Gas Dynamics
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Module - 13
Lecture - 26
Prandtl-Meyer Function, Shock expansion theory and its applications

Hello everyone, welcome back. We have come to a point where we can directly go for expansion fans and shocks both together used around flower bodies cases. So, that that particular theory of using shocks and expansion fans is called shock expansion theory, and it is very commonly used in all kind of books. So, that is a very nice way of representing what the flow field will look like, and now we are equipped with solving such problems, okay.

So, we already looked at one such problem yesterday; that was flat plate flow over a flat plate. We did not fully solve the problem, but we almost solved the problem fully expect for I did not do any numerical work on it. Yeah, it is a little more tedious; today we will look at a little more complex problem.

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We will go to the board. This is the function of Prandtl-Meyer function for every Mach number plotted against Mach number here. I am showing it for three different gamma values here. What we want is we have to look at the function for any particular gamma and see what it looks like. This is the one particular gas; for us most commonly is air. So, we look at this green curve first; that is for gamma equal to 1.4. All of them start at m equal to 1 onwards which we know because this function Prandtl-Meyer function is valid only for Mach number greater than 1; it is not valid for less than 1.

So, we have the case where it starts at 1, and it goes all the way up to 100 I have plotted. We can go all the up to 3000, 4000 if you want; most common Mach numbers never cross 20, or anyways if I pick any particular curve let us say we will pick this green curve. We see that there is a lot of change initially up to around Mach 5 or something; all of them have a lot of change, and after that they slowly steady up. This is the specialty of hypersonic flows when Mach number is above 6 or 7, when you see that most of the variables will start tending to become a constant; that is we will deal with more if you are doing a course on hypersonics, but we will just leave with supersonic flows.

Ideally I have to zoom in only from 1 to 5 and show you what it looks like. I am just showing this because I want you to note that they all go to an asymptote somewhere far way; after sometime they will all go to some constant. These numbers we already introduced last class; we said that for gamma equal to 1.66 I am finding that it is this value; they are 90 degrees. We did this in class there itself. And when we go to gamma equal to 1.4 that goes to a slightly higher value and gamma equal to 1.3 it goes to even higher value. It is just justified the way I look at it. When I change gamma I am changing the compressibility of the gas. If the gamma is lesser the gas is more compressible; we dealt with already so many times.

As the gas becomes more compressible it goes from when I go up along this way in the curve I am going more compressible gas. When I go more compressible it is also more expandable; that is the way you want to think about it. If I am going from one Mach number to some other Mach number the flow can turn a lot more and expand and fill the region more if I go around the corner. We did that particular example for m tending to infinity last time, and we found that for gamma equal to 1.66 we found that it just turn 90 degrees and stopped while the other cases it went farther.

That is basically it is going to the same Mach number, but it is filling more space. So, it is more expandable similar to looking at more compressible gas; that is one way of looking at things. Other than that what I have to see is only other aspect is every curve is completely monotonic. There are no kinks anywhere; it is just going to increase always. If I want to increase Mach number my nu will increase monotonic function. This is going to help you in solving for this if you are ever solving an inverse function for this. All I have to do is just write a bisection code and it is very easy to converge on this as one helpful thing about this. Now we will go back to solving problems.

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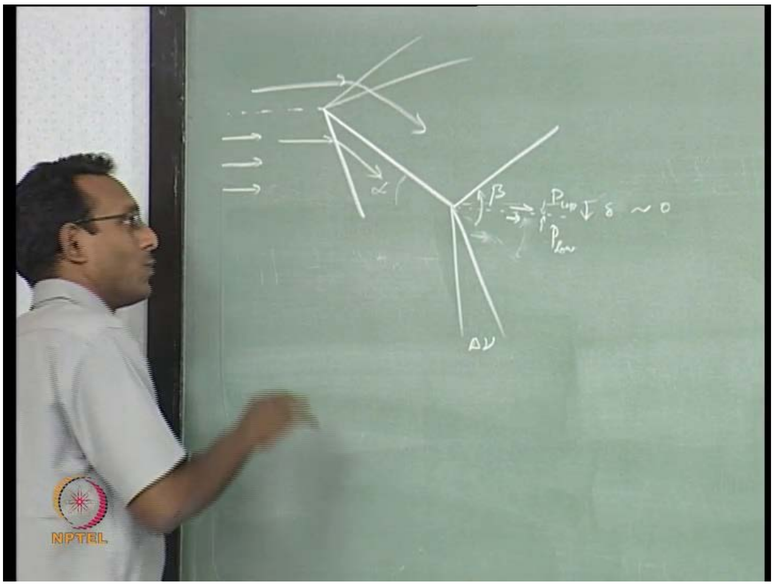

