**Gas Dynamics**

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**Module - 14**
**Lecture - 30**
**Convergent-Divergent Nozzle**

Welcome back. We were discussing flow through Convergent Divergent duct. I already gave you what will happen inside we just have to into little more detail, I told you already that this particular thing for it to get into your brains, I have to going to it three loops, four loops then only it will get into your brain. We showed you a picture first then I explained it in one round, we will go through that a little more detailed with more plots and figures this time, and probably a animation also.

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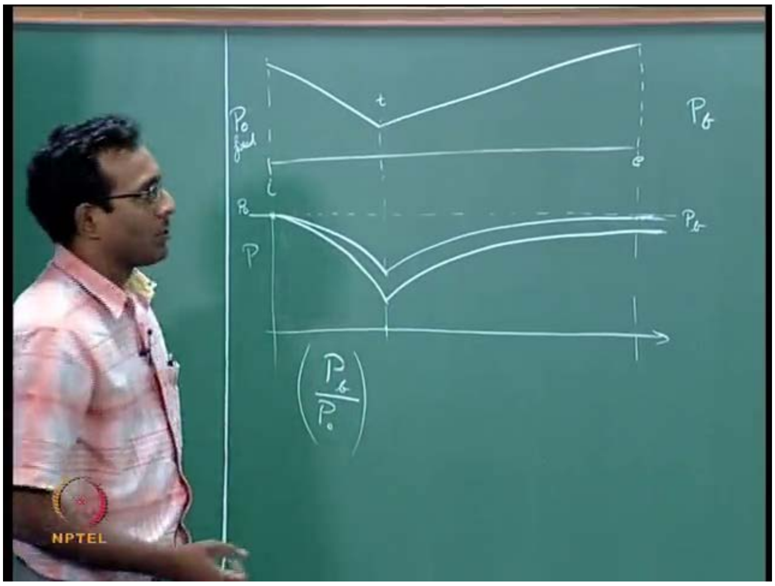


We are having some convergent-divergent duct and we are saying that we going to maintain p naught as a fixed value. And back pressure p b is what we have going to change, we have going to slowly decrease the back pressure starting from equal to p naught value. And we start with p v equal to p naught, you we going to have no flow and when I slightly decrease the pressure there, immediately the pressure differential we will derive the flow.

And when the flow starts we know it is going to be subsonic, we go it was stagnate just before this it will be subsonic and subsonic flow convergent duct it will increase an area, increase in velocity that is it is going to increase the mark number. And after that it will still be subsonic I said it is a very small pressure differential it is not going to get to sonic condition here. And after that it is again going to go subsonic and lesser more lesser subsonic value, lower mark numbers it is going to reach finally.

And if I look at what happens to pressure, as a function of distance you know that from here to here the flow axial rates and p naught is maintain constant and it is very low subsonic values, we can even think about it has, Bernoulli's equation. And you can tell that when velocity increases pressure decreases, but even if I think about compressible flows and Mach number increases pressure decreases for isentropic flows, we are still using isentropic flow.

So, I am going to get something like this, from inlet condition to the throat condition I am calling it throat I am not putting a star on top of it only when I put star it is choked condition, from here again I am having subsonic flow and it is diverging duct. So, my flow decelerates again and it is going to some other value. Now, I have to be very careful about what it will be the final value compare to the initial value, initially I am going to say this is close to my p 0 stagnation condition. And the final value cannot be equal to p 0 it has to be slightly less otherwise there is no flow just be careful in drawing these pictures that is it, this is particular p b value we are thinking about and after that the pressure is the same. To maintained that pressure only there this flow going through this nozzle that is a situation.

Now, if pick up let us say this is my p naught I will draw dotted line here, if I pick a slightly lesser p b say this value, then my flow axial rated a little more it is start from the same it want to go to a lower value and it is going to go here. I will showing all this really nicely we have to know that the scale of this I would not put any numbers here. All these will happen really small values. Now, depending on whatever is my p b to p 0 ratio the final pressure in same them in the middle will be decided. So, to decide this problem what matters is just this ratio of p b to p 0 or we can say that it matters only if I think about p 0 and p b these two numbers, that is what matters for this problem. And typically, we write it as p back divided by stagnation pressure, this ratio decides this problem.

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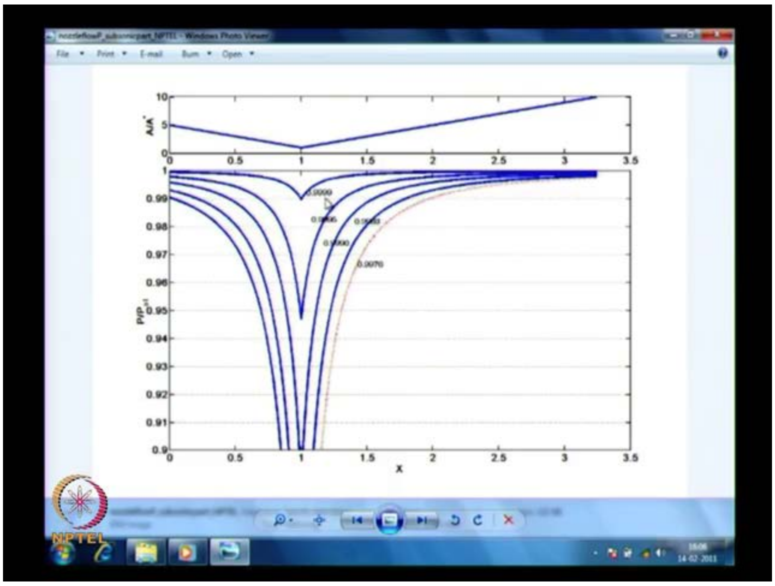


So, next go to the screen I am showing here, the nozzle which I have calculate it the pressures for, I am showing you here a by a star where star is the throat condition here, where it is 1 a by a star is 1 at this location this number does not mean anything really the x axis is just some distance. We going to see that it is starting from a by a star of 5 and goes to a by a star of 1 and then it goes back to 10 at some location.

Even if I stretch this nozzle along the x axis, the flow will not change except for the plots will also be stretched along the x axis that does not mean anything for us we would not worry about it. Now, if I look at the p b by p 0 value I have to show you what is the plot is, it is the pressure divide normalized by p naught 1 that is the initial stagnation conditions somewhere, for outside. And from there we are plotting for different locations as if I am having pressure measurement along this nozzle all over that is what I am plotting here.

If there is a small change, then this throat section is going to have a pressure of 0.99 I am thinking about 10 power minus 1 10 power minus 4 drop from 1 it is a very small change, in p b by p naught 1 value, and that is giving you a 0.99 p by p naught at throat location. But, if we look at all the other places most of the other places it is almost 1, it probably be ending in 0.999 that is what it will end up with that, if I go to a 0.9996 then I am going to have much see for a drop it is going down very fast up to here, and I just wanted to zoom into this region.

So, you have seen that from 0.9999, 996 it has drop from 0.999 at the throat to 0.946 or something at the throat. It is dropping more, seriously and make it 9990 it is going even below 0.9 I am showing it very clearly to here. And if I keep on doing this I decrease it by small amount that is a point where it becomes choke that the throat, that particular calculation happens to be happening at 0.99976 this is happens to be your first critical pressure, we were discussing critical pressure.

This is the point where the throat location is first choked I am slowly decreasing my p b and it is getting choked at that point. We will go deal with numerical example, probably another at in may be next class as of now, we will just assume that this is what happens we will calculate how to I will figure out how to calculate these numbers and these plots some there of it. Now, once we understand this we can proceed further I will go back to the board and extend the plot further below I will extend the plot further below.

