Gas Dynamics
Prof. T. M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 2
Lecture - 3
Thermodynamic Processes, Mass and Momentum Conservation Equations

Hello, everyone. Welcome back, last class we were looking at perfect gas as such we gave definitions for perfect gas. We told we can have calorically perfect gas or thermally perfect gas. Both of them will obey the ideal gas law plus along with that if it is a calorically perfect gas we are going to say C p and C v are constant. And if it is thermally perfect gas, we will say they are functions of temperature in our gas dynamics course why are we talking about this. It is simply because we want to use this ratio of specific heats gamma a lot we will start using it a lot. And that is decided by this particular assumption we are going to talk about calorically perfect gas predominantly we will say the gamma for the gas does not change for our gas in this particular course. But later, if you ever go to high temperature gas dynamics course where they will talk about changes in C p and C v. Because the gas now can vibrate in such things there you will have gamma variation with temperature which can be taken into account.

But we will keep it constant for now; it will work for most of the common problems. So, the next thing we want to do is we should be able to plot any process on a state diagram. A state diagram is basically we are trying to plot say pressure versus volume as the axis vertical axis is pressure horizontal axis volume. We want to plot say a process an adiabatic process, isentropic process. Whatever process we are thinking about constant temperature process, isothermal process any process we want to plot we should be able to draw. When we are thinking about it this whole concept actually was started sometime back by Gibbs. Again he was the first one who started giving plots of functions as the way of explaining process or a path. I made a small mistake last class I told that Gibbs was the father of modern thermodynamics it is not true it is actually thermodynamics. The classical way Gibbs just talked about equilibrium based thermodynamics not non equilibrium thermodynamics which is the modern thermodynamics we would not talk about that here anyways.

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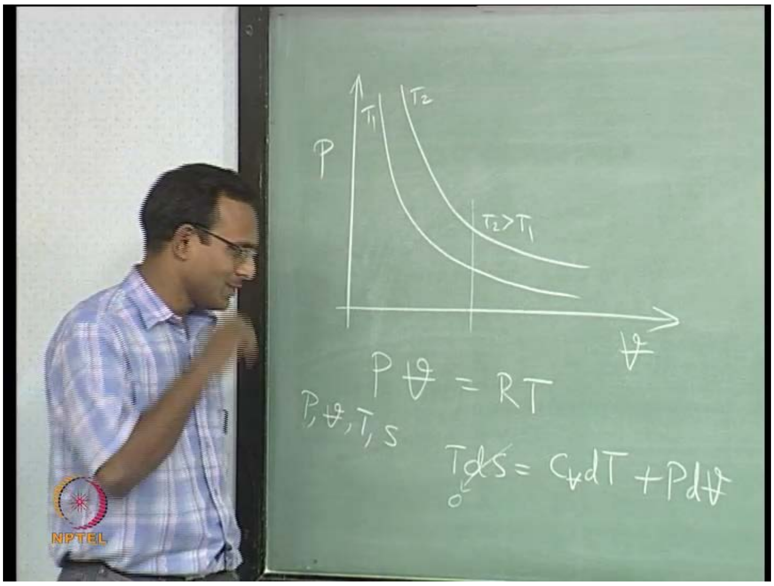

So, now we will start with I am plotting a process on the state diagram which is basically it could be 2 variables any 2 state variables I am picking pressure and volume. Now, I am using volume per unit mass specific volume. And now here let us say we will start with constant temperature process. How will that look like it will be a hyperbola there are 2 types of hyperbola which one rectangular hyperbola. We are talking about something like this how did we get to this point? This is a constant temperature line, how did we get to this? We are going to use ideal gas equation I am using this particular relation. And then I am getting this of course, this is equivalent to p equal to rho R T this v is inverse of density. By the way you should be comfortable with various forms of ideal gas equation, anyway I just used this for constant T P v is a constant I get to this curve.

So, for some other temperature it should be another line let us call it T 1 and T 2 which one is higher temperature. I have to ask them which one is higher temperature curve T 2 is higher. How do you know this for a given volume easiest way to do this fix the volume find out which one has higher pressure, higher temperature will have higher pressure. So, I can directly draw and tell this one is higher temperature I can easily say this you should know this kind of curves. Later we will just start talking about various processes and I want you to plot it in different state diagrams. This is just one particular set of variables chosen for axis I could have chosen temperature and entropy or pressure and temperature any such variables I could choose.

