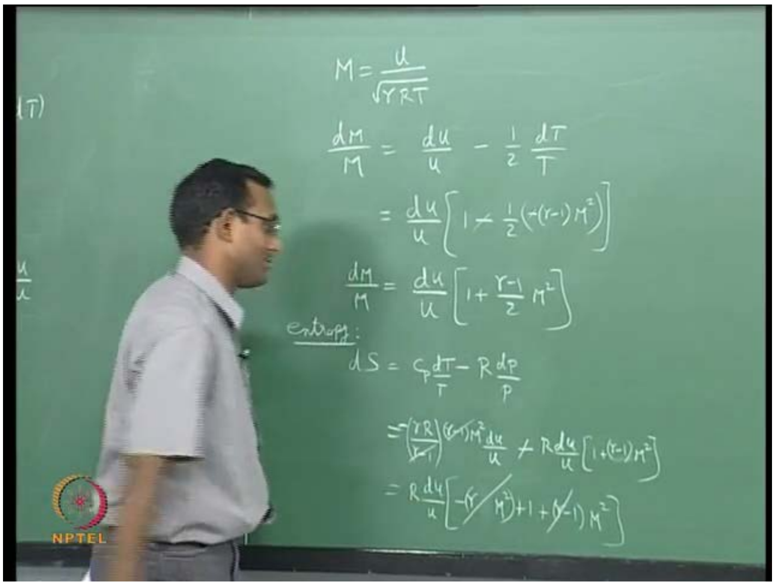

Let us find out that next what do we have M equal to u by square root of gamma RT. So, now, I can tell that I will just go a little faster dM by M equal to d u by u minus 1 by 2 dT by TI am assuming you can do that one more intermediate step just differentiate this and divide by M you can do that you will get to this particular form.

Now, I will substitute my dT by T in terms of d u by u here. So, I am going to have d u by u pulled out common 1 minus ½ times dT by Tt will be minus gamma minus 1 M square times d u by u this and this will get canceled it will become plus.

This is the term I have now if I look at it 1 plus gamma minus 1 by 2 M square you are getting. So, dM by M equal to d u by u into 1 plus gamma minus 1 by 2 M square this is the next term you are having (refer time: 33:00) now if I think about it for a given d u I can get a dM now I can get every other variable P rho T everything I can get if I know what is the change in velocity.

I have think about one more thing entropy because some of the changes may not be allowed we need to have d s greater than 0. So, now, I want to find d s by s in terms of d u by u that is the next term we want to do actually d s in terms of d u by u is also fine we want d s in terms of d u by u how do I go there we know d s I have to put entropy here entropy d s is equal to we have.

So, many forms I will write a form that is convenient for me Cp dT minus R dP by P actually it should be dT by T I will write it like this you can bring it to this particular form this is how you get the isentropic relations remember long time back we did pressure to temperature isentropic relation we got this form and this is when you integrate and put d s equal to delta s equal to 0 you get to that particular relation within P and T that sitting here.

So, I will start from this particular relation now what should I do I want to get everything in terms of d u by u start substituting everything else inside here. So, Cp in terms of R I will expand gamma R by gamma minus 1 times dT by T in terms of d u by u will be minus gamma minus 1 Msquare d u by u I put a minus in front of this bracket and I have a minus R dP by P dP by P we already have an expression there is a minus there. So, I will make it plus d u by u times 1 plus gamma minus 1 there is no by 2 here gamma minus 1 times M square this is expression I get from here.

Now, if you look at this particular term gamma minus 1 gets cancelled. So, if look at the remaining terms I will bunch out common terms R times d u by u is common look at the remaining terms there is a minus gamma R M square minus gamma M square is remaining and then I will look at here I pulled out R d u by u. So, reaming thing is plus 1 plus r cannot be inside right I pulled out R already R I pulled out already here.

So, it is minus gamma M square plus 1 plus gamma minus 1 M square this is what I have if I look at it minus gamma Msquare plus M square will get cancelled now it looks like this is one term actually this is just one term and I cancel that term.

