Gas Dynamics
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Module - 14
Lecture - 27
Quasi-1D flow with area variations, Choked conditions

Hello everyone, welcome back. I wanted to move into area change during flow today, but just before that I will finish off with something in expansions fan, just we will go over the same piston analogy video again, but we would not do the video; we will just look at it at the end.

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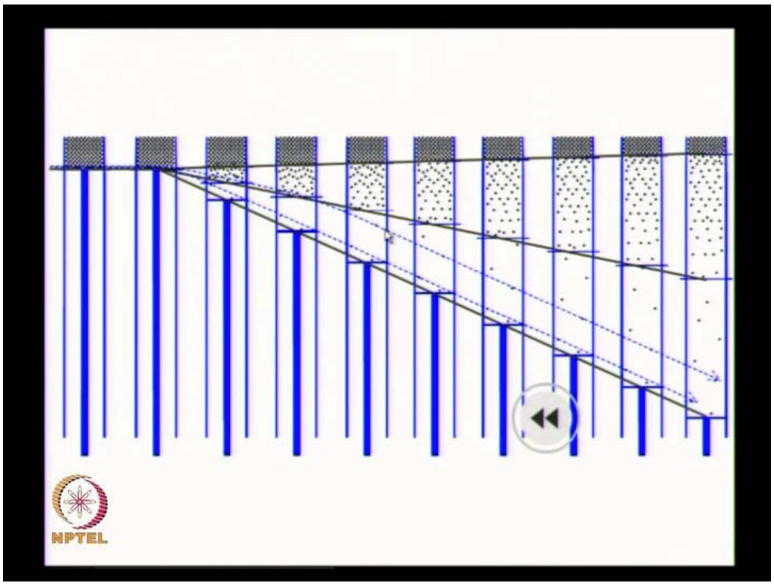

If you go to the screen we have seen this piston analogy already, I have definitely shown this to you some four or five classes earlier. What I wanted to show now is this streamline drawn here. We said that it is tracking of a particular particle especially this one and this one along as a function of time, and that becomes streamline if you look at in 2D flows, okay. Now what I want you to see is they are starting very very close to each other; that is, the gap between them is very small.

If you go back to understanding fluid mechanics the stream tubes, can be considered as the streamlines in this case in 2D flow, right. The stream lines will perform the stream tube in 2D flow. The stream tube is really small in area and a cross section here, and after the expansion they have become really wide. Basically telling that the flow has expanded; that is one way of looking at it. Another way of looking at it is during the expansion fan, the stream tube area is changing, okay, and the steam tube area is expanding if you look at it. Inside the expansion fan the stream tube area is increasing with along the flow direction.

What this means is as it is increasing I can now say that if my flow is supersonic, you remember this from long back we said that if there is area change; if the flow is incoming and supersonic and the area is increasing, we are going to have higher Mach number. That can also be seen from inside here if we think about the stream lines as stream tube walls; that is the extra information I wanted to give here in this particular expansion fan example. We saw that similar thing is happening in compression wave the oblige shock also, but I cannot show you what is happening inside a shock there. Here it is big region I can show it you very nicely.

In the other case the area between the stream lines will be very large before and after the oblige shock the stream line stunt they will be much closer to each other. I cannot show you anything more than that in here I can show that; in here I can show that the stream tube is expanding during the expansion fan, after that it is staying constant cross section between them, okay. This is one final thing I wanted to show before we jump into area variation. Since, I already discussed what happens to area variation in supersonic flows, I can now give this information already. This completes ours discussion on expansion fans and shock expansion theory, all that together.

Now we will move on to flow through ducts or flow through some channels with area variation, typically compressible flows. So, Mach numbers are going to be high enough; that is the condition we are going to work with, okay. We will move on to a new problem. We will start with flow with area variation. I would not go on derive all the equations again; we already derived most of it. I in fact listed it out in your notes somewhere where it is all nicely written in one page. So, just you just go back to that page and look at it whenever needed.

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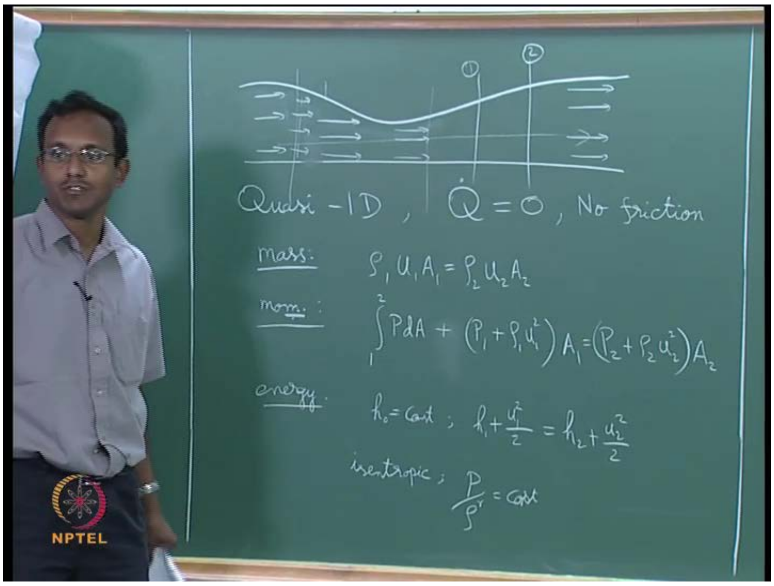

So, what we want to do now will be consider some duct flow through a duct with some area variation. I am picking something like a convergent divergent nozzle; flow through this is what we are going to study. And of course, I have to revisit all my assumptions as before. I am going to say that I am going to use quasi 1D assumption, where we say that the flow is going to be going only along a particular direction, and I am not worried about the perpendicular component velocity. Ideally, if I think about the real flow it is going to flow along the wall; it cannot flow any other direction. I am going to assume that we will not worry about those; we will just say the flow is just going straight.

