Gas Dynamics
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Module - 14
Lecture - 31
C-D Nozzle Numerical Example, Multiple Choking Points

Hello everyone, welcome back we started doing, going through the procedure of how to solve problem in Gas Dynamics, 1D isentropic gas dynamics through a convergent divergent nozzle. I discussed how to solve when it is simple case of isentropic subsonic flow fully, I do not want to go through that I believe you can solve it by yourself. All you need to do is A by A star calculations, now a better 1 to do will be a case where it is non isentropic somewhere in the middle.

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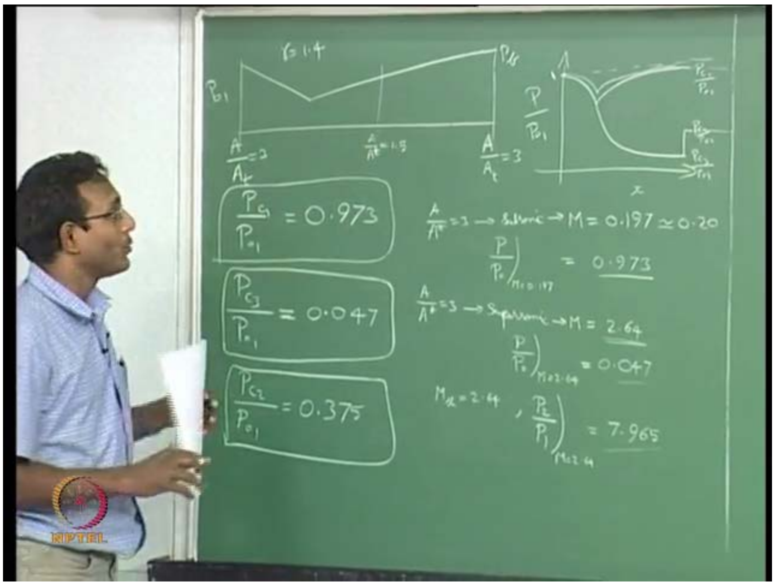

Even if it is subsonic here becoming sonic and going supersonic all through here, we know that A star is this value and I am given area at every point. I know A by A star everywhere, which means I can solve this problem immediately, if it is fully isentropic problem. I just have to go to isentropic tables A by A star will give me that particular mark number it will tell me the P by P naught.

I know the P naught, so I can get pressure here t by t naught. So, I will get temperature here, whatever any location even if it is here. Only thing you need to think about is given

A by A star am I having a fully supersonic flow or a fully subsonic flow in the divergent portion. Depending on that I will have either I may going to go past if I plot P by P naught 1 versus distance.

I could have a curve that is going like this, this will mean I am using A by A star going subsonic solution, if it goes like this I am using A by A star going supersonic solution. These are easy problems to solve and you know that this is going to reach up to P critical 1 by P naught 1 P critical 3 by P naught 1. And if there is a shock at exit then I get P critical 2 by P naught 1 and P naught is somewhere, where that is your P naught P equal to P naught; so it is P by P naught equal to 1 that particular condition.

These are the cases if I have to solve a problem along this particular line, then I know that my throat is choked. Then I will say that is my A star, I know area everywhere in my nozzle then only I can calculate flow inside the nozzle. So, A by A star I know I will get subsonic here, subsonic math number on this side, if I am going over this particular curve then I will use A by A star and get subsonic math number. Once we know the math number I can get pressure at temperature at any other property I want, if I assume that it is coming here sonic and then going supersonic all the way till the end.

Then I am going through this particular line, if that is a case then I have to use A by A star, A by A throat will become A by A star for you, because throat is choke. After that all I have to do is find the supersonic solution for that particular A by A star at that location, and then you can solve the problem form them. Once I know the mark number at the location isentropic flow tables I will get P by P naught, t by t naught whatever else, you want can be obtain once you get pressure at temperature.

So, I can do everything here a more critical problem to solve will be the case where, when I am in between these in between critical pressure 1 and critical pressure 3. So, we will pick up that particular example today and start solving the problem. So, I have pick the particular nozzle where I am going to say I am starting with A by A throat equal to 3, and ending with also A by A throat equal to 3.

I am not telling anything about whether the throat is choked or not I am just telling A by A throat is this, it may be that my pressure at the exit is very low very close to P naught and I am not even below P critical 1. In this case my pressure distribution will be more like this, if it is something like this then my throat is not choked, that could be a case which is what we dealt with last class towards the end.

So, in today's class we will just assume we are going to go in between critical pressure 1 and critical pressure 2 and we want to solve the problem; we will just pick up that particular case. So, I am going to assume gamma equal to 1.4 for this problem, I am going to say there is some P b back pressure and some P naught 1. And since, we already discuss that if I want to change P b by P naught 1 ratio I could either decrease P b or increase P naught 1, we would not worry about what experiment really I am doing all I worry about is I m getting a particular ratio P b by P naught 1; I will just work with that for now that should be enough to do stuff.

So, I do not know what pressure I should use for this particular set of ratios of a by areas. So, what I am going to do is first find the critical values P c 1 P c 2 P c 3, how will I find P c 1? P by P naught 1, critical 1 how will I find that I am going to assume that this is getting choked here. And then I am going to have subsonic flow all through tell here, which means this A throat becomes my A star. It is choked which means now I can use this A by A throat as A by A star at this location.

So, what I will do is I will go and find A by A star equal to 3 subsonic solution, that will give me a mark number from my gas tables or compressible flow tables, isentropic flow tables. I am going to get a value 0.197, and I am going to approximate it to 0.20 because my tables have only accuracy up to 2 decimals in my math number. I wanted it to interpolate, but then I found it is too difficult I will just use to a 2 decimal accuracy, now I have to go and find P by P naught for that math number.