So, the reaming thing is 1 minus M square that is the only thing left. So, this is equal to 1 minus M square times R du by u very important result from now on it is simple I just have to find what is going to happen with each variable simple thing to do.

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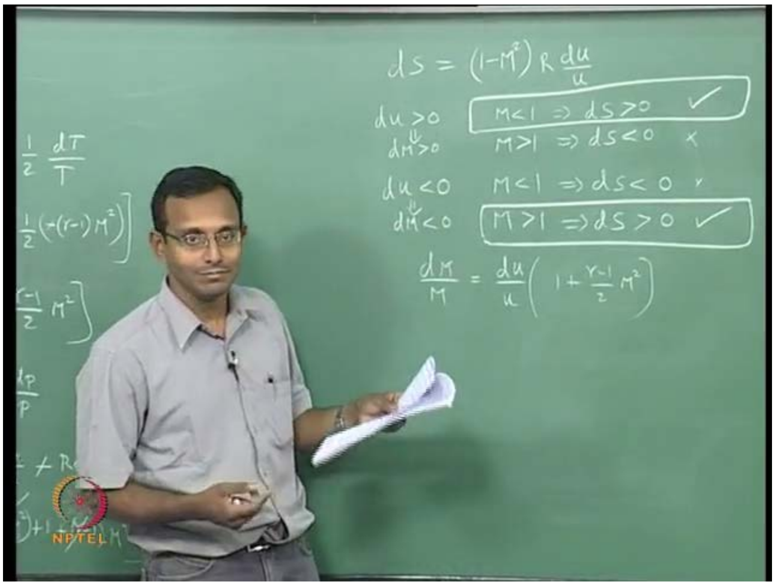

I have I will write it again here d s equal to 1 minus M square times R d u by u. So, suddenly we are seeing d s which is sign across M equal to 1 very important thing. So, now, let us consider a case d u greater than 0 if M is less than 1 then I will have d s less than 0 M greater than 1 plus d s greater than 0 this is what I am saying. So, the top case is not allowed this one is allowed we know only cases where entropy increases will be allowed in nature.

So, I am going that direction and similarly if we look at d u less than 0 if d u is equal to 0 all that d delta d terms will all be 0 d rho by rho will be 0 dT by T will be 0 dP by P will be 0 d s by s will be 0 if there is no change then I do not need to discuss any of this that is all it is saying we will consider there is always change there is no d u equal to 0 case now if I think about M less than 1 this is negative and M is less than 1. So, this is positive. So, I am going to get d s greater than 0 and the other one will be less than 0. So, here this is not possible and this is possible only d s greater than 0 are possible.

So, now I am finally, seeing that if my mark number is less than one wait I am having something wrong somewhere what is wrong I did whole thing wrong where is it wrong.

Student: d u should be greater than zero.

D u greater than zero.

Student: So, d u is positive.

M less than 1 d s is positive I did the whole thing wrong and you should have stopped me much before you are stopping me very late finally, anyway we are going back to this whole thing whatever inequalities I wrote everything is wrong if M is less than 1 then this is positive d s greater than 0. So, this is greater than this 0 this is less than 0 and so, this is less than 0 this is greater than 0 this is all it should be I just realized it did not match what I believe in Fanno flow. So, now, I am right we will leave it like this.

So, we are finding that d u greater than 0 that is I am going to have increase in velocity if my mark number is less than 1 if my mark number is more than 1 I cannot have increase in velocity. So, this is not possible this is possible when my mark number is less than 1 I cannot have decrease in velocity I can have only increase in velocity only when I can have decrease in velocity only when mark number is more than 1 these are the two things we are seeing which means I want to look at it in terms of d m by M also where is my dM by M formula dM by M equal d u by u into 1 plus gamma minus 1 by 2 M square if I look at it like this term is not going to be ever negative it is 1 plus something and gamma is always more than 1 it will never be negative which means whatever is the change in mu is the change in M.

