Gas Dynamics
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Module - 14
Lecture - 29
Convergent Nozzle, Divergent Nozzle, Convergent-Divergent Nozzle

Hello everyone, welcome back. We were discussing Convergent Nozzle Flows, I just want to go to a point where we can do some analysis with it and then we will move on to convergent. Actually I want to go a little bit over Divergent Nozzles and then Convergent Divergent Nozzles. So, moving on we are going to go for convergent nozzle.

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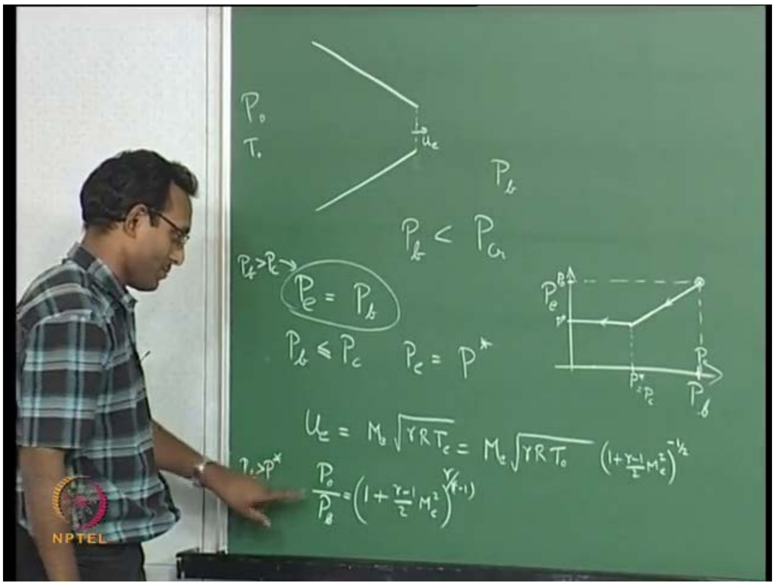

We already know something about it from last class, we said that if I have a particular P naught here and T naught here for a given gas, and I keep on changing my P back. The back pressure then if I keep on decreasing this below my P 0 value, then there will be a flow starting. And if I keep going further and further the velocity at this point will keep on increasing we said and we said that at some point at a particular pressure. This particular system will get to a choking condition, that is the exit will go to a m equal to 1 condition and we said that after that whatever, we did with the back pressure if I decrease the back pressure, further no change will be felt on the inside.

What if I increase my back pressure at that point, if I increase my back pressure that will be felt inside. So, this is where many books do not explain things very clearly in compressive flows books. They say the flow gets choked here and that means, that any change here will not be propagated upstream, it is not really true, if I increase the pressure here immediately, this will go and chocked that will still happen. So, what is happening here only expansion fans do not go through and compression fans goes through not like that say I go to a point, where I am at the critical pressure.

And at that point if it I send an expansion fan, expansion wave, that is I am sending decreasing the pressure by a little amount. If I decrease the pressure by a small amount then this outside ambient atmosphere is going to send out an expansion wave through this. When the expansion wave tries to go through, this it cannot go through this point and so, it is getting blocked and so, whatever the book says is all correct.

But, when I increase the pressure to slightly above the critical pressure, what will happen it will send a compression wave. As I said before moving compression waves typically, moves slightly supersonic. If it is only one very small change it is almost isentropic compression wave, where we said we were very careful by saying compression waves are slightly more than speed of sound, expansion waves are slightly less than speed of sound.

So, I will use that argument here and say that when we are at critical, if I increase pressure that one single compressive wave that is produced will go a little inside and once it goes in it is all subsonic. So, it just goes in and changes all the flow filed when all the flow field changes, there is no more sonic here. If I was already working at critical condition and I increase the pressure a little bit the whole flow field will become subsonic nowhere will it be sonic ever or supersonic. It will never go supersonic ever anyways, it will go end up like that.

So, this is just a feel for the flow you have to get, after sometime we will give you more complicated nozzles and you should be able to handle what will be the flow field inside kind of problems. But, that is the simple physics for this. But whatever the books say is also correct in a way by saying, if I go P b far less than or P b less than, P critical some critical pressure, where it just choked. If I have such a back pressure then it is going to go to a point, where whatever expansion wave that goes it will never cross this, because there is m equal to 1 sitting there. That is going to be the case, even if I increase the back pressure slightly and it never crosses the P critical that wave also will never cross and go through.

There will be changes happening in here, we are not currently interested in solving the problem outside of this nozzle ideally, we could solve that problem also. And in fact, depending on whatever is the pressure here, there will be supersonic or subsonic flow outside of this area, we will deal with that after a few more classes, where we go and solve jet flows from nozzles.

This part will be a jet and we would not worry about that jet currently, we will look at jets in 2 3 classes later. So, as of now, we will just deal with this part inside the converging term or converging nozzle, if you want to call it. So, we are going to say if I keep on decreasing my back pressure from P 0 keep on going down. There will be a point where my nozzle exist, just gets M equal to 1 condition that is the choking condition that is a critical pressure that particular back pressure will be called the critical pressure. If my pressure goes below that back pressure goes below that value then the flow inside will not see any change, that is the basic statement for convergent. We have this physical feel for things, now I just want to say calculate the velocity at this exit plane u at exit, if I want to calculate or P at exit P at exit is easier.