We did this flow over a flat plate last class where we said that I have some angle of attack alpha, and I said that there will be a shock here. This flow is coming parallel to my board I am assuming. There will be a shock here, and on the other side I told you to draw a dotted line which crosses changes this, and we said that the flow above is going to see an expansion fan somewhere there. There the flow comes here, turns around and goes like this; while this flow that comes here will turn suddenly and go parallel. And we already said that at the end the flow will go at some angle such that I will draw this angle, okay; let us call this some delta.

I am drawing it downward. The reason being the flow has deflected downward by the plate. If you think about it from aerodynamics point of view this is the airfoil at positive angle of attack. It is going to produce lift which means that should be a downwash. So, I am going to say that there will be a small downward component to this flow after it leaves that point; that is the way I am thinking about it, which means this top flow is going to see a compression wave here with respect to that incoming flow direction there is so much change, and that change will be a shock out there.

And with respect to this flow with respect to this line there is again so much change. That will be an expansion fan sitting here. This is the overall flow field I am going to get. We did this last time; of course, we just told how to solve this. We never solved it really. I just told that across this region I am going to have velocity vector directions parallel, which means I should turn from this angle through this expansion to the same delta downward and the same thing on the top. The oblique shock will make it turn only up to that point; that is what we said.

That is not enough; we also have to tell that the pressures across this surface p upper and p lower must be exactly the same; only that particular condition it will be coming to equilibrium, and that will be the final condition. We said that the two unknowns in the problem are the beta angle for the shock and the final Mach number you are going to reach here or the delta theta for this expansion fan. Those are the two variables or delta nu I will call it for the expansion fan. These are the two variables we have, and if you use these two conditions pressures must be equal and velocity vector direction must be the same. We are going to get these two conditions solved; that is the basic idea.

If you solve this problem using these conditions and it means iterative procedure. Initially I have to guess a delta and based on that solve it. If I do this procedure iteratively, eventually I will go to a point where I can find this delta, and I have solved this using my computer, and I found that this angle is almost 0. So, for simple practical purposes we can say that it is 0. If you want exact number it is of the order of 10 power minus 39 degrees; it is very very very small numbers. So, for all practical purposes we will just say the downwash is extremely small in supersonic flows; that is what we are going to get, typically that is what will be the case. There is one such problem we have solved.

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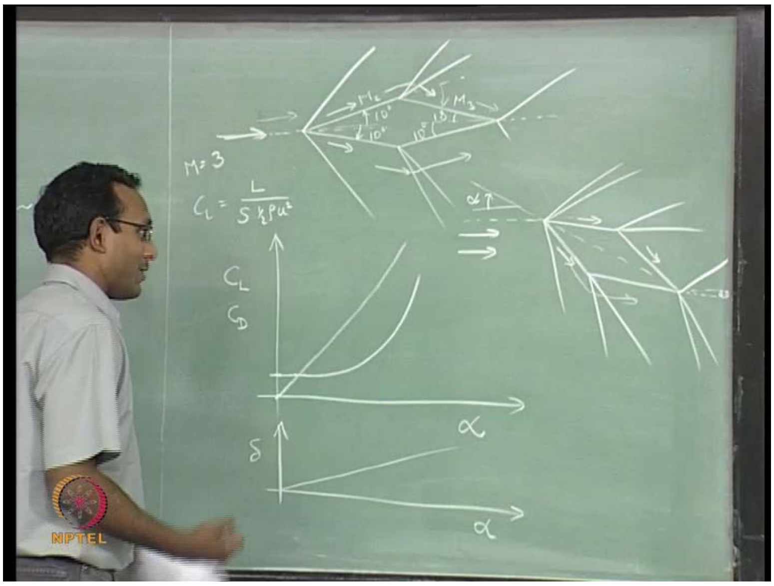

We will look at the next case which is a little more complex. I am going to take a diamond airfoil standard airfoil talked about in all gas dynamics books. I am going to say it is very symmetric in all directions. It is 10 degrees in all direction; I am going to pick such a case, okay. Now what do I want to do? I want to just solve for a given angle of attack, I want to solve for the flow field around this. Currently, I have drawn it such that angle of attack is 0; that is fine for now. I am going to say the flow is incoming like this, and it is going to go along this cord line. So, the angle of attack is 0. What will this flow field be? We are using this shock expansion theory, very simple to use.