We will look at a few of them today, this is just one particular case predominantly we use only these 4 variables pressure specific volume temperature and entropy. You could think of it as volume also I am not worried about that intensive or extensive I am not worried so much. The curve will just stay the same it may be scaled a little bit depending on whether it is intensive or extensive. So, these are the 4 variables predominantly used. So, we need to know whatever be the axis out of these 4 any two I should be able to plot any particular process. What will the process look like? If it is a constant pressure process just horizontal line somewhere volume increasing or decreasing we will decide which way it goes. It could also have a constant volume process; it will be a vertical line. Now, what if it is constant entropy? We have tried everything else constant temperature, we have constant pressure; we said was horizontal line constant volume was vertical line constant entropy. It will be a curve, how does it look with respect to this curve that is important for me perpendicular as in you are talking something like this. No I do not think.

So, this one you gave an expression you gave a plot based on expression give me the same thing for that case. That is what you want really you have to be mathematically correct always it will be a log curve, not really what should it be wherein P and v? We want to find a relation between p and v for a constant entropy process. How will I get to there? We want to set somewhere d S equal to 0 that is a clue I am giving you we have to go back to T d S relation. Gibbs equation I am going to write this I will write this relation now I will say I want isentropic process. So, I am going to set this equal to 0. Now, I have a relation between temperature and volume, But I want pressure and volume what should I do rearrange things? I have to rearrange this temperature in terms of pressure that is what I need to do. How will I do that? I want d T in expressed in terms of d P then I can integrate this as the overall thing we are looking for how will I do that. So, I have to take a derivative of this what will that look like this is what it should look like.

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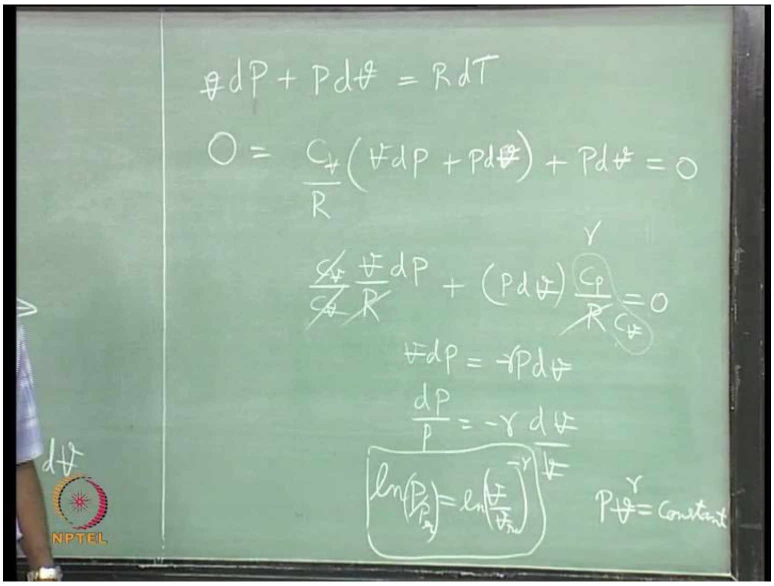

$$v\,dP + P\,dv = R\,dT$$

$$0 = \frac{C_v}{R}(v\,dP + P\,dv) + P\,dv = 0$$

$$\frac{\cancel{C_v}}{\cancel{C_v}}\frac{v}{R}dP + (P\,dv)\overset{\gamma}{\frac{C_p}{\cancel{R}\,C_v}} = 0$$

$$v\,dP = -\gamma P\,dv$$

$$\frac{dP}{P} = -\gamma\frac{dv}{v}$$

$$\boxed{\ln\left(\frac{P}{P_R}\right) = \ln\left(\frac{v}{v_R}\right)^{-\gamma}}$$

$$P v^{\gamma} = \text{Constant}$$

Now, I have to replace this d T with this expression and substitute it here that is all I need to do. So, I will go back and write I will put this side as 0 d T I will put this R below that. So, I will just get d t. This is the expression I have now it is getting to be simpler what will I get it p d v can be coupled together. So, I will have v d P by R or this is C v also sitting here C v v d P by R plus p d v multiplied by 1 plus C v by R which is what 1 plus C v by R will become C p by R. I hope you know this should not be that difficult to get and this is equal to 0. Now, look at it R will never be 0 I can remove this right multiply by R should not change the expression. So, this is the final form I have I will rewrite this a little bit I will take this C v divide by this. So, I will get divide by C v on both terms this will become your gamma that becomes your gamma. So, I can write a simpler expression take one term to the other side. So, let us say I will keep v d P minus gamma p d v get to this form.

