Gas Dynamics

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Module - 14
Lecture - 28
Quasi-1D flow with area variations, Geometric Choking, Numerical Examples

Hello every one welcome back. Last class we derived the area Mach number relationship I told you I will show you the plots later. So, we will start this class with that and then we will go to mass flow rate expressions.

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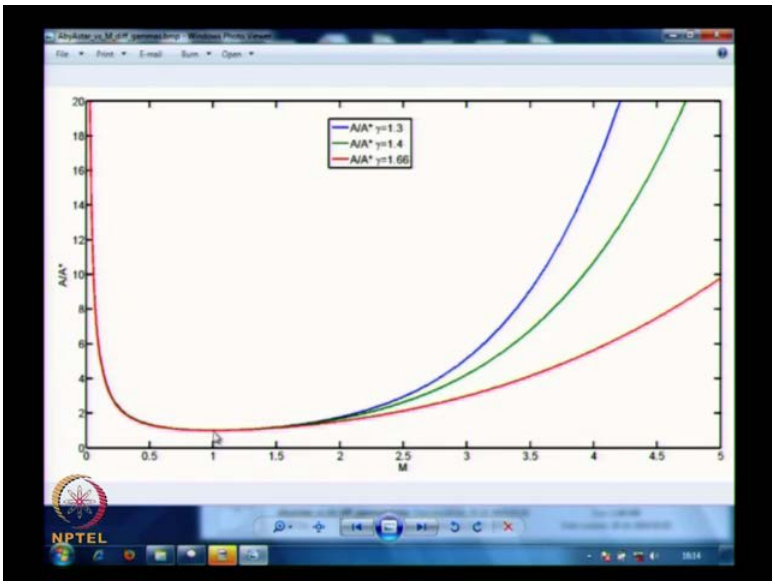

Let us go to the screen. Here, I have plotted A by A star as we derive versus the Mach number, for different gamma values. Commonly used one is you gamma equal to 1.4 which is the green, the first thing we want to note is Mach number 1 is somewhere here and that is the lowest point on all the plots, all the three of them have A by A star equal to 1 when m equal to 1 by definition that is right area is star area when m equal to 1. And when we see on the subsonic side, there almost the same there is not much change, almost up to m equal to 1, there is a small change out here if we zoom in, but we would not worry about zooming in right. Now, there almost the same number, they have a huge difference only one above 1.5 Mach number. Till that time A by A star for which ever table you look up, the numbers will almost be the same and after that there is a huge difference in numbers fine.

If you look at one more thing the stiffness of the slope here, is much higher in subsonic range for very low values areas used to be changing, drastically when I change Mach numbers slightly, but when I go somewhere here Mach 1 area does not change at all. Or somewhere between 0.9 to 1.2 is not much of area change for my stream line at all that is what I see here; almost no change, there is a small slope and almost nothing, and when I go very far, to start seeing that thus a huge difference in these gamma values.

Now, we can look at effect of gamma variation If I look at gamma variation gamma equal to 1.3 is more compressible gas compare to gamma equal to 1.66. So, I am going to say that if more compressible gas is also more expandable gas, that is what we discuss last time and we did expansion. We are going from low Mach number to higher Mach number, which is actually an acceleration, which is expansion of the flow the way we looked that it in expansion fact.

If we are expanding the way as, more expandable gas expands to higher area, easy way to explain the plots. We look at it that way more expandable gas, expands to a higher area for the same change from m equal to 1 we think about it. Now, we can easily plot this curve it is not very difficult to plot after that point. Mainly the stiffness of the slope is very high in subsonic compare to the supersonics simply because in supersonic I can go from 1 to infinity in subsonic I can go only from 1 to 0.

All that variations should be hidden inside this span that is why its look like, it is going like this it is just functionality in math, we would not go in to more details than this that is en half, that the main thing you need to get out of this is when we are using A by A star there is always two solutions for whatever gamma we pick. It is, so happens that gamma equal to 1.66 it got cut off in this limits, it will still go to A by A star of 20 at probably Mach number of 10 or 12 percent we would not worry about that part currently. But, you should always know that there is always double solution for A by A star if I have a particular A by A star it can have a subsonic solution and a supersonic solution. That is the main thing you need to get out of this will be very useful when we go and look at solving problems, given A by A star now, will go back to the board.

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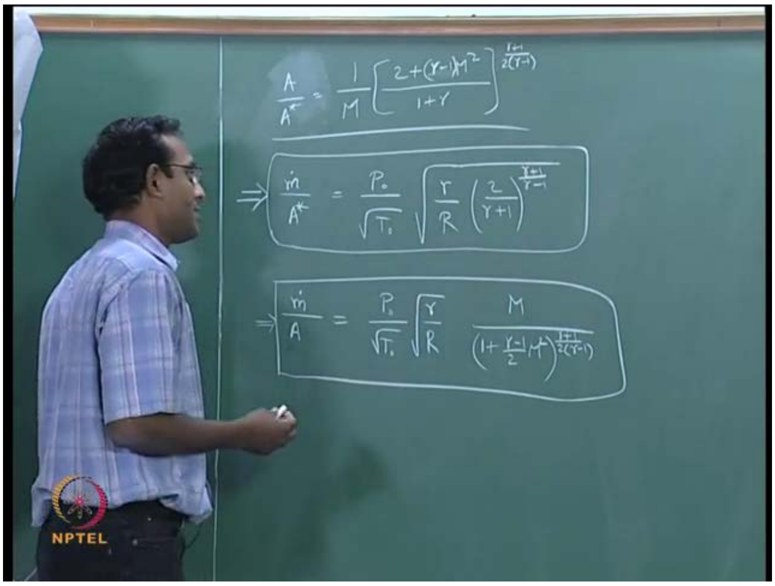

So, whatever we saw on the screen now, was just this expression and we stopped last class with the mass flow rate derivation. This is one of the forms of mass flow rate derivation and what we are saying here, is a if I know the throat area and the stagnation conditions and the gas I can tell you the mass flow rate through the throat, fine. And of course, you can derive this not just at throat conditions, but at any condition I just told you that I did not give you an expression for it, I will just give you the expression I will not derive in class here.