So, we will do this if it is subsonic then we can say that every pressure wave will talk to the next wave and we can say that the pressure at the exit will match exactly the pressure at outside back pressure. If pressure at the exit, if my mark number at the exit is less than sonic then we will have P e equal to P back always. Now, if it is choked, if I have a case P b less than or equal to P critical that is the point, where it becomes sonic.

Then my P exit is equal to P star for that P naught that is all it will be it cannot change anymore at the exit plane, if I measure pressure it will be only P star corresponding to that particular P naught value. which you know how to calculate right, it is just a 1 plus gamma divided by 2 to the power gamma by gamma minus 1, that is your P naught by P star, you can get that number. What is the number for gamma equal to 1.4, we did this in isentropic flows P 0 by P star will be 1.892 or 1.893 somewhere.

That it is a nice number to remember, I told that time also. It will keep coming up several places in practical applications it is nice number to remember at least for gamma equal to 1.4. So, if I have this condition how will I put it in plot form say I am going to put P at exit plane versus P back, I am doing on opposite experiment I am not increasing the P back remember that, we are decreasing P back from P naught.

So, I will mark this as P naught value. So, when this is the case, what is my P exit, if my P b is equal to P naught what is my P exit, 0 no it is wrong, 0 is a wrong answer same it is same as what same as P back. So, P e is same as P back. So, it is a linear relation and. So, it will be the same value here this is the point there, if my P b is decreased further what will be my p back. It should be the same value wait P b P e, I have to mark the P 0 here. So, this is my actual point here at that condition.

Now if I decrease my P b, further this will also drop it will be equal. So, I am going to draw a ideally, I should have made the scales equal, I did not in this case. Ideally, it should be a 45 degree line up to a point, where my P b e is equal to my P star which is my P critical P star is my P critical in this case. If that happens then this side also this will be the P star, after that what happens. After that I still keep decreasing my P b say my P b is here currently, what will be my P e at exit say P star. So, it just goes along this line after that it just goes along this line.

So, in my experiment the way I am going to do it P b is decreased keeping P naught constant, it starts from P naught keeps decreasing up to P star and it stays constant all the way up to P b equal to 0 conditions. Even if I evacuate the outside, there is nothing changing inside here till that point that is what I will see. So, simple curve for this case expressions are not very difficult to write it is just this you can write the expressions, I am not going to worry about that path.

So, a more interesting case is the velocity, how will I find velocity given back pressure basically, I have to write u exit in terms of velocity in the in terms of P b, that is my goal. So, if I think about it, I can write u e in terms of mark number that is easy to write. So, I will write u e as m exit times square root of gamma r T exit, where now I can rewrite this T exit in terms of T naught, I will just write equal to.

This is same as minus 1 by 2 sorry, I have pulled out this from this T naught here, T naught by T e is equal to this and. So, I will get to this particular form I can write it like this. But, still I have not solved the problem, I am having M e here I am having M e here, but what is given is only P b. So, I still have to solve this problem how will I solve it, I have to write P b in terms of P naught, P naught by P b, actually I have to write P naught by P b in terms of mark number at the exit that is the basic idea, what will be the mark number at the exit, if I am somewhere here.

P b is greater than p critical, I did not write that here P b greater than P critical I am going to have this condition, when P b is less than P critical or equal to P critical, I am going to have P e is equal to P star, this is what I have. So, if I consider the case P b greater than P critical P critical happens to P star. So, I will just keep it as P star. What will this function be in terms of mark number at the exit, it will be this function this is a standard isentropic relation, I can use this because I have P e equal to P b what I can tell is P naught by P e is equal to this for sure, it. So, happens that P b is equal to P e. So, I can say P naught by P e is same as P naught by P b. So, I can write this expression remember that always, now I want to rewrite this in such a way that I can get M e equal to something in terms of P b by P naught that is all, once you do that, you will get the answer immediately.

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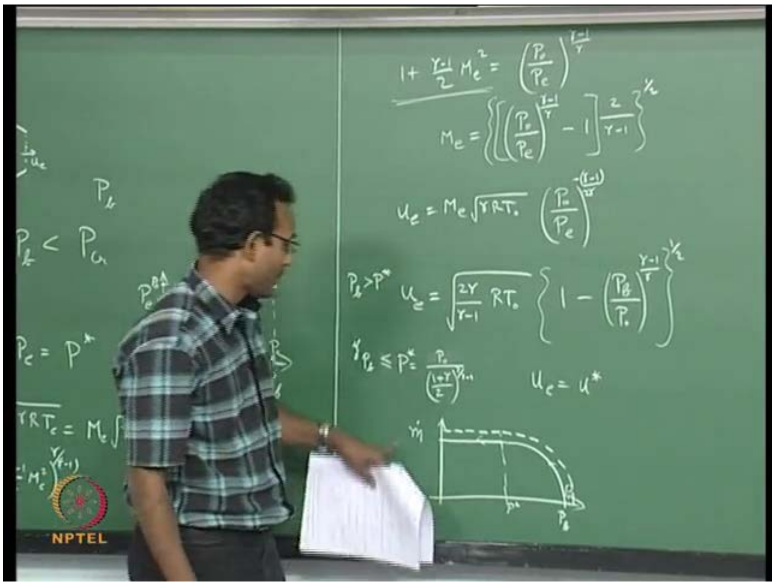