I am going to say, I will draw this dotted line along that point; that top half surface is different from the bottom half surface. I am going to solve only the top half; that stop streamline is going like this, and the wall is going into the flow which means there will be a shock. I am assuming that there will be an attack shock right now. Of course, you have to go and check whether this angle is low enough or not; that is something you have to do. I will just give this as Mach number 3 currently.

For definitely for ten degrees at Mach number 3 it will be an attached shock; even for Mach number 2 it will be an attached shock. And here also I am going to do the same thing. Since, it is symmetric I know the beta values this way and that way will be the same. After I have solved this part I am assuming that you can do this, because we have done this kind of problems already in the earlier classes, simple oblique shock same to single turn case very simple to solve.

The next thing we did only last class; sharp corner how will I find? What is a net angle change here? It will be the 20 degrees total; it was going up 10 degrees, to going down 10 degrees. So, it is totally 20 degrees change. So, I have to go find this Mach number; let us call as M 2, from here I have to go to some M 3 such that it has turned 20 degrees. So, I will probably have an expansion fan like this which will cause this flow to turnaround to this point, and it will go along this wall now. I am assuming you can solve this problem, because we did in an example last class; otherwise, it will take a lot of time solving the whole set of problems. Same thing on the bottom; flow comes here, turns around and go straight this way.

Finally since the flow is very symmetric I can just draw the exits angle to be 0 exactly zero angle of attack; it should be 0. Now the flow is coming like this, and it has to turn that way which means there should be a shock. I can find that angle because that will be again 10 degrees up. Similarly, the bottom side also will have a shock. This is the problem you have, okay. Now if I want to find the lift or drag on this body, very easy to do now. Once I am given the initial conditions the free stream conditions, thus they call it the aerodynamics. I say it is one atmosphere and 300 Kelvin or something, and I have given the initial Mach number.

I can solve this whole problem, and I will come to some nice answer. I can give you lift value exactly, right, because I know the pressures here. I have given the lengths let us say. Then I can give you the exact value of pressure, because I know the area. I am assuming 2 d airfoil; I will do it for one meter length of airfoil. So, like that I can find pressure here, pressure here, pressure here, pressure here. Will this airfoil have drag at this angle of attack? Two people say no, one person says yes. So, will it have drag or not? It will have drag, why? Okay, will it have lift? It will not have lift.

Why? It is very symmetric problem, okay. Whatever pressure is here same pressure here; same pressure is here and here. So, it is equal; there is no lift produced. Lift is a force going upward; drag is going along the flow direction. So, will it have a drag? It should have a drag, why? This pressure is higher than this pressure, why? I am going from here to here through an expansion, simple enough logic; that is enough to show this. From here to here I have gone expanding two to three which means pressure three is less than pressure two.

If I look at the vertical this component of the area then I will see that this is going to push and that is not going to push back as much. So, there will be a net push this way; that will be your drag. So, this particular body will have drag even at zero angle of attack which is not the case for the previous problem flat plate, where we assume it is very thin flat plate, no frontal area. So, if I start doing this drag polar as they do it in aerodynamics, typically they represent C L and C D which are just C L as given to be lift dived by the area of that surface airfoil, the plant form area of the airfoil they take typically into half rho u square waves, the dynamic pressure.

If I do this and similarly for drag C D will be D divided by the drag force divided by plant form surface area times dynamic pressure. If I plot those as a function of alpha the angle of attack, currently I have drawn the picture only for zero angle of attack, okay. We know that the very first point for drag is here; what is it for lift? It is here; this is what we know as of now. Let us draw another picture. The power of this shock expansion theory is now I can just keep drawing this picture for every angle of attack. Let us draw for some arbitrary angle of attack. I will draw this as my angle of attack line, and it is still the same 10 degree thing, same wedge I am using, and I am going to call this is my angle of attack. This angle is alpha.