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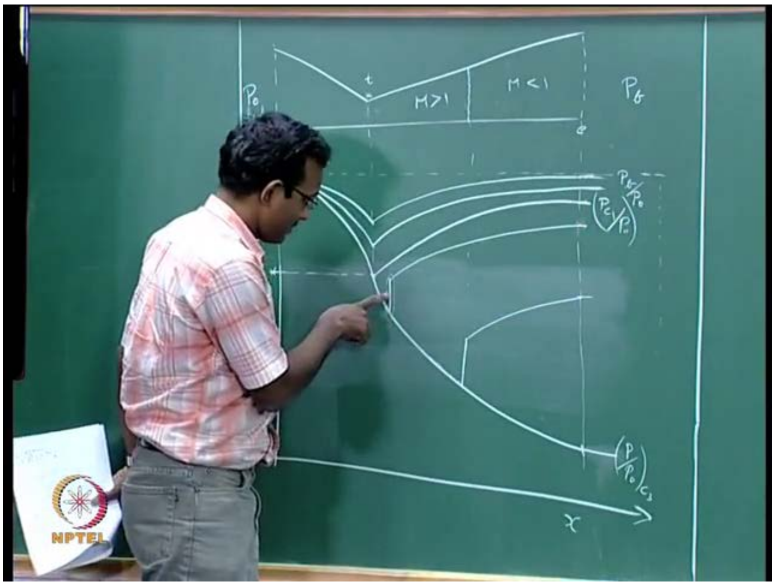

This is again my location x and I set p here, we are normalizing it p 0 we will keep it that way in which case this will become 1 that is it no difference p b by p 0 1 is what I am changing. So, I have decreased and there is a point where it is critical let us say, this is my p star value. So, this is my star condition from here it is going back to this is my p critical 1 divided by p 0 this is since we are plotting ratios here.

Now, after this we said that the flow wants to be supersonic and if it is fully supersonic. I will draw that curve also, if it is fully supersonic then at the exit the pressure will be very, very low. This is your critical 3 we have said p critical 3 divided p naught that is what I have put there. And anywhere, inside the flow is not fully isentropic flow we said. So, what we want to do is if I go through the process of starting from p b by p naught equal to 1 and going down slowly I will go to point where, suddenly the flow becomes supersonic partly and then it realizes that it cannot reach that pressure.

So, it goes through a shock becomes subsonic and then goes on a subsonic line. We already know that if it is subsonic flow and then diverging duct it is going to increase in pressure similar to these curves, we already have subsonic solutions. Now, this is what the flow does it is once it is below critical pressure 1, it is going to take this supersonic path for sometime or some distance in this case, and then it is going to jump across to subsonic solution using a shock non isentropic flow again.

Only this 1 plot is non isentropic really just that one region everywhere, else it is isentropic from here to here p naught 1 and after this it is p naught 2 really right because, this is a drop across the shock for p naught. But, the way we plot it typically, we normalize the even the other curve, the pressure down streams also with respect to p naught 1 that is where we have to be more careful and say p divided by p naught 1.

So, that in case of the shock being existing, we have to be very clear which p naught we are using. if I use p naught 2 here that curve will be slightly different, we just want to keep single normalizing factor that is p naught 1 for us. Now, I go for even lowers pressure then this value, that is say somewhere in here then that will take supersonic path for even longer time jump up to a subsonic solution and from here it is going to go each that value something like that it is going to pick some such solution.

You should know that the p 2 by p 1 jump is going to be higher and higher as it goes down here, that is also true why is that, why will the p 2 by p 1 b higher as I go along x Mach number is higher because, and I go from here throat is choked now, if it is choked then I am having if I have pressure very low, I have to always give this constraint, if the flow is choked I cannot help for sure what did the down strain Mach number is unless I tell you the downstream pressure.

Now, we have decided that the downstream pressure is below pressure critical 1 which means the flow just downstream of the throat is supersonic. Otherwise it can never reach that, that is what we saw it will take this particular path if it is fully subsonic, that will not reach any of these values. So, it will take a supersonic path up to some particular location let say, we will talk about this particular curve somewhere here there will be a shock till here it is m greater than 1 after that it is less than 1.

At this location it is non isentropic only this one spot, we already discussed that shock is a very thin layer and we will assume that it is a point change in my full x dimension. So, up to this point the flow is supersonic and from this point, on at this point it jumps for supersonic to subsonic and from this point on it is subsonic this side, that is what we will deal with this problem simple enough to deal with. And once, we know that it is subsonic after that the diverging flow, we will we will increase Mach number sorry decreased Mach number further increase pressure and go to meet the correct p b value.

If I decrease the pressure further then the shock goes more out, what does that mean now, my flow if the shock goes more out the shock it is at a higher area, higher area means from throat m equal to 1 have become supersonic and it is diverging duct it is going to increasing Mach number much further go to a higher Mach number there. And, so the pressure drop will be higher pressure jump will be higher not a drop it is actually, increasing pressure across the shock.

Now, let us think about this and give a physical field for this, I decreased pressure here why should the shock go more out, just from physical point of view not understanding, pressure needs to get higher I am not going to think about that way I am thinking say I was already operating at this shock location and from here I want to go here. Say, my shock was initially here, I decreased pressure and my shock went here decreased back pressure keeping p naught the same and my shock went more downstream, can we give a physical field for this.

I am going from here to here pressure is decreasing at the exit that is what we are looking at pressure is decreasing at the exit. And why will the shock go more downstream, Mach number comes only after the shock moves, till that time it is have same Mach number why will the shock moves my question, the way I am going to think about it why will the shock moved. What is that more freedom to go.

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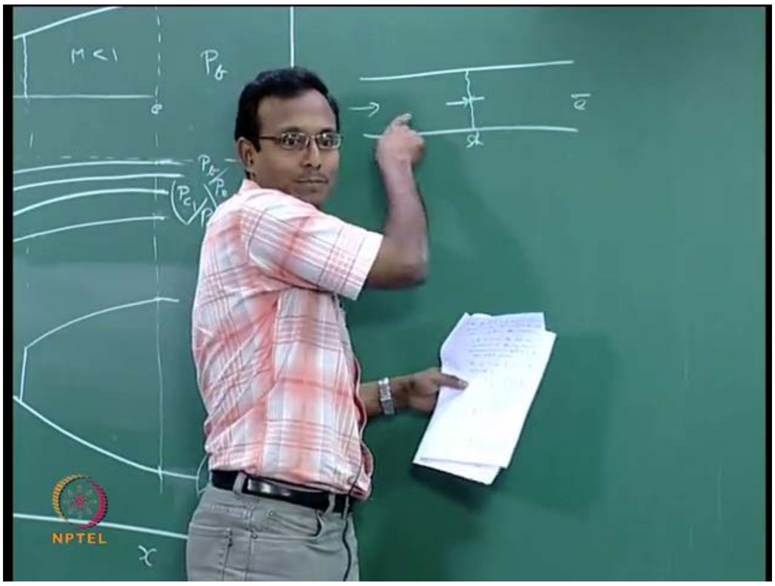

Lets think about it slightly differently, that is probably one way of looking at it, we had this is other example in flow in duct, constant duct with a mark with a single shock sitting at a particular location long time back when we introduce normal shocks, stationary shocks moving flow we said. The way we thought about it was the shock had it is own speed and that was mark the exactly with the flow speed.

And, so it was standing there, that is a nice way of thinking about it we said that it is as of this is flow is like a trade bell and the shock is running on top of it and it is equals speeds, so it maintain constant that is nice to think about. Now, suddenly if I send a expansion wave from here, what will happen this is an expansion wave this is actually a shock. What will happened if send on expansion wave, this is subsonic flow it will go again sit and reach it, when it reaches what is shock it is really a bunch of compression waves together.