So, which is one plus gamma minus 1 by 2 M e square was equal to P naught by P e to the power gamma minus 1 by gamma, now I just rearrange this. So, I will get M e is equal to this minus 1, it will become into 2 divided by gamma, gamma minus 1 sorry. So, it will be P naught by P e gamma minus 1 by gamma minus 1 that whole thing multiplied by 2 by gamma minus 1 to the power half. This is what I will have, now all I have to do is substitute this M e in the u e expression, we already have.

Now since I have this form my u e expression also had this 1 plus gamma minus 1 by 2 M e square, I will just directly substitute it in terms of this. So, I will have u e equal to M e square root of gamma r, T naught into P naught by P e whole power gamma minus 1 by gamma to the power minus half. So, it will become 2 gamma and I will put a minus sign here, just leave it like this. This is what I am having now, I have m e equal to this I will substitute this inside here, If I do that I will get to a final form, I will just write the final thing. You guys go figure out how you got it 2 gamma by gamma minus 1 r T naught 1 minus P e by P naught gamma minus 1 by gamma power half.

Actually, I will write it as P b here, you know that P b and P e are equal here, we wanted to write the exit velocity in terms of back pressure. But, we know that when I use this P b is equal to P e and it is going to be the same. So, we have written it like this now, if I have a special case where this is for P b greater than P, critical P star. If P b greater than or equal to sorry, less than or equal to P star then I cannot do this where P star I have to write it as P naught divided by 1 plus gamma by 2 to the power gamma by gamma minus 1.

I take as now this happens to be this function happens to be 1.893 for gamma equal to 1.3 to get to that form. Now what will this value be I can of course, substitute this inside there and say that this is P star, if it is u e. It will just come out to be P star by p naught, I will just you can substitute this to be 1 plus 2 by 1 plus gamma. It will just come out to be 2 by 1 plus gamma, I can simplify, it further and I will get a even smaller expression, which I will leave it, u e is equal to u star I will write here, you then find that value it will come out to be u star exactly.

It will not be anything different. So, we will leave that to you for now the main thing is I can now, tell after this point that my density does not change my mark number does not change. So, my density does not change or temperature does not change beyond that point and my velocity is the same. So, my mass flow rate is a constant after this point, when I say my mass flow rate is constant, if I plot my mass flow rate M dot versus P b again back pressure.

If I do this again remember that, we are doing the opposite experiment P b starts with P naught value and then we are going to go decrease P b from P naught all the way to 0 that is the experiment we are doing. Initially, when I have p b equal to P naught mass flow rate is 0, after that the mass flow rate keeps on increasing and when my P b is less than P star after that it is staying constant.

So, I will draw it like this, it will look like this now my experiment is going this way, I am starting with P b and I am decreasing and it is going to go like this. This is what I will see this is why people say that, If my flow gets choked my mass flow rate cannot be increased anymore. It is a standard statement in all books just be careful that, they are having an assumption there P naught is constant in this whole thing, we said P naught is constant. If I change my P naught say I have a new P naught and I do the whole experiment a higher P naught and do the whole experiment again. I will probably end up with this curve this is again for another constant P naught.

It will do something like this that is what I get. So, we need to be thinking about, what is held constant anytime, you are making a statement, you should know what are all your assumptions based on that you have to work. So, now, if I go and do the opposite thing, if I keep my P naught increasing keeping P b constant, what will that look like if I keep on increasing my P naught keeping P b constant.

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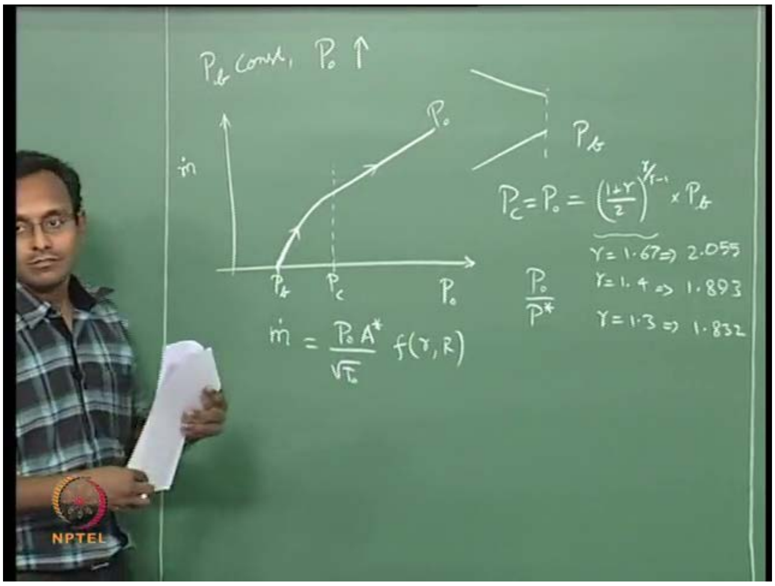