Now I will tell this also means that dM less than 0 this means that dM greater than 0 that is if I see that my mark number is less than 1 it can only increase mark number if my mark number more than 1 I will first remove the cases which are I will just underline the case which are possible in nature only these two are possible in nature if I think about mark number greater than 1 it can only decrease if my mark number is less than 1 it can only increase dM is greater than 0 it only can increase which means my flow has a general tendency due to friction that it will always go towards M equal to 1 that is the common thing you need to think about my flow has a tendency to go to M equal to 1 naturally M equal to 1 is a special situation like this.

Now, in this whole analyses did I ever say anywhere that friction is. So, much this whole analyses whatever we did till now up to entropy I did not use my momentum equation at all right, but I got to a point which is possible which is not possible and all am I doing something wrong somewhere not really because there is a connection between I go back to this momentum equation .

I can replace this dP in terms of d u finally, I will find that d x dictates d u I do not know what my d u is as of now that is given only by d x and my tau all my the shear stress at the wall those are the things that decided as of now it looks like we can find all variables just from looking at this I cannot find any variable unless my d x is given because I need the value of d u to tell the new value of velocity I can just tell that trend general trend, but I cannot tell anything specific .

It so happens that the equation seem to be almost coupled they are all independent equations before look at it we had 6 equations and 6 unknowns mass momentum energy state entropy and mark number definition these are your 6 equations in 6 unknowns what are the unknowns pressure density temperature velocity mark number and entropy those are your 6 variables so you need all of them to actually solve the problem now the only thing left is to solve the problem I need to find d u that is where my d x comes in. So, I will go back and look at my momentum equation since I have it on the board I will start from there

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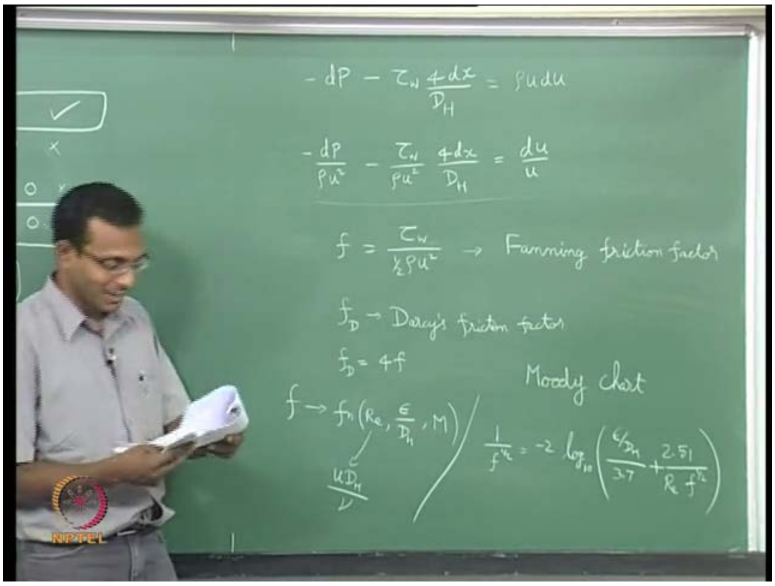

I will start from here. Now, I will define one new parameter called friction factor is defined as tau all the shear stress divided by dynamic pressure ½ rho u square standard aero dynamics based definition if you use this definition this particular f is actually called the Fanning friction factor this is different from another friction factor used in incompressible flow wall people who are doing hydraulics they have a different definition for this and they call there f as fD actually I call there f as fD which is actually Darcy's friction factor the relation between f and fD is fD equal to 4 f that is the connection between these two .

It so, happens that the friction factor f is actually a function of it is actually a function of Reynolds number the surface roughness of the wall with respect to hydraulic diameter epsilon is a surface roughness and it is normalized by hydraulic diameter and the mark number it so happens that this function is not very strong function of mark number it is extremely weak function of mark number. So, what we will say is I will use the charge used by incompressible flow people because mark number is not very serious function my f value will not be very much of.