What will happen to it all the compression waves, the last way alone will be negated by this expansion wave right. There are, so many waves, let say there are 100 waves inside that shock. I sent one expansion wave inside now, it is only 99 waves it is not 100 waves. So, what will happen that moving shock is no going to go slow because, it is not, so many compression waves it is one wave less. So, what will happen, treadmill problem easy to understand, what will happen the treadmill speed, treadmill speed remains the same, but the guy running again is running slower. So, he will go that way.

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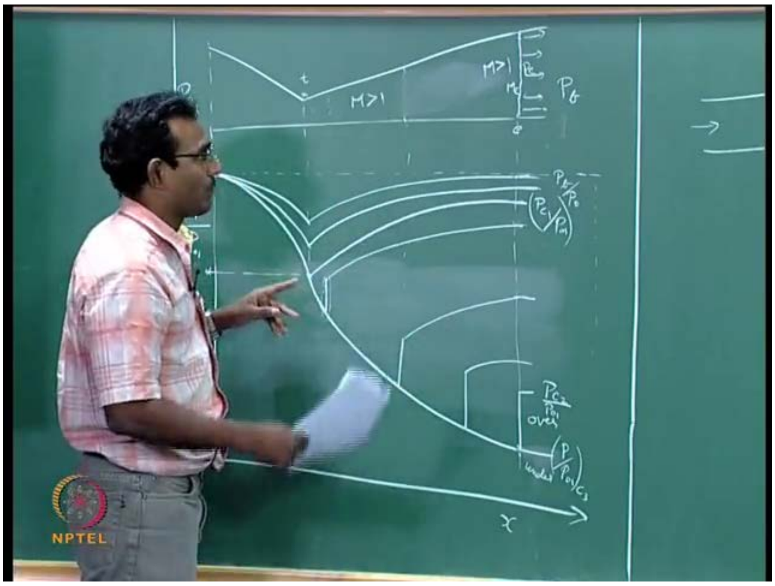


Now, we will go back to this problem apply the same logic here. I am going to say there was some particular back pressure when shock was here. Now, I decrease the pressure here, the flow was subsonic downstream of the shock fully, this whole region it was all subsonic. Now, since I decreased pressure the change always causes a wave, right the stadium we change the gate the flow they changes a wave width there is a wave goes through.

So, that decreasing the peruse since, I have to an expansion wave on the way through and by the way if you change it to suddenly and you are doing high speed emerging you will see, that wave way will travel through all the way. And till that time the shock will not respond to that, till that time of propagation of the wave of this shock will not respond to that. Now, we will assume all that is just high speed flow and the unsteady for not present current world because, you are thinking coastal steady problems.

We are making changes, but we will wait for infinite time after that, we would not worry about what is really happening during the unsteady motion. So, I am going to say I decrease the pressure downstream. So, there is an expansion wave coming in and weakening the shock which was originally here and because, of that the shock is being pushed by the flow of more downstream, but as it was more downstream the Mach number changes and because, of that the shock finds a nice place to set.

It goes there shock becomes stronger and stronger in the Mach number of shock gets higher it can run faster, but remember the flow is also getting faster, it will go and sit at a new point where it is going to be equal that is where it is sits. So, if I decrease too much the shock will go more out, this is one nice way of thinking about, it this kind of expansion and compression waves moving around helps you a lot in understanding and getting a physical field for the term.

So, I have got to this point now, if I decrease further what will happen again another expansion have comes through makes the shock we correct this particular location, it will go more downstream. I keep on going like that. say I reached this point there will be a point where, the shock will go to the exit and just outside it is meeting the correct pressure and that we said was p critical 2 by p naught 1 and again b very careful and marking p naught 1 always.

That is are normalizing factor, there could be p naught two in case of non isentropic problems where there is shock sitting inside my divergent or it does not matter for p critical 1 or 3, but I anyway just for consistency I just put p 0 1 everywhere, matter is mainly for this other cases where there is a shock sitting. So, if this is the problem, now there is a shock at the exit and immediately, after the shock the flow is meeting the pressure that is outside it is all matched perfectly nice it is just going to set there.

So, the full flow inside now we are at the exit shock at the exit I will rearrange this again, shock at the exit I am going to assume 1D flows. So, the flows going to go straight here, this region there is a shock here, it is m greater than 1. In fact, m equal to m exit at that shock location. And I am going to have subsonic flow just our after the shock and the pressure is matched p exit is equal to p b if you know to all these cases, up to p critical 2 that pressure will always be matched.

There will be a point where when I go any more outside the shock if I decrease the pressure further below the p critical 2 the shock will be pulled out further or the shock will be pushed out by the flow that is a another way of looking at it right, we said the treadmill business shock will be pushed out by the flow a little more. We will go and study that after finishing this nozzle flow problem, we study what happens just outside a nozzle that creates a z flow.

The next thing z flows, once you finish this we will discuss what happens outside this part. Let us just go through this problem currently as of now, we just say that the region between p critical 2 and p critical 3 is the region where, the nozzle has a p exit that is less than the p b value. That is the nozzle has expanded the flow more than required, that is over expanded case, the pressure is to low compared to what is needed, it did not meet the back pressure till here it the met the back pressure here, it does not meet the back pressure.

So, this is your over expanded region from p critical 2 to p critical 3 or p critical 3 it is perfectly expanded, perfectly expanded fully supersonic flow inside the nozzle. I can even say that from p b equal to p naught till p b equal to p critical 2 my flow is perfectly expanded. Because, it is meet the back pressure right, but typically when people talk about Seri nozzles they just talk about fully supersonic flow inside and they just talk about from below p critical 2 onwards when they talk about over expanded, perfectly expanded and under expanded.

Even other places, even above p critical 2 everywhere, I matching the back pressure exactly p exit, p at the exit matches the p back pressure p b, but this region is our over expanded, and this is your under expanded this is what you will have. So, this is just explanation now, I go back to our nozzle plots which we had all that is time we plotted up to above p critical 1 values what we saw before in plot, I will go back to this screen, we will look at remaining plots.

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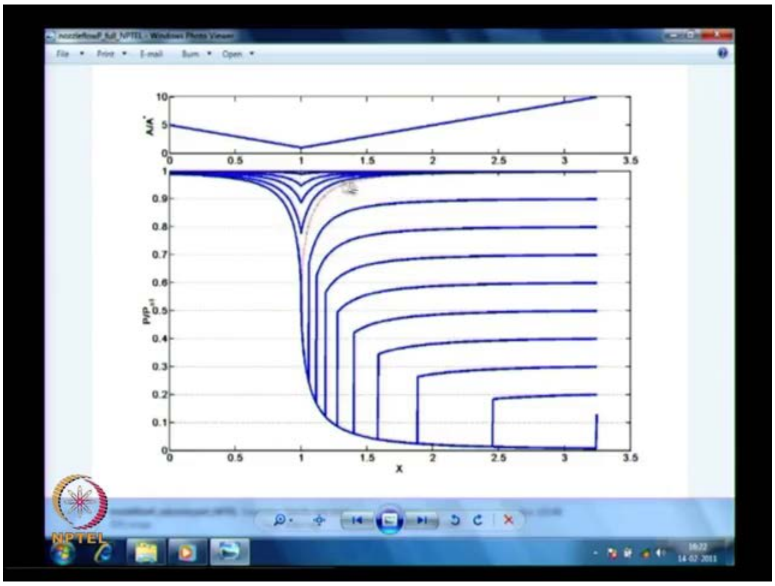


We will look at this particular plot, where it is again a by a star it is a exact same nozzle I am speaking and I am having this as the distance, same thing here distance and p by p naught 1 now, I have to be more specific p naught 1. And all the previous curves I showed they are all here this region. In fact, I showed you that the dotted line also which corresponds to that 0.9976 number that is also matching.

