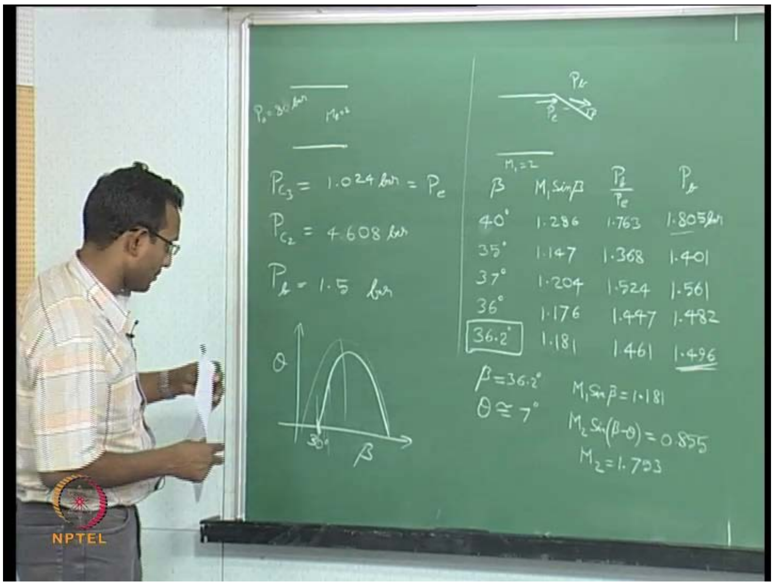
Gas Dynamics
Prof. T. M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 15
Lecture - 34
Supersonic Jet, Numerical Examples, Shear Layers Flow

Hello everyone, welcome back we were beginning to solve numerical example of supersonic jet problem.

(Refer Slide Time: 00:20)



We said that we have a P naught of 80 bar and M exit is 2.0, and for this particular condition we found that P critical 3 was 1.024 bar which of course, will be equal to the P exit at the end of the nozzle, when it is fully expanded. So, I just said it is equal to P exit, then we also did P critical 2 and we got a number 4.068 bar and so we wanted to pick a case where there will be a over expanded jet. So, we said the value should be something in between these two, to not to have problems with mach reflections at the center axis, we pick a case closer to this end.

So, that our shocks will be just clean crossing like an x, so I pick the P back pressure of 1.5 bar, so now our job came down to I have to find that particular angle beta for this shock, which will make sure that P back is the pressure of downstream of the shock. And upstream is your P exit, this is what we have to find out that particular, toady I make mistake it is 4.608 not 068, thank you for that 608 that is the correct number. So, we were discussing last class and we said that, theta beta m curve looks like this, for any oblige shock for particular mach number.

And now we have to look for some solution, reasonably close to that pressure, because it is not very far 1.5 somewhat close to 1.024, which means am most slightly going to be in this weak reason. And so I have to guess some beta value inside this range, and we said that this value was 30 degrees, how do we know that, that is the sine inverse of 1 by 2 that 2 is our mach number, that is your mach angle. So, I want to guess some value inside here, if we go look at mach two condition this number will be around 63 or something peak value, we would not worry about that, currently will just look at numbers something in between.

Let us pick a value of beta 40 degrees I could pick 50, but I know the answer should be close to 30, so I am going to pick something closers for easy conversions, so I am going to pick beta value 50, also I said 40 not 50, I will pick a value 40 degrees. If I pick a beta value immediately I can find my M 1 sine beta, that is normal component for that particular oblige shock, which is what matters to find the P 2 by P 1 across the shock. So, I will find that, and I know my M 1 value is 2, which is the exit mach number at that end of the nozzle.

So, M 1 sine beta happens to be 1.286, I will take this number as my mach number and go and look at normal shock tables, I will find that P 2 by P 1 or I will call it P b by P exit here, I will find P b by P exit that happens to be 1.763. So, now I will find my P exit, multiply by P exit I will get P b, P b I will find that number is 1.805 of course, it is in bar, so that is the number I am going to get there. I see that the pressure achieved by this oblige shock is higher than the P b we actually have which means, the shock angle is too strong, it is too normal, it is more normal, normal is higher strength shock we want to make it less angle, beta should be lesser.

So, next value will guess 35, I know 30 is no use, P 2 by P 1 will be 1 for 30, if P 2 by P 1 is 1, then my P b will equal to 1.024 that is lesser, so I want to pick a number in between 35. So, for this M 1 sine beta is 1.147 and I go look at normal shock tables again gamma equal to 1.4 table I get that value 1.368, and my pressure happens to be 1.401, so

I am close, now I know the answer is in between these two numbers and closer to the bottom number, I want 1.5 it is like 1 4th from here this way up.