It is going straight like this everywhere, and when there is no space they will readjust to go something like this, and in here it is going to go something like this. And when I am out there, it is again going to go like this. This is our assumption, quasi 1D assumption. We are going to use that so that our derivations will become a little simpler, okay. When I say quasi 1D I am just going to say it as stream normal direction velocity components are neglected; stream wise components only. It is not really components; I am going to say all the velocity is along stream wise direction which is for me in this direction for now.

I would not worry about area variation; one side is more than the other side and all that. We will just assume. For me locally I am going to consider some perpendicular line and flow is always perpendicular to it; that is a simple way of thinking about it, easy to convince myself that way; that is easier to convince, okay. And of course, I am going to also neglect some things. I am going to say heat transfer in or out of this flow is 0, no heat transfer adiabatic I am going to assume currently. And I am also going to say that there are no friction effects. We have been assuming this all this time all of these assumptions till now. We want to start removing them one by one after sometime. Eventually, we will consider a case where these assumptions are not really valid; that is towards the end.

Now if I think about mass equation, if I look at the integral form, the way I am going to start with this is I am going to say pick two sections, section one, section two; it could be any section. It could have been here and here or here and here whatever section, just some two sections one and two. This is going to write the integral form. This is something you should remember from before rho U A is a constant; that is the mass flow rate through the duct momentum equation. We said that this will have a special term. It is going to look some, I will write it in a nicer fashion than this. It is going to be something like this, and we said that if it is a quasi linear term then I can neglect this, currently I cannot; it is no more a constant area duct.

We said its area may be changing. Initially we said constant area duct I can say P plus rho U square is a constant. Now I cannot do that anymore. I had to be more careful about which momentum equation I am using. This is the correct one to use energy. This is going to be similar, because we said heat transfer is not present. So, is going to be h naught equal to constant, but writing in an expanded form it is going to be h 1 plus u 1 square by 2 is equal to s 2 plus u 2 square by 2; that is going to be this form. We will also assume that the flow is isentropic, extra assumption, okay. Now if I want to solve this flow problem from one section to another section and I want to see how flow properties are changing.

Now I am in trouble, because I have to do this integral every time if I am using this set of equations. It is not easy to use; of course, what is an equation when I say isentropic; what is that? P equal to? Okay, we can write it in several forms. Let us say I will use this form, you guys hear this. This is one of the forms; you can use this form also, any of these forms, okay. So, you can think about using any of these forms, but it is more difficult to solve this problem from the integral form of these equations, but these are the correct equations to use if you want to solve the problem. Instead of this now we will start using the differential form which is a little bit easier, because I am gone to look at any section what happens just immediately next to it; a very close line just next to it. So, the change will be extremely small, very small d A which is what we will start looking at next, okay.

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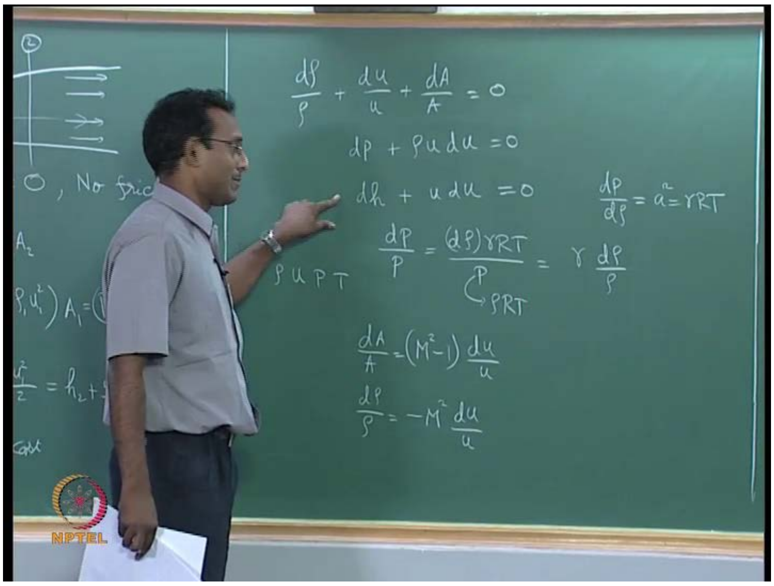

And I am going to go back and use whatever expressions we already derived. This was our mass equation in differential form, and we had this and this mass momentum and energy equations. We had different forms. And I wanted to write the isentropic relation in this form; how will I write that? What will this be? d p by p, anybody directly remembers it? If you do not remember this, the easy way to derive it is d p by d rho is what? For isentropic a square which is gamma RT. We will keep this form which is very nice form. Now I want d p by p. So, what do I do? d P is d rho times gamma RT. I will keep it that way, okay. So, it will be 0 times gamma RT divided by P.