P naught increasing, I am using up arrow for increase, you just get used it now of course, I have to use P naught as my axis. And here let us say again I am looking at M dot mass flow rate, through that particular nozzle, what will that look like that is my basic idea, we have to think differently now. I am again starting with P naught equal to P b and I am increasing now if I decrease what will happen the flow will go reverse, we do not want that kind of problem. We will deal with that some other time let us say this is my P b, we will just start with this and below that P b, we would not talk about it I would not say it is 0.

It will have a negative mass flow rate, we would not talk about it, but after that it is going to increase mass flow rate will keep on increasing. I would not say it is going to go linear it will go somehow some angle. I am not talking about, what is happening here it is not 0 here, it will be going down somehow some other way, you would not worry about that problem. Negative mass flow rate means, flow is going in the opposite direction mass flow rate will keep on increasing like this, till there is a point where my choking condition happens how will I write that in terms of P naught, when the flow chokes what will be my P naught value.

I am having this particular flow P b is fixed flow is chocked at this point and P naught has continuously changed, when will it first choke. If you remember for gamma equal to 1.4.528 of P naught is equal to P b or instead I do not want to think about that I will just say P naught is equal to 1 plus gamma by 2 to the power gamma by gamma minus 1 times p b.

That particular value will be the value here, I will just call that my p critical here. So, that is my p critical till that point mass flow rate is going to go something like this after that what happens to mass flow rate. After that it is choked till that point, it is not choked after that it is choked does not mean my mass flow rate is constant it. So, happens that when it has choked, I can write this relation, we did this some 2 classes, before we can get to this particular form.

So, now, I am going to say m dot is proportional to P naught that is it is a linear relation if P naught increases M dot increases, it is going to go like a straight line after that. So, now, my new experiment was my P naught is increasing continuously and it is going to go slightly non-linear initially and then it is going to go linearly. This will be the curve, you can go figure out what should be the functionality in terms of P naught, I will leave it as an exercise, you can get to this plot it is not very difficult to get that.

So, I just want you to have a feel for this that is the main ideal now, we need to know these numbers for different gamma values. So, we will just give it anyway P naught by P star for gamma equal to 1.4, we said this is 1.893 for gamma equal to 1.3. This value is 1.832 and for gamma equal to 1.67, this value is 2.055, these are the numbers we have now of course, you can go and figure out with compressibility what happens in here.

This will have a same explanation as we gave for expansion fan the flow accelerates faster versus slower, I will leave that as an exercise now you can go find out that with more compressibility, you will find that it need not change p star. So, much P star will be closer to P naught that is what you will see anyways, I will leave that as an exercise for you can get through that. It is not very difficult to bother any other questions on convergent nozzle shall, we move on to divergent. We will go for divergent nozzle once.

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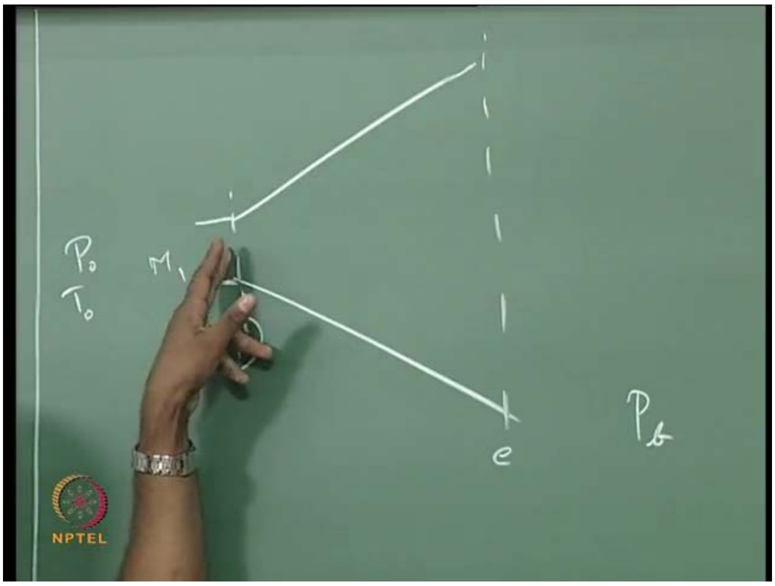

We would not talk too much about divergent nozzle, there is nothing much to talk about here, I am having some duct and initially my flow is coming with P naught, T naught and some M 1 at this location is my 1. And I am going to say this is my exit there is some back pressure, if I think about this problem there is it is not very appealing problem. It is just a very boring problem, if you think about it, if my incoming mark number is subsonic. And it is a diverging duct we will go back to our table and we will say that subsonic diverging, it will decrease mark number. We have already solved such problem 2 classes back think last class right, we solved a problem like this.