Now, I will just put a d P by p now you can give another expression for this if I integrate this I will have log p ideally. It should be p by p reference I will put some p reference. I will put p R as p reference is equal to log of v by v reference to the power minus gamma. I hope you are not getting confused with that expression I will write it closer it will get to this form now you of course, if both sides are log. So, I can just take exponential of this function, we will get to a simpler form which will give me the answer as p v power gamma equal to constant where the constant happens to be some p reference v reference to the power gamma. That is what it will come out to be, it could be any value you just pick some number. Now, I want to plot this expression I want to plot that expression back on the same plot. So, and here I want to plot this for a given temperature constant line p times v is a constant.

For the other case p times v power gamma where gamma is more than one right for air it is 1.4 it is never less than one for air it is 1.4. So, it is more than one. So, if I pick some particular condition let us pick this v times p is a constant as here for the same p. If I want to say from here I move to some other volume then say I double the volume. Then my pressure should be half that volume should naturally occur if you have drawn the curve correctly let us assume I have drawn it correctly. So, I should get half the pressure it is close now if it is the other case what should happen? Pressure should be even more even lesser why p times from here I am going to a new condition p times v power gamma I double the volume. But 2 power 1.4 is the factor I have to take into account now. So, that will be even bigger number.

It will be roughly 2 to 2 root 2 or something 1.4 somewhere 1.4 times this 2.8 times this sorry 2 root 2. So, I am going to sit somewhere there. So, that big number multiplied by p is a constant let us say I started here. Now, my pressure should be much lower to compensate for that for the newer volume that is what it will be equivalently. So, my new point is here. So, my new curve is going to go like this with respect to the other curve. This dashed curve is your s equal to constant we started with that we said d S equal to 0 which means s does not change that is how it starts we came to this point. So, now, I have isentropic like now I can tell that if we are having isentropic flow. Then on a p v diagram it will look like this with respect to a constant temperature passes. We have to know how it looks between these two, there is a difference you should be able to do this whole exercise for a new variable.

That is the idea overall. So, now, this is for say s 1 I want to draw for some other entropy. Let us say s 2 which is higher than s one of course, the curve go will be shifting one way or the other let us say we are looking at along v you go to the left or right when s 2 is higher than s 1 goes right why anything you say I want reason. It may be correct or wrong I do not know somebody can give an explanation for left also no explanation. Let us think about this, can you give me a better explanation if it is above or below does curve moves up or down in terms of pressure? You have to basically think about what happens to entropy if I change volume that is how you get to answer. What happens to entropy if I change volume if I increase volume? What should happen there is more space for molecules to move around which means there is more disorder possible is that entropy is related to disorder molecules are more happy if you think about it that way.

They have more space they are more happy they are not constrained themselves as smaller volume. So, entropy will be higher if I increase volume. So, I would say moving to the right entropy increases. Next curve will look like this by the way the curves should diverge more as they go down. They will diverge more these matters in enzymes if you are thinking about enzyme cycles they use this to produce useful work. There are different cycles which use this to get different useful work enzymes work with different cycles. Brighton cycle, lotto cycle, diesel cycle whatever they are going to think about all these it depends there this curves matter. So, I have to know what kind of curves they are all now this is the simple thing we just did p v diagram next thing.

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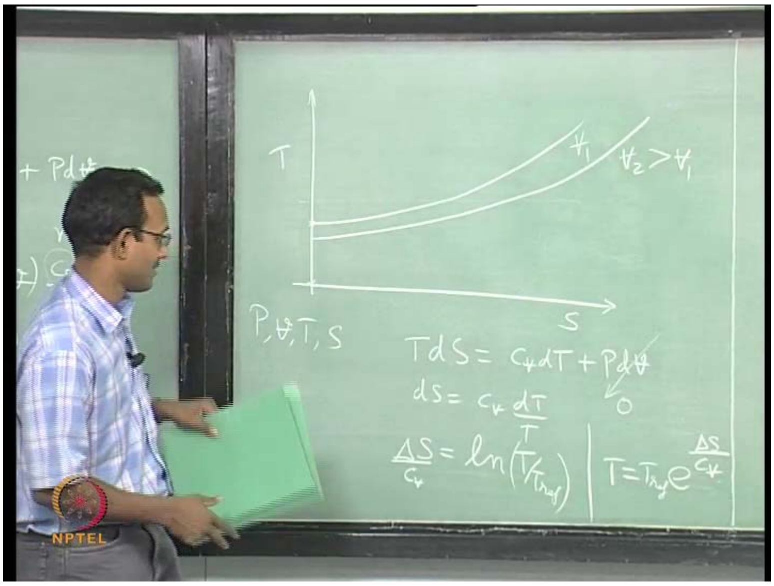

Let us say we pick a T s diagram temperature and entropy are the variables now. So, the standard thing we had those 4 variables right of which if I want to talk about constant temperature process of course, it is a horizontal line. If it is a constant entropy process it will be a vertical line simple step no point drawing it, what if it is constant pressure or a constant volume process? That is the next thing. So, how will that look like where will you start, how will you start if pressure increases constant pressure curve will be like this. Why it is probably correct I do not want you to just guess give me a expression.