Now P can be rewritten as rho RT. I am using ideal gas law already inside. Now RT will get cancelled. I am going to have a form gamma d rho by rho. d p by p is gamma d rho by rho; that is the form we will use here, okay. So, we have set of variables; what are all the variables we have? Rho, u, p and t; these are our variables. Of course, a is also changing, but that is the condition we are supplying to the flow; that is not a flow variable really. As a condition we are giving to the flow. So, these are the equations, four equations and four unknowns can be solved. We just have to manipulate this to get to a point where we can use it better.

And of course, I will just leave it to you to think about it a little bit. I will just jump one step, but we have derived something like this already in long time back; quasi 1D when I introduce I derived some of this. So, I will just go and write expressions from there directly. If I look at that then I will start with this was one of them. And of course, this you cannot get without d rho by rho actually. I should have given probably d rho by rho first. This M square in this d A by A expression comes actually from d rho by rho in here.

That is why you go to that point, and this d A by A is coming from the original mass expression. You can just link all of them together like this, okay; you can derive this. We have derived this already; I do not want to derive it again. We will just use this, and of course, once I know d rho by rho I can use this isentropic relation, and I get d P by p which will just be gamma multiplying this, very simple, okay.

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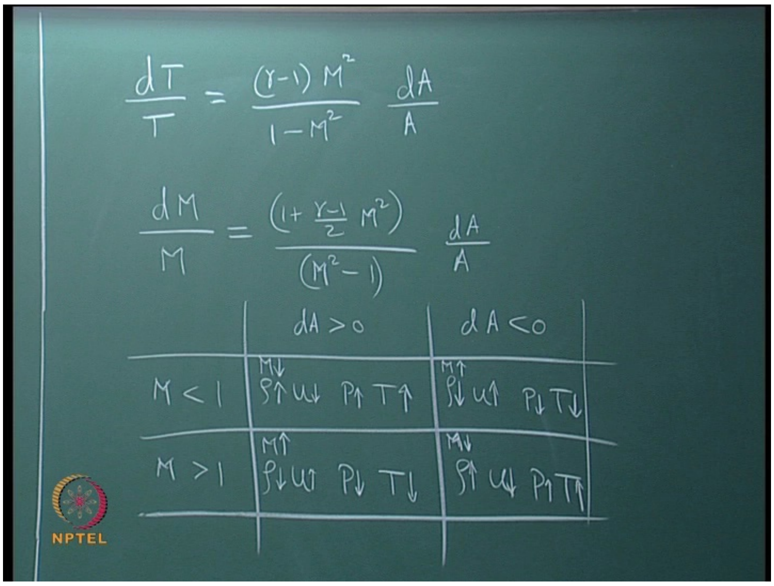

Now better expression we want to get will be d T by T from energy equation which of course will come out to be, I will leave that as an exercise for you to get to. It is not very difficult to get, okay. We will have it in this particular form, and there is only one more left which is probably the most used for analysis. How does Mach number vary when I change area? That is what this gives. And we have already used this in telling if I increase area what happens, if I decrease area what happens and all that expression we have already tried before.

We will anyway go over this once more because this is directly applicable to flow through ducts with changing areas. So, if I look at that again I am going to draw that table the way we have already done, M less than 1, M greater than 1, and I am going to say d A greater than 0 and d A less than 0. I believe you have already drawn this plot; I have drawn this table sometime back definitely. Now we just want to look at those four variables we just listed, density, velocity, pressure, temperature; what happens to all of them? So, let us speak a case; it is just to remind you what is suppose to happen. Oh, I will also put M so that it is easier.

I will keep M on top of all of them; of course, we have expressions for all of them. We just wrote expression for every one of them. We will just look at this. M square always positive, and it is just going to be some number for any particular M. But here if M square is more than one this is going to be one sign positive. If it is M square is less than one it is going to be opposite sign which means if my Mach number is supersonic, it is going to be positive sign. If it is subsonic, it is going to be negative sign; this is what is the critical thing. If I look at area increase, my Mach number will increase if it is supersonic; that is M greater than one that will make it positive. If this is less than 0, then this will also be less than 0.

So, I am going to pick a case d A greater than 0 which means this is positive, and I will pick mach number greater than one; that will make this positive. So, the overall thing is positive. Mach number will increase; d M is positive; that is this case. What if it is M less than one? This will just change sign. So, I can immediately write that this will be decreasing; Mach number decreases there, okay. Now let us say I am considering a case where Mach number is more than one. This is positive, and my area d A is less than 0; this will also be less than 0. So, I am going to have Mach number decreasing for supersonic area decreasing case; supersonic area decreasing case that is this. I am going to have Mach number decreasing, and of course, you can prove that it will be the other way in here.

It so happens that when Mach number increases velocity increases. We have already seen it as a numerical example recently, and we have also seen it from looking at the expressions also you can tell the same thing. I will just go through the remaining ones a little faster. You can also use d u by u expression which I believe I have here. I can use this also to tell the same thing. If supersonic if area increases velocity increases. I can do that also. And supersonic area increases velocity increases; it can be told from here also. Now since I am adiabatic I can now immediately tell that if velocity increases temperature drops; t naught is constant.