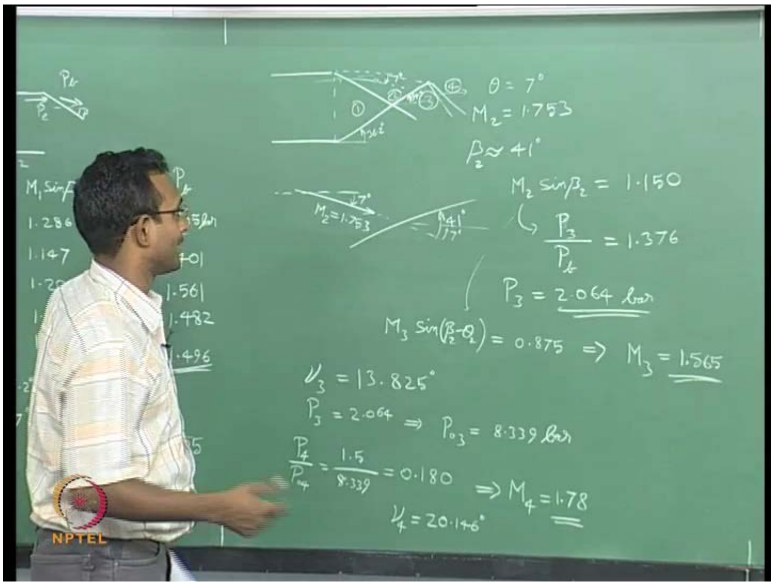
So, I will pick a number something close to that 1 4th from here will be 1.25 added I will just make it 37 degrees, will try 37 degrees and see what happens, that 1.204 and then, next number is 1.524, so I am getting 1.561. So, now I see the answer is between 35 and 37, logical thing to try 36 number is 1.176, 1.447, 1.482, so the actual answer is something in between these two very close to 36 degrees. So, tried this 36.2, just guess I tried this I know the answer should be closer to 1.48 then 1.56, then I am getting 1.181, 1.461 and this is 1.496, and I do not want to write it any more than that, I stop that.

Because, my theta beta M curves which we have, we do not have so much accuracy in our theta values, when I go for beta values of this case, typically we get numbers something like M equal to 2, M equal to 3, M equal to 4 like that. And the beta the resolution typically is every 2 degrees and I am thinking about 0.2 degrees here, which itself, I cannot read from this kind of chart. So, I know really have to go to this accuracy, I should be able to stop at 36 and say it is close to 36, I just went a little far and when I wanted to find, and I am going to say this is my beta value.

Now, of course, next thing I want to find is the how much is the velocity deflected function, how much this turn by, so I have to go find out that value from that chart and I am getting that value to be 7 degrees; it is rough estimate. I will put this approximately equal to sine value, it is just the rough estimate, I do not have resolution there to find this from the chart. So, now I will go do this whole analyses, I will go through that full oblige shock calculation for beta equal to this value and M 1 equal to 2, so my M 1 sine beta is 1.181, M 2 sine beta minus theta is 0.855.

How did I get this number 0.855, normal shock tables looking at this number as the incoming shock, find out the outgoing shock that should be the number put here, outgoing mach number. And now I have to get the actual M 2 out of that point, I know my beta and theta I can get M 2, that M 2 value happens to be 1.753 which means, I started with M 1 equal to 2 and my M 2 happens to be 1.753 after my oblige shock. You done this kind of calculations already, so I am just going a little faster and also it involves lots of alterations, so I just went a little fast.

(Refer Slide Time: 09:21)



Now, I want to go a little further into the jet, I have this particular , now I know that I have a shock which is having an angle 36.2 degrees, which is turning the flow by only 7 degrees very small angle change that is all it is turning by that angle change is 7degrees that is all it is doing. I know my pressure here now, this region 1, region 2 I know the pressure there, which is our P 2 which happens to be equal to P b back pressure, I want to find region 3. How will I find region 3, I just have to go for, I know my mach two there, I have a shock which will turn it back to parallel to the axis, what will be that angle again 7 degrees.

So, in this case I am given theta equal to 7 degrees, as the input and M 2 is given to be 1.753, now I have to find M 3 here, again oblige shock we have to just go through the whole process. This is a easier 1, because now I can go look at the theta beta M chart for 7 degrees and 1.753 I will just interpolate between two curves, and I will get to a number and that happens to be beta approximately equal to 41 degrees. Now, I want to be very clear about what this 41 degree means really, I have a flow with respect to horizontal having 7 degrees this is the flow direction, and my mach number here is M 2 equal to 1.753.

And with respect to this flow vector direction I have a shock which is 41 degrees, remember the beta is always with respect to the stream line direction, from there it is 41 degrees. I have to draw it such that, it is with respect to horizontal some value, let me find that number from here, how will I find that what is this angle, same as this angle, it is parallel lines ones one line cutting through, I believe this I did this geometry stuff long back in 6 degree, 6 7th grade somewhere there. So, it is 7 degrees there, total is 41 this part is 7 degrees, so the remaining is your answer 34 degrees.

So, actually if I want to draw the shock here, this angle is only 34 degrees, but what the flows is 41 degrees, so now I just have to go and find pressure here P 3, again simple stuff I just have to find M 2, I will put beta 2 here, because this is the second shock. M 2 sine beta 2 that value is 1.150, this will from normal shock tables give me P 3 by P 2 or I call it P b both are same, P 2 equal to P b any way. That value coming from this particular mach number normal shock tables, 1.4 gamma 1.376, since I know my P b immediately get my P 3 it is 2.064 bar, this is the pressure at that point.