Since, the whole flow is subsonic in my P critical 1 condition all the subsonic completely isentropic flow, I can just go use isentropic tables at m equal to 0.197, that is going to give me a value 0.973. So, that becomes my first critical pressure ratio, this my first critical pressure ratio and how I got it does here, just next to it this is how I got to this point. Now, I have to do next thing P critical 2, it is easier to go to P critical 3 and then come to P critical 2. So, we will directly jump to P c 3 you will now why where the way we solve it you will know immediately.

So, what we do in critical pressure 3 I am going to save my pressure this is going to decrease continuously all the way to supersonic solution up to here, which means subsonic solution became sonic at that throat and became supersonic continuously. So, now, I can again say that my A throat is my A star and A by A throat is equal to 3 will now become A by A star equal to 3, but my flow supersonic here. So, I just have to go and use A by A star equal to 3 supersonic solution and go look up tables and you will get a math number of 2.64.

And corresponding P by P 0 for that mark number 2.64 is 0.047. So, I will just wait that number here, because that will be the pressure ratio if I am having perfectly expanded supersonic flow in that nozzle. In this given nozzle remember that this for this particular nozzle with A by A throat of 3, it will not be working always. By the way what will be the math number here, at the inlet of my duct? The answer on the board by the way, it should be equivalent to A by A star of 3 subsonic solution 0.2.

That is what it should be, when I am this condition it is going to be 0.2, even when I am at critical 1, it is also going to be 0.2. It goes from 0.2 to 1 and then decreases back to 0.2, if it is critical pressure 1. Otherwise, it is going to go from 0.2 to 1 and from 1 it is going to go increase to 2.64 at the exit, depending on the conditions. If I want to find any intermediate condition what should I do? Find that area whatever be the area let us say this is 3 right, let us say this is A by A star of A by A throat of say 1.5.

Then all I have to do is since I know it is choked at the throat A by A star is equal to a by throat that is 1.5. Go to isentropic tables, find the math number, find the pressure ratios, find the temperature ratios, get multiplied with P naught or t naught you get to the answer. So, I would not go through the details of it here, but the next problem. I am going to solve the full thing, where you will go through this detail also will go look at that case any ways. I still have not gone to the full problem critical pressure 3, I have to mark sorry, 2 we have done 3 already.

Critical pressure 2 with P naught 1 that ratio, how will I get it? I am going to say it is going to fully supersonic up to the exit, at the exit there is a normal shock with that particular supersonic math number. And then immediately after that P b is matching P exit that is my critical pressure 2. For that to happen I should have a math number 2.64 at the exit and that should be a shock at that mark number.

So, all I have to do is m shock equal to 2.64, I already did P critical 3 that is why I did this before the other one, t critical 2. So, once I have done that now just I have to find from here P 2 by P 1 at m equal to 2.64, if I find this number that happens to be 7.965. This is all for gamma equal to 1.4, I am solving, I am getting this particular number 7.965. And if I have to find the exit pressure now, after the shock what will it be I will take the previous exit pressure which is this value 0.047 in to P naught multiply it with P 2 by P 1. That will give me my after shock pressure.

So, I am going to multiply these 2 and that will be my new exit after the shock, if I multiply this with this 0.047multiply with 7.965. I am going to get a number 0.375, that is my critical pressure 2, these are my numbers. This is what I am having critical pressure 1 is 0.973 critical pressure 2 is 0.375 critical pressure 3 is 0.047, this is what we are having. It is easier to calculate 3 before 2, because 3 is needed for 2.

So, will typically use this particular order in calculating nozzle flow problems, we want to actually started our problem, I wanted to solve a problem, where there is a shock sitting in the divergent portion. Now, we know what should be the pressure? Pressure should be the pressure at the back pressure to P naught 1 ratio should be somewhere between P critical 1 and P critical 2.973 and 0.375 in between this numbers.

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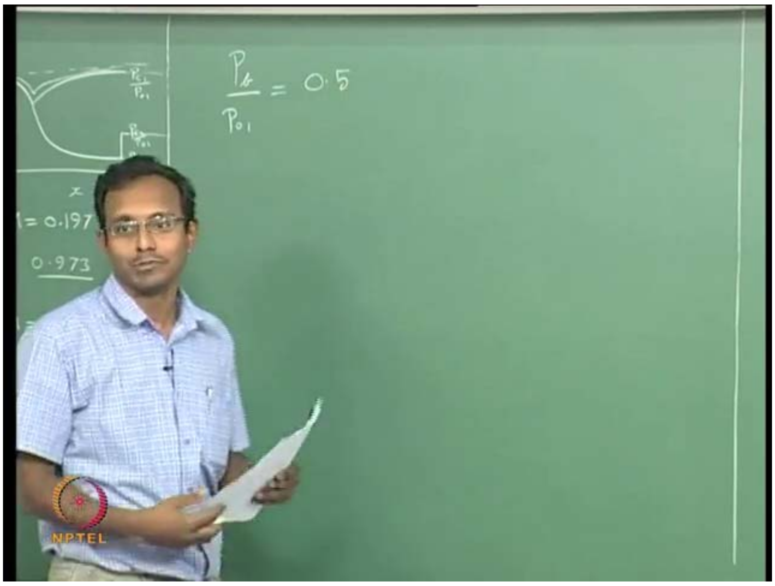

And just for the fun of it I pick the number 0.5, simple enough it is definitely in between those 2. And now we want to solve this problem.