This may also be useful m dot by A I am writing not A star at any area I can write the same thing, not very difficult to drive you just derive it by yourself. This is the other expression, what this says is at a given area given the stagnation conditions the gas and the Mach number. I can tell you the mass flow rate through that area, it has the way to look at this expression which you can easily derive if I just write m dot equal to rho 1 u 1 A 1, where A 1 is R A is here, just express rho and u.

In terms of stagnation conditions you will get to this functions it is not very difficult to get at all and I do not want to waste around 20 minutes deriving as in our class here. Now, the main thing we want to think about I am not interested in this function, this is useful typically when we are solving some problems, but I would typically use only this function very, very useful function, if we can remember it is nice for you. At least you should remember that m dot is proportional to P naught A star divided by square root T naught multiplied by a function of gamma and R at least that you should be able to remember, very easy to remember at least that step. Now, if I look at it mass flow rate is proportional to now, may be I will write that again here.

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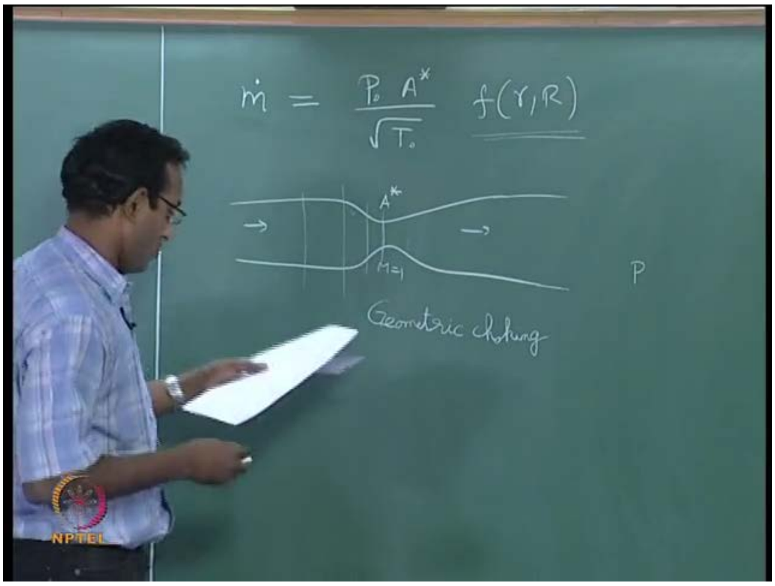

Mass flow rate is proportional to P naught A star by square root T naught in to some function of gamma. And R this is very important thing we are going to look at it from this way from now, on this way I am going to say it depends on the throat area it depends on stagnation pressure, directly these 2 are direct relation and with temperature it is inverse relation is it logical. Yes it is if I make my gas colder then it is more dense, if it is more dense I can squeeze it through the same area, they can be more mass can go through it. So, mass flow rate will increase, it is the way to look at it.

So, it is logical if I increase pressure, again I am increasing the gas density. So, you can go through that same area squeeze through it or if I increase area of course, I can have more mass flowing through it then also I can have more area curve more, more mass flow rate coming from that for higher area, these are the logic we are going to use. Now, you want to look at it as, I have a given gas a fixed gas, so these properties typically do not change.

Now, if I want to increase area increase the mass flow rate, I can either increase area increase the stagnation pressure or decrease stagnation temperature, there is more to it than just this I will introduce this when we go to non isentropic flows. Where these things can change, even there I can find that there could be some effect on mass flow rate we will come back to it after I introduce some more concepts in this itself.

Now, I will go back to a duct. I have some duct with some area variation, we remember that we are still only quasi 1D by naught thinking about the flow in vertical direction of course, there is a vertical component of flow we are going to neglect that, we still say that the flow is going along the axis. Only thing we have to take all take care of it is local area variation that is only thing I need to take care of. If I think like this I go back and look at this formula I know that this is my A star, this area is the A star and we already proved that.

Mach number can become 1 only if this area is a minimum and not the maximum, which is what we did last class I believe right before deriving this formula. So, we are saying this is the minimum area and I am calling it A star now, and let us say I have achieved Mach 1 here. Now, that is the special situation if I have Mach 1 here I do not care what the flow Mach number is here and flow is going like this, flows going like this I do not know whether there supersonic or subsonic here, as well as here.

All I know is it is Mach 1 here because I said this is A star if I have such a situation. Now, I am going to say one special thing, if I make some changes in pressure here downstream, then we always been discussing gas dynamics if there is a change that change is communicated through the gas only through pressure waves. Now, immediately let us say I increase pressure, what will happen it is like dumping more people in to a stadium, what will happen immediately they will squeeze and sent compression waves all across.

So, there will be a set of compression waves coming this way, what will be the speed of those compression is they are going to come at local speed of sound at every point. If it is coming to this point it is going to travel at speed of sound local at that point corresponding to T star and gamma and R square root of gamma R T star and the speed of sound here, that is the speed at which the wave goes this wave.

But it, so happens that the velocity here is exactly equal to that value M equal to 1, which means, the wave is going to come and sit down at threat mill and just keep running for ever. Threat mill belt is going this way that is your flow velocity is going this way and the wave is running on top of it the other way and they are going to be travelling at the same speed the way of never crosses this point, what does that mean. Now, I am going to say if I make any changes downstream of a choked condition this is the first time I am using the word choked here.