So, whatever velocity is this will be the opposite arrow, okay. Now we can also show from momentum equation, and in fact even we also already derived this for whatever be the Mach number this M square is always positive, which means velocity if there is a change, density change will be opposite of that, because there is a minus sign. If this increases, this decreases; you can always tell that. If velocity increases, density decreases. So, I can draw the same thing for this. And now we also did this d P by P and d rho by rho; d P by P is directly proportional to d rho by rho by a constant. So, I can also tell it will be the same direction.

So, I can populate this whole table. The main idea is you have to just get used to it. I will just tell you a simple way to think about it; p rho RT will always go the same direction, u will be the opposite direction. U will go the same as M; u and M will go one way, p rho RT will go the other way in any of these blocks if you look at; that is what will be happening there. It is a easy way to remember; p rho and t whatever is in your p equal to rho RT they are all going to go on one way. And flow related thing Mach number and velocity will go the opposite side; that is what will happen.

Now you just need to remember only one thing; how does Mach number vary when I change area? If you remember that you can tell anything else, because you are remembering the other statement p rho RT go opposite of u and M; it is easy way to remember this. Now all we need to do is go to a problem flow situation and look at what happens there. I will redraw this picture again with some other picture next to it.

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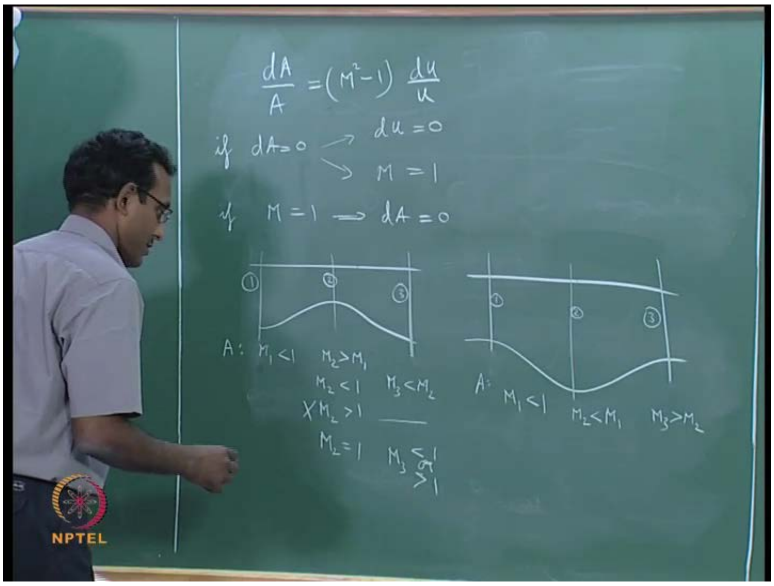

Oh, before that I wanted to revisit this d A by A expression M square minus 1 times d u by u, okay. Now if I say that there is no change in velocity for a given Mach number, then my duct should have constant area; that is what this statement says now, as one inference I am going to have. Another inference I can say now is if d A equal to 0, what all can happen? If d A is 0 what all can happen? One thing could be d u is 0; other thing could be Mach number could go to 1. These are the two possibilities. When I say d A is equal to 0, what do I mean? I am talking about a case where I am saying there is no area change in my duct; does it mean that I am always going to be Mach one in a constant area duct? Not really.

So, I cannot use this meaning all the time, but I can tell one thing for sure if M equal to 1 definitely d A is 0 irrespective of what happens to u; that I can say for sure, okay. This is a just one way thing. So, this is one thing I can say for sure. If Mach number is one definitely d A is 0. That does not mean if d A is 0 Mach number is 1; one of the common mistake in simple books. If you go pick some simple books not well written books, you will see these kinds of mistakes happening there. I can give you a crazy example for it; I liked the example from one of the books so I can use that in here. If I pick an example like this and another case where it is this; I am going to have d A is 0 at these two places. Both these places, the slope of area changes 0, right, because the slope is 0 here. It is as if locally it is a constant area duct here, same thing in this place.

So, now I am going to label them one, two and three. Here also we will do the same thing one, two and three. If I have this let us pick different cases; I can have a case of let us say A, I can have a case where M 1 less than 1; what can happen now if M 1 is less than 1? M 2 is more than M 1 is all I can say really, because subsonic flow we already wrote the table; subsonic flow this will accelerate. So, I can tell that M 2 greater than M 1. Now what all can happen? Let us pick a case where M 2 is greater than M 1 and goes to Mach one let us assume. I could have two cases. So, I will say M 2 less than 1, and I can also have a case where M 2 greater than 1, I can have a case M 2 equal to 1.

I can have different cases like this; something must go wrong in this. What will happen here? One of them should not happen. These are three possibilities. I can say that M 1 is say 0.5; it can go to 0.8; that will be this case. Here I am saying it is going to 1.2, here I am saying it is 1, okay. One of them should not happen, which one? The second one cannot happen, why? Because directly going from subsonic to supersonic and that will be violated in your table if you look at it. Say, I am having 0.5, it is going to 1.2; let us pick this case; this cannot happen, why? We will go to that table. Let us say I am having 0.5 and I want to go to 1.2, and I am having only a converging duct d A less than 0.