So, you can just go and solve this problem nothing great in case this is supersonic, I will still go to the same thing supersonic flow diverging means, my mark number will keep on increasing. If my mark number keeps on increasing again I will be having simple problem no change nothing like supersonic and subsonic mixed together. It is only supersonic simple problem again similar way to solve all I need is a by a star I will know A by A star here, I can just find the mark number and solve the problem. Very simple problem it will be called we did such problems already, I would not go and solve these problems separately. There is nothing special about a diverging duct. So, we would not deal with it more that is it.

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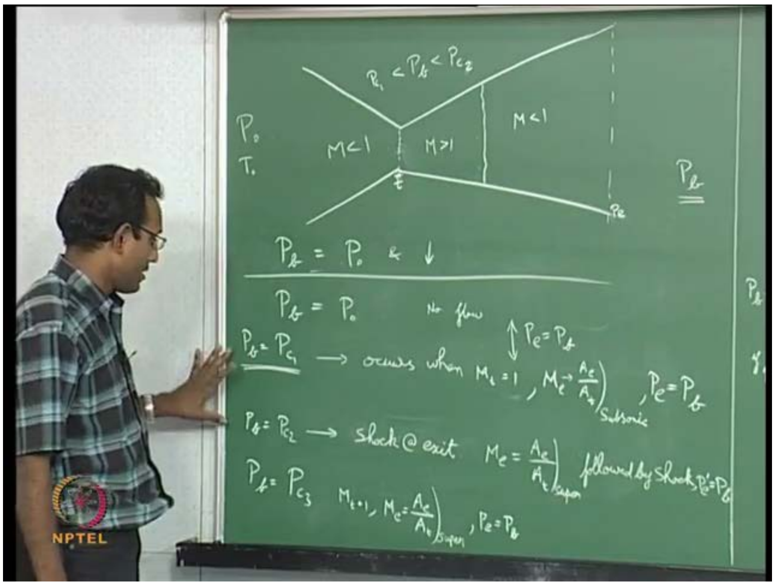

Now, we will go for a more interesting case convergent divergent nozzles, look at convergent divergent nozzle, I am going to think about any area variation. I am still assuming quasi one dimensional steady flow and all those standard isentropic kind of assumptions we will still keep those assumptions. We may find that some of them may break down, we will go to that after some time, if I think about this. We can go through the same analysis, what we did some 2 3 classes before, where we have introduced the term choking, we will again say it is a duct with area variation where there is a minimum area. If I keep on increasing my flow rate through this by decreasing my back pressure I mean, I have to call a back pressure here and this is my exit plane and I call this my P exit at the exit plane. If I keep on decreasing my back pressure keeping my P naught constant have increasing flow till a point, where the minimum area gets choked.

We did this already, I just do not want to do that part again. So, and after that now my flow could choose several paths, that is where that is the special thing about this problem, we already dealt with this a little bit, we said when m equal to 1 here. Now my flow here we said that we did this M 1 M 2 M 3 kind of example. Some 2 3 classes back where, we said m three can be subsonic or supersonic, we do not know when m equal to 1 here M 2 equal to 1 that is a special case. We are going to deal with that particular case, now we are going to say I do not know what it was, But, now we will start seeing, what this really does when I say my experiment is going to be P b equal to P naught and then decrease, that is what I am going to do.

I am going to decrease this after this I am going to start using these arrows more often you know that is the easiest way to write increase or decrease. So, now, when P b equal to P naught no flow, simple case no fun there is no flow, we are gas dynamics people we want high speed flows anyways. So, after that we start decreasing back pressure what will happen the flow slowly starts happening everything, similar to whatever we get before this will be the only point this will we will call the throat section is the point, where I will have maximum velocity and when it has the maximum velocity that will be the 1, which will reach m equal to one first, because that is the highest.

So, it will reach the sonic conditions soon, once this reaches sonic condition the pressure at that point decides the flow inside this. It is actually a difficult thing to explain unless I go over some loops. So, I am telling you some statement, we will go and give a rational for it come back and visit the statement again. That is the only way to explain this problem, because the way we explain this problem is by first time observing nature, what it does and then understanding this problem. So, now we will go through this in a simple system. I am going to say what really happens then we will go and see, if it really happens in nature, I am going to say if I pick, we will cut off that top portion, we will just look at from here. P b equal to P naught no flow simple case, now I will introduce some special condition where I am going to say P critical 1 P b equal to P critical 1 this occurs, when m at throat equal to 1, when I do this particular experiment. I start will P naught and decrease P b continuously, when I do that whenever this happens that particular condition is called P critical 1, I say 1 because they are more critical pressures possible now when this happens it.