Now I am having flow coming in like this. What will this; I will again draw that dotted line for ease of solving the problem. I am seeing that the net change will be alpha plus 10 degrees for this. If I assume for this Mach number the flow as still attached shock, then I can draw an attached shock here, okay. And we already told that in detached shock problem we cannot solve the problem analytically; it is not easy to solve. We have to go for solving the full lines of equation and get the solution, because the problem has subsonic, transonic, supersonic parts in it.

We will avoid that particular problem currently, okay. And on the other side I have chosen the angle of attack such that I am going to have expansion that side on the top. So, it is going to have some expansion; it looks like it is a small angle. So, I have drawn a very thin expansion fan. What is going to happen after this? The flow is becoming parallel here, and it is again seeing another expansion 20 degree expansion. So, I will draw another expansion here.

I have come up to here. Let us go on to the bottom side. It is gone here, it is going along this. Now what will this see? Wall is going away from the flow. So, it will be an expansion this way, some expansion fan. It is going to turn from here to this direction. Now these two flows will meet, and I told you that the downwash is going to be really small. So, I know that it is roughly going to be close to 0, close to 0, angle of attack with respect to original flow direction. It will go almost like flow direction when it leaves the airfoil, almost.

We will still have that small delta downward just for completeness sick. I can call it 0 also; that will force the problem. We do not want to call it 0. Let us say we find the angle so that I will match the pressure and match this delta value for both sides. What is it going to be on the top side? It is going to be a shock, and the bottom side it is going to be an expansion. It is going below horizontal, and it is going to come back to horizontal. So, it is going to be an expansion here. This is the flow situation you are going to have.

This is a general picture. It will keep on changing depending; this may become a shock if I have lesser angle of attack. So, depending on the case it will keep on changing, and when this becomes the shock this will also become a shock; we will see that. It is a very symmetric airfoil it will become. So, this is just a general picture. You can in fact write a code to solve this problem for a given angle given airfoil, but of course, I have done this code already. And we will just tell you what it does, but tell me from intuitive feeling, what you think will happen to the lift? It has to increase with angle of attack, why? Because this top is going to have expansions, bottom is going to have a strong shock, but there is also an expansion remember that.

There is always going to be expansion at the bottom. So, I am going to say based on this; top pressure is definitely lower than the bottom pressure. I go back to this zero angle of attack; zero angle of attack it is going to have same pressure top and bottom. So, the lift was 0. If I have any angle of attack, what will happen? The angle of deflection on the top wall is less than angle of deflection in the bottom wall. In fact, the bottom wall angle increased, top wall angle decreased which means we know automatically that the shock downstream pressure will be lesser there compared to here, which is the proof for my producing upward force.

And can you prove that it is going to have drag? Yes of course, again the same kind of system. If I look at this particular problem I am going to say this is going to produce vertical force that is my left. Horizontal force is produced directly from here I can say, aftershock pressure here is very high, and there it is after two expansions very very low pressure. The drag is going to be very very high so that I can directly say from here. Will it cause a moment? Aerodynamics people are interested in moment across airfoil. Will this airfoil start tumbling around? Will it rotate about the centre let us say? That is the next question.

It will rotate. In fact, it will rotate like this, but you can go and prove it if you go and solve this problem for one particular angle of attack. You can prove that the pressure here is higher than the pressure here; pressure here is higher than the pressure here, all that you can just prove it. Actually this will also have force like this, this will be lesser force compared to this. So, it will start rotating like this. This will send a force like this; this will send a force like this. If I consider this surface and this surface that will be a net force like this. This surface and this surface net force will be like this but a smaller quantity.

Overall there will be a moment, and this net force will be the sum of these two forces that will be like this. This component is the lift; this component is the drag all that is sitting inside that. You can solve this problem. I would say I am going to give you exercise on the web where you can solve this whole problem. If you look at what it does, lift is going to increase slowly while drag is going to start slow, but increase drastically; this is what will happen, okay. I will show you actual data for this 10 degree with Mach. I think it is Mach 2. I have plot of that, will show you after some time.