After that, you can see what p b by p naught 1 I used really, this is directly from here I used 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1 actually I used these numbers slightly above 0.1 I think it was 0.13 or something for this particular nozzle gamma equal to 1.4 case, that is the second critical pressure. So, if you see, what is there happening I have plotted for a linear nozzle, straight line nozzle it is this curved does not changed beyond that point.

The subsonic part does not change why; obviously, this point gets choked and whatever downstream happens nothing gets affected in the out stream of that mark 1 condition, this is choked condition nothing of strains changes beyond that point. So, this curve is the same for all these curves starting from this red dotted line all the way till all these curves, to the left of x equal to 1 is all the same every curve is 1 on top of the other.

After that, I am going to have different curves, after that if I pick this p b by p naught 1 has 0.9 then it goes to this up to here, a shock is sitting at this location and from there it jumping to some other pressure and it is going through this, going through the remaining part of subsonic solution. And you can see as I decrease my back pressure the shock is going more and more downstream, these vertical lines correspond to 2 pressure values at the same location which basically, means that I am having a shock.

So, it just jumps and keeps going like this and the shock goes on the way to the exit. Now, I will ask you a nice question here, if I look at this particular case it looks like the delta p is really high here, while for a higher Mach number the delta p is lower and even at the exit which is supposed to be the highest Mach number of the shock, it is the lowest delta p is that logical is that logical.

Student: Shock is moving.

Yes, shock is moving one case to the other, but the delta p seems to be getting lesser and lesser, is that logical delta p in this case happens to be p 2 minus p 1 for each across the shock for each of those cases seems to be lesser is that logical.

Student: Back pressure.

Back pressure, is continuously decreasing which means my strength of my shock should increase right, that is what we have been discussing till now. In fact, the Mach number at shock is higher, we have seen that the delta p is lesser and I know for sure there is nothing wrong in my calculations, is that logical can you explain this the basic question, when we say the strength of the shock. What do you mean by strength of the shock, strength of the shock increases, that is why we answer likes, what is that?

Student: To what extend it can make the flow subsonic in the sense.

Up to what extent it can make the flow subsonic

Student: (( ))

Probably we are talking about the Mach number jump the delta m we are talking about.

Student: Yeah.

We will look at that the next plot I have is delta m base plug, this pressure versus distance now, I have a mark number. We will hold that for now may be you are right, pressure based on answer is what I am looking for it, so happens we will go to the screen again.

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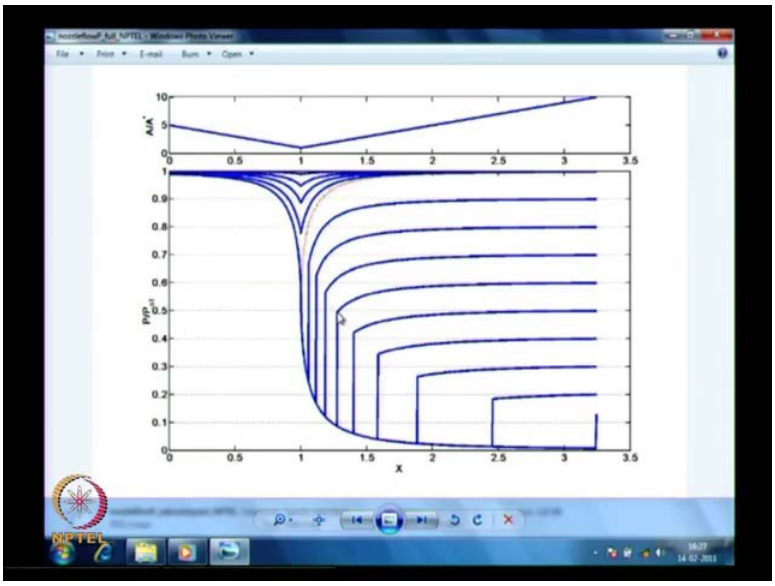

It, so happens that from here to here there is a huge pressure jump, I am at say 0.24 or something it is going all the way to 0.67 or so. And here, if I look at it, let us find the numbers say 0.25 and 0.68 or something like, that it will be of the order of 2.3 or so p 2 by p 1. Let say, I go here I will pick an lesser number, lets I will pick this 1 this is 0.1 to 0.5 p 2 by p 1 is 5.

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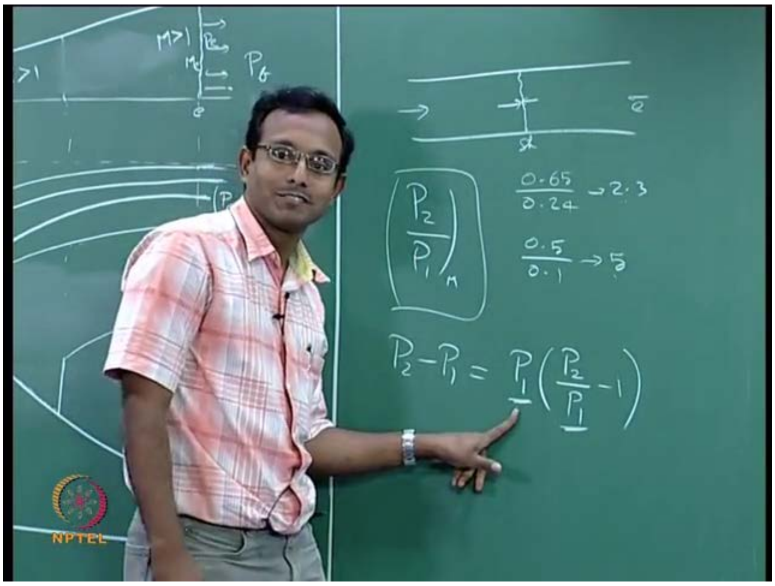

All this time we have been talking about pressure, the pressure jumps, but what we really wrote on the board for strength of a shock was really p 2 by p 1 at that mark number. This is the strength of shock, really the strength of the shock increases for that mark number. In fact you can see that in the plot also already I gave you two numbers, one of them was 0.65 divided by 0.24 and the other one was 0.5 divided by 0.1 this is at higher Mach number and this is going to give you an answer 5 this is going to give you something 2.3 or some. I am not just generally calculating in it something between 2 and 3, that is all of that order. So, you are already seen that the p 2 by p 1 has increased from there to here. So, it is matching with whatever we have been discussing till now, the strength of the shock increased, but I mean by strength of the shock is only p 2 by p 1 and naught p 2 minus p 1. If I look at p 2 minus p 1 this is actually, p 1 multiplied by p 2 by p 1 minus 1 this is what it is.

It, so happens that this number increases, but I am multiplying by a very small number p 1 keeps on decreasing, with Mach number increasing because, a shock goes to higher and higher Mach number the pressure on the supersonic curve, we will go to this for this plot I can even go to the screen, but this is better.

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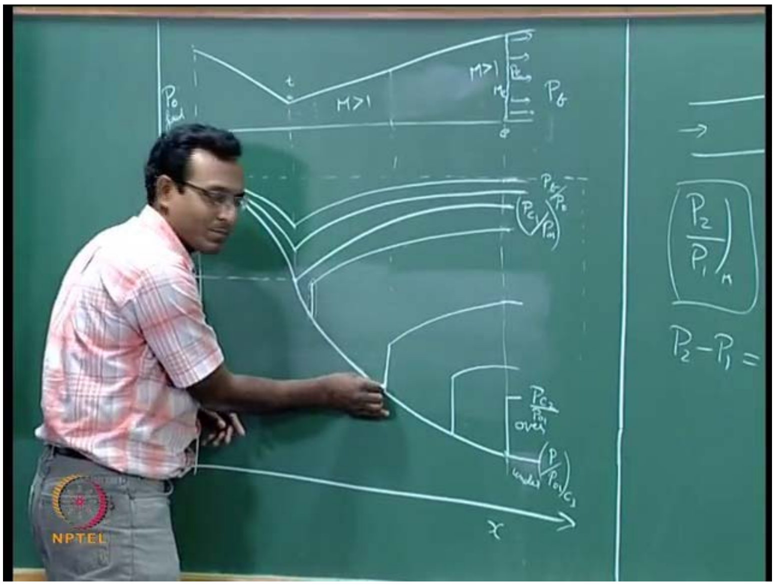


When I go along this supersonic solution, my pressure keeps on decreasing. So, at every shock location the p 1 this is just before the shock that pressure, is decreasing as I go more and more downstream. That is why it is look as if the plot in the we just go to the screen now.