We know that my outside pressure back pressure here is 1.5 bar, pressure in the region 1 or at the exit of the nozzle was 1.024 bar, from here it went up by a 0.5 bar more, from there it again went up by 0.5 bar more, 1.4 became 2.0 again 0.5 bar more it went up. And I want to continue the calculations, so I have to find out the mach number at 3, M 3 sine beta minus theta beta 2 minus theta 2, again from the same normal shock tables I will get this information also, and that is 0.875, which tells me that my mach number as 3 is 1.565. That is I started with mach number 2 after the first shock it became 1.753 after the second shock it became 1.565 those are our numbers currently.

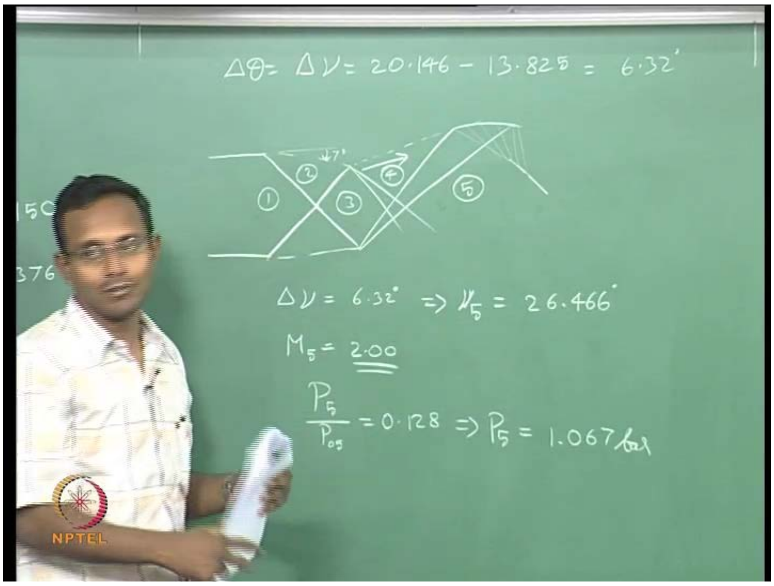
Now, you want to the calculation even past this point, now I know my dotted line if I extend for my jet boundary, when the shock reaches there, after that there has to be some expansion fan here such that, the pressure in this region 4 is same as my back pressure, the outside pressure. So, from 3 to 4 I need to have expansion fan, simple calculation after that we know expansion fans are easier to calculate, I have my M 3 to be 1.565 which gives me my nu 3 to be Prandtl-Meyer angle nu 3, 13.825 degrees.

I am going to look at pressure at 3, 2.64 and I want to find the final mach number such that, the pressure will be equal there, how will I find the mach number rom knowing the pressure, have to say that that region is isentropic. If I have to say that region is isentropic, but I am gone through shocks in a middle it cannot be the same P naught as before, I have to find the local P naught, how will I find local p naught I know the pressure and I know the mach number.

So, I have this pressure and I know the mach number M 3 there, from this I will get my P naught 3 as 8.339 bar, now you can see from 80 bar it came down to 8.339 bar finally, that is what you having at the end, we started with 80 bar P naught. Now, we continue from here I know that across this expansion isentropic relations, so P naught 3 is equal to P naught 4, now I know P 4. If I know P 4 than I know P 4 by P naught 4, P 4 by P naught 4, P 4 is 1.5 bar divided by P naught 4 8.339, this value happens to be 0.180.

So, now I will go to isentropic tables, isentropic compressible four tables for gamma equal to 1.4, it is going to give me a mach number at 4 as 1.78. How much did it turn the flow by how will I find that, again I have to find Prandtl-Meyer angle here, so I will find nu 4 for that mach number, that happens to be 20.146 degrees, this is a number I have there 20.146 degrees. So, now, I can find out what is my actual change in angle delta nu equal to delta theta that is our Prandtl-Meyer expansion.

(Refer Slide Time: 17:57)



So, I am going to say my delta theta equal to delta nu equal to 20.146 minus 13.825, which happens to be 6.32 degrees, you have around 6.32 degrees there. So, if I draw my picture beyond that point, initially this turning angle was 7 degrees, now I am turning lesser 6 degrees, it is not turning by the same amount, it is turning lesser 6 degrees and this expansion fan goes beyond that after that we know. So, I have solve for region 1, region 2, region 3, region 4, now I want to go to region 5, I want to go and solve region 5 next.

How will I solve region 5 it is experiencing an expansion fan started from here with this particular pressure condition, as the main information given by the expansion fan 3 to 4 which means, it will operate with the same delta nu and the same delta theta. Because, it is the symmetric system, if this pressure is different from that pressure, then it will do crazy things currently we are taking it as symmetric system.