I have a construction in my flow I am going to call at the choking of the tube, choking of the duct and I am going to say, if there is a choked condition here, any change downstream of it, will not be communicated to the upstream of it that is the main change I am going to look at. Does it mean that if I completely close the duct, that information will not go through not true.

If I completely close the duct immediately that information will be going through, why there is no mass flow possible here there is no mass flow possible there has to be some strong waves that are coming in typically. You will have a supersonic moving shock wave coming in all those thin compression waves are small changes, if I completely block this, then we have done this problem already end of a duct we close a duct suddenly, what will happen there is a moving shock. If there is a moving shock then that can travel at supersonic speeds it will go through this point that will happen.

So, when we talk about this kind of problems and say downstream does not affect up stream we are talking only steady problems, not unsteady problems no un steady phenomena here. In a steady flow situation I am going to say that any change downstream, will not affect up stream, but by the way it is oxymoron in a way I am saying in a steady condition there is a change. Change is no more steady right change is something that change is at sometime T equal to something and after that it is something new means it is already unsteady any ways let us just keep in that way.

Any information from here that is travelling along all the flow directions, will never change anything up stream of this throat condition. Throat and that is the condition where we are going to call choked because information does not go upstream of it, nothing can go upstream they all get choked at this point. That is the meaning they got out of this of course, chocking comes from somebody chokes your throat that is the way they got the word from here.

And this particular thing is called, since there are other ways of choking a flow I will call this the geometric choking, this is a geometric choking they are other ways of choking the flow will come to it when we got more non isentropic things. I already gave you an hint that is somewhere here, it is hidden inside here will get back to that later that formula has, currently we are saying if I come to area decrease to A star then that can have a choking condition. There could be other cases like may be P naught and T naught can be adjusted to have different choking conditions, we will get to that after sometime. Now, I want to understand this a little better.

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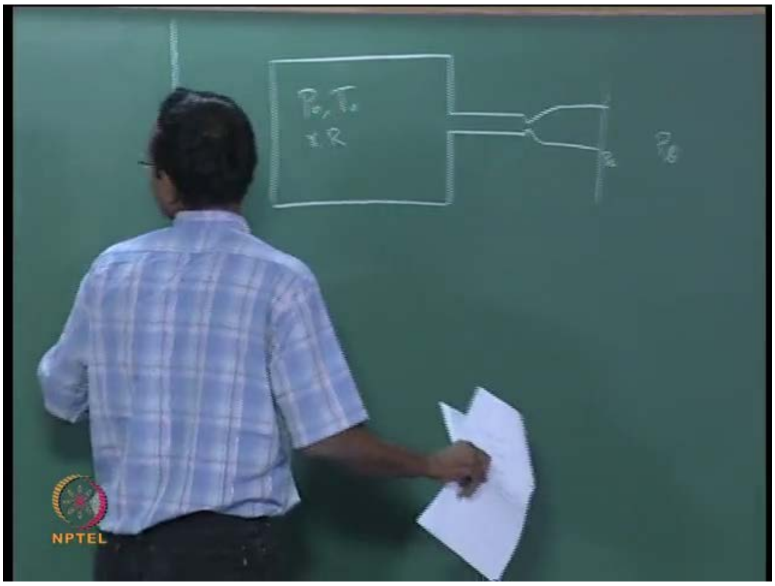

So, will do something I will set up an experiment to understand this a little bit, when I leave this we just go to the I will take a small tank, not a small tank a really big tank with P naught, T naught and the gas are fixed. Now, I am going to have a small hole in it and a duct form it and I have a choking condition in that duct and then it can have any area variation across. The way I am going to think about it, to explain the chocking better I am going to do this I will label, this exist plane pressure to be P exit and the other one to be a back pressure will keep it like this.

Initially this pressure is very, very, very high; and let us say my back pressure is equal to my stagnation pressure and I start with, if I start in that condition what will happen through this duct, some variable area duct what will happen through this. Finally, P b the back pressure is equal to P naught the stagnation pressure. Pressures are equal there is no flow anywhere. So, there is no fun, if I increase this p b to something higher what will happens flow goes reverse and I want to study that problem its equivalent problem.

So, we will study the opposite problem we will decrease the p b, right we will decrease the p b the back pressure it has assume there is a bigger tank in this side and I am decreasing the pressure slowly in that tank. If I do something like that now, the fluid here sees that the pressure here, is less than pressure here should be there is unequal forces on it. So, it starts moving that side that is how the flow starts accelerating. And we are currently not interested in studying the unsteady part of it we just still studying only the steady state phenomena.

So, it will finally, reach some study state that will be some continuous flow and I am assuming that this is big en half that that P naught never changes, reasonable assumption if my mass flow rate is very, very small the volume is really big it will take infinite time for it to pull out all the volume mass from all this big volume. So, pressure may be maintained the same and it will come to equilibrium where, the pressure is matched by the inertial force out, that will be your steady state.

Now, that need not always reach Mach one displace, it will be some flow area decreased and then it increased that is all I have. Now, if I go to a case where I keep on decreasing this pressure, then the requirement on the sides is much higher. So, the fluid will have to travel faster and faster the unbalance is across this fluid element is very high, it will want to travel faster and faster to manage that unbalance in to kinetic energy, this just convert all the unbalance force in to kinetic energy.

Since, the pressure is much lower will go to a point where the velocity will become higher and higher, what will be the point where the velocity will be highest. Since I started from subsonic flow, velocity will be the highest first at this least area point, we did this table thing long back area increase area decrease that kind of things subsonic supersonic. If you go back and look at that table you will see, that the this will be the first point where super sonic condition will be achieved.