How will you start? You have to go through the same process whatever we did here the whole thing we will start with again we want somewhere we want to say. Let us we are picking constant pressure. So, we want to say somewhere d P equal to 0. And we want the variables to be s and T for me to integrate what will I start with T d S equal to that is the relation we are going to start with. But this has volume of course; I can just take a short cut now. And say let us draw a constant volume curve let us do that right now, because I have drawn I have written this expression will say constant volume curve is what we are plotting? First we want to make this equal to 0. Now, I will just integrate this I that will give me a relation between T and s what will that look like this is the thing.

So, I can get something I will write it like this or equivalently I can write this as T equal to some constant T ref times exponential of this function. Now, you can tell exactly what the function looks like. This is easier on starting with some T reference it will have some s value that delta s is basically s minus s reference. So, it will have a T reference s reference and starting from here the curve goes like an exponential growing curve. This is how you should think about it. Now, this is for one particular volume, we said volume constant one particular volume v 1 what if I have another volume? Let us say v 2 greater than v 1 what will that curve look like increasing volume increasing the volume increases entropy. So, which way will the curve shift to the right or to the down whichever way you think about it? We will think of it as going to the right if I pick a particular temperature and increase its volume molecule finds that for the same energy. I am given more space I am more happy, it is not constrained; it is not tight not having enough space more happy that is the easier way to think about entropy.

If molecules are happy entropy is typically higher. Just have this feel for it, it helps you in explaining things easier. This is typically the case or I am saying the curve shifts downward when I increase volume or curve shifts up when I increase density. They are equivalent right this volume we can equivalently write it as volume per unit mass that will be 1 by d, I can plot it that way also it will just come out to be that. So, this is one case this is simple case, now we want it actually we started with constant entropy constant temperature we finished constant volume. Now, we have to think about constant pressure, how will I solve this I have to rearrange this equation such that I will have d S and d T. But that term should be a d P what will that be we already wrote that yesterday.

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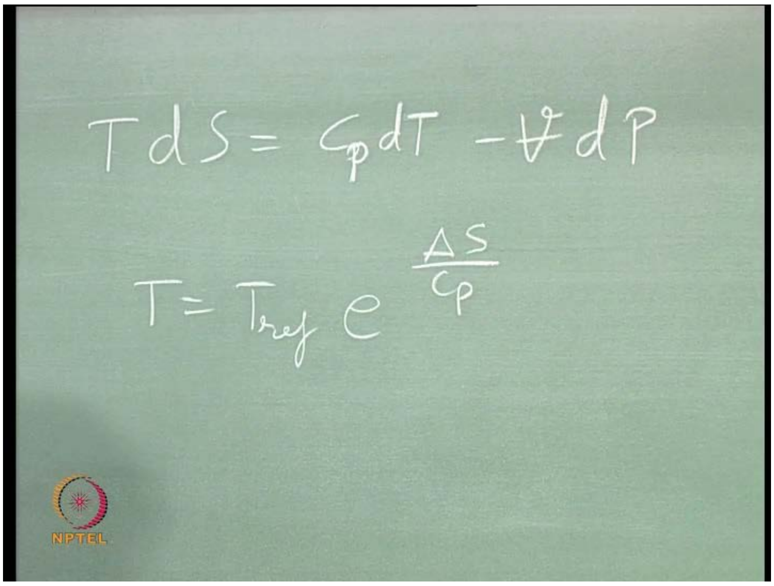

$$T\,dS = C_p\,dT - v\,dP$$

$$T = T_{ref}\, e^{\frac{\Delta S}{C_p}}$$

T d S can be written as d h minus v d P, how do you get to this from this expression I will just go and substitute for d v from p v equal to n R T actually p v equal to R T. Just use that put it in here, we will get to this form whatever we did before at the beginning of the class the same thing we are doing here. Now, we can solve this problem I do not need to go through the whole process I believe you can integrate it. Again you will get to this is what you will get, if you go through the same integration like in here simple enough we will get to this form. Now, what will you do? What will this expression look like? This is again exponential this is again exponential, but I want to compare this curve. And that curve constant volume curve and constant pressure curve C p is higher than C v. So, the exponential factor will be lesser.