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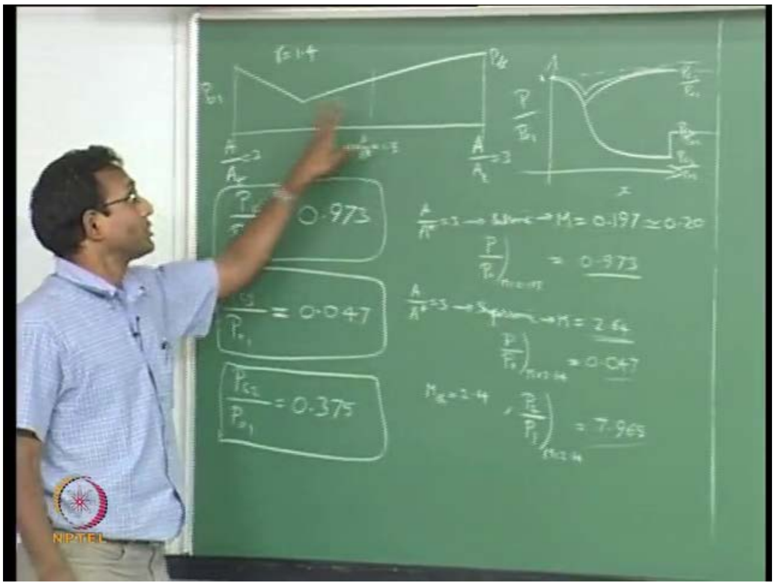

And all we know as if now is there is a shock definitely in the divergent portion, from all our discussions last 2 classes and the experimental video, we saw we know that the definitely there is a shock in the divergent portion. Now, we will say that we want to find the location of the shock, I am not interested in finding the exact x location or the axial location, all I care about is the area at which the shock is standing.

Once I know the area I can just go and look at the geometry of my nozzle, and tell where that area occurs in my duct. So, we will worry about area at the shock location, that is all we want to find that the only way we can find this happens to be by guess work, because unless I put a shock at a particular place I cannot tell what will be the pressure downstream. And I have to solve the whole thing from left to right all the time, if I am doing that I cannot tell what this will be unless I know this? So, how will I think about matching the pressure here, I will have to adjust this, this way, this way till that pressure matches the back pressure, back pressure is given to be 0.5 already 0.5 of P 0 1. So, I have to keep adjusting this till I get to the correct pressure, this automatically calls for iterative procedure, I have to go through iterations.

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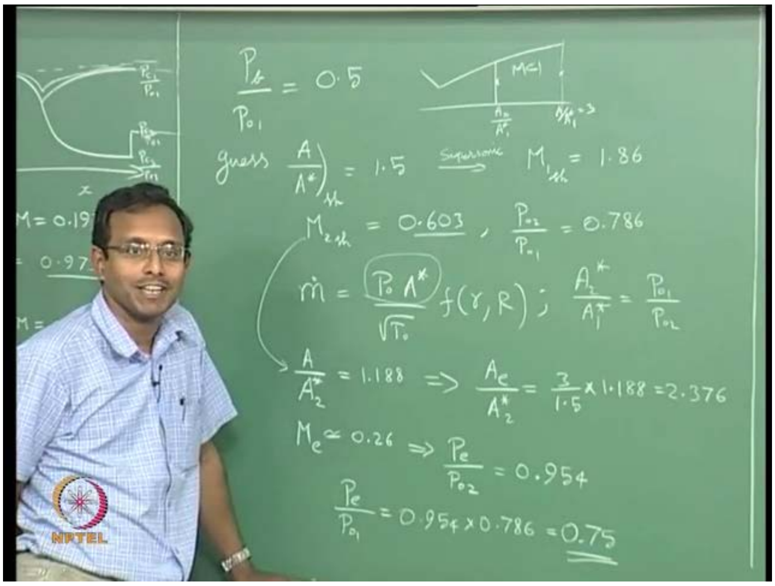

Let us say I guess my A by A star for shock to be 1.5, just a guess because it should be somewhere between 1 and 3, because 1 means throat 3 means the exit. It should be somewhere in middle I picked roughly half, so I go and use my supersonic A by A star value. To find what will be the supersonic math number, I am going to call it m 1 shock at the shock the math number, just before the shock that value that happens to be 1.86. Now, I go to normal shock tables at this. take this number go to normal shock tables and that is going to give me m 2 shock as 0.603 and P 0 2 by P 0 1, this is 0.786.

There are so many ways of solving it I track only P 0 because 1 side track P 0, if I know mark number using that P 0 mark number, I can go to isentropic tables and calculate my pressure at the exit. I am going to use that method now, I could of course, do pressure ratios P 2 by P 1 and then tell that the mark number based pressure ratios. P 2 at previous area to pressure at previous area to pressure at new area that ratio can also be found that is too much work, this is simpler this is a shortest route I think.

Once, I have this number now I am actually located I will just draw the picture here it is easier, and somewhere here I am currently located at this point, because I have calculated every property that I need. We know the t naught also by the way t naught 2 by t naught 1 is 1 in a shock. So, I know every property at this location now I have cross the shock, now I know that the solution here is subsonic m less than 1.

I have to go all the way to this exit where by A by A star is equal to 3, wait be care full A by A star 1 is equal to 3. Why? Since, my P naught dropped my A star would have increased, that is where you have to think about again. I know my P naught as dropped to roughly 78 percent of the previous value, which means for the same mass flow rate to go through I know my functions looks something like this.

Function of gas, gas is not changing, so you would not worry about it my t naught does not change across a normal shock, t naught and A star. Now, I know my A star dropped sorry A star increased P naught dropped and A star increased to keep the same mass flow rate that has to happen. So, now we have to work through the whole thing again, there are several ways of working through it I can of course, find the new A star just from this relation.

I can find my new A star in with respect to old A star, how can I do that? A 2 star by A 1 star is equal to P naught 1 by P naught 2 I can use this also to maintain same mass flow rate this has to be true. I can use this or I use something much more simple, because I just go to direct tables and just look up numbers. I know my mark number at that section, once I know the mark number at this section I can go and find A by A star corresponding to this value.