So, I am going to say I will use this friction factor Darcy's friction factor typically in all in compressible flow books they will give you something called a Moody chart moody most likely somebody else I do not know his history right now mostly somebody else who actually gave it in a platform and incompressible flow people have given an expression for this f d there are 2 3 formulas they are all called colebrook formula there are 2 3 versions of it whichever one you want to use you can use they will all give roughly the same curve there will be small this way that way variations they all will be giving the same thing. In fact, the Moody chart is a plot of that colebrook book colebrook formula.

It so happens that it is an implicit function where the function looks like this I will just write the function here 1 by f power ½ is equal to minus 2 log to the base 10 of epsilon by dH divided by 3.7 do not ask me why it is just a correlation plus 2.51 I do not think it is a correlation may be they did something else which I do not know about times f power ½ again .

Where this Reynolds number I will write here is based on hydraulic diameter and viscosity of fluid u DH by mu where mu is your dynamic viscosity d is dynamic viscosity or kinematic I just get confused with that dynamic I think is correct kinematic I think is mu it that other way this is kinematic. So, this is kinematic viscosity and mu is where without the rho is dynamic viscosity. So, anyways so you can write it like this now if I look at the plot of this it is going to you would have seen Moody chart anyway I will just draw the plot here we have only 1 or 2 minutes we would not go any more than this.

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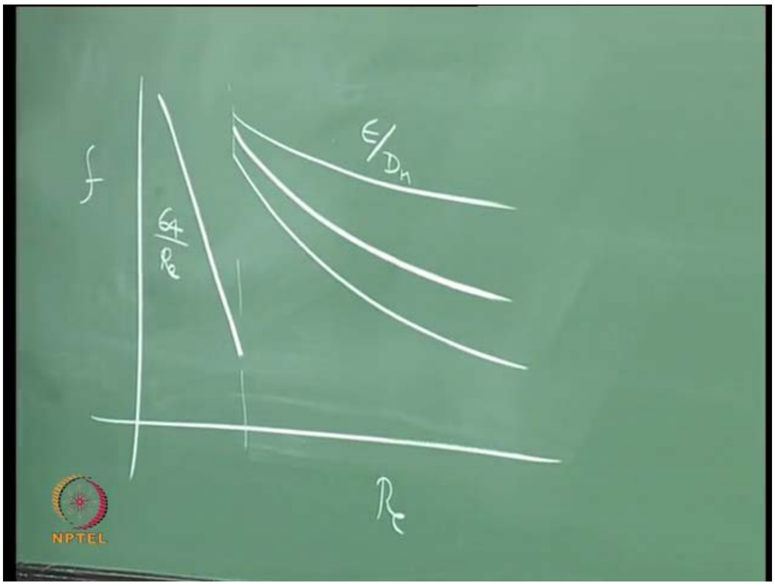

If I draw a plot of this Moody chart I did not want to put a Moody chart up for you guys you can go find this it is actually a log plot it is friction factor f versus Reynolds number for Reynolds numbers less than 2000two thousand or 2300 the transition for tube it will just go straight like this and suddenly it will go up after transition because turbulence increases friction and its going to go down something like this and depending on various values of epsilon by d can be any of this curves each of them is one particular epsilon by d I am just drawing qualitatively this is log plot both are log axis and this particular curve is 64 by Re if I write it as 64 by Re I am thinking about Darcy's friction factor there are some books which will give you 16 by Re they are talking about fanning friction factor defined the aero dynamics way that is what it will be.

So, depending on which book you are talking about it will change slightly is get use to this now after this we have just go substitute this f into the momentum equation I think we will do that next class we would not do that it will take some time I have to rearrange the terms and get it to a nice form I will take a little bit longer since the class time ended today we will stop here we will get back to that class yes.

Student: (Refer Time: 49:46)

No it is just d s its not d s by s it is d s by s it is just d s because it is having units of r it is just d s that is how it suppose to be its just d s d s is same units as r. So, it is going to be that if you want you can make it d s by r and it will become non dimensional less automatically yes.

Student: (Refer Time: 50:10)

This f is somewhat related to the skin friction corrosion which we use in aero dynamics its coming from there. .