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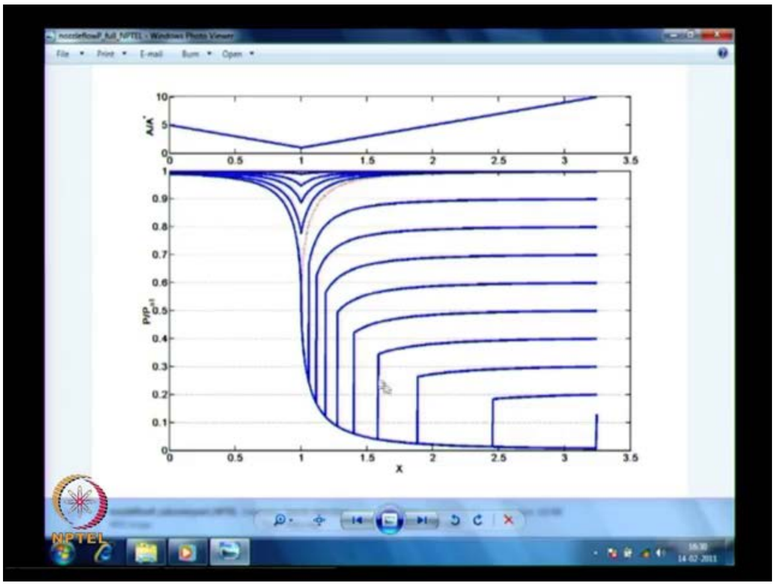


If I look at this plot it looks as if, then delta p is lesser yes delta p will be lesser that is primarily because; the p 1 itself is really low. From that low value jumping up to that high value of enormous job done by the shock, if I think about this particular case it is probably 0.1, 0.1 going to around 0.18 is like 18 times increase pressure. While at this particular condition it is roughly 0.1 going to 0.5 that is 5 times increase pressure has being increased by 5 times roughly here around 18 times here. So, it is increasing much more while it was the very first curve we had it was around 2.4 times or so. So, it is increasing drastically, that is what is the real truth? Now, we will go to the next plot, we had just Mach number discussion anywhere.

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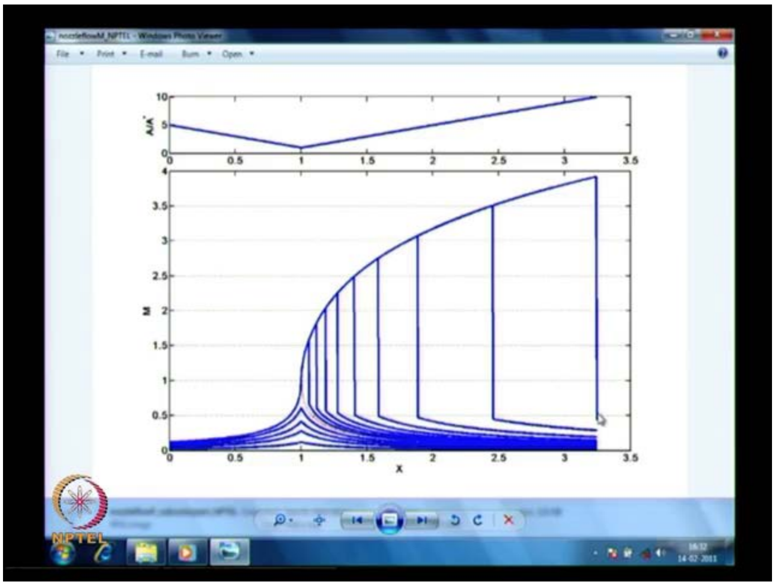

This is the Mach number plot. Now, whatever she was talking about is actually, true right the strength of the shock is actually, related to how subsonic can you take the flow. You think about it that actually, let explain this again the same nozzle plot above a by a star versus distance along x axis and here, I am plotting Mach number at every location versus x axis. For the same set of p b by p naught 1 I use to plot in a previous case.

If you look at for very low p b by p 1 values sorry very close to one values all the curves are here, it is going through fully subsonic solution and then it going back to low subsonic, it increases closer to sonic goes back down to low subsonic. There is point where it becomes sonic, when goes through this path the dotted red line is our p critical 1 pressure p back.

If I go anything above that and anything below that pressure my Mach number increases, it goes supersonic, jumps across the shock to subsonic if you can see that m equal to 1 line has here, it is jumping across from there to here to subsonic and then it is going subsonic solution decreasing Mach number further. If I decrease pressure further it is going to go to even higher Mach number and then from 3.5 let us, take it is going to drop to 0.4 or so. It is dropping really high, while in the initial cases it is something of the order of 1.6 to 0.7 or so rough numbers.

This is jumping only a little bit and when I go to the highest case I have that is probably 3.9 dropping to 0.4 or so something that of order. So, it is dropping really huge amounts, it is means the shock strength higher, that is way of giving the definition right we got one definition from here, which was a shock is considered to be stronger if it is starts from high Mach number and drops to really, really low mark numbers how low can it go in Mach number that is why we have thinking about shock strength also that will also work. That is this picture. Now, I want to go beyond what is given in ordinary books and I think I want to show you one more before that in animation.

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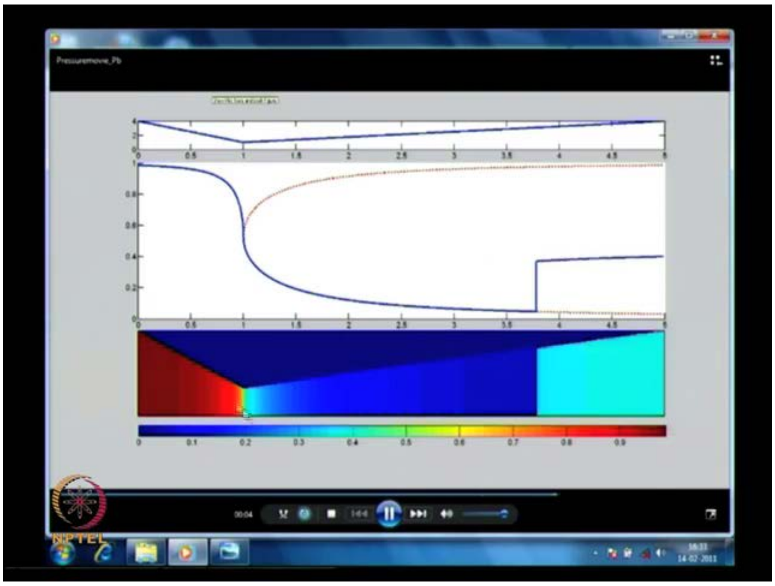

This is the same thing Just I have added animation of pressure, color coded, red is high pressure, blue is very low pressure and green is somewhere in a middle. So, what we are seeing is very high pressure, coming to some green is your throat condition choked and then it is going subsonic and the shock is moving as we go, this is what really happens inside your nozzle. We saw moving of this from an experiment I think last class or the class before ending here.

We start with fully red which is almost no flow and then it is becoming more and much more and more low pressures, to center becoming supersonic that blue is also in supersonic conditions very low pressures, and the shock this jump is the shock really and that is moving more and more downstream. This is one nice thing, we can of course, plot other things like temperature, Mach number everything similar they are look the same almost.