So, I will use this same delta nu here again, which will mean that my angle of 6.32 up will come back to 0 again, so it will turn back to 0 at that point. So, my delta nu is again equal to the same value 6.32, this implies that by nu 5 should at increase or decrease nu from 4 to 5, nu will decrease why I am going to ask why even you said increase, it compressing from here to here, 4 to 5 it is expanding. Because, it is experiencing this expansion wave from here to here it is still expanding, this experiencing an expansion wave, so actually what should happen to my nu mach number increases.

So, what happens to nu, it is a monotonic function it is going to go same direction and it is going to increase, so I am going to say this will further increased, so my nu should be the previous nu for 20.146 plus this 6.32, so I am getting a number 26.466 degrees. Now, I want to find P 5 for that I need M 5, which is easy to do from Prandtl–Meyer angle I can just go to my Prandtl-Meyer function tables, and get to a mach number. Mach number happens to be 2.00 which means, my P 5 by P 0 4 and P 0 5 were the same again, I will put P 0 5 here P 0 3, P 0 4, P 0 5 all are the same, because these processes are all isentropic processes P naught does not change.

So, I can find this ratio for this mach two conditions is 0.128 and that is going to give me my P 5 as 1.067 bar, so have you ever seen this number before, last class when we are solving this P critical 3, we got this exact same number 0.128. We were looking for P by P naught for M equal to 2, one more thing thought is we started with M equal to 2, we ended with M equal to 2, that is how it just keeps going you have to go up and down up and down across there.

Ideally I should get back to 0 to 4 I did not get 0 to 4 I got 1.067, this is because of all those round of errors I had and accuracy in interpolation and all those problems will be there. Ideally will just go up and down up and down across this, if I solve for one more cell exactly it should go through the exact same process again, but after this point it is more difficult to solve the problem. So, we will currently stop at this stage, this itself is too much for this particular course by think about, it is an undergraduate course I do not need to solve after this point, I just thought let us go a little more beyond any other book.

So, we can solve all the way up to one full cell of my expansion compression cell, so will solve at this point we would not go beyond this, because after this what I have to do is find out the strength of these compression waves. And tell me where it is going to meet at finally, form an oblige shock that region is a very non layer region and it is not easy to explain it to people right now, so we will avoid this whole thing, if I do this I will at explain five classes, we will ignore that further, it is not worth it.

So, this is a same process I can follow, if I want under expanded jet which will start like this, exit pressure will be like three, condition three from here I will just go and suddenly say cannot go pass this point currently, I do not know how to solve for compression waves coming together and forming shock, we will leave it there for now.

Student: ((Refer Time: 24:04))

It is not point, we started with 80 bar P naught, how did I make mistake somewhere.

Student: ((Refer Time: 24:17))

0.1 of this may be I did 8, so it looks like I did 8 bar and not 80 bar, maybe I made a calculation mistake somewhere, if that is 8 bar then I want to see the other P naught, other P naught is going beyond 8, it cannot increase I do not like that, so I am not sure of this part right now, maybe I will go and check again. 8 into 0.1 is 8, should be more than that, so I made a mistake somewhere 0.024 does not seem to be right.

Student: We will get 1.024

We get 1.024, so I have used 8 only it looks like, then how did I get a P naught more than this value that is my next question, it should not be going beyond that, it is possible that I had made some calculation mistake somewhere. Let us assume that this is 8 bar for it to give this, it looks like 0.128 into 8 will give me 1, 0.125 into 8 is 1, so it is closed, so I agree most slightly I used 8 bar instead of 80 bar, I made a mistake somewhere in my calculation.

So, from here I have come all the way up to here, and when I find my P naught 3 here this looks like it is more than my initial P naught that can never happen. So, most slightly somewhere along the way my beta approximations are hurting me, I have approximated to 41, I have approximated the previous one 36.2 where is it, somewhere here it is 36.2, this kind of approximations are hurting me. And I have used theta approximately 7, if I use 6.5, may be it could have be better I do not know right now what it is exactly, I did not go through this from my numerical code I just use my charts, and so it is doing this kind of result.

If I go and use numerical codes, probably I will get a little better result than this, I wanted to do it as if you guys can sit and solve in your rooms. So, I used only whatever equipment you are having access to and that comes out to be some number like this, and that is the reason I think most slightly it is having a huge error 8.339, should actually be slightly less than 8. Because, each oblige shock will decrease P naught by a small amount, as wondering why it was dropping from 80 to 8, it cannot drop from 80 to 8 to slightly less than 8 it is what I expect.

Maybe it should be 7.9, 7.8 kind of number, but because of this small errors, it is glowing up to some higher number currently, I rounded of the wrong direction. So, when you are getting results like this of course, you have to now think about whether your calculation are accurate enough. Now, you can say that you are error band is really wide, we are getting results more than what is possible theoretically, so that is not possible to happen ever, there is some error somewhere in that.