So, happens that the mark number from 0 increases increase and increases goes to mark 1 and then it again decreases, decreases, decreases goes to some particular exit condition M e corresponding to a exit by a throat, subsonic solution goes through to subsonic solution that is what I have. After this if I decrease by P b further it cannot change pressure anywhere here. Because, this area is increasing subsonic solution, I am not going to have any good solution, which will get to matching the pressure at the exit by the way I have to write here. P exit is equal to P b for all these cases by the way even in between these 2 cases P exit equal to P b that is for the cases, where P b is equal to P naught or slightly lesser to P b equal to P critical.

Everywhere in between this whole range of P b values, I am going to have P e equal to P b for cases, where m throat is not equal to 1 then I have to now think about what will be the exit pressure exit mark number that is decided only by P b equal to P e, which will be same as what we did in converging nozzle sometime back it will be the same function same way of deriving it, I do not want to do it again. So, next interesting case if it cannot, if I want to decrease pressure further P b, I have decreased pressure P b, but P e has to decrease. But we said it is isentropic flow it cannot choose a subsonic solution anymore.

Because subsonic solution will give you some mark number less than 1, Because, this is a highest mark number possible in the diverging turn. So, let us pick the supersonic solution, what will happen if I pick supersonic solution, if I go slightly supersonic across this somehow, because I want to reach a lower pressure. If I go slightly supersonic then the diverging duct in an isentropic flow is going to tell me my mark number keeps on increasing. What will that lead to it will give me some mark number at the exit corresponding to a exit by a throat. If that is a case, it will have 1 particular mark number only it cannot have any value that is decided by the geometry of my nozzle. It cannot have any value it feels like. So, we will pick that special case say whatever flow happens let us say I somehow magically went to that particular back pressure. Such that pressure is equal to P at exit and my flow is fully supersonic in this region that particular condition is called P c 3, critical pressure 3, where I am going to say again.

M throat is equal to 1, I am going to say m at exit is equal to a at exit divided by a throat supersonic solution and I am going to have P exit equal to P b the special case, I have left a gap here, that is the whole range of values, if you are missing. We said my experiment was P b started with P naught and it goes all the way down to 0, that is the experiment I am doing from P naught to P e P critical 1, I know my full nozzle is going to have only subsonic solutions. And at this point my throat section chokes, which means below this always it should be choked.

Once it choked and I keep on decreasing pressure, the choking condition will be choked only it cannot increase any more, it will be choked, because I am keeping P naught constant nothing will change. So, below this whole region my nozzle is nozzle throat is choked, now I am going to look at this is the next isentropic solution possible, there is nothing else in the middle.

In a diverging duct I can start with M equal to 1 and end up with a fully subsonic solution or a fully supersonic solution all the way up to here nothing else. So, now, it looks like isentropic assumption has to be removed at this point I cannot keep using this anymore. Because I want to force my nozzle to have a pressure something in between P 1 and P c 3, I want to have some other value in between these 2. Then suddenly my flow cannot use any isentropic path, it has to go in non-isentropic path it.

So, happens that the flow chooses to have a shock in the flow, it will have a shock somewhere inside in the nozzle. Now if there is a shock in the nozzle, this is for the special case, where P b is greater than P c 1 and less than P c 3, I have to go and refine this more to P c 2. I have not given you what P c 2 is, but it has to be P c 2, I will tell you what I will have m greater than one here of course, M is less than 1 M equal to 1 at this point, I will just call it star condition and across the shock of course, M is less than 1.

So, what my flow is doing is if I have a pressure that is in between these 2 as in my back pressure the flow chooses to go on this path for sometime it goes well below the back pressure. And then increases, it is pressure across the shock and then increases very slightly in the subsonic portion to come and match this pressure. It goes through a non-isentropic path to match the back pressure exit pressure equal to back pressure, that will happen inside in between this region.

Now, I will define what P c 2 is this is the path, this is the point where my shock sits at the exit plane, I will just put shock at exit this particular board is having lot of information, we will go and look at this again, but we will go and look at it in a different order after sometime. But, anyways this is having too much of information currently, what happens is m at exit is going to be the same supersonic thing A e by a throat deciding supersonic solution followed by a shock.

And then P exit prime equal to P back pressure, when I put P exit prime, it is downstream of the shock, shock is a infinitesimally thin region we said. So, there are 2 pressures at the exit, now the shock sits at the exit and I have pressure before the shock and after the shock, before the shock, it will be very low the supersonic condition.

After the shock pressure has to be high, it is a compression wave. So, pressure will be high and that pressure matches with the back pressure, that is the way I am going to look at this. Now I just gave you a long sermon in a way right. I just told you this is what happens now I want to give you a little proof that this really happens. So, we did some experiments in the lab with convergent divergent nozzle, we would not worry about how we are seeing the waves in the flow currently, we will deal with that towards the end of the course what we are doing currently is a shear and emerging.

We will look at the movie soon, what we want to show is the particular movie is going to talk about only the region P b equal to P critical and above it is. In fact, slightly above this as an above in the sense it is P b is less than P critical and it is trying to go all the way up to P c 2, my P b is going to be somewhere between P c 1 and P c 2. That is the kind of condition I am going to show you a movie off now. So, let us go to the screen.