Now also if I plot the delta as a function of alpha, what should it do? Say, I plot delta versus alpha. Delta is downward angle; what should it do? Delta is the downwash angle; I am giving you a clue. What will happen to delta as I increase alpha? It should increase, why? Lift is increasing, simple proof; lift increases so downwash should increase, okay. The way I think about downwash is like you are climbing stairs. What you do you kick that particular stair down so that your body goes up; that is what the airfoil is doing. It is kicking the air down. So, that it goes up; that is lift.

This is the way you think about it when you talk Newton's laws, same Newton's law working here. The airfoil is kicking the air down so that it goes up, okay, its action and reaction if you want to think about. So, you will see that this is going to increase, but I already told you that this is going to be extremely small number. I have not given you solid proof yet; I am going to show it to you in one minute.

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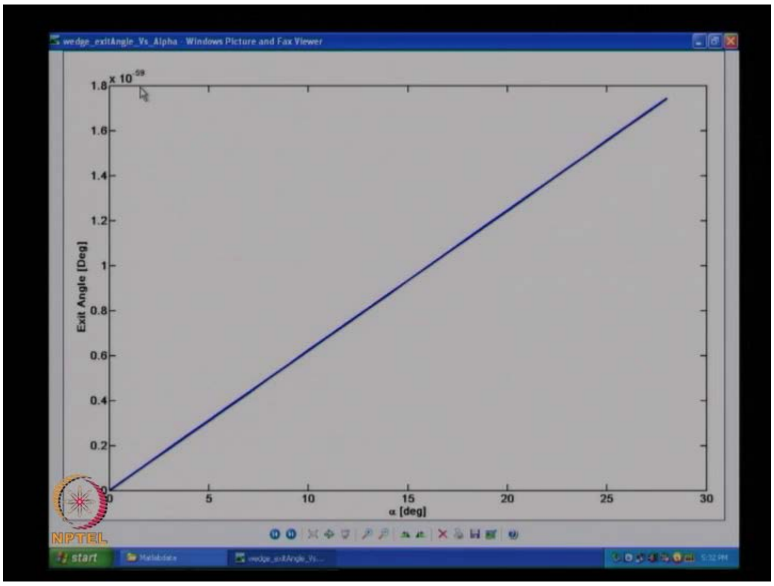

So, let us go to the screen; I am showing you the proof right there. I am plotting the exit angle the delta, the downward angle the downwash angle if you want to think about versus alpha the angle of attack for a 10 degree in all direction wedges for a flow of Mach 2. What I am seeing is number is increasing slowly. In fact, it increases almost very linearly, and its quantity is 10 power minus 59 in degrees, extremely small change. For all practical purposes I can assume that this delta is 0. It looks as if there is no downwash, but it produces lift. So, you have to be careful about this. It is actually a very small value that we neglect this. Ideally there will be a downwash if you are getting lift; Newton's laws cannot go wrong as of now. So, this is one thing.

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The next I want to show is this lift and drag curves. They are not very linear; they are going to go up. The drag increases much faster. This green curve is the drag while I forgot to label them, but I have the curves here. The green curve is the drag; we know that zero angle of attack it is going to have a drag. That is what this jump is. And the lift is going to be 0 at zero angle of attack, and it is going to increase almost linearly after sometime, and then it becomes non-linear after sometime. You can talk more about why it becomes non-linear; you will suddenly see that the flow is jumping from shock to expansion; all that will start happening.

This is where it starts going non-linear; around here is when the shock becomes Mach wave and becomes expansion. The top surface shock, we will just jump. And the numbers are crazily high here c D or c L. I think there is probably some mistake here; I think I did not divide by the correct dynamic pressure, with wrong units probably I did it. But the curve is this. I think the angle this number is too high for this to be. It cannot produce more than the dynamic pressure kind of lift and drag. So, I think there is some mistake. Probably it is absolute values of lift and drag and not c L and c D; I made a mistake there.

It is for Mach 2 one atmospheric pressure 300 Kelvin flow; that is what I have. So, this kind of problems I can solve with shock expansion theory. When I told that at around 10 degrees the flow behavior changes from almost linear to non-linear here; what I was talking about was this particular condition. I will draw especially here separate picture.