Someone look at only this column, and I am going to say my Mach number is less than 1 when I start. Mach number keeps on increasing which is a good thing; I am going from 0.5 towards 1.2, but when it reaches 1 I should not look at this row, but I have to jump to this row and look at this one. Because I want to go from 1 to 1.2 I have to use this row. If I have the same area variation, it is going to decrease Mach number again. So, that particular case cannot happen. So, I have written it there, but we would not think about that particular case at all. Now let us go back and look at what happens to three; this we will not consider. This will not be a possible case.

What about when M 2 is less than 1 which is only case that is possible? What will happen to M 3 in terms of M 2? M 3 less than M 2 and what about M 3 with respect to M 2 in terms of this one, this particular case and M 2 equal to 1, cannot say. Now you are at a situation where I do not know which table I have to look at; which row of the table I have to look at? I do not know, because this is M 2 equal to 1. We have only rows that correspond to M 2 less than 1 or greater than 1. All I can say is M 3 will not be equal to 1; can be less than 1 or greater than 1.

I cannot tell which one it is going to go; information is not sufficient as of now, because we are on the border between the two rows data there the table. We have to figure out what can we done. This is one system I have. What happens in the other system? Let us go there and see what happens there. I will again consider the case A for this duct if I pick this duct, again we will pick same M 1 less than 1. We have not done case B M 1 greater than 1 yet; we will go back to that M 1 less than 1. What will happen to M 2? It is a diverging duct subsonic; it is going to decrease Mach number further M 2 less than M 1, and here it is converging duct. This is already subsonic; it is going much more subsonic, say 0.5 became 0.2 further subsonic.

Now it is going back to converging duct. All I can say currently is M 3 is what greater or less than M 2?

Student: Greater than M2.

Greater than M 2; unless I am given exact area values I cannot go and calculate what is the exact Mach number. Currently, we would not worry about how to calculate it; this is what will happen. Can I have any other options here for M 2? Like we had in the other case we cannot have; there is only one option there. This is the only thing that one can happen there. Now we will go to the next case. I do not have space here I will go and write here

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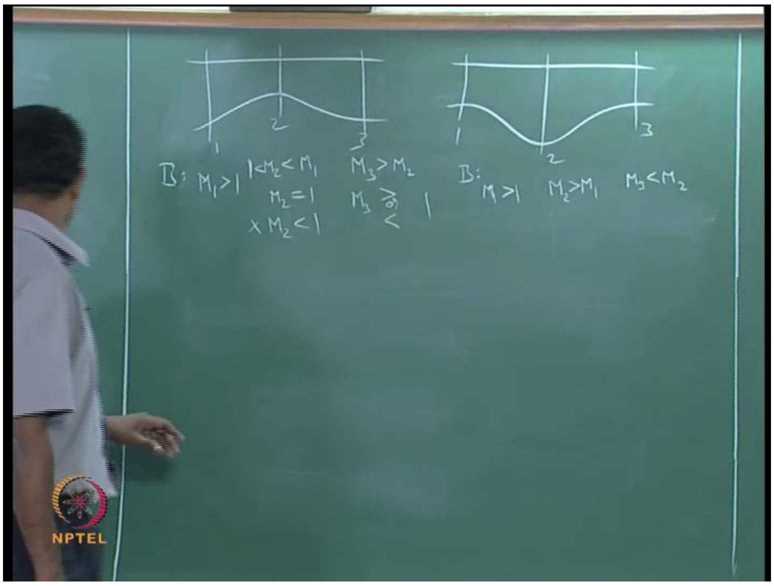

Have this, and I am considering case B where I am going to choose M 1 greater than 1 already. I have M 1 greater than 1, and I am having a converging duct here. If you go and look at the table it is going to go in the direction of M 1 equal to 1 or Mach number equal to 1. By the way a simple thing; if it is the converging ducts it is going to go towards M equal to 1, if it is a diverging duct it is going to go away from M equal to 1. But this may get confusing after sometime if you do not remember which is towards which is away, you will get confuse. Instead remember the other statement also. I like this statement; anyways you had to get used to something in your compressible flows.

I am going to say it is going to go supersonic; it is going to go towards M equal to 1 which means 1.5 will go towards 1; maybe it will become 1.1, I do not know. All I can tell is M 2 less than M 1. Let us consider a case where it is this. Another case I will say M 2 equal to. I will specifically say that it is something like 1.1 in this case, and here it is exactly equal to 1. Can I have a case again M 2 less than 1? This particular case is not possible; because again similar argument as what we said here one case is not possible. Similarly, I can say that there also it will not be possible. It has to cross and suddenly area change should change the other away for it go cross Mach one.

Now I have these two cases. If it is M 2 greater than 1, what happens to M 3? Supersonic flow area increases; what happens to it? This will also increase which means M 3 is greater than M 2. Now if I look at M 2 equal to 1, again similar situation arises. I cannot tell what it is going to do, because I am not anywhere on the rows of the table. I am in between two rows of the table. I can have M 3 greater than or less than 1. This probably not a mathematical symbol; am just using it any way, could be any of those cases. I cannot tell for sure what can happen. What about the other system? If I have case B here M 1 we said greater than 1 I am going to pick. Am picking M 1 greater than 1, what happens to M 2? Supersonic flow diverging is going to increase M 2 greater than M 1.