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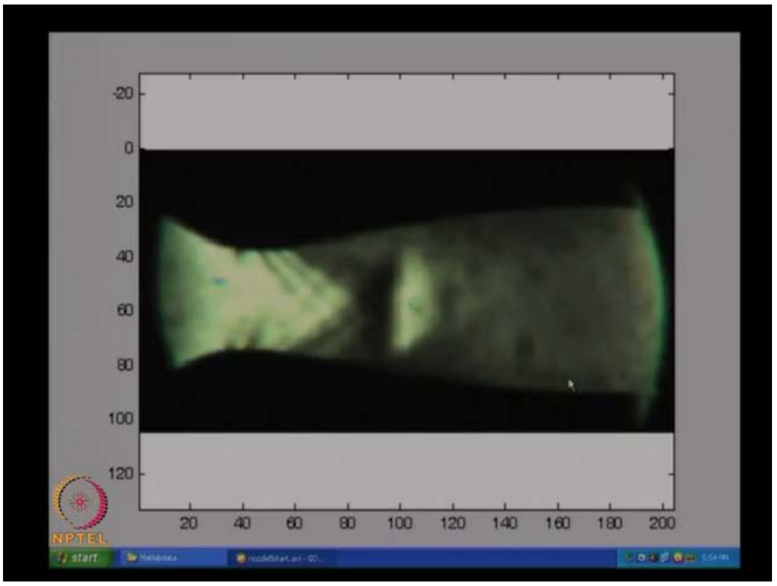

The emerging was done with a c c d camera and it is not very high resolution, you can see this edges coming up like this, we will leave it that that is for now. We can see the nozzle shape here, it is this curved converging portion and then it is diverging out this way going out like this and the same thing on the top also it is a symmetric nozzle. And what you are seeing here are just edges of my glass window across, this is the region I can see, I cannot see outside of this using my flow visualization system.

Now this is my throat section and we can see some waves like things going here it. So, happens that in this particular flow visualization, if it is the bright region, it is expansion and if it is a dark region, it is a shock. And that is what we are looking at here, what we are seeing here is, there are very bright region here, this is my expansion region I am sitting with. And then after that there is 1 discontinuous region out here, which is very very dark that happens to be the shock for this particular problem, currently we are picking a case where, I have gone a little supersonic and then there is a shock sitting here, that is the condition I am living.

Now, we will run this movie and we will see that when I change this P b by P naught to much lower values, if I decrease my back pressure keeping P naught constant what happens to this 1. So, it is changing and it stopped for a little, while and it is again going to change further a shock has moved from here to here. I will pause at this point to show you some more stuff, if you look here now, it is expanding further in this region. And we did this shock expansion theory sometime back where, we said that, if flow around a expanding corner will send out expansion waves and you can see those expansion waves also in here.

Whatever, we drew on the board you can see those coming on the other side, it is a inverted problem it is going to expand the other way, we are sending expansion waves, you can see all of them. These waves are visible primarily, because there are small bumps in the machining, it is not very, very smooth machining, there is some bump and the flow picks it up that is what this is. And if you go and look at each of those waves you can find the local mark cone angle and you can get the local mark number at that location.

So, it gives you a rich information, if you use this kind of flow visualization method it gives you lot of information, now currently the shock is sitting somewhere here of course, the emerging was it is just a preliminary emerging. I just picked it up, because this is all I could get in this time, now the shock is currently sitting somewhere here exit of the nozzle is around there. We will continue with our process of decreasing my P b decreasing further. Now you can see that the darkest region happens to be this the movie quality is not very good, otherwise you can see this going out here also the lighting was not done very well in this image, but otherwise you can see that it is a shock is somewhere in this region.

And that is roughly the exit after that, we were not interested in the problem, we were interested in studying whether the shock interacts with the boundary layer on the nozzle in this particular case it is not interacting. So, much with the boundary layer and the nozzle, so we stop this particular nozzle experiment after that for this particular nozzle other things, I will show you some other day when, we are interacting with boundary layer the towards the end of the course.

But, the idea is when I keep my P naught constant and decrease my back pressure, I am having a shock forming somewhere form here and it just keeps going down down down till somewhere here by the way, if I did the opposite experiment continuously increase my back pressure. It will go the other way the shock will go the reverse direction now I will tell you the truth about this video, I did not really do decrease of back pressure. But what I did was increase of P naught keeping back pressure the same, in both the cases it will do roughly the same thing.

In fact, exactly the same thing as long as I think about P b by P 0 constant, what matters for this flow problem happens to be P b back pressure divided by P 0 the stagnation pressure, only that matters for this problem. So, whatever I told about this flow field all this time is all correct statement nothing is wrong in there right. But, the actual experiment was done by decreasing P b by P naught by increasing P naught and not by decreasing P b that is the difference there.