So, we will come to a point where this is sonic now, after that if I keep decreasing this p b suddenly, we end up with the problem we just discussed on the other board, will have a situation where this pressure drop there are expansion waves coming in here. And it cannot cross this point, it is just running constantly on the threat mill it never cross the condition gets choked, the flow gets choked to that, which means the flow upstream of a does not know that the pressure decrease more. So, it does not want to change in this section.

So, we have come to a point where the mass flow rate through this has come to a fixed value it cannot increase any more, all this time it was getting higher and higher because the pressure differential was higher, at some point it will not be any more. And after that they call this flow choked and it cannot go any further increase, this is what is commonly explained in all text books, but I always want to show this formula again and again.

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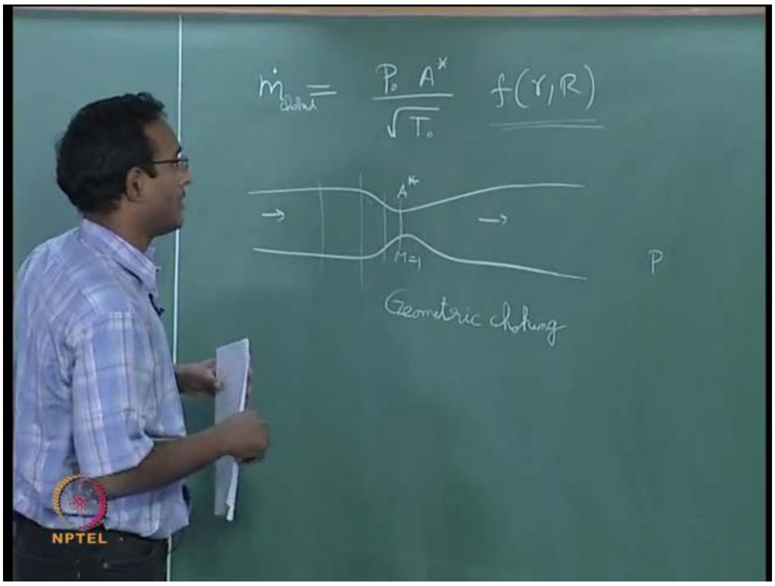

This formula shows that mass flow rate is proportional to P naught divided by square root out T naught times A star of course, also a function of the gas. Now, what I want to say is, if I kept the stagnation conditions constant then there is no other way of changing mass flow rate beyond choking condition, this is the choked mass flow rate if you want you can put a subscript here choked, choked mass flow rate. Once, I have choked condition by mass flow rate cannot increase beyond this point, that is the situation I will you give you another example again to explain choking condition.

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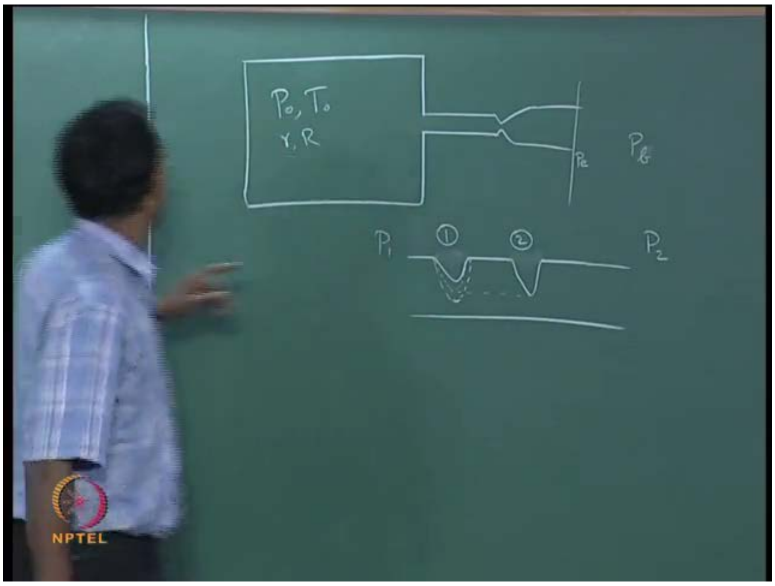

I have a straight duct, I have on both the ends pressure P 1 and P 2 by have such a situation in fluid mechanics this is called the poiseuille flow or poiseuille flow whatever depending on whether you pronounce the letters or not poiseuille flow. Now, if I have this flow you can of course, based on the viscosity effect across here, we can tell that there will be a particular parabolic profile and all, but we are an in visit gas than x world. So, what will happen here, I am going to have a condition where there is no boundary layer, no friction across the walls.

So, it will just keep on flowing how much ever mass we want, we keep on increasing to whatever value you feel like that is the special case, instead if I look at that problem it is a little more complex. So, I will change the problem a little bit I will say, my duct now has a choking condition in the middle, as in somebody bumped through the tube somewhere, that tube cross section decreased and then it is increasing again let us say I have something like this.

If I have this case now, again and once the pressures P 1 and P 2 are sent across the tube immediately flow will want to go to how much ever acceleration possible because just nothing retarding it no friction in our world, gas dynamics in visit gas dynamics. So, it will keep on increasing, but we go back to our previous thing again I am going to say, the first point where it will go sonic is this point. Once it is goes sonic here, this fluid on this side does not know in any way that the pressure here is much lower than what they can supply.

So, after that point the flow gets stuck it I got choked and it will never go beyond that, but that being said I should also say that the flow here can still keep changing, there is more expansion waves coming here the flow downstream of this choking condition can still keep changing nothing upstream will change. This is just I am discussing all this just to have more physical feel for the problem, once you understand choking we can go and talk about more stuff beyond that choking condition.

Now, I will have another special case, I will take the same thing, but I am going to put another bump let us say, I will call this one and this 1 2 something like this. Now, I have two bumps we can label at whichever way in your naught does not matter for now I had two bumps. And let us say, I have possibility of adjusting these, imagine them as something like a valve I can close the valve more open the valve more. In fact, every valve was a choking condition in the flow if you think about it that.