I will go do that from here I go to isentropic tables and I will find A by A star, now I will call it A star 2, because this would not match here previous A by A star. If I find this number that number happens to be 1.188 does this proves that my A 2 star increased A 2 star is higher than A 1 star. Can you say that? Yes, you can. Why? Previous A by A star which we started with those 1.5 at the same location, we want to moved, yet it is a same location.

That is say that across the shock, this is my A 1 star really A 2 star is lesser sorry A 2 star higher the denominator is higher, so my ratio is lower, I have gone to this point. Now, the same A star will go from here all the way to here because the flow is isentropic here, nothing is changing same mass flow rate isentropic, so it has to be the same. So, now, I can use the same A star I have to find the A by A star at the exit, which is what does are going we want to match that with 0.5 of P naught 1, that is the goal for us.

So, I will I find that I have to find A by A star at the exit, how will I do that? What I actually have to do is A at x by A star 1, I know because this is what I guessed, I guessed it to be 1.5 and I am going to 3 from there. So, similarly I have to work through the numbers, so how will I find that if this is 1.5 that will be 3. If this is some other value I have to work through that again, so what will that number be this is going to tell me my A exit by A star 2 is going to be 3 by 1.5 in to 1.188.

So, let us find this number that is going to be 2.376, once I know this I know A by A star at the exit, and I know the flow is subsonic at the exit. So, well I have to do is go look up A by A star and subsonic tables isentropic, find the subsonic solution for naught number at the exit. So, that is going to give me a mark number, I will put an approximately equal to sign here, because the numbers are not exact I dint want to interpolate. It is close to that is 0.26 roughly, so this is going to tell me that my P exit by P naught 2, this what my mark number isentropic tables will give. That number P exit by P naught 2 happens to be 0.954, this why it comes have to be. So, what will be my P exit by P naught 1, how will I find that, multiply it this 1 with P 0 2 by P 0 1. I have to multiply this and this, then I will get this is the standard thing know ratios being multiplied that is gas dynamics people we always love that.

So, as gone be 0.954 into that number 0.786 that is going to give me a value 0.75. If my shock happens to be at A by A star of 1.5, then I am going to have an exit pressure of 0.75. At what does are target 0.5 I did not meet it. So, what should I do? What should be my next guess for my A of shock? Where is the shock line located? What should be my next guess? Area higher, area lesser, area higher why?

Student: we have to decrease the pressure.

We have to decrease the pressure at the exit expansion fans will come in we can the shock the shock will be thrown more out. That is one way of looking at it, if you do not like the explanation let us go to the screen.

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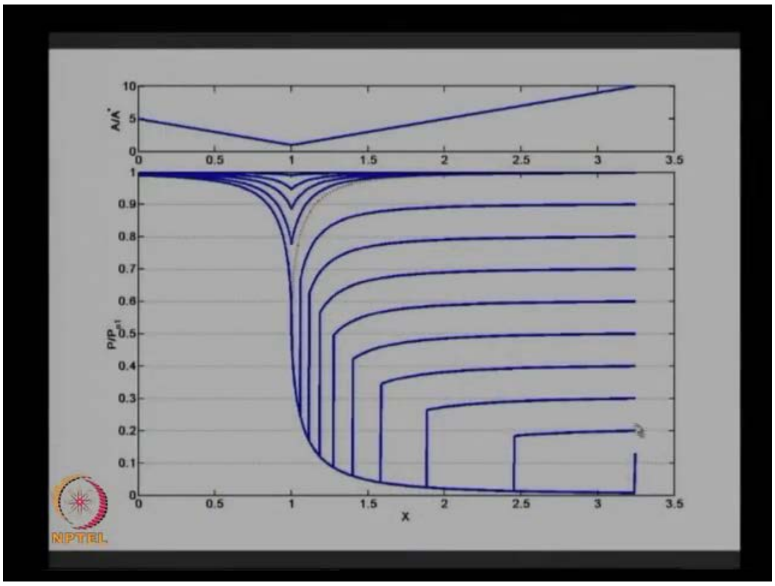

All I have to do is just remember that as the shock goes downs stream the pressure at the exit drops, this is only thing we need to remember. If you remember this, then things are easy for you automatically, I did not want to choose this exact nozzle, because you can just use this plot and give me answers. I picked a nozzle with exit area as A by A star of 3, this happens to be 10, I just picked some other case. So, all we know is if I want to decrease pressure my shock as to go more downstream.

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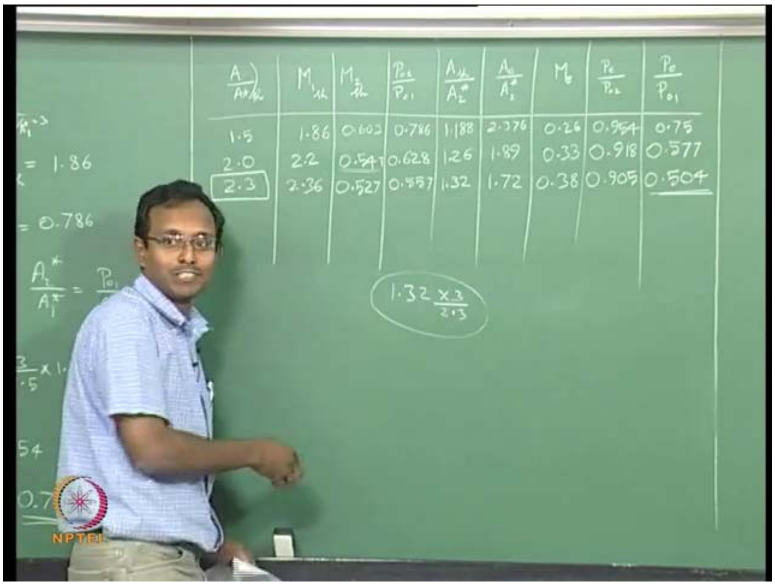

Now, will go back here, since it is iterative process we have to go over a particular table, I will call it a shock by the A by A star shock. M 1 of shock M 2 of shock P 0 2 by P 0 1 a shock by A 2 star exit by A 2 star M exit P exit by P 0 2 P exit by P 0 1, these are the things you need to calculate in this sequence. So, that you will get to the answer, so I will just write whatever we just calculated it in this table for the first round 1.5 1.86 0.6 0.3 0.786 1.188 2.376 0.26 0.954 0.75.