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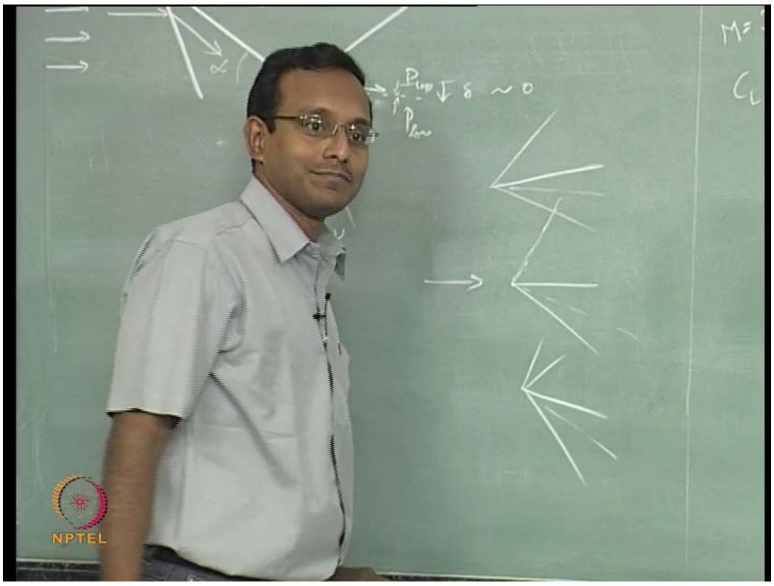

I am going to look at the top surface of my wedge; when I start close to zero angle of attack there is going to be shock. There will be a point where exactly at 10 degree of attack, what will happen? This top surface will become parallel to the incoming flow direction. When that happens this is your 10 degree angle of attack. What wave will be there on the top? No wave; I do not like that. There is always wave, and we call it the Mach wave, okay. If I think about the continuous change of angle of attack, what is really happening is this oblique shock is going to become weaker and weaker and weaker. Eventually the beta will become equal to mu; that is where it has the weakest oblique shock the Mach wave.

If I go any lower, that mach wave does not change. That stays there, but there are more Mach waves downstream which eventually forms your expansion fan. I want you to feel that. That Mach wave stays there. As I go more and more angle of attack this way, there will be more and more waves coming downstream, and that whole thing forms your expansion fan. I do not have an experimental facility to show this exactly happening, but I eventually plan to have one video like this. It may takes some two, three years for me to form such a experimental setup and do this experiment, not an easy experiment to do, but it will be fun to watch this happen.

The shock slowly becomes Mach wave that will be here. After that, that original Mach wave stays. There will be more and more Mach waves coming out as if the fan is opening out; you know this Japanese paper fan. It is opening out from one line; that is going to happen there in here. That is your expansion fan they call it. So, that is going to happen as you increase angle of attack. It is more fun to watch this. You should start feeling this kind of flows. If you can feel that, then I have done my job teaching you where is this flow. Now I want to pick up some other flow situation, more complex stuff.

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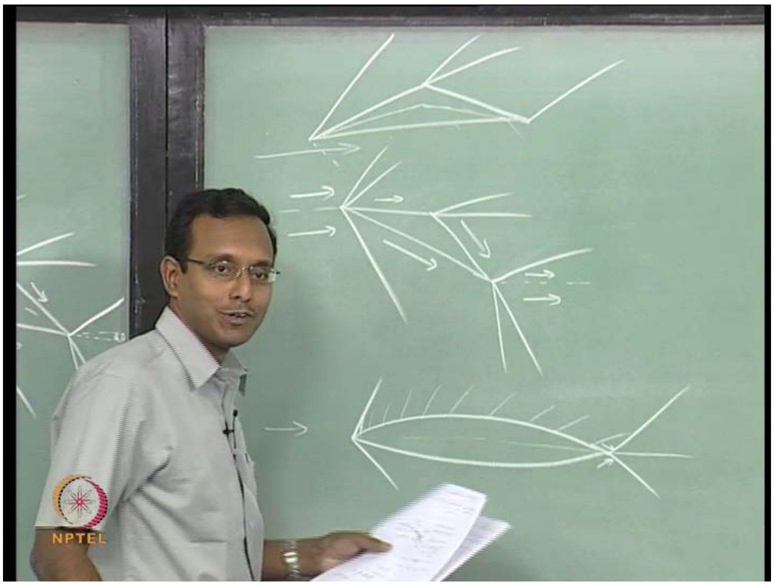

Let us take a case where it is half of the diamond airfoil. Will this produce lift, why? From aerodynamics point of view, it has a camber; from aerodynamics point of view this has a camber. Cambered airfoil will produce lift; that is if I think about the centerline at every cross-section the centerline is going to go like this. In fact, it is going to go like a straight line to straight line if I pick up some airfoil like this. It has to produce lift because it is a cambered airfoil. Will it? Let us see.