And of course, that means it is definitely more than 1; we do not need to worry. It is going to be 1.5 becoming 3, something like that, and it is converging now. It is going to go in the reverse direction M 3 less than M 2. We would not worry about what is happening from 1 to 3. We would not compare M 1 and M 3 as of now. It is not a serious problem that way; we would not worry about it. Any other possibilities here; no other possibilities exist, okay. The nice thing about this particular example is I am going to say here also d A is going to be equal to 0, here also d A is going to be equal to 0.

In this case whichever be the condition I pick, I am never going to get M equal to 1 in this kind of a duct, but in this kind of duct I have possibility of M equal to 1 happening; that is what we need to think about. It is something more than just d A equal to 0. It has to be area constriction. It cannot be area opening more to something else; that is one more thing you have to look at here. This is called chocking of the flow, because they are constrained of duct; we are chocking the duct that is the idea. If you have a pipe and I am crushing it in the center that is called the chocking.

Now you are chocking the duct in here; that is why it is called chocked flow. If it goes to a situation where Mach number at that point is sonic, it does not ever happen in the other configuration. Both are going to have d A equal to 0, and as per this I cannot tell anything for sure, but we have to look at more than just what it is. It has to be a chocked flow for it to go to M equal to 1. Now after this we want to be able to go to a situation where I can start solving whatever flow field in different situations; for that I should be ready to extract more information out of this.

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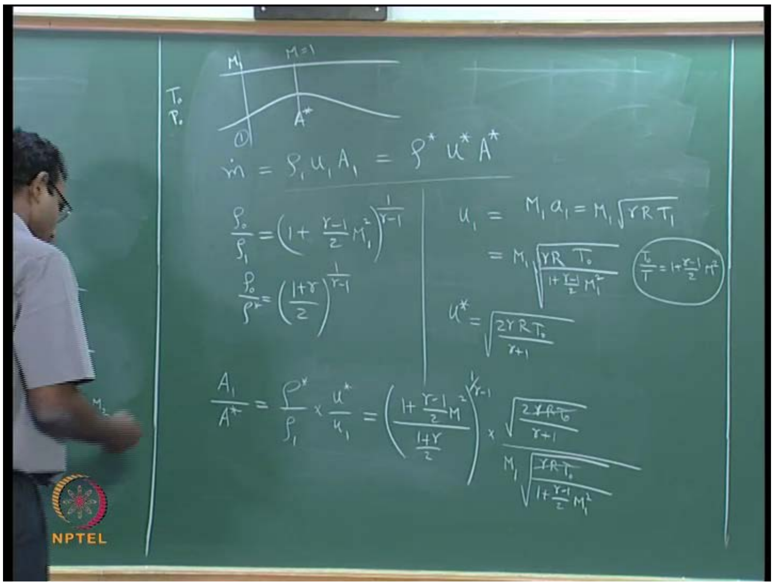

What we have going to say now is I have a duct with some chocked area. Since, it is some chocked area something special I am going to call this is A star. I know that it is chocked area which means my Mach number here is 1, because that is by definition chocking for me. You want to say the least area point and it is chocked I am going to say. We will go and redefine that chocking more after we have done some derivation. As of now we will just assume that it has to be the least area or the minimum area possible, and we will assume that Mach number is 1 there. There is also a possibility like I will go back here. There is also a possibility that from here I start with say Mach 0.2, it went to a higher Mach number, but it did not reach.

It need not always take this path; it can also take this path. Let us say it did not take this path currently. We are considering a duct which has this particular path going through; that is the path you are going to consider. Now our job is to go and find let us say section one. I want to know the mach number for this area, whatever; area A 1 is given let us say I want to find the mark number M 1; that is the overall idea, how will I find it? So, now I will start using the integral equations and trying to get to some point where it will look like I know how to solve this problem. I have already assumed that it is an isentropic flow, and I am starting to solve the problem, okay.

I am going to pick this M dot mass flow rate equal to, I am going to write it like this; I am gone to pick two conditions, one and star condition. I am going to say I know that the flow is chocked here, and I am going to say also know isentropic flow T naught and P naught do not change. I know that also; for this whole flow fields somewhere I have measured it. I know something's about the flow. I know that the flow is chocked here, and I know T naught and P naught for the flow. These are the information I know currently, and we want to solve every other point in the flow; can we do it? Let us see how we can solve this problem.

So, I am going to look at only this set of expressions. I want to rewrite them in such a way that I will eliminate all the variables. I know Mach number here. Let us say I want to call it M star. M star will be 1, and here this is some M 1. I want to write all these variables or the ratios of them in terms of M equal to 1 or M star and M 1; I want to write these two. So, let us start with what we know already rho naught by rho is equal to what; in terms of Mach number what it will be? 1 plus gamma minus 1 by 2 M square to the power 1 by gamma minus 1; this is what I have already. So, now, if I say I have a special condition rho naught by rho star, this is going to be equal to M equal to 1 will simplify to 1 plus gamma divided by 2; this is what I have.