Now we will go back to the board I will rewrite, what we did in the previous board again with some more proof from nature. We can now substantiate our claims that this is how the flow happens. Of course, we can substantiate it more even, if I give you pressure data from experiments and that are matching our theory. That is also 1 more way of doing the same thing, which even I do not have currently. So, I cannot give you that, but just believe me, that is correct.

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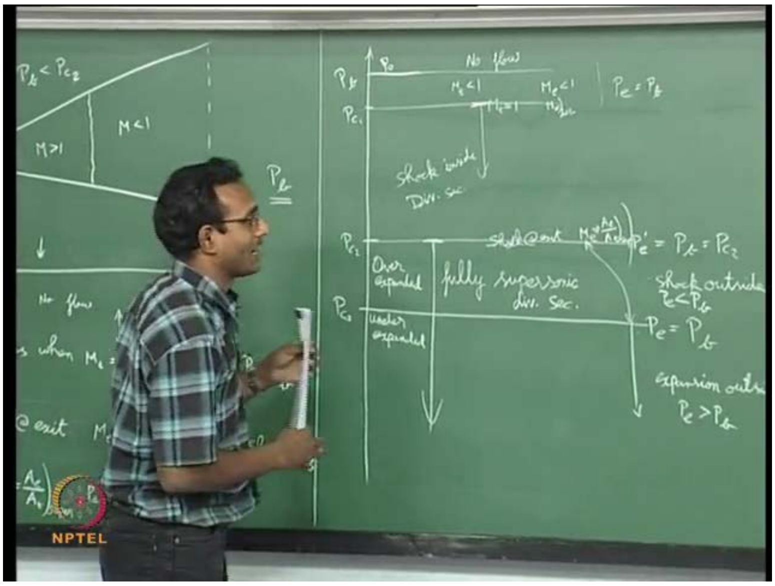

Now, we want to go in more detail I am going to have a pressure axis going to call it P b axis and the absolute 0 is somewhere very down, we are not interested in absolute 0. This is my P naught P b equal to P naught condition at that particular line no flow, I just have to write no flow and I want to write it no flow.

Now I will keep on decreasing, it there is a point, where there is P critical 1, if that is the case then the main change happens to be M throat equal to 1, at that line before that M throat less than 1. That is between these 2 lines somewhere there. Now I can still say in all these region P back equal to P, I have to write it as P exit equal to P back. And 1 more thing, I can say is m exit less than 1, I will write it before P back because I have a bracket there M exit less than 1 between these 2 lines of course, at the top line no flow.

So, M exit is 0 at this point it is less than 1, but I can give you a design value for it M exit design value which, we already looked at it will be related to A by A star A by A throat subsonic solution. That is what it will be that mark number, now after this point for all these cases M throat is equal to 1, that never changes. Now after this case, Now I have a P critical 2, I am trying to draw a relative scale of what these critical pressures are looking like.

They are going to be far away out here, that is how it will be, I will show you plots of things later next class probably, I will draw this line. Now I will say 1 more thing shock inside my diverging section that is the specialty between P c 1 and P c 2 and at this line shock at exit. That is the special thing about this particular P c 2 line shock at exit and the post shock pressure P exit prime is going to be equal to P b here equal to P c 2 and of course, mark number at exit is going to be A e by a throat supersonic.

Supersonic case and after that, there is a shock M e prime will be subsonic across shock, if I go a P c 3 that will be somewhere here, if I look at P c 3 line fully supersonic divergent section, which ever happens at this whole region is fully supersonic divergent section just after P c 2 everywhere. Divergent section is fully supersonic from this point onwards and here from this point onwards all the way down M throat is equal to 1 from that point on all the way down that is what we have.

Now we just have to label this region and this region and we will go discuss more after sometime. I am going to say P exit this particular P c 3, we already defined it is fully isentropic supersonic solution. P exit equal to P back here, but anywhere in the middle it is not P exit cannot change, because the flow in the divergent portion is fully supersonic nothing changes. So, it will come out to be whatever exit mark number the same value all the way up to here till this region, everywhere it is exactly the same mark number.

Nothing changes in this whole region. In fact, even further down exit mark number is all the same now what happens here is shock outside and here expansion outside of my nozzle exit. That is what I have here, I have shock outside of nozzle exit here, I have expansion outside of nozzle exit. I just have to give you separate names for these regions. This region is called under-expanded sorry, over-expanded this region is over-expanded nozzle this region and this will be your under-expanded how will I label this is over or under-expanded easy way to think about from this point on it is fully supersonic divergent section.

So, at the exit mark number it is going to be exactly, that value supersonic value decided by A e by A throat. So, if I am at this line P e equal to P b. If I am anywhere above P e is going to the same. Because M e is the same all the way here, but P b will be higher it is means, I expanded too much P e is less than P b here, this region that means, the nozzle expanded too much the flow expanded too much than what is needed.

So, it is over expanded and then here, P e is greater than P b, it has not yet expanded enough. So, it is under-expanded. So, it has to expand even outside of the nozzle that is this case, we will go deal with the pressure values how to calculate it what it looks like what how to calculate the curves and stuff next class, any questions on this see you people in next class.