Not many people have studied what is the actual flow inside a valve, it will look like that it will go choking and then it will go supersonic and then come back to subsonic will talk about all that later. And here, if I have a such a situation where will it choke, in this case again I am going to say P 2 is for less than P 1 it will choke at 2 first and after that does not care this will be some high subsonic math number, this will be again some lower, lower value here and here high subsonic sonic, sonic this could be anything subsonic or supersonic depending on whatever has happening here we would not worry about it now will go back to nozzle flows problem there we will deal with that again.

Now, I want to adjust this a little bit I want to say now my 1 is more like this, there both equal now, what will happen. So, little more tricky thing, depending on how the problem started it could do different things, but the way I want to say ideally it can chock here and here, if there is no viscosity, if there is no friction in the flow then it can chock at both the places exactly the same math number one will happen. But, by mistake I increased it slightly I close the valve little more and it goes through this I have this condition.

If this happens then only the 0.1 will get choked and that 2 will become un choked now suddenly, all this time this was at Mach 1 this was increasing high subsonic as a it is going toward Mach 1 0.8, 0.9, 0.99 and then it became 1 after that this is still 1, but this 1 now, will start decreasing this will start decreasing to lesser and lesser values 1 will now become 0.8, 0.7 whatever it will keep on decreasing if I keep on closing this further and go to this point.

One more thing you can already tell from whatever we discussed today is the mass flow rate will keep on changing in here but it will start changing only after some specific time what is that point, when will it start changing. Let us assume that P 2 and P 1 are such that P 2 is for less than P 1 I have already said that I started with 1 at this point and 2 at this point. I let it go for steady state and then I am looking at the flow, what it will look like it will be choked at this point, that will have some particular mass flow rate that is the beginning I am having.

Now, I slowly start closing this valve here, I am crushing this pipe here, by keep doing this what will happen to mass flow rate just after I started crushing, nothing should happen why A star is this, that controls the mass flow rate. If at this point there is a choking condition, then mass flow rate should be the same for the whole duct mass cannot escape anywhere on the side that is the rule of at pipe right. So, it cannot change anywhere, whatever mass flow rate is imposed here is a same mass flow rate here also anywhere everywhere.

So, now, I can say mass flow rate does not change, until that chocking point changes in the duct that is important here. So, there is a point where both 1 and 2 are choked, if I slowly progressively keep on crushing the pipe at 1 there is a point where it will get choked twice. After that if I keep crushing more, then suddenly this point becomes un choke and then this gets choked and what will happen to mass flow rate now, it will reduce it will decrease why because now my A star has got in going down further. I am assuming in all this that my P 0 upstream of this does not change ever that is my assumption in here I am assuming that my P naught and T naught are not changing, gas is not changing all that assumptions. So, I am going back again.

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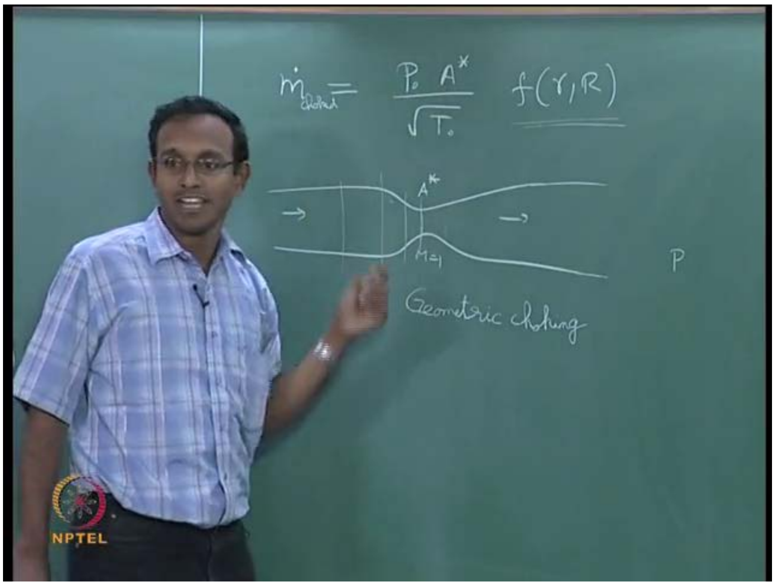

To tell this particular point when 0.2 was choked A 2 star never changed, but when I had 1 and 2 choke A 1 star equal to A 2 star then also this formula does not change. P naught and T naught maintain constant, gas is the same only thing was mass flow rate proportional to area. Now, when I had A 1 star allow an A star and other one is just becoming A 2 now no star, then I will go to a point where this is becoming A 1 star which is continuously decreasing, if that is decreasing this is decreasing. So, you will have that kind of a special situation.

So, you should start thinking what is a happening to your duct flow? If you have any problems like this. Now, this is all to give you an idea of what happens in a flow through a duct when they are choking points in the flow, are star conditions Mach 1 conditions, sonic conditions. In this whole thing I have assumed some set of things, I assume the flow is isentropic which is not always the case. I am assuming the flow is at least adiabatic, as isentropic is much more stricter I am assuming P naught and T naught are constant in this case currently.

I can have a case where P naught is changing, but T naught is not changing that is the special case, are T naught can also change that is another special case, we are not dealing with any of that right now, will go back and deal with it in few weeks time will soon deal with may be for 6 classes are something will go and deal with that particular problem. Of course, if I change those you can; obviously, see that if I have choking condition in the flow then if P naught changes m dot changes accordingly from this formula directly.