So, temperature will be lesser for the given entropy. So, I have to draw the curve it may start here T reference and s reference are the same. But the curve will go different path it becomes more confusing with this label in the middle that is your v 1. Now, this is your s 1; this is still an exponential function, but with lesser power right the exponential factor is slightly lesser. So, it will be like this p not s, thank you. It is constant pressure line not constant entropy line p 1. Now, if I go for say higher pressure p 2. What will that curve look like? Same logic entropy should increase when I increase pressure what happens to entropy decreases? Why molecules are pressed together? They are coming closer they do not have enough space think of it. That way it cannot have more disorder they have to become arranged more, because there is not enough space less disorder.

So, entropy should decrease. So, the curve will shift this side p 2 greater than p 1 will be than curve. These are going to be useful for you when you are going to think about. What kind of process my gas molecule is going to go through gas? Fluid element is going to go through then we have to think about which process did it go through. And it will help you a lot Of course, now I can start thinking about various permutation combinations of this with different axes. Of course, I can now pick p versus T and plot the remaining 2 constant variables that is another curve I can get I have to go through the same kind of process. We have been doing till now, but let us say we would not do that in here let us say you do it as an exercise just go. And solve this whole thing not difficult to do just go through the math once it is useful.

So, I will assume that you can give me such curves. From now on more useful curve will be trying to plot pressure versus temperature axis. The remaining 2 constant curves volume and entropy constant curves plot that also it will be useful for you. Pressure versus temperature, volume curve will look like what constant volume curve straight line raising as temperature increases should know that how do you get that p v equal to R T that is how you get that. So, you have to think about that how will you plot it each one, you just have to go through the same process; you will get to it. So, that comes to the end of review of thermodynamics for us. Till now we have been reviewing thermodynamics Of course, I just wanted to go through a little bit of derivation like this.

So, that if you need it in the future you can go and derive it anytime. The next thing is review of mechanics it will not be really review of mechanics which is going to start thinking about what are the laws of mechanics which we will use in gas dynamics. What will, what are the laws of mechanics? What is that conservation of mass? Yes once that starts and now people will just start telling everything from fluid mechanics. Conservation of mass, conservation of momentum, conservation of energy all of them should be conserved any other laws from mechanics.

There are others Newton’s laws unless there is an external force things do not want to move. And you are going to think about rate of change of momentum is proportional to the force. Actually it is going to be equal to the force if you think about rate of change of momentum mass to energy inter-conversion as a nuclear reaction if you need it you can keep it. But typically people do not consider if you are in astrophysics where gases come closer.

And then suddenly there is nuclear reaction nuclear fusion like in our sun hydrogen becomes helium and helium becomes carbon. And all that if such things are taken into account if you want to take such things into account. Then yes you can astrophysicists deal with the same set of equations. But they will have that extra term nuclear reaction term in that for stars. We would not deal with it, we will just say there is no nuclear reaction. So, there is no energy to mass conversion for us if you have in your specific problem. And you want to deal with it yes you can, equations are such that you can always drop out terms if you are not thinking about that. So, the set of basic laws of mechanics we are going to use are inertia things do not want to move unless there is some external emotes.

Then we will start talking about if there is a force there has to be some rate of change of momentum if it is a torque Then there is rate of change of angular momentum we would not deal with swirling flows right now in our problems. So, it is just straight flow no torques for us of course, there can be shear stress need not be a torque component. Other than that mass has to be conserved momentum has to be conserved energy has to be conserved these are your various mechanics laws. We are going to this next thing; we need to think about as control volume, what is a control volume? Any specified region an enclosure for our system a reasonable explanation. But there has to be some intelligence into it after that I can pick any volume or my system can have any shape. Depending on what I want to do depending on how my analysis can be complicated or simplified we have to choose our control volume.

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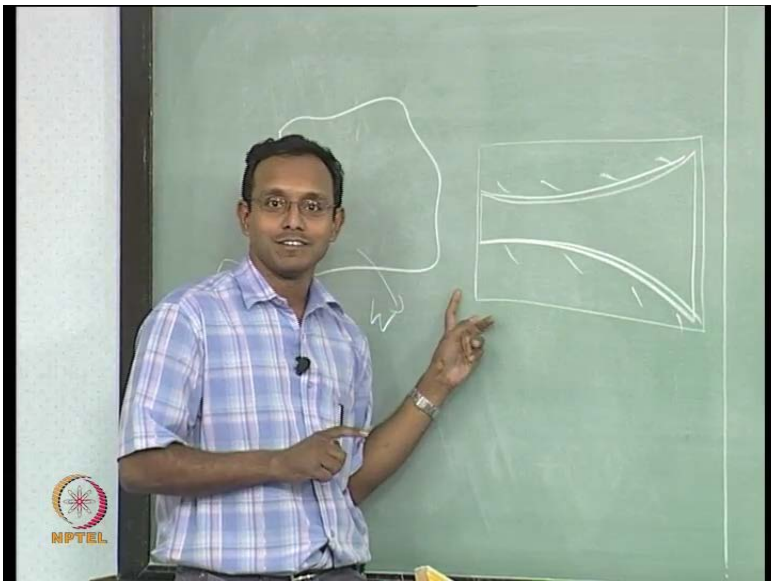