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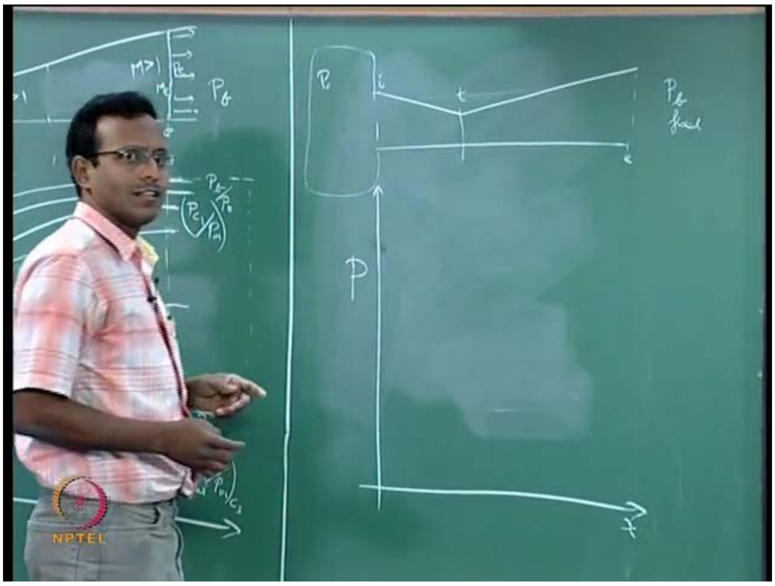

Now, I want to you to think little beyond this I want to look at the same nozzle problem, I am going to keep back pressure fixed now. This is my exit condition, this is my throat condition and this is my inlet and there is a big tank here which will be my p naught. These are standard model for our nozzle flow right. Now, I am going to say that I am keeping my back pressure fixed let us say, the nozzle is exiting into atmosphere, and atmospheric pressure does not, so much. And I am going to have a big tank now, the tank is being pumped in compressed more and more, there is more air pumped into this tank as we think about. So, if p naught increases what happens to this nozzle that is the other question we have to go into look at.

I will tell one simple answer, if I plot it as p by p naught versus distance, the plot does not look any different, it looks exactly the same. This is the reason by most books give you this plot. It does not matter whether I keep p naught fixed and vary p b or keep p b fixed and vary p naught, this plot does not change. That is the important thing there. So, most of the books will tell you this p by p naught plot is function of x, and they will just talk about one particular case which is keeping p naught constant and decreasing p b.

And this gives the wrong impression that if I keep on changing back pressure only I will having Mach number go downstream, that my shock go downstream that need not be the case. I can have the opposite problem if I want to decrease my p b by p naught let us say, that line right we use to draw this kind of plots, that line can be decreased by increasing p naught keeping p b the same. Even if I do that, that will drop to some other value. If that is the case also you will again go through the same set of plots, but it is more fun to look at this plot, as just pressure versus x is more fun to look at the font that way gives you a little more inside into what is happening in a duct. Let us go back one step.

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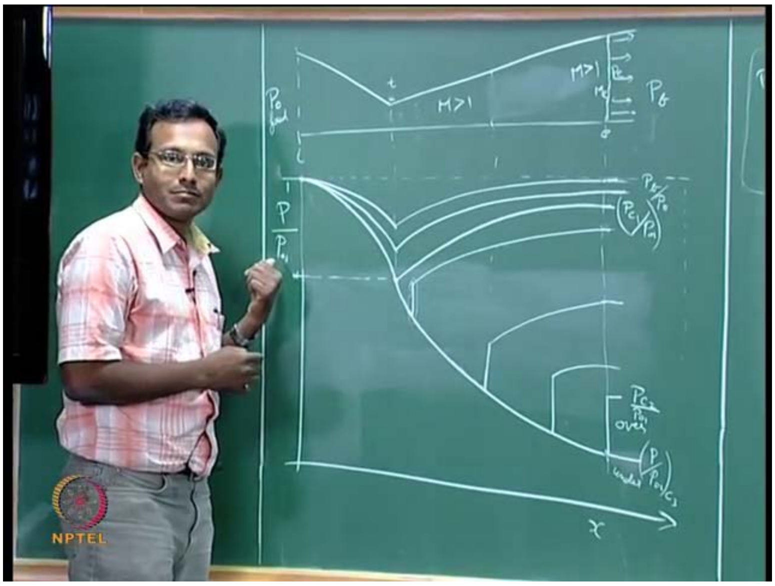

What if I do the same thing here, instead of plotting p by p naught I just plot pressure versus x what is this plot look like. It will not look any different because, I am just divide this whole thing by a constant that will just change the scaling, does not change anything else. If I multiply this whole thing, this whole plot every point on the plot by a p naught 1 that is just equivalent to multiplying this 1 by a number p naught 1. And similarly, if it is say 0.8 p naught 1 0.5 p naught 1 for my gamma right 1.4 it is 0.53 something. So, it is going to be 0.53 of p naught 1 that is all nothing changes it just scales this plot, that is not the case when I change the denominator.

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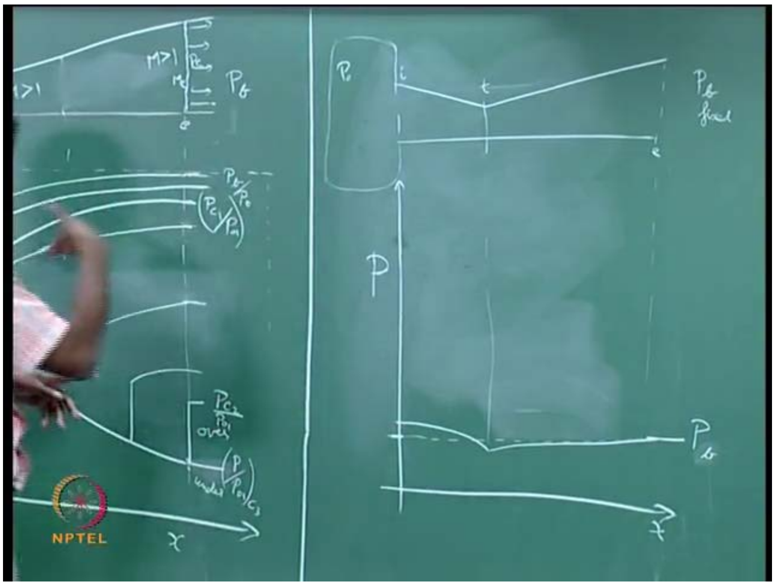


So, I will go and plot it again I will just plot pressure versus x in this case, if I do that. The first thing we have to says I am fixing my p atmosphere or my back pressure, this is always I want to call it back pressure and not exit to back pressure, that is a fixed value in my whole system. I am starting with this p naught and I could decrease it if I decrease this reverse flow in the nozzle, we would not consider that case initially we will just go above this if I go above this we know that from p naught the pressure decrease and then increases to p b, which means it should be something like this, it will go below and above. So, if I think about it like this what have I done from the previous plot.