And it is must slightly, because I approximated beta from looking at a chart which does not have so much resolution, my grid levels are like 2 degrees, am resolving it into 0.2 degrees, 0.5 degrees kind of numbers, not very good that is where we had problems. Now, I will stop jet discussion right there, all this time we did not think about viscosity in my gas and I said that, if I have viscosity my jet I have a picture here, so I will just use this. My jet boundary is something like this and then, it is going out like this and then, it is turning like this, and I said that it will go back down and it will go back up, again have the same cell structure, this whole structure keeps repeating.

And I said it will repeat forever how many ever cells like this, but if I have viscosity present, then this particular line along which the fluid is going, it has a fluid element below and it has a fluid element above. We know that fluid element at two is moving with whatever velocity corresponding to mach number 2 some value, it will be something pretty high, not going to be very small. While the outside is suppose to be stagnant gas just sitting ideal, if that is the case, it is going to be a very high velocity gradient at that point.

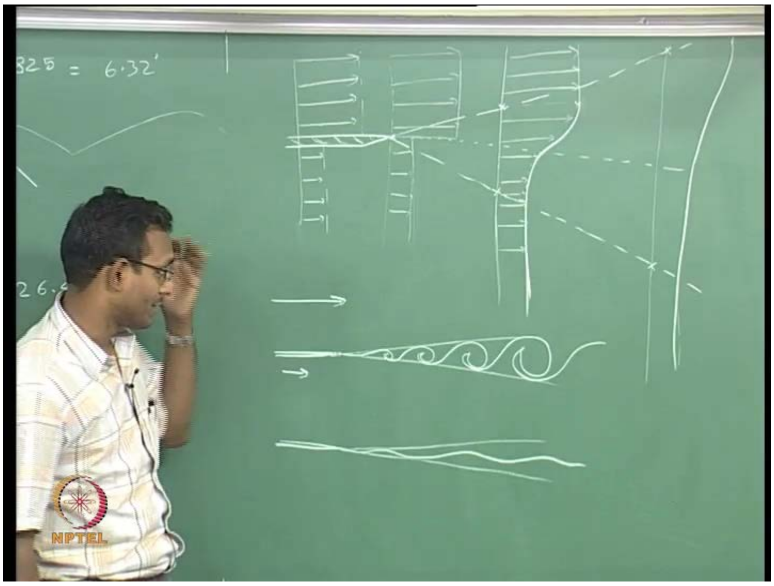
If there is such a high velocity gradients at that point, you can now immediately think about vorticity being present there, vorticity is curl of del cross of V vector. So, immediately I can now tell that there is a non zero votricity present there, in real life if there is concentrated vorticity present somewhere, immediately viscosity vector will come in and distribute this votricity equally all over the place. It is like you put a drop of ink in water, immediately defuses out to all the regions finally, you do not see any ink in water.

The whole water is having all the ink dispersed everywhere equally, till that time the diffusion happens, similarly here viscosity diffuses the vorticity, the only place where you will see serious amount of vorticity is this edge, jet boundary along. So, you are going to have that special case called a shear layer, where is that one layer of fluid elements, where you going to have very high concentration of vorticity distributed along that line. And because of that, we are going to have viscosity in real life, now I am no more in my ideal gas dynamics world, I am trying to include my viscosity in my gas dynamics.

If I include that then you will start having some momentum mixing happening across this region, I do not want to go disturb this picture any more, will just go a separate picture. This has nothing deal with compressibility any more, I will just say it is just fluid mechanics, viscosity does not care about really so much of compressibility, viscosity is just something that worries about high concentration of vorticity somewhere. And low concentration of vorticity some other place, that is not like by viscosity, think about it that way.

One place very high vorticity concentration, other place there is no vorticity, immediately vorticity will be taken one point and given to the other point, it wants to make every where equal vorticity, which will eventually come down to every where equal velocity distribution. If there is any velocity gradient, there is some vorticity that place, that is what we are trying to achieve finally, that is what it will end up with as a final situation. That is the job of viscosity in nature, it is diffusing velocity differences or momentum differences among fluid elements of different layers, that is it is job.

(Refer Slide Time: 31:27)



So, if I look at a simple problem in fluid mechanics, ideally I should not be teaching this here its fluid mechanics course. I am suppose to be teaching only compressible flows and I will anyway I give you this, so that you have that background when I start talking about supersonic compressible shear layers. If I have this velocity as something high, this is my velocity profile there, and this velocity being something low and these two fluid elements meet, when it comes out.

And I assuming that the plate is extremely thin compare to the dimensions I am interested in very thin plate, I do not need to worry about the thickness of the plate, when it comes out at this point, I am going to still have this same velocity profile. If I think about vorticity in this region, there is no vorticity at all in this region, because all the velocities are exactly the same del cross will become 0, same thing on that side, except for that one line the midpoint, the center line. That will be the only region where you will have non zero delta del crossing, that is where you have going to have vorticity distributed.

And all the fluid elements are going that way which means, I am going to have over all some convention of this vorticity distribution, as it is going that way. While it is getting convicted, viscosity comes into play and it wants to take that vorticity and distribute it every direction, up and down all the direction. Every other place it is 0, only one guys having all the vorticity viscosity does not like it, so it will just take it from that person give it to the next person, immediate next person.