We will deal with this later, one simple example you can always tell us there is a choking condition suddenly there is a shock, what happens T naught does not change, but P naught change. So, now, there can be a problem, if P naught changes then my area A star should change to handle that much flow rate or else, suddenly mass gets accumulated inside the back we will go and deal with that kind of problems after some time. I want to worry about this whatever statement I have written here about a shock, will come back and deal with it when we are going for discussion on supersonic flows.

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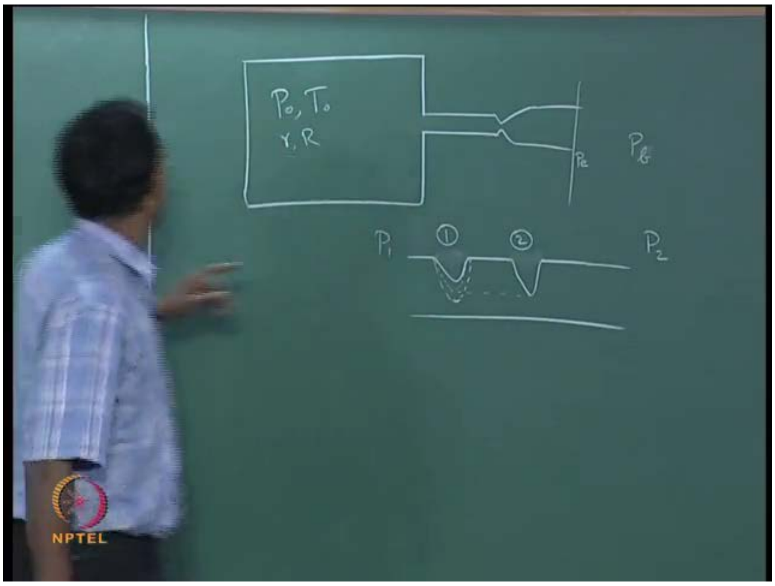

I also told you one more thing on the passing naught in this problem is, flow can choke many, many times. It can choke more than one place in a flow that is all you know right now, you do not know for sure whether it can choke 10 other times further, but I know it can, we will get back to that problem have after some time. Of course, there are some conditions provided I have to tell something more I am not going to tell that right now. Just look some numerical examples of how I can use this A by a star to calculate the flow situation in a duct. What are problem we just did can be numerically calculated if I know the exact area variation. So, will look at one simple case.

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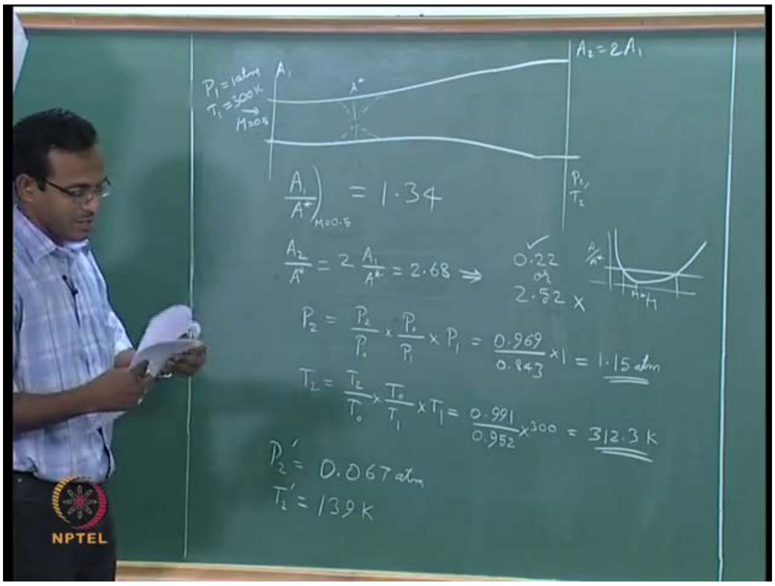

I am having some duct I would not want to draw a good duct whatever is some duct this is A 1 and at this section A 2 equal to twice A 1 that is all I would say, there is a duct where area becomes double across. And I am going to say the flow incoming inside this is Mach 0.5, pressure is 1 atmosphere and temperature is 300 Kelvin. We know this and now, we want to find the properties at this point. The location 2 I want to find P 2, T 2 how will I do this.

Not very difficult to do all I have to do just go and find that see the problem is based on area variation, which means the only function which can give me Mach number form area variation is A by A star as a function of the gas and Mach number, which is what we looked at the beginning yet today. So, I am going to find A 1 by A star for this Mach number that is given for m equal to 0.5. If I find this that number happens to be 1.34 I just go to tables A by A star is a typical table value listed in compressible first tables and you can find this for this Mach number this is the number you get.

Now, immediately I can tell that the choking condition did not happen anywhere inside here based on the duct shape this is the minimum area as per the duct shape. And I can tell that A star is not inside the duct of interest, does not matter I do not need to find A star value currently, if I want I can also find out even A 1 if I am knowing the area then I can tell I do not know the area we just leave it A 1. I can find A star from here. A star is

A one divided by 1.34, but what I am over interested is A 2 by A star how will I find this.

I know that this is equal to 2 times A 1 by A star, this is true by the A star is fixed value for the whole flow. So, I can just tell this now, I know A 1 by A star to be 1.34. So, I am going to get 2.68 here, but also I know once, I know A by A star I can find Mach number from this, we just did the plot of this function today and if I go look at tables I am going to get Mach number to be 0.22 or 2.52 as we know given A 1 by A star I can have subsonic or a supersonic solution, we had this kind of a plot today a by a star versus mart number M equal to 1 is here was something like this.

So, for a given A by A star I will have a subsonic solution and a supersonic solution, these are the two numbers for our value 2.68 which one will I pick for this problem is a next question. If I know that my duct is like this now, we go and use our area variation versus supersonic, subsonic table we had before 2 by 2 table we had before it is a subsonic Mach number and area is increasing or will happen to the Mach number it will decrease further, if Mach number decrease further then it will never go supersonic. So, I know my solution should be subsonic solution.