I am going to say the flow here just go straight. I am having zero angle of attack. It just go straight; if at all there is any wave it will be a Mach wave and that will be everywhere. Remember that Mach wave does not happen only at corners; it happens from every point in the flow, okay. Even from somewhere here there is the mach wave going this way and this way. We are ignoring these Mach waves. Mach waves are just the wave; one point talks to other point, right. We said that at the beginning of the course. I am going to have a shock here, an expansion here and a shock here; this is what I am supposed to have.

Will this produce lift? Difficult question to ask. It so happens that the pressure here will be higher than the pressure here, pressure here will be lower that the pressure here. What will be the net effect? I have to actually sit and calculate. It so happens that this will produce lift, because this pressure will be much less than this jump. This decrease will be much more than this increase. So, it is going to eventually have an upward force; that is not the only thing this will have. This will have a huge torque downward. It will have a negative torque on; it will have a moment on it airfoil, because this pressure is high, this is some middle, this is low. So, this force will be like, this force will be like this.

Overall it is going to have a torque at about around this center. It is not a good airfoil to use in any aircraft, but this kind of flow field you have to get used to solving. If I have any angle of attack, how will you solve this? If I have something like this same problem as before I will just draw a dotted line there, and I am going to consider this flow as it comes straight and turns in. So, there should be a shock here, and then flow goes here. We will wait for that point. We know that it is going to have almost zero angle of attack 0 angle at leaving point. So, it should be an expansion here, and the flow goes like this.

In here I have drawn it such that it has gone below. So, there should be an expansion fan here, flow goes like this. Another expansion here, flow goes like this. Finally, there should be a shock, because the flow has to come here, turn and go this way again. The whole set of things happening here. I have drawn it such that this shock angle is 90 degrees; that is very very wrong, 90 degrees with respect to streamline; that will never happen. I am assuming attached shock still. So, it is going to be less than 90 degrees with respect to incoming.

So, this is more like a flow field. So, you should be comfortable in drawing such flow fields around different bodies; that is the idea. I will draw one more which is more common airfoil shape if you think about high speed bodies. If you do not want to have sharp corners on the top you will have it smooth; how about like this? But it will still have sharp corner in the front. So, this is your airfoil shape. Let us assume that it is symmetric currently; if it is not symmetric it is more complex problem to solve. What is going to happen in the front? I am still assuming that the Mach number is high enough that I am going to have attached shock.

We will go for detached shock case, say, in two, three minutes or probably five minutes and there is going to be a shock here, oblique shock at the front. Immediately after that I am going to have smooth expansion corner, right. It is continuously smooth expansion wall all the way till the bottom, other end other tail. So, it is going to send expansion waves from here everywhere continuously, and if you notice I am drawing very carefully making sure that the slope of this line is going that way. Slope of this line is tilting that way; expansion fan should be drawn that safely.

Now at the end what will happen? The same kind of flow will come from this side. They will both interact with each other to go straight line at the bottom along the centre which means I have to have shocks at these points. This should be the case if I think about what the flow field looks like. This is all again shock expansion theory can give you pressure data all around the airfoil. This is why high speed flows; aerodynamics people use shock expansion theory a lot. This is application of gas dynamics directly there, okay. So, if I have any high speed flow around a body I can immediately use shock expansion theory to give pressure around the body always, which means whatever we just did, right, lift coefficient, drag coefficient.

I can give you moments on the airfoil; I can give you all kinds of data like that. I can tell you what are all the features I will see. Now there is only one small thing which I have did not tell you till now which is what happens when this expansion goes and touches that shock, alright. This expansion from here from the smooth corner, that wave will go and touch that shock. When it touches what happens? We will leave that out for now; we will come back to it probably at the end of today's class or next class, and same thing here.