And it just diffuses out from there and everybody eventually has same amount of vorticity, if there are so many fluid elements, they will all have almost 0 vorticity, it goes still there. Now, immediately some distance downstream, I will draw a dotted line going slightly downward will come back to that later. I will have a velocity profile looking something like this, far away the values are same as before, even here far away the velocity values are same as before, but now there is a whole bunch of fluid elements which have a velocity gradient.

Of course, still the highest velocity gradient is at the center line, that is what it is still, but I have drawn at the center line as dipping slightly, we will worry about that later, I am currently assuming that the pressure for the fluid above and the fluid below are the same. That is typically the way people think about shear layers, I think about shear layers there is you are going to think about pressure are the same, static pressure are the same. Now, eventually it is going to look something like this, and in the middle the velocity profile is changing.

If you think about at the center typically it will be half the value or the average between that value and this value, it should be the midpoint typically. And if I go further down, there will be another region where it will be far away it still the same, far away it still the same, but everywhere else this is having that vorticity. So, it is going to look something like this, it has distributed much more uniformly, now if I think about drawing a boundary for these things.

I have to define something as my boundary, what we do in boundary layer will tell that at the edge the velocity is same as far away, I will use the same concept here, when I look at far away on the top that value same as some value here. So, I am going to call that my edge, that point is my edge which means, that corresponds to this location here in physical space, this is just a profile remember. And similarly when far away here it is some value and as I come closer it is deviating and going away at some point, that point I will call as my edge again, I have mark two points.

Now, I will again go and do the same thing for the next profile at some other the downstream location, somewhere far away out here and out there it is somewhere there let us say, it is all only symmetric I may not be going same growth rate, if you are interested in that. What will it be at this first point, there both at the same location, there are merging, because at that point every where the velocities are the same, from the top there all the same just when it is one point above, same thing one point below everything is same as below.

So, it will be almost that point both the axis will be meeting at that point, so now I will draw a dotted line through all this, dashed line through all this. And similarly here, this particular region or the band of fluid elements I am considering, is the only region where viscous diffusion or vorticity redistribution has happening. Outside this I am going to say the velocity gradients are almost 0, same thing below almost 0 velocity gradients, so there is no vorticity.

So, this particular region is called the region of shear forces or the shear layer, this layer is the layer in between the two regions where there is not much of viscous forces, so this inside in between region is called the shear layer. And if you go do more of a fluid mechanics course, we will start seeing that, now I will go for a different picture I will draw very thin splitter plate. And I am going to draw more realistic shear layer growth, that looks more like this it will not be typically that, so I wanted to talk about this going down dotted line typically, when we meet the pressures the same.

The velocities are different, the P naught for the top and bottom are the different values, which one is higher upper one is higher, guys moving at a higher velocity for the same static, so it has more energy. So, eventually it is so happens a starts pushing it down, so the dotted line is the actual boundary between the two fluid particles, let us say that the top is oxygen bottom is nitrogen, this lines will be the line separating nitrogen and the oxygen. And am still assuming that there is no species diffusion across that, ideally every diffusion happens across that, in real life diffusion always happens; diffusion is a natural process to make things equal everywhere.

Now, if I look at case like this, if my shear is not very strong that is my delta V across these two layers, the top and bottom the velocity difference if it is not very high, then it will just be growing like this with velocity profiles just going simply like this. But, if my strength of the shear is higher and higher, it is I am saying the top is moving much faster than the bottom, if I pick such a case. Then I will have a situation where the strength of vorticity is so high, that there will be some non-linearity is coming in fluid mechanics, vortex roll up will start happening.

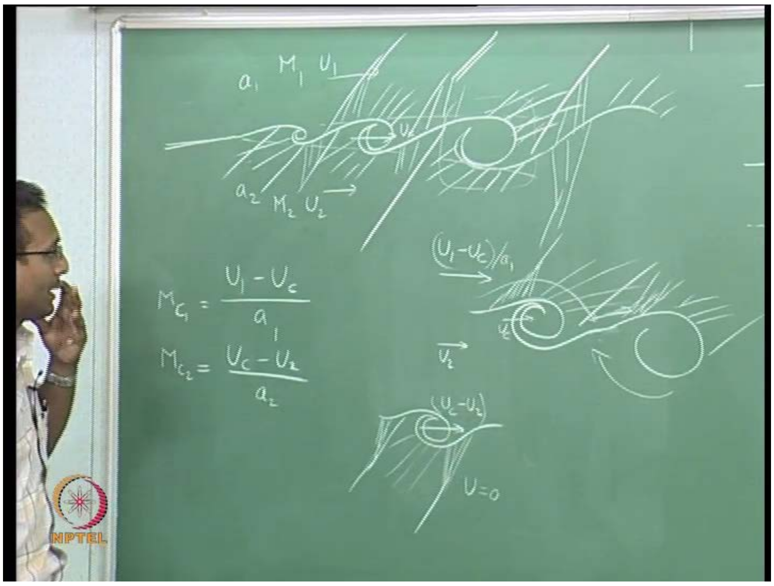
You would have seen this in so many places, I am just going to draw this, this a nice picture I can draw you would have seen this in so many places, fluid mechanics typically you will see something like this. What is really happening is I am assuming the top is going faster, bottom is going slower that is my assumption, otherwise it will not be turning down like this. Otherwise it will just be a flipped up image, it will just go turn the other away, if this is bottom is going faster, it is going to have something like this.