So, now if I take the ratios of these two, I will get rho by rho star which is what I have here, okay. If I use M 1 here this will become rho 1; that is the way I look at it. And here

I have used M equal to 1; that becomes the star condition, this is what I have. Similarly I can do for u 0, okay; u 1 in terms of Mach number is what? U 1 in terms of Mach number, M into speed of sound; so, it will be M 1 times A 1 which is written as M 1 square root of gamma RT 1. Now I still have to write T 1 in terms of Mach number. That is going to be M 1 square root of gamma R. I will go and write a separate expression here T naught by T equal to 1 plus gamma minus 1 by 2 M square. I have this here.

I want to use that inside here. I will take T there and the whole expression below. It is going to be T naught divided by I will extend the square root further 1 plus gamma minus 1 by 2 M 1 square. This is the expression I will have. Now if I want a u star expression that is just substituting M 1 equal to 1 in this. So, that is going to come down to square root of 2 gamma RT naught by gamma plus 1, okay. You can simplify to something like this. Now you want to simplify it much further. I will substitute the ratios of these such that I will get A 1 by A star. A 1 by A star I want to substitute. If I do this, I have to take the remaining things there rho star by rho multiplied by u star by u. Actually I have to write it as u rho 1 and u 1, and we just have to substitute these functions inside there.

And it is going to be rho star by rho 1 will be this divided by this will give you rho star by rho 1. So, I am going to write it as 1 plus gamma minus 1 by 2 M square divided by 1 plus gamma divided by 2; the whole thing to the power 1 by gamma minus 1. This is the first thing multiplied by u star by u 1 which is this divided by this where some of them will get cancelled. We would not cancel any of them as of now. We will write it as such and cancel it later square root of 2 gamma RT naught by gamma plus 1 divided by M 1 square root of gamma RT naught by 1 plus gamma minus 1 by 2 M 1 square; this is what I have. Now of course, I can directly cancel gamma RT naught, gamma RT naught. I am going to get something like this.

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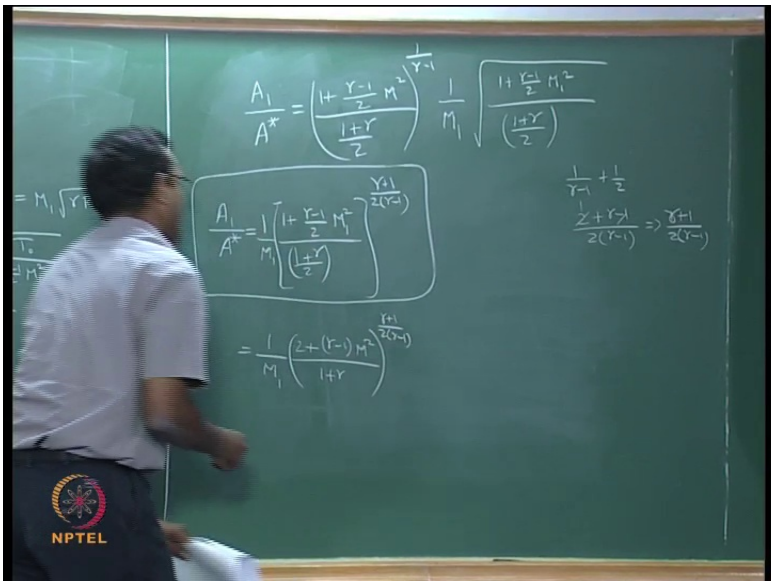

Now I just have to simplify this whole thing. If I look at it, the expressions are almost the same in this bracket and in this square root terms. There is just a 1 by M 1 which will be separate which we will put it in the front. The remaining things if I look at it, they are almost the same thing. If I slightly rewrite this, I will get it as 2 plus gamma minus 1 M square whole divided by 2;; then that by 2 will get cancel it this divided by 2. If I look at here, it will be the reciprocal of this in the numerator, and here the reciprocal of this in the denominator with the square root here. I can rewrite this expression as this whole thing to the power half; that is what will happen here.

I will go and write it like this 1 plus gamma minus 1 by 2 M square by 1 plus gamma by 2 to the power 1 by gamma minus 1. I also have a 1 by M 1, and the remaining terms inside the square root I will combine them as one and rewrite it in a nicer fashion 1 plus gamma minus 1 by 2 M 1 square divided by 1 plus gamma by 2. I write it like this. Now if I look at it, this and this are exactly the same and they are multiplied. So, the powers will be adding. So, I will just do that 1 by gamma minus 1 plus 1 by 2, and it is going to be 2 plus gamma minus 1 by 2 times gamma minus 1. This is going to be gamma plus 1 by 2 times gamma minus 1; this is what it comes out to be.

So, I will write my A 1 by A star expression. It is going to be 1 plus gamma minus 1 by 2 M square divide by 1 plus gamma divided by 2 whole to the power gamma plus 1 by 2 times gamma minus 1 with a 1 by M 1 in the front. Now I will have to put M 1 square here. Now I have our expression where I am going to say area is a function of Mach number. This is not an ideal expression for us, but this is the only thing we can get right now. I can say that area with respect to critical area; I am using the word critical for the first time, this is the area at chocked condition.

In compressible flows they use the word critical a lot. I am going to say the area divided by the critical area; will this be more than one or less than one? It will always be more than one, because A star is defined to be the least area in your whole duct. So, this will always be more than one; this can never be less than one this function. So, that being said will it automatically be so? Even if it is subsonic, I can say when it is supersonic Mach number square yield by Mach number, it is going to be very high power; that is easy to say that it is going to increase. Can I tell that that is the same when it is decreasing when it is subsonic? It is so we will look at the plot after sometime, but that is what will happen there.