Now, after this I know I made a mistake in my calculations, so anybody has gas tables here. No, so you are going to be my gas tables and calculator it has gas tables with the end. I know I made a mistake there because I did not change the areas accordingly. So, we know that I have to guess higher area, so let us get guess an area 2.0. So, I guess an area 2.0 I am going to use A by A star of 2.0 and get a supersonic solution that is going to be 2.2 and if I go to normal shock tables that is going to give me 0.547.

And if I go look at the same normal shock tables I will get P naught 2 by P naught 1, now 0.628 after this I have to go and use this number M 2 of shock to go and find my A by A star. If I use that I am getting a number 1.26, now I have to find from here a exit, now you guys need to calculate and tell me for that 1.26 multiplied by 3 divided by 2, 3 is the exit area 2 is the actual area.

Student: 2.8.

Which we guessed 1.5 times this at.

Student: 1.89.

1.89. From now on I want people to tell me numbers, 1.89 A by A star I want to find my mark number from there subsonic mark number. What is that?

Student: 0.39

0.39.

0.33. Now, for this mark number I want P by P naught same row I just have to look at numbers, if you have a reciprocal just tell me the reciprocal will invert or tell me the actual value.

Student: 0.918.

0.918, now I have to multiply the 0.918 with my P naught 2 by P naught 0 1 0.628, multiply this with this while be the number 0.57.

Student: 0.576 sir.

0.577. That's fine, 3 decimals is probably 2 accurate may not be that accurate, when we calculate stuff 2 decimals are really enough. So, yes we have moved in the correct direction when we increase area you are finding that the exit pressure as dropped. We have to decrease it a little more I do not know how much should decrease lets guess 2.3 or we will guess 2.3 now. So, if my A by A star is 2.3 what should be my mark number, so I am looking for tables.

Student: 2.36

2.36. Now, we go to the normal shock tables 2.36, I want the downstream mark number and P 0 2 by P 0 1.

Student: 0.17.

0.

Student: 527.

527 and the P naught 2 by P naught 1.

Student: 0.557.

557.

With this mark number in isentropic tables, now I want A by A star 0.527 A by A star for that.

Student: 1.32.

1.32 now I have 1.32, I have to make it 3 from 2.3. So, in to 3 divided by 2.3 I want that number.

Student: 1.72.

1.72, now we go back to isentropic tables A by A star giving mark number.

Student: 0.38.

0.38 and for that condition I want P by P naught.

Student: 0.905.

0.905, that multiplied with 0.557. So, next number you guys understanding the procedure that is the same sequence you keep on going.

Student: 0.504.

0.504, since this is close enough will stop here, I do not want to keep spending time you know the procedure this close enough to my starting condition of I wanted to be 0.5 of P naught 1. So, I am getting this close, so I am going to say my shock is suppose to be setting at 2.3 it is lucky guess for me I could have guess 2.5 and I would have gone a longer process I just guess close enough, if I guess 2.29 I would have hit it exactly, is just close enough for us.

Any ways, but remember that you have to convert the areas accordingly this is the mistake I made in my notes, and that is why you are doing the calculations. Now, I realized it while I was talking about it to you guys here, so you are prone to make, so many mistakes here because you have to look at different tables and if you have a full compressible flow stables you will have different gammas tables.

Sometimes we will use gamma equal to 1.67, sometimes you will use gamma equal to 1.4 ideally you have to use a same table all the time and then you have to use isentropic tables normal shock tables. So, there are so, much of confusion possible yeah and instead of subsonic, you will use supersonic things can go wrong. So, you have to be very, very care full you have to be more alert mentally prepared for doing this problem. So, we will do more exercises I will put up some exercises on the web anyways. So, it is just practice now of course, you can always go and tell I do not want to hit 0.5, but I want to hit 0.25 as a problem.

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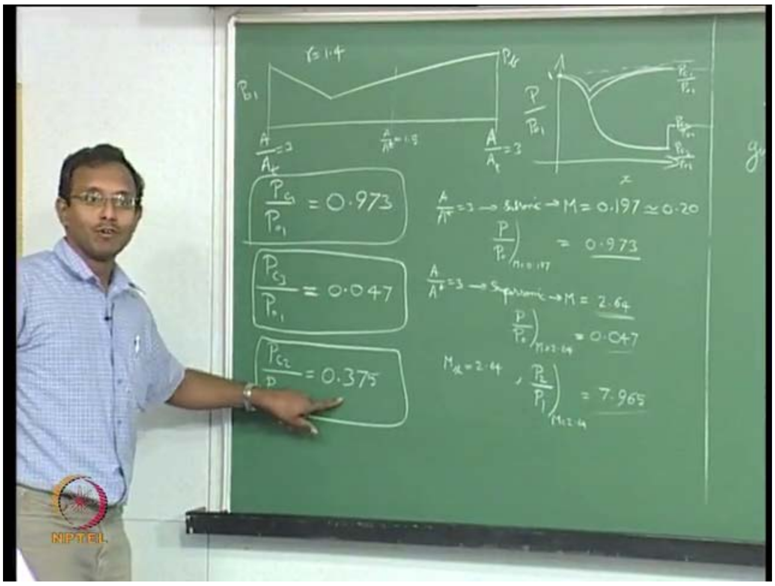

If it is 0.25 is it. If it is 0.25 I am an over expanded region remember it is below P critical 2 I cannot use 0.25. So, choose say 0.6. And solve the problem again if I choose 0.6 and I want to solve the problem again new problem P exit by P naught should be 0.26 0.60.