Now, I will tell you the main thing about this compressible shear layer, the same thing happens in compressible flows also, viscosity does not care about compressibility, they are independent aspects of nature, they do not want to worry about each other. There will be a non-linear effect if the things are too strong, we would not worry about that part right now, well a now is this can be independently thought about without worrying about compressibility effects as of now.

I can tell you that this kind of role up will happen, if there is enough difference between the velocities to begin with, if the velocity difference is not too high, then I will draw another picture it will still grow like this. But, if I see that particular center line like whatever I have drawn here it will just be something like this, it will just do this just oscillate it does not rolled up yet, which just beginning to become one step. If they are almost close to each other, not much of shear there, not much of shear layer there, shear layer growth will also be really small and all that will go much slower.

I just talked about simple shear layers right now, will now think about compressible shear layers, and particularly I want to talk about these cases where large scale structures are formed will pick such a case. That is I am going to say one is going extremely fast compare to the other, then I am going to form these large scale structures, typically some rotating eddy like structures, they are just going to be rotating as they are moving forward. When this one moves forward to become this, there will be one more coming from here to form something like this, I just replaces every now and then, this a particular rate at which it is going to be shut. There is so much to it, we want talk about all that here directly.

(Refer Slide Time: 42:01)



Now, people wanted to understand this in compressible shear layers something like this, and I am going say that is my shear layer, and I am saying that it is having some large scales structures that are growing as they go out. And the vorticity strength will keep on decreasing for all of this, because its spreading out more, it is not going to be rotating as fast as the first one. Anyways when something like this happens people started discussing, I have a mach number there, what will happen because of such a structure moving inside my high mach number flow, that is how they started thinking about, and the real is there is another velocity here.

So, there is another mach number here, by the way the temperatures may not be equal here and above, so I will call it a 1 a 2 and I will put a M 1 and M 2 which of course, means that I am going to have a U 2 here and U 1 here, velocities and I know P 1 equal to P 2 as only thing I can tell right now. I have some things like this, now they started giving one velocity to this eddy or the large scale structures, they called it the U c convict to velocity. And based on that, they started discussing things they going to say that U 1 is some high value, U 2 is some low value, U c is some intermediate value, because that is caused by this two differences.

It has to be something in the middle, which from your velocity profiles also that this value will be roughly close to the middle average of these two values. So, they started defining things like relative velocity of the fluid with respect to this structure, how will I find that U 1 minus U c and I want something like mach number to figure out, whether there is going to be compressibility effects on this. So, I will divide this difference divided by that particular speed of sound, which matters for the top divide by a 1, I will call this M c 1.

Similarly I can get a M c 2 for the bottom structures as U c minus U 2 divided by a 2, I can get this also another definition equivalent shock, if I think about it U c is more than U 2. So, if I look at it there are the structures moving at velocity U c roughly, and the supersonic flow is going faster than that, so what will it cause, it is like some object is moving with some velocity. And other one is moving much faster, and we are in compressible fluid it will form a shock, it should form a shock in front of this somewhere here, or if you think about it.

I will just take one particular structure like this, I will draw that picture along something like this, and I know the velocity here is U 1 minus U c, if I sit on this structure it will look like U 1 minus U c is the velocity here. And the mach number corresponded that will be this divided by a 1, now that is the mach number of the flow here, and what will happen it is like flow around an airfoil or something, some such object, it wants to go around this. So, it is going to see set of compression waves here of course, they can come together to form a shock later, after that it is seeing this turning, why that is induced velocity because of this what rotation also.

So, there will suddenly start seeing expansion fans here, and immediately after that, it is seeing a point where it cannot go anymore this line, why it cannot go any more, the next guy is sending fluid this way in. While this guy is sending fluid out this way, they gone to find a intermediate locally something like a stagnation point there, it is not really stagnation point with respect to these two structures. It is like a stagnation point, over all this also having a convicting velocity U c which we have neglected, we have removed, so we can see this particular thing happening here.