Now instead of having this divided by 2 and stuff I can rewrite this as one more statement 2 plus gamma minus 1 times M square divided by 1 plus gamma whole to the power gamma plus 1 by 2 times gamma minus 1. This is one more form you will see. This is probably easier; they are doing some Huron coding for this function, or I will take this divided by 2 as a square root in here for this whole thing; that is another way people write it, different ways of writing. Whichever form you remember is good for us.

This is called the area Mach number relationship, very very useful in calculating Mach numbers at different points in your duct. This is the area Mach number relationship. The next thing we want to do is go back and substitute it in this expression; I will just use this expression here. I will erase this. I want to find the mass flow rate through my duct. I will erase this, not very difficult to do. I will not use this rho 1, u 1, A 1, more difficult.

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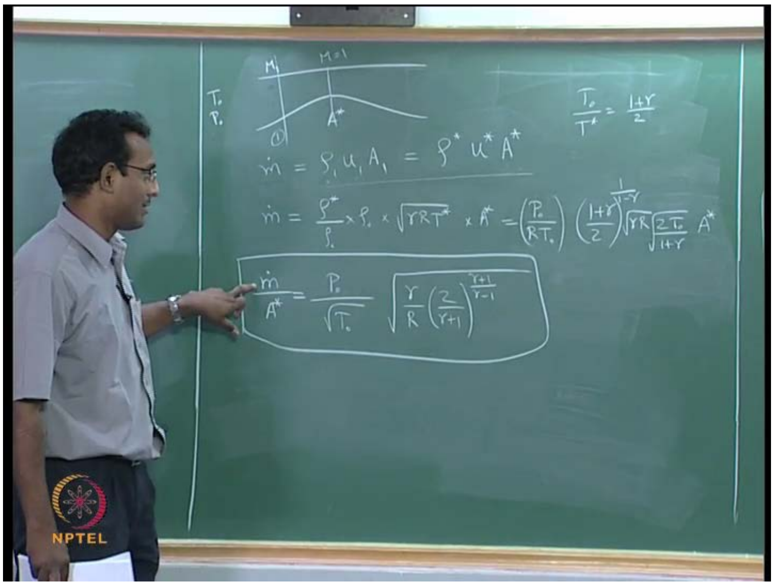

I will go use this. M dot is going to be equal to I will write everything only in terms of P naught and T naught, the stagnation conditions. I will just pick this rho star by rho naught times rho naught into u star which is nothing but for M equal to 1, right. So, it is just going to be square root of gamma RT star. Now I have to expand T star further and multiply by A star I will keep it right now, one more line I will write. Rho not is P naught by RT naught into rho star by rho naught we already had an expression. I just erased it. It is going to be 1 plus gamma minus 1; for M equal to 1 it is just 1 plus gamma divided by 2 to the power 1 by 1 minus gamma; that is the function I have.

Of course, I put 1 minus gamma here instead of 1 by gamma minus 1, because it has to be reciprocal of that. I have already taking care of that; that is my rho naught multiplied by rho star by rho naught. Now I have to do this square root of gamma R. I will keep that as it is square root of T star; T star in terms of T naught. T naught by T star is equal to 1 plus gamma divided by 2. This we already wrote some time back. So, I want T star; that will come out to be 2 T naught by 1 plus gamma square root of that multiplied by A star. This is the expression I have. Now if you see nothing really gets canceled except for this RT naught and RT naught here inside the square root.

Everything else stays as it is, and probably I can simplify this 1 plus gamma to the power 1 by gamma 1 minus gamma and 1 plus gamma to the power half. Let us say I will leave it to you an exercise, and I will just write the final expression. If I write the final expression in a form which is very common, I will take the A star and put it in the denominator here M dot by A star; that is going to be given as P naught by square root T naught. I am writing it in a particular form multiplied by square root of gamma by r 2 by gamma plus 1 to the power gamma plus 1 by gamma minus 1. You can end up with this form; it is not very difficult. You will end up with this form after some time. It is not very difficult; you will just go through the math. I will just say you know how to do that math.

Now what is the mass flow rate at section one? If you think about it must be the same as that at this section, right, it is a duct. It is said it is one duct and it is just area changing; mass flow rate here must be same as mass flow rate here; another proof this sentence here. The expression written here M dot is equal to rho 1 u 1 A 1 is equal to rho star u star A stat. This is going to help us solve problem faster. I have this expression. I can of course go and do the same kind of derivation for rho 1 u 1 A 1. I can do that and there will be a better expression than this. There will be 1 by M 1 somewhere and then there will be 1 by 1 plus gamma minus 1 by 2 M square kind of terms will be sitting inside here; that can also be done.

And I will give an exercise of that form, and I will put it up on the web for that purpose. That can also be derived, and that means I can get mass flow rate. If I know the Mach number and the critical area, I can get mass flow rate through any duct. Of course, you should also know the stagnation conditions pressure and temperature, and of course, the gas the gamma value you should know. These are the things you need to know to get to mass flow rate through the duct. We will go look at more expressions and the functional form of A by A star, what happens to Mach number when my area A star changes and all that in the coming classes. See you people next class.