So, for this problem I am going to say I used logic and it is going to beat this. This is some time needed, the problem is decided by the shape of the duct here. And so I am going to pick my Mach number at 2 is 0.22 now, the remaining things are easy what I have to do is I have to find P 2 write I can write P 2 as P 2 by P naught in to P naught by P 1 in to P 1 gas dynamics people love this ratios this in always, so get use to this it is not difficult.

We know this ratio is given in gas tables this is inverse of the same kind of ratio P by P naught I can find these 2 numbers and P 2 by P naught for Mach number 0.22 happens to be 0.969 and P 1 by P naught for Mach 0.5 happens to be 0.843 into P 1 is at 1 atmosphere. So, this gives me a answer of 1.15 atmospheres, find I never found A star I never found P naught, I could it is just going to waste if I do not go through this ratio is business that is why I gas dynamic people like this.

Otherwise, what should I do, I will use P 1 by P naught find P naught, then use this expression into P naught to find P 2 that is more calculations here I avoided all that just founded whatever needed and nothing else, that is why people likes ratios a lot. Same thing I can do for T 2 and I will just write numbers for this 0.991 divided by 0.952 into 300 that is going to give me 312.3 Kelvin, this is how I solve this problem.

Now, we want to look at when can I get the other solution I have to think about, what is happening in here, I will just tell you this you do not know enough background to get this point I will just tell you that, if I have my star conditions somewhere in here, inside in here, if I have my star condition here. So, this is my A star, at this location my duct has star condition.

Then now, there is a possibility that this Mach 0.5 increased it is Mach number to 1 here and after this point if it becomes supersonic then increasing area will mean my Mach number will increase, if it was sub sonic after this point then the increasing area will mean Mach number will decreased then the answer will be subsonic, when I could end up with a supersonic solution or a subsonic solution. We still do not know how to get there all we know is both are possible, if my chocking condition is between the section 1 and section 2, that is all I can tell you right now.

If that is a case, let us say if I go a subsonic part there is nothing change in this problem. So, we have already solved that problem we already come to this answer, even if there is a star condition in the middle. If that is not the case, if I said that it went to star condition M equal to 1 here and after that some how became supersonic just downs to not and this increasing area, then I am going to end up with the supersonic solution that is considered that case and see how we can solve this problem very simple I have to just go through a same procedure instead of 0.22 for Mach number at 2 I will just use 2.52 and solve the same problem.

I believe, I think you can solve that problem by yourself any way I will just give you a final answer if it supersonic cases. I will put P 2 prime if it is the other case, if I use other case I am going to get an answer of 0.067 atmosphere and T 2 prime if there is a supersonic exit solution, we are going to get the 139 Kelvin it is a same procedure you are sort to find some other ratio for P 2 by P naught and P 2 by P naught calculate that star you will get this answer.

You would not worry about this. Has of now, we are not told how you can choose the solution from subsonic to supersonic, if I had a situation here that the pressure is too low at the exist, if I ever had such situation here then there is a possibility that the flow may want to choose to go supersonic to match the pressure here. That is the main idea you are always think about, fluid wants to Mach's the pressure conditions one of the serious constrains in gas dynamics energy constrains on my gas that is how it was works.

Now, I will ask the other question why it is always pressure can it be temperature, say I have my exit condition at very, very cold conditions. Let us say I will pick the same examples since I have numbers here, if I have the same conditions with that throughout at that point and I am going to consider here, temperature for supersonic solution happens to 139 Kelvin. Let us say I maintain this temperature at 138 Kelvin, when my fellow want to go supersonic not relay flow does not care about the temperature it cares only about pressure.

So, if my pressure happens to be this 0.0 whatever as 67 mille at mille bar roughly, if I choose that particularly that pressure here then the flow will want to go supersonic at this point, that is what we have going to think about. Temperature does not affect, so much fluid communicates through pressure waves not through heat waves or whatever only pressure waves. In fact if you send heating into the flow also, it creates pressure waves which is how it communicates again we will go that after some time.

So, temperature is not a serious constraint in solving flows in our invicid also I have to add invicid gas dynamics, things may be slightly different when it is viscous gas dynamics. Now, we can move on to next thing we have as of now discussed flow through duct with area variation, we gave area Mach name relation we know how to use it in flow situation I gave in subsonic and supersonic solutions. Now, the next thing I need to do is move on to some special cases which are more commonly occurring in common application let us think about that one.

If I look at them to understand this flow behavior it is better, but we go in a particular sequence, we are going to go start with a convergent duct. Once, you understand a converging duct how would the flow will behave then we will move on to converging diverging duct of course, I can also talk about just a diverging duct. Diverging duct is no fun we already done something with that it is no fun anyways, whatever we did today the first example of numerical duct is the, is diverging duct problem.

It moves away from M equal to 1 not much fun there, more fun when it is crossing M equal to 1 and we have to figure out what it will do. So, we will pick a case converging duct.