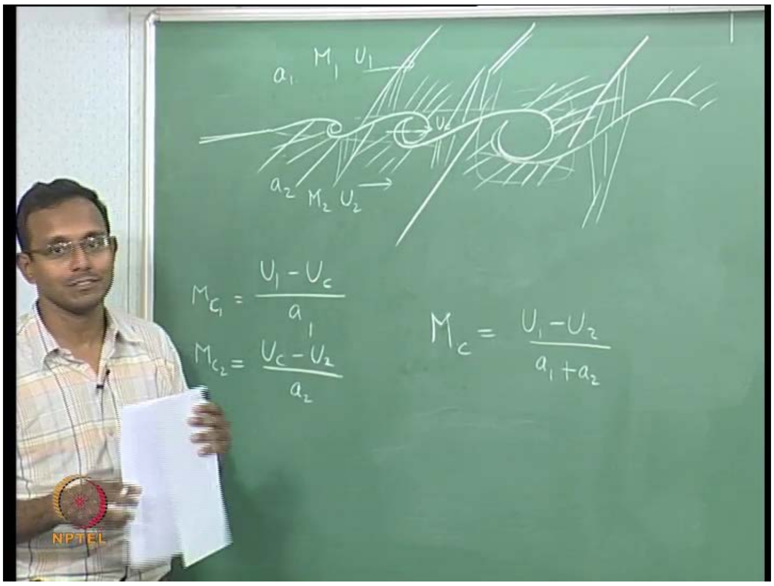
So, what will happen it is expanding becoming higher velocity and suddenly it has to stop, so if it has to stop there has to be a shock again sitting there, so that it will immediately turn out and go out that way. And then, the whole procedure starts again, it is again seeing things there will be expansion, compression initially compression and then expansion and then, another shock all that kind of stuff is start happening there. This whole structure keeps repeating like this, same thing on the back on the bottom, if I look at the bottom side I am going to say the velocity here is, I will just draw this as U 2 as the velocity here, this is travelling at U c.

So, if I look at this it looks as if, I have a body that is shaped like this, which is travelling in a velocity U equal to 0 flow medium with a velocity equivalent to U c minus U 2 equivalent to this. As if I sit on this fluid, which is actually travelling a due to velocity, if I sit on that fluid I will see that, this structure is travelling faster than it which means, it is like a aircraft travelling in supersonic flow, this way ((Refer Time: 48:02)). Now, what will happen, this is a body going like this, it is going to start forming shocks here, set of compression waves forming shocks and then, after that the body turns around in a expansion.

And then, the next shock and I set up compression waves again forming much stronger shock, this whole sequence starts happening here, if you ever happen to experience a shear layer and supersonic flow, you will start seeing these kinds of waves. I am drawing it as if they are very strong, this decided by what is the actual difference between this, if the difference is very large, then you will start seeing that, these waves are not very significant, they become more and more insignificant. This is how your shear layer structure happens, so if I look at the whole thing we will see, if we have a high speed camera, will start seeing this on the top and this on the bottom all together on here.

So, I will just draw all of them together, shock expansion, shock whole thing together form a shock and then, expansion again expansion will be more higher angle like this. And there will be a shock and again bunch of compression waves going in connecting with that, and the next expansion starting here, on the bottom this fluid is going faster than the outside, so there will be a shock set of compression waves. And the whole bunch of expansion waves and then, another shock and the structure repeats like this, shock set of compression waves and this structure repeats.

(Refer Slide Time: 50:05)



If I ever have a high speed camera, I will remove this thing which is confusing, I will have this whole thing happening there, if I have a high speed camera and I watch this, I will see that there will be a set of compression waves, which are just moving like a train one after the other, as it goes down stream and this keeps on happening. And the same thing on the bottom, it will be just offset by a small amount, then it will be going along with this structure again following this. This is the way the shear layer structure happens, we can discuss more about this, I can tell you that the pressure has to match at this point and because of that, you get a pressure matching condition at this point.

We can talk a lot more about this, and they came up with the single convict mach number using these two definitions, and that happens to be U 1 minus U 2 divided by a 1 plus a 2. Because, this there is just a little bit more, so I will just go 2 minutes and stop after that, so they found the convict mach number based on this, actually it is based on these two formulae plus pressure matching condition, I can get to a formula simple enough; if my gas is the same top and bottom, I can get to the thing.

Now, people started discussing this particular function and they said that, if this number is less than 0.5, then I am having only these structures only two dimensional structures, I am assuming that the, this thing extends all the way out and into the board fully, just two dimensional flow field. But, when this goes beyond 0.5, this two dimensional structures starts to break down and becomes three dimensional, if you want to think about it from fluid mechanics point of view, the velocity differences so high.

When I say this number is high, U 1 minus U 2 is very high which means, my vorticity is very strong and vortex break down occurs, it is no more just a 2 d vortex it just breaks down and becomes stream wise vortices. And those stream wise vortices cause lot more mixing, if it is stream wise vortices, then there will be like rotation like this as they are going, and the next one will be rotating the other way as it is going. And the next one will be rotating counter clockwise as it is going, clock wise counter clock wise, when I look from the front along the flow field.

This is going to help you in mixing, this is what they study as mixing layer and supersonic flows, this is going to take this bottom fluid up, and the top fluid down and just mixes. As it is you can see that it is doing that by taking this fluid down into here, and this fluid has gone into that gap, by it is much more if I go for higher convicted mach number, where it will go three dimensional structures. And there are special research papers where they do this, and the recent paper I have experience was 2003. Anyways will stop at this point, next class onwards will go for non isentropic gas dynamics, any other questions you have, see you people next class.