If I think about a general system all this time I have been drawing some box and said everything inside. This is my system what I have done is put boundaries around and that boundary encloses some volume inside. That volume is my control volume, why is it called a control volume? I am thinking about only whatever is happening inside it. And now I will just think about heat interaction, work interaction. I am putting only one kind of arrows remember that we talked about it last class where we said q in is considered positive w out is considered positive. We just keep this convention it can be any convention; let us fix this I have something like this. Of course, I could think about there is q 1 in q 2 out then this q will become just the resultant of those two.

The difference between those two it whichever it is something like that can be talked about for work also we are currently going to neglect shaft work. And other things saying we are having a simple compressible substance there will be no shaft work concept. We would not deal with a fan sitting inside are stirring my gas heating it the joules very first experiment remember which talked about energy heat and work are same he said. They are both forms of energy that experiment we would not do, he did it in water by the way we are thinking gases. So, control volume I can pick any control volume for analysis math allows you that. But some control volumes are easier to pick than others like if I have a flow through some such duct I could pick a control volume like this everything inside this wall can be a control volume then.

And then now I want to study how much mass enters, how much mass leaves? What is the force interaction at the surface? That is all simple. What is the shear force? What is the normal force? All the interactions with the surface can be talked about in this problem. This is one way of picking up control volume typically pick a different control volume something like this. This is a simpler control volume to work with. So, depending on how you pick a control volume your problem may becomes simpler or complicated. It is just that if I pick the first control volume I choose the inner one. Then I need to know pressure and shear force interaction all over everywhere inside in the other one I do not need to know. So, this is a better control volume to choose to simplify my problem. Of course, now I have to give some number here for normal force shear force everything at the boundary. I have to talk about work and heat interaction for this control volume this is a better control volume to choose.

Because now I have to prescribe only normal force, normal flow everything simple easier than this other thing where it is a complicated shape. And I have to give normal force in this direction normal force in this direction normal force in this direction and all that its more complex problem to choose. So, there is some kind of what you say a personalized way of choosing it is an art a in engineering, we still have art picking a control volume is still an art. So, you have to know which control volume to pick. And that will change your analysis, the way you solve the problem currently we would not go. And pick any control volume for a particular problem, we will just derive general laws conservation of mass is what we will pick first.

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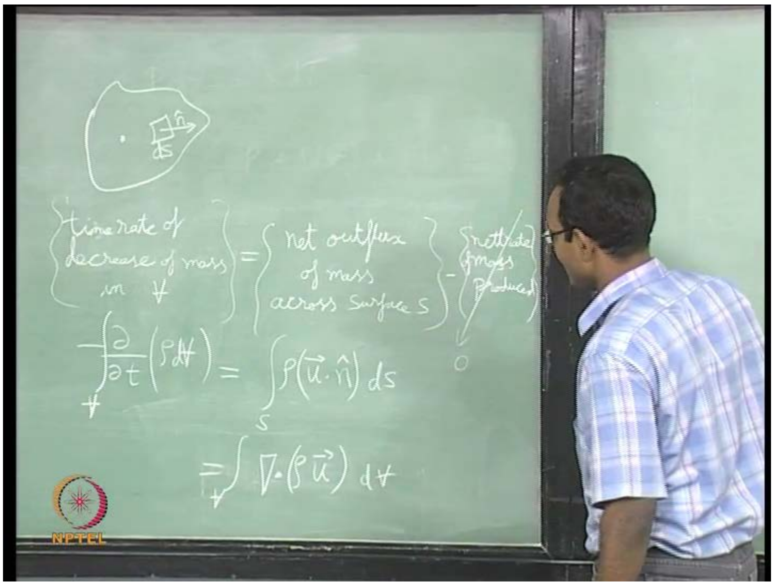

So, I am going to pick a random control volume and I am going to think about let us say it is a 3 d volume. I cannot draw 3 d on a board. So, imagine that this is some balloon like thing where I have drawn this square on the balloon surface bulging out. This has a normal vector outward and I am going to say it has I have to be careful here with this s. Till now we used d S as differential entropy this is just differential area on the surface it is also s. We will keep it that because it is a surface area. So, just know what it is we will never get into trouble, because soon we will get rid of this s from our problem. Now simple enough like what we talked about with energy last time, we are going to think about conservation of mass energy is conserved mass is also conserved. So, simple enough way to think about it I am the control volume somebody gives some money I keep it or I send it out. Now, we are thinking about what is the net money on me that is the kind of analysis we are thinking or I may be printing money it is a possibility.