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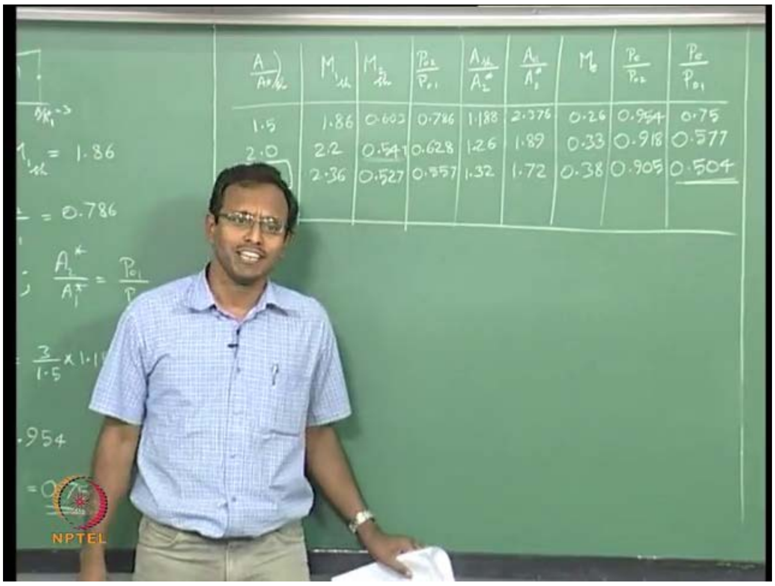

if I want to do I have the first columns already ready for you I know the answer is in between these 2 and very close 2.0. So, I will guess something like 1.95 and that may hit the answer immediately, but you have to do that one more column, the one more row of the same thing this is the procedure used in general. And that is exactly what I used to get the whole plots which I am showing all this time there is a computer program which is doing this exact procedure, that is the way you have to do.

In fact, it is good thing if we go and write you are own code to solve this problem, then you will understand the details of it. After sometime you will be comfortable with gas dynamics problem solving, it is fun to solve this. Now, we will assume that you know how to solve any problem in convergent divergent nozzle, is there any other questions you have which you may not know how to solve? The only thing I have left out happens to be the remaining cases.

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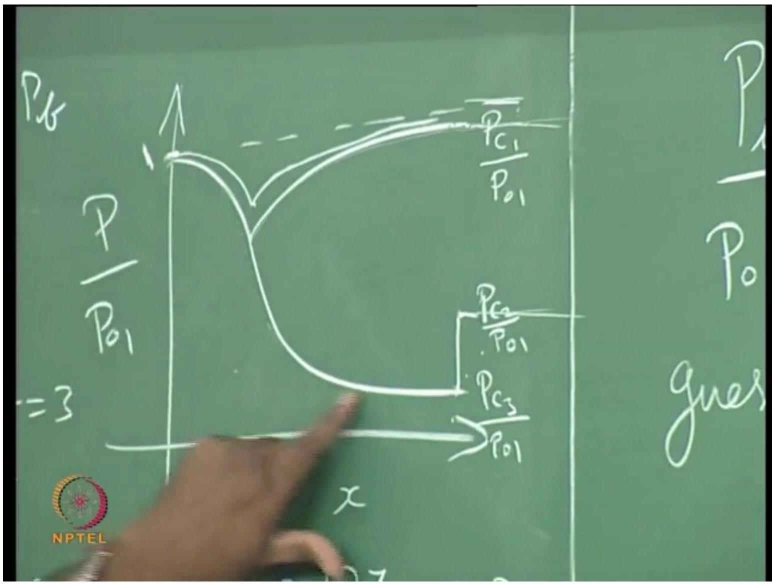

We have solved a problem P naught 1 to P critical 1, last class not with numbers, but with just ideas. This class we did problem from P critical 1 to P critical 2 and at P critical 3 we have solved, only cases which we have not solved or region from P critical 2 to P critical 3 and below P critical 3. It is so happens that for all those cases inside the duct it is exactly the same pressure profile.

So, I will solve the same thing as P critical 3 and it will be and it will be the same everywhere inside the duct, ideally the remaining portions over expand and under expanded had a talk. After I introduce jets to you guys because only outside the nozzle things will change, inside the nozzle it is exactly the same as critical pressure 3 condition. If you know that you know how to solve the problem, now I will give a more complex problem double throat nozzle.

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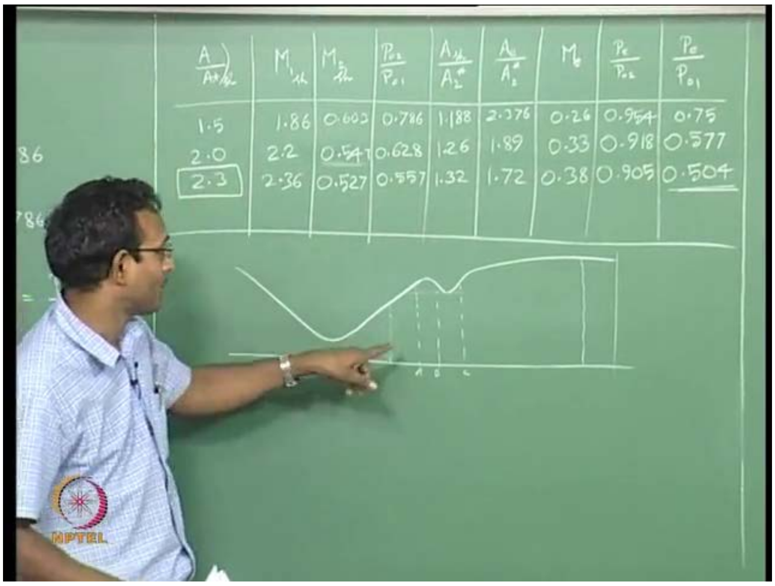

I am having one convergent divergent nozzle and it is having a small bump here and then it is going like this. Let us say I have a nozzle like this and I want to solve the problem in this. And let us say the other side is just straight does not matter I am just worrying about area variation because this is 1D problem analysis, anyways we pick such a problem. There are 2 throats we know that it is going to choke at minimum area as long as I am giving enough pressure difference across it.