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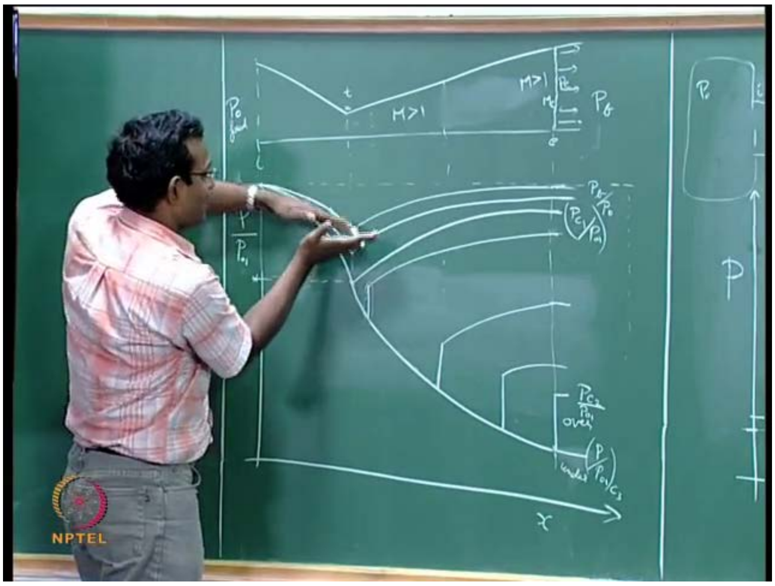

Let us say it was this case, this particular curve that is pick say I had that particular p b divided by p naught has the other curve there, what I have done is I multiplied this curve by a p naught, such that this curve has been scaled up a little bit. And this point has been make to reach this dotted line because, p b is a particular value we are fixing p b.

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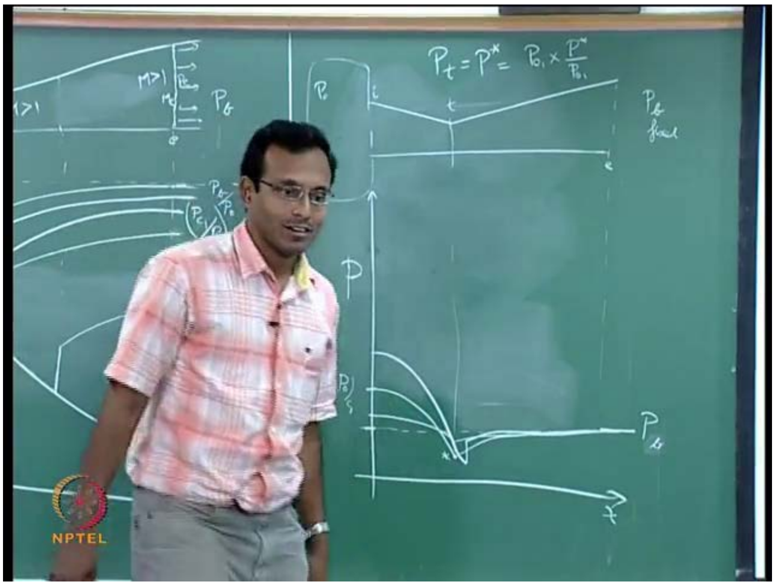

So, it has been make to reach this dotted line that is all I have done, just came here. Similarly, if I increase it further we know that it is going to go drop more and then go back up again, to reach the same p b. And let us say, this is the point where I get this star condition. So, this will be my p naught critical 1 now, I have to call it p naught critical 1 not p b critical that is a difference because, I am doing the opposite experiment I am not changing p b now, I am changing p naught that is a main difference and it is going to go through this path this is what I will get.

Now, of course, I can keep on doing this further and you will see that, now there is one more one more thing I want to talk about, if I increased p naught any further the choked condition remains choked. What happens to the p star, the star the pressure at the throat. P star will be the same, an increasing p naught. So, p star should increase why p at throat is equal to p star which is equal to p naught 1 into p star by p naught 1 we know, that this is a constant this happens to be your 0.528 for gamma equal to 1.4 that multiplied by, p naught 1. So, p star should increase.

So, if this is my critical pressure 1, the next case is going to be higher and then, we know that it is if I go above pressure critical 1 p b by p naught will go below my first critical pressure, which means I am going to go a supersonic jump across with a shock and go subsonic. So, it is going to go like this, jump across and then go along to subsonic solution to reach that back pressure. It looks more confusing beyond this point. Let's go to the screen again.

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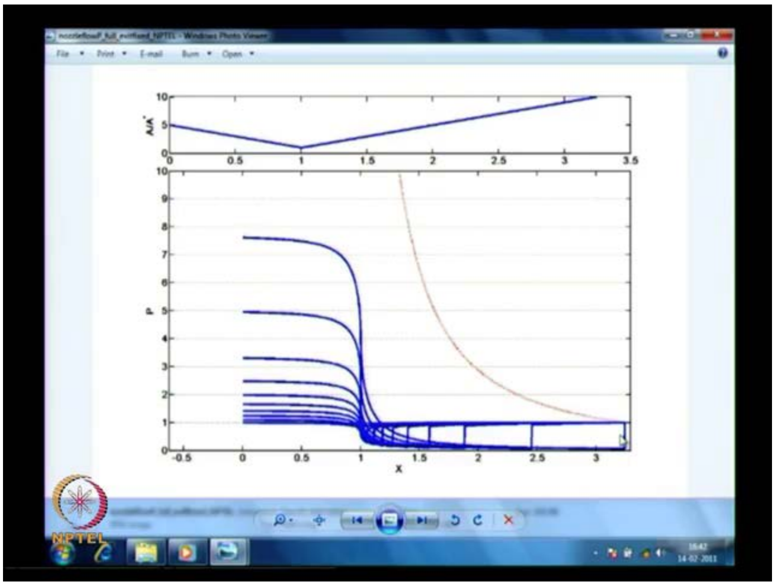

Look at this particular picture, where I am having the same nozzle I have just plotted it differently now, we are keeping the exit pressure exactly the same, as the back pressure. And the flow wants to meet this pressure over there happens, but we are saying as it starts with slightly higher value, it goes down and goes back up it is all merged inside here you cannot see very clearly, but say about 2 or so. It is going to go become supersonic have a shock and then go through the subsonic path. And as, I increase my p naught now, my shock is pushed more downstream. Now, I go back to my shock and expansion, explanation again we will go this picture.

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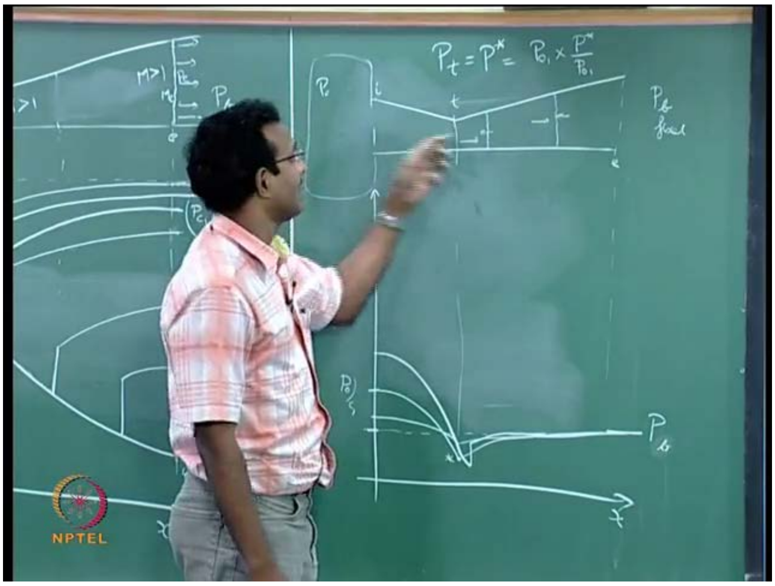

I am going to say that there was a shock pressure, here is exactly the same, but here I am increasing pressure. What do that mean the flow strength has increased. So, the treadmill is running faster even though the shock does not want to change it is strength. This is the opposite weight of looking at it, treadmill is running faster even though that person does not want to change his running speed.

He is still running this way, shock is continuously trying to run against the flow and the flow is matching it if it is staying constant speed, constant location, but now I am increasing p naught the strength of this flow is higher it just runs through. It is like a stampede right, if there is a lot of bulls running through and there is some frog trying to run against it is not going to happen, it is just going go through that way and it goes to some other location.