Then I may send out more money than what is coming in may not be reality. But it is possible to do that in theory right that is what we are thinking about. So, we are thinking something like that. So, my conservation law should be thinking about in these directions, how much is given in how much is being stored inside. And how much is sent out is there anything produced from inside that should also be taken into account. In a way I can simplify this into 3 terms instead of 4 how much comes in, how much goes out can be told as how much net went out right something came in something went out. The difference is the net went out easier way to think about it.

And then some money may be getting accumulated inside in this case some mass may be getting accumulated inside or think about how much is getting accumulated inside. That is one more thing is there any mass being produced inside. If we are not thinking astrophysics we do not need to consider that if we are not having radiation forming mass. Then we do not need to worry about source of mass in our problem. We will typically not have that situation there could be a special case where I have say a solid sitting inside occupying almost no volume suddenly it burns to become gas. Previously I did not want to consider the solid in the form of gas, because it is solid now I want to say suddenly gas is produced inside this volume.

Then I may need to take that into account that is a special case it may happen in a propellant burning inside a rocket motor one special case it may happen. But there they take into account how much mass is entering depending on how you draw your control volume. I can get around this problem if I draw my control volume with a boundary around that solid. Then I get out of this problem then the mass is added from outside of the control volume into it. Then I will not have a source term there is no mass source inside that is why you have to be it is an art of choosing which control volume you like. We will get back to this problem I am just going to say the conservation term it is going to look like time rate of decrease of mass in the control volume be is going to be equal to net out flux. How much money am I giving out that is the total money I am loosing from me it is the same thing in here mass net out flux of mass across surface s where surface is the full surface of my control volume.

The full area I am considering I will keep this just in case somebody wants to work in astrophysics. Again I will tell it is 0 a soon I will tell minus mass produced net rate net rate of mass produced inside the control volume. In case somebody wants to think about it this way for our practical problems in gas dynamics this is always 0. So, we will set it to 0 now of course, it is just a review of mechanics. So, you should know some fluid mechanics already where you would have derived this in more serious manner we will just look at what is this term? And we will just go a fasted time rate of decrease of mass. So, I am going to put in negative gradient in time. So, it will be minus dou by dou T because it is rate of decrease minus dou by dou T of mass inside volume will be rho times it has to be capital v now.

This is the net mass inside the volume of course; I have to put a integral over this for the whole volume. So, I have to think about it more carefully and make it d v and put an integral outside. This is what it should be rho d v is any small volume inside. And then I am integrating over the whole volume of my control volume this is what will come to it this is just the first term. Now this is going to be equal to the out flux, how do we define out flux? It is related to u dot n it is related to u dot n it is not just that rho times u dot n times d S where this s is surface area as of now. And now this is just for a small elemental volume I want to integrate this over the elemental area sorry small elemental area on the surface of my control volume. I have to do this for the whole surface. So, I will integrate this over the surface now you will just go and ask mathematician call Gauss how to convert this. So, that both are looking like volume integrals.

One is surface integral other is volume integral gauss has a vector algebra based theorem which will just directly link this to volume. The net flux of a quantity out of a surface is going to be equal to the net divergence inside that volume enclosed by the surface. This is the wording of the mathematical statement they are talking about. So, I just write that in the mathematical way it is del dot rho u that is your divergence term we get to this form. So, now, it is easier to look at this expression they are all integral over the full volume d v. So, I will combine these two take both the expressions to one side combine this make it equal to 0 and look at it that we will write here.