Let us assume I am giving enough pressure difference that it is choked at this condition, otherwise there is no fun in this problem. We will consider the case where there is some little fun in this problem. So, we will consider this particular case, how will I solve this problem? If it is fully sub sonic solution I am going to say it is going go through us A by A star, I am assuming it is choked here. If not also I can solve the problem similar to what we talked about last class towards the end, same procedure can be applied here.

That is not interesting enough, so we will go to a case where there is shock in the divergent section, now for finding the exit conditions. I will follow the exact same procedure as we followed for single throughout nozzle, and find the area location at which the shock should be present. Exact, same procedure will work expect for this small range of areas, there are multiple locations for the shock possible. That is the only special thing, if my pressure is such that the shock is suppose to be here it will here, there is no difference.

If my pressure such that the shock is suppose to be at this big area it will be here no problems, then this will be not there no problems. What if my pressure is such that the shock is suppose to be some area here could it be here, here or here where will it be there are 3 places. I will give them A B C 3 possibilities, I have gone through the calculations and I found that the A by A star at which the shock is supposed to be located for me to satisfy the exit conditions happens to be that particular area.

Now, I have 3 possibilities, the area happens to be this height that is the area where my shock should be standing. Now, I have 3 possibilities for this where will it stand, now we have to go and think about it a little more. it is so happens that shock is not either, I am giving you a new information here shock is not stable in a convergent duct. It can be proven and it is related to unsteady flows we want to worry about it right now, can be easily proven not very difficult to prove, but we would not worry about it right now.

We will just say that shock is unstable in convergent duct, which means B is not a possibility I still have A or C both are divergent ducts. Now, which 1 will it be now it depends on history, it is related to histories is whatever history of the shock. If I am slowly making the shock move from here, say I am increasing P naught keeping back pressure constant I am going this way from here as shock is moving continuously.

Then it will go and reach it is point A till the shock goes to this area I keep on increasing P naught it will be sitting in this side. If it needs to go higher area than this maximum point here, then it will jump to back point and from there it will keep on increasing area further to there. Incase I have push my shock here, and I am decreasing my P naught or increasing my back pressure which ever then my shock will start coming back. When, comes back now, it will not jump from that location directly to this peak.

It will not it will just keep on going on this convergent divergent portion back this way, will keep on going this way till it goes to this point. And then if I decrease it any little any lower P naught decreasing a little bit, it will jump to a lower area and that is only possible in this section, so will jump to this point. That is what really happens there shock wants to find the area where it can sit, so that it matches the exit condition pressure that is the idea for the shock, it will find whichever is closest. So, it could be A or C, if the shock is here and it has to come to this area.

It will come to C if the shock is here, that will go to A. What of my shock is at this peak, at this location? And I decrease my P naught it will still to A, that is a closest. If I am here at this location shock is at this point, this line vertical line here the minimum of the other second throat. If you think about if it is here if I increase my P naught only it will go to that point because see right, it is going to higher area it more downstream. If I think about the simple convergent divergent nozzle, if I increase P naught it goes downstream.

So, it will go that way to reach that point, if I decrease my P naught then it will jump to this solution somewhere here. It will not reach A. If my shock is anywhere in the second divergent portion and I reach that particular P naught it is going to reach C. If it is in the first divergent portion and I reach that particular P naught for that area, then it will sit in A. So, it is related to history of the shock, now you can think about what if I oscillate up and down, then my shock will go like this up to the point jump to that go there. When, it comes back it comes all the way till here and jump from here directly to here. I will keep on doing this, if I keep oscillating my P naught or my P b up and down across this point it is all fun to think about. Next thing this is a serious question which I asked long time back and we still it did not answer very well.

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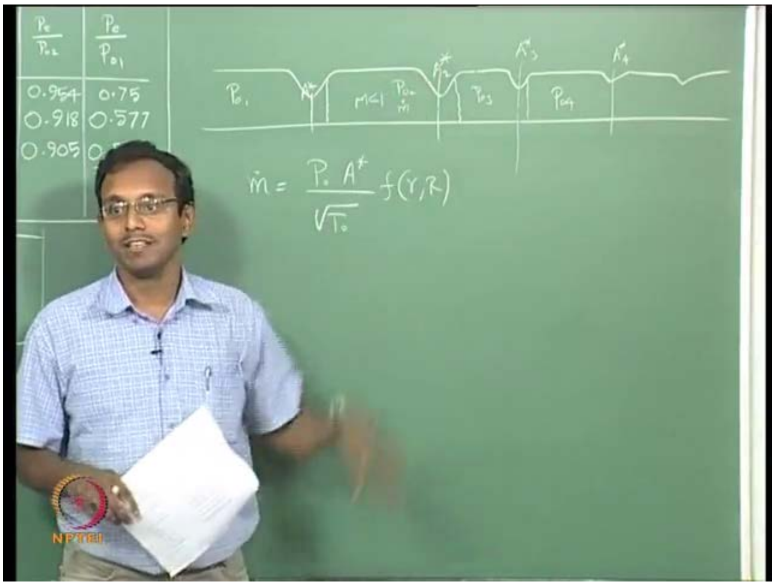

I am having a duct that is something like this, how many times can I choke a floor when in duct? That is the question. So, single duct and I want to choke the flow as many times as I can, how many times can I choke the floor? That is the next question, I want to ask.