And where this will match nicely, till that time it will just keep going this way. Why will it match nicely here suddenly the shock strength is increasing, area is changing shock strength is increasing, it is running a little faster. And the flow strength will decrease because the pressure here is decreasing. So, they will come to a nice compromise here, and they will sit at that point, that is how it goes to the new point.

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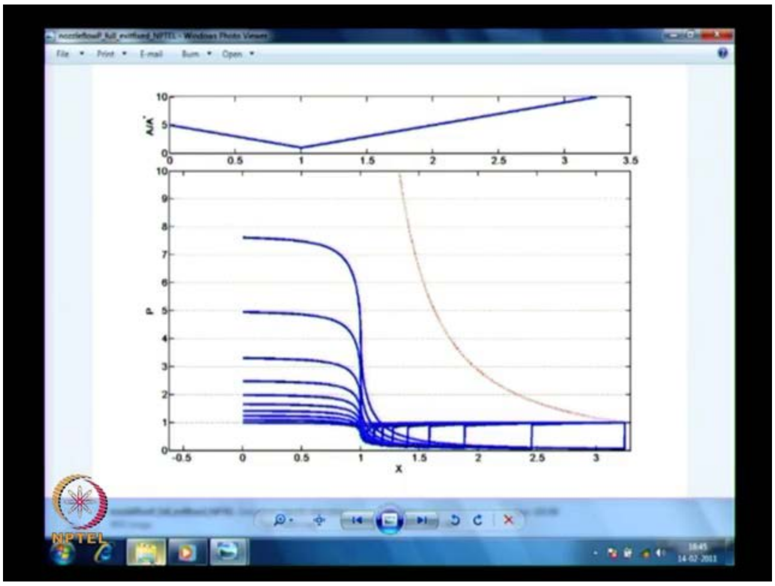


And of course, beyond that point this we go back to the screen, beyond that point I do not want to plot too many functions here, it will just make this just get crowded. I have drawn one case here and the one case where shock is almost at the exit. We will keep on drawing this forever and I have drawn that completely, perfectly expanded case here. As your dotted line here, where we are seeing the pressure for it to meet will be somewhere very high above the screen probably and it is going to come down and go down like this.

If we look at this x equal to 1 location which is our throat location, the pressure is continuously increasing, after it gets choked it is. In fact, even for sometime it will be decreasing and then it will keep on increasing above that, it keep on increasing at this point this is the pressure p star. If we look at that number from there it will be roughly half 0.75 half of that is 0.375 it is slightly more than that. So, it will be this is my x equal to 1.5 p star is higher it is above atmosphere here p star.

If I do this particular experiment of keeping my exit pressure fixed, back pressure fixed not exit pressure, back pressure fixed and I increase my p naught on this side. Then it is going to keep on it will decrease my p star for a little bit and then it will keep on increasing my p naught p star. Because, my p naught is increasing. Why will the pressure at the throat decreased for initial cases, initially it is subsonic where, there is very first case it is no flow.

So, pressure is same as atmospheric pressure, after that it is some flow, but it is all subsonic it will go below and then increase to atmospheric condition. That is the reason it is going to decrease initially, from there it will decrease after that all the curves will go up. Only if you have plot that are you will see it nicely I will just show you an animation in next slide.

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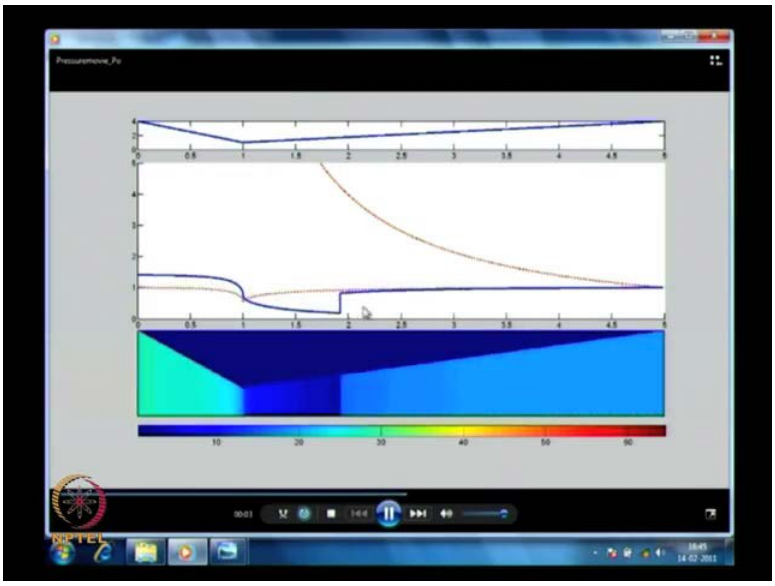

This is the other animation. This is the other animation, where I am plotting one by one in each of these, this dotted line of red corresponds to the p critical 1, this dotted line of red corresponds to p critical 3. And as this pressure increases you can see that the shock is formed and it is moving more and more downstream I am plotting 1 at a time. And this is showing you the same thing with pressure as the pressure increases here, the shock is being pushed out slowly. That is what this animation is supposed to show you, this just gives you a feel for what is happening in there. So, now we are at a point where we can move on to something else, but instead I am going to talk about, how to calculate these pressures and mark numbers, if I am given a particular nozzle geometry.

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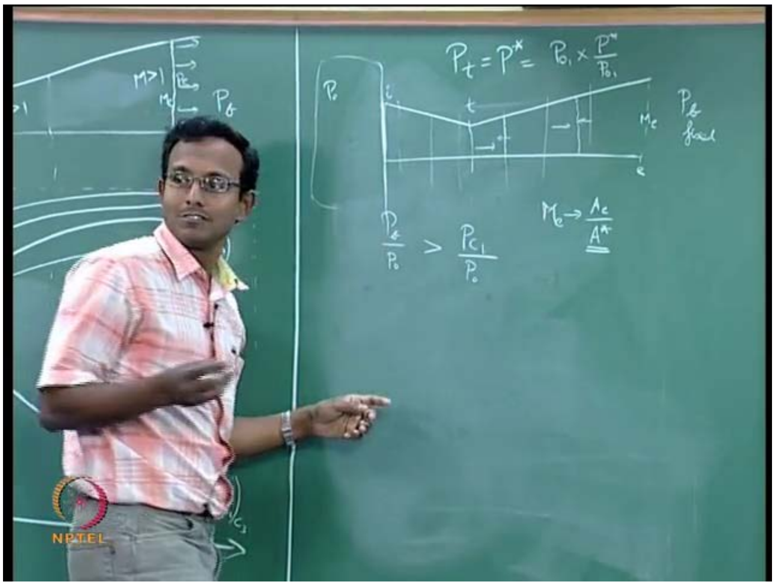


Given a nozzle geometry and given a particular p 0 and a particular p b how will I find the pressure distribution along distance. That is what we want to do next, and there is not enough time today, I will just give you the subsonic thing very subsonic, pressure p b by p 0 greater than p critical 2 by p critical 1 by p 0. This is very simple case we know for this particular condition, flow is always subsonic. Flow is fully subsonic, easy to calculate I know my area here and I know my it has to reach this pressure finally, I know my p 0. I will find p b by p 0 here it will match the pressure.

So, it is going to give me my m at exit. Once, I know m at exit I can use m at exit will give me my A e by a star, mind your a star need not be same as a throat because, it is not yet choked. So, I will find my A star for this particular flow. Once, I find that I have different areas in this nozzle all I have to do is find this A star and go and take any particular area A by A star will give me my m at that location.

Once I know Mach number distribution everywhere, I can find pressure everywhere, why I know my p naught. So, I can get from my isentropic tables for the given Mach number I will find the pressure. This is simple way to solve this problem, we will deal with the other cases because, it needs iteration. The remaining cases we will deal with later.