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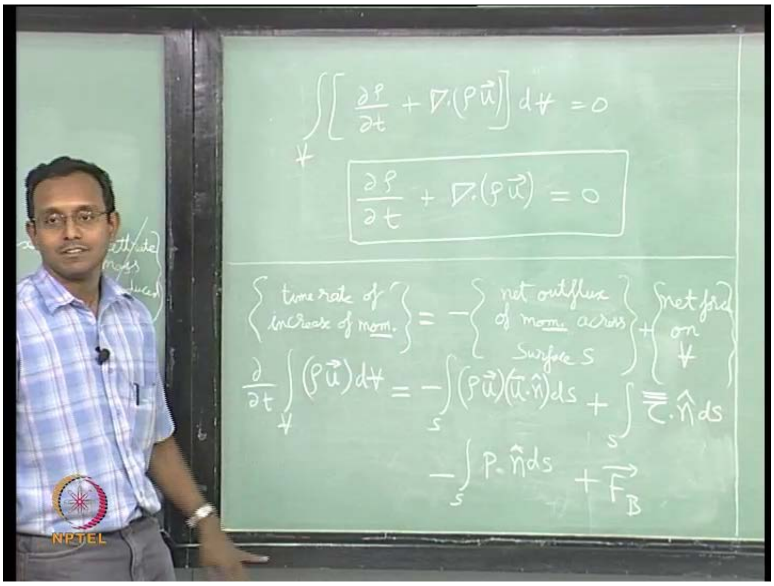

$$\int_{\forall}\left[\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{u})\right]d\forall=0$$

$$\boxed{\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{u})=0}$$

$$\left\{\begin{matrix}\text{time rate of}\\ \text{increase of mom.}\end{matrix}\right\}=-\left\{\begin{matrix}\text{net outflux}\\ \text{of mom. across}\\ \text{surface S}\end{matrix}\right\}+\left\{\begin{matrix}\text{net for}\\ \text{on}\\ \forall\end{matrix}\right\}$$

$$\frac{\partial}{\partial t}\int_{\forall}(\rho\vec{u})\,d\forall=-\int_{S}(\rho\vec{u})(\vec{u}\cdot\hat{n})\,ds+\int_{S}\overline{\overline{\tau}}\cdot\hat{n}\,ds-\int_{S}P\cdot\hat{n}\,ds+\vec{F}_B$$

This is what we will get. Now, I am going to say something more this volume can be any volume. I can pick my full control volume or quarter of my control volume half of my control volume 1 by 1 millionth of a control volume it does not matter it still valid. So, the only way that this always equal to 0 will be if the integrand itself is 0. The quantity inside this square brackets must be 0 standard logic people use to go from integral equations to differential equations. So, now, I will just make this equal to 0. This is your mass conservation equation this is the differential form of the mass conservation equation it is also called as the continuity equation. It is talking about mass is going continuously and so, on. We would not worry about continuity and at least just mass conservation equation is a simple way, easy way to understand it.

The next thing we need to do is conservation of momentum I will just begin it I do not think I have time to finish it. If I think about momentum the same way now we will start using laws of mechanics again. And tell the net change of momentum net rate of change of momentum of my control volume is going to be equal to the net forces. And is related to how much momentum is already being sent out, because of some velocity carrying it these are the 2 things. So, we will look at it like this, we will write this time rate of now I will use increase of momentum time rate of increase of momentum of course, momentum has three components I am just writing it in vector form. So, it does not matter for me to solve the same, it is one full vector which is 3 equations together is equal to minus of net out flux. Net minus of net out flux of momentum across surface s why should it be negative because it is going out momentum should be dropping. But we are writing a term saying it is increase in momentum.

So, there will be a minus sign let us just to take care of that part that is not the only thing if there is some force acting on my control volume. And there is no out flux let us say there will be momentum produced. So, that will be the next term net force on my volume v that is this is a general form now I just have to go. And write the expression in a similar way like that to get to the answer not very difficult. So, we will just write the one step and then we will stop there we will pick up from next class. So, if I write this first term it is going to be this is your flux term again we said u dot d n times d S times the quantity which is being transported previously it was rho. Because that is the mass per unit volume now, we are thinking about momentum per unit volume that is being transported.

So, it will be momentum times this flux quantity that is your net flux minus of the net out flux we have this remaining terms.

We are just thinking about net force on the volume I could have normal force shear force. They are all surface forces or I could have body force, those are the various things. If I think about surface forces this is your shear tensor dotted with the normal to the surface that will give you force along that plane of that surface that is that integrated over that d S. Next thing is it is a big expression. So, it is 5 terms. So, we just talked about shear force on the surface next is normal force typically pressure is acting from out inward. So, if I think about force acting it is going to act inward.

So, it is going to carry momentum inward. So, that should be taken into account correctly n is always pointing outward. So, to take that into account I have to put minus n. So, that this is my net force I have to think about it correctly minus n will think about it correctly pressure is acting inward and vector is going outward that is this. And then the last term I will just leave it like this we would not typically use this term. But if at all I have a body force let us say gravity is inward then I have to think about this that is this term. So, this is that full expression, we will start simplifying it next class onwards. See you people in next class.