Let us say initially, I did not have any of these bumps, I just had one C D nozzle and then a straight line duct. I said some pressure and I put a P naught and I know it became supersonic after this point or you want I just worry about choking. It choked definitely, I given P critical 1 for this throat it choked, after that if I wanted to choke again, what should I do? I wanted to choke again in a sense I want this choking to be present and this choking to be present, what should I do? How will it choke twice? Let us not worry about tries 4 times 5 times whatever, how will it choke twice? I can choke twice, how? I believe I told this long back once, just as a passing mention you just have to figure out when anyways. How will it happen?

All I have to make sure is M dot equal to P naught A star by square root t naught function of gas. This is what I need to think about keep remembering this formula, mass flow rate is the constant in a continuous duct. So, as long as I am thinking about this particular thing, if my P naught does not change in this whole floor. Then I am assuming that my flow is either fully subsonic or fully supersonic after the first throat, if it is fully subsonic.

Let us say then and I am isentropic, I said right fully subsonic, isentropic then P naught does not change t naught never changes, A star will be exactly the same. If A star is exactly the same I can put the same dent everywhere, in my duct how many ever times I feel like. And it will just go subsonic to sonic, subsonic, sonic, subsonic, sonic, subsonic, sonic, subsonic, I can keep on doing this for ever. That is one possibility, it can happen by the way, if you are having a supply line of gas.

We have a big tank and you having after long distance some flow facility, there will be so many valves in the middle. Each valve is like a construction in the flow I can have several choking's. That is one particular setting, where the flow is completely subsonic or just sonic no supersonic. Even if you make a small mistake, one of them will become chocked and everything else will become un choked right that is a very critical situation.

I have to make sure that all the area are exactly same, even if one of them is slightly smaller than all the others only that one will choke everything else will be un choked. The very special case typically it is be impossible to achieve it, ideally I can achieve. How many ever choking's I want? If it is fully subsonic and just sonic conditions alive allowed, let us think of other case, much more easier case to think about. I am going to have A throat here sonic I put A star here and then there is a shock here.

Now, what will happen? Flow subsonic on this side, if I just think about this problem flow is subsonic here. And from further down I can make it choked here, that easy to do I can make it choked here. If I make it choked here what should be the condition, all I have to think about is this is some M less than 1. I want make this also choked what should I do pass flow rate should be constant.

So, I know my mass flow rate through the first throat and I know there is a shock some where I know exactly P naught 2 by P naught 1. I know my P naught at this local location and my m dot that has to go through this with that P naught. Based on that I can find my A star from there, so if I make a dent in this pipe up to a point where this A star achieved here. Then this will be choked again, again I can have subsonic or supersonic.

Subsonic I told you already that it is very, very unstable, I cannot maintain how many are subsonic solutions let us assume there is another shock after this. We will make it numbered P 0 1 P 0 2 now I have P 0 3 that is a possibility, now if I decrease my pressure further downstream P back really low. Then I can still make a choked here for this P naught I just look at this problem from this point on downstream, if I make my P b low enough for this particular P naught 3.

Then this condition may get choked right, I just look at it as a module all this point is all ignore, currently I am just going to see this is only the portion that is left I have P naught 3 in my duct. And I can choke this for a given mass flow rate, I know the mass flow rate I known the P naught I known the t naught I known the gas. Only A star is left, I can now make a new dent A star 3 we have to label A stars also, A star 1 . So, now, I can have another A star typically since P naught is dropping continuously my areas will keep on increasing.

Now, in all this I am doing one more thing which I am not telling stressing to much yet, we will see if you are not noticing after one more. If I have P naught 3 and I want to have a shock here, then this will become P naught 4. Now, I do the problem again, I will look at only this problem there is a P naught. I can form a A star 4, I can make it choked as long as my P b is low enough for this P naught 4. Now, that is the statement I receive repeated 3 times now, as long as P b is low enough.

So, now, I will go and answer my own question I said how many times that you can choked a duct, flow through a duct. One situation is for a particular P naught and P back flows fully sub sonic only some points it is sonic. Then it can be any number of times, but if you make any small mistakes only one of them will be there it is very unstable. Typically, it is impossible to achieve practical situations.

So, we will think about the other case where there is a shock now it is very stable, shock in divergent portion very stable. So, now, if this is the case as long as my P b is very, very low and my P naught is P naught 1 is pretty high. And I can go to a point, where I can choke it, how many every times I feel like, as long as I maintain my mass flow rate constant. Of course, you should know that there is only one particular value at which this will choke, this particular values it cannot work for all conditions.

My area should be more than at least a minimum value, if my A 3 star is anything is less than whatever value we get from here then only that will be choked. This previous 1 will be un choked. This one may still be choked, I do not know that depends on the P naught value, so much dependent on P naught. Now, I will give you a complicated exercise like this also, so that you can have a feel for this. In all this discussion I have assumed that, there is no friction in the duct.

In real life if there is friction in the duct, there will be P naught loss because of friction also because flow has to do work against friction. So, my flow energy the pressure energy is decreasing, if my pressure energy is decreasing then it cannot go through that small area it may need more area. There will be a point, where that area will be equal to the duct area and in the even the duct will get chocked. That will your limit. So, it cannot be infinite numbers, there will be a specific number for a given P 0 1, that can also happen right.

So, typically you can choke flow 5 or 6 times, but it could be infinity if you assume friction. That is where you will be I think I have discussed reasonably enough convergent and divergent ducts and multiple convergent divergent portions. Now, the next thing we need to go is what happens just outside the exit of the nozzle. That is your jet problem now, from now on you assume we are having supersonic jet exciting, subsonic jet is no fun fluid mechanic people would have taught you enough.

There will be a shear layer form and if this is in visit flow, it is just a straight line jet going, there is no fun in it. We are in visit world, so we will go supersonic because there is some more fun in. So, involved there we will consider supersonic jets in to stationery coefficient atmosphere that will be the next class, see you people next class.