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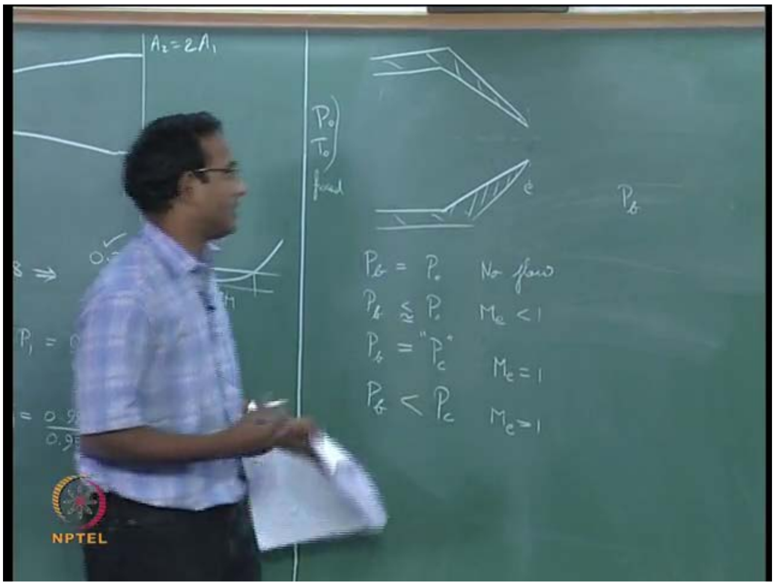

I would not worry about what it is doing after that, it is just converging duct I am not worried about what it is doing after that. Some such philosophy, there is a duct and it is coming to converging and this is a lowest area point, the exit is the lowest area point, this is my exit point and let us say there is some back pressure also there. Similar convention P e and P b and of course, I still have a P naught somewhere far into the flow P naught and T naught.

Let us consider a case where these are fixed values, I do not change stagnation conditions I am considering this and I am going to do the similar experiment as what we did before with variable area duct. Now, I am more specific about one particular variable area duct, I am not saying everything about how the area is varying all I know is it is converging only, it is not going to converge diverge, converge diverge it is just simply converging monotonic. It could be just a straight linear variation of area or it could be parabolic exponential whatever area variation I do not worry about it right now, some area variation.

If I have of course, we have to consider all the cases if my P b equal to P naught, what will happen no flow. We do not worry about the case where P b is more than P naught because that will have reverse flow it is flow from outside into the duct we would not worry about that kind of problem now. We can look at it as an application later we will pick a case where P b is less than P naught, but only slightly less than P naught I will put approximately equal to less than this is not a pure mathematical symbol I am using it.

If that is the case then definitely I am going to have a slow flow here, subsonic flow here, I can say that M exit is less than 1. Now, if I keep on decreasing P b further M exit will keep on increasing. Once it goes M exit equal to 1 what can happen, when M exit becomes 1 P b goes below I do not want to put that particular value let us say this is called a critical, some P critical I will just call it this if I call P b is less than some particular P critical value with in quotes, there will be a point where M exit is equal to 1 then I have to put equal if I say equal there I have to put equal here.

And my back pressure is such that pressure exit is going to be equal to P critical then I am going to have a Point where it just became sonic. Now, we again have to think after this when it becomes sonic at the exit, if I decrease pressure further what will happen it will again be expansion wise trying to come through this to tell this high pressure region that there is some more space available you can go there, but as the expansion wave comes here it is again going to be running on the treadmill thing velocity is going that way, wave wants to run this way they just go stagnate there nothing moves in anymore.

If that is the case then I am going to have all the waves getting chocked here and nothing going through I have just stopped all the waves there and so it is called chocking right we have stopped all the waves there nothing goes through. If that happens what can I tell about the flow inside here, nothing changes. So, now if I have a case where P b is less than my P critical, what will happen nothing changes I will still say M exit is equal to 1 only it will not change because if it is any other value.

So, it becomes supersonic then I will have a problem of somewhere else there should be sonic because this is subsonic highly subsonic it is coming from stagnation condition it is going to sonic. If I say it is expanded more then I am needing to have supersonic here, but supersonic cannot happen at any other place other than the least area. So, there is no other option for the flow than to just say M equal to 1 at the exit and nothing changes after that. So, this is a special characteristic of a convergent duct of course, you can find expressions for it which we will do probably next class if I begin that it will take lot more time to finish this problem.

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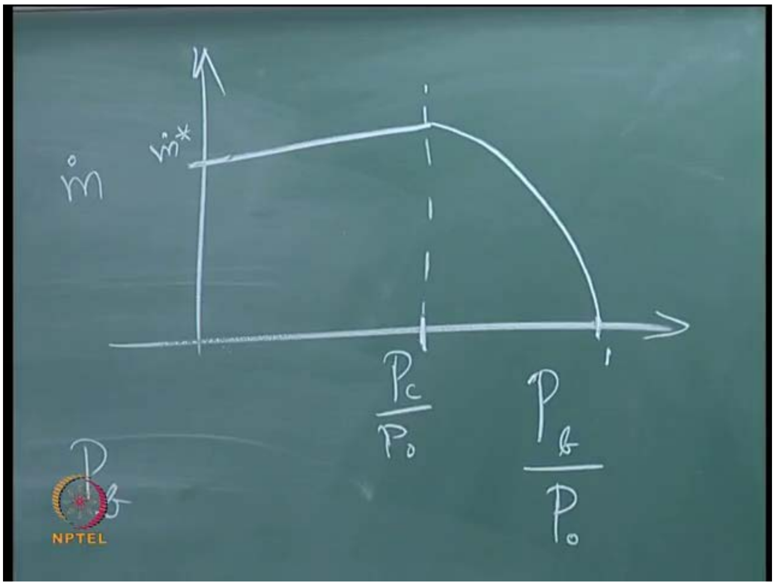

But, only thing I can draw right now is a semi plot M dot versus P back pressure divided by P naught we will come back to and discuss this once more, if I have P back pressure divided by P naught this way increasing on this. When I start at 1 my M dot is 0 after that it increases at this point it is my P critical divided by P naught, at this point onwards it is just a fixed mass flow no change in mass flow rate after that, that will be your M dot star it will just come out to be there ideally it is not decreasing it is supposed to be a constant from here. It looks like it is decreasing, but it is not it supposed to be parallel to the x axis, we will deal with more of this next class, see you people next class.