This expansion will go and collide with this. What will happen at this point? Expansion and then shock coming from bottom and hitting. What will happen there? You have to think about these cases. It is probably easier to use piston analogy and explain that; let us not use it right now. We will come back to it after sometime, okay. But this kind of flow field drawings you should be able to draw pretty fast, one more complicated case.

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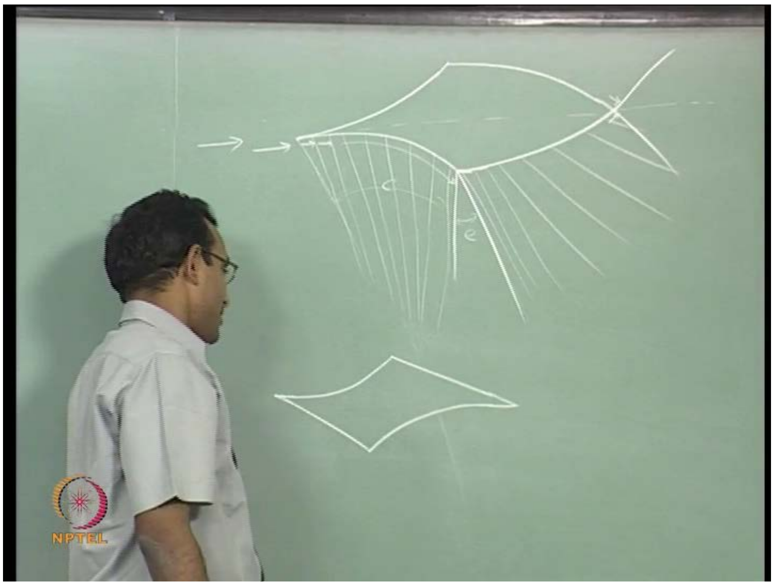

This is something I typically I give in my interviews. If I am never asked to ask question in gas dynamics I will ask such airfoil shapes which are probably not practical, but let us say I created an airfoil like this. I am going to say it is going to have a cusp like very thin wall at the front; flow is coming exactly parallel to this. And let us assume currently that it is symmetric, it need not be. I can have all kinds of variations here; let us pick such a case. I want to see what the flow field looks like. Again I am going to use shock expansion theory here, the craziest airfoil. Since I am saying it is symmetric, and I do not have top space in the board I will look at only the bottom. It is a smooth corner transition, smooth compression.

So, I am going to have a set of Prandtl-Meyer waves which are coming out from here, and I already told you that these waves since they are all compression waves they will all get together eventually to form a shock here, but let us not worry about that right now. It is not coming from shock expansion theory. So, what is going to happen is the flow is coming here, and it is slowly turning the air along the wall all the way, and it is going to come to this point. And it is coming out at some high angle this way. Immediately I am telling it is going to 0 angle there. What is that? That is an expansion fan, okay.

And we know that the last expansion wave there is a last compression wave there is also Mach wave which means my expansion starts immediately there, and this whole region will be my expansion fan. This whole region is my expansion fan; this whole region is my compression; this whole section is my compression fan; this is my expansion fan. Immediately after that I am again having expansion surface. So, it is going to go this way, and the first wave for that expansion will be the last of this one. This is the shared one. Unless I tell you that there is a straight portion it starts from this point. I currently did not tell you there is a straight portion.

So, it starts from there and immediately starts going out like this. I am giving you as complicated case as possible. After this since the flow on the top is also very symmetric the flow will come like this and meet at the corner. This is same as the previous ending like the wedge problem. It will go parallel to it finally, and you will find that there will be shocks on both sides. We have not figured out what the shock will do when it interacts with this expansion here; we have not done that part yet. You cannot do something like this.

Now of course, I can give you more complicated cases something like any shape like this, whatever airfoil shape. I can draw anything; in the interview that is what I do. I just go there and start drawing whatever I feel like; something like this should be very easy to ask a question, okay. So, you can start solving for such a flow let us say. I would not worry about it; you can solve it. Let us say that this angle is parallel to incoming flow; this is parallel to incoming flow and you just solve this problem. Now we will go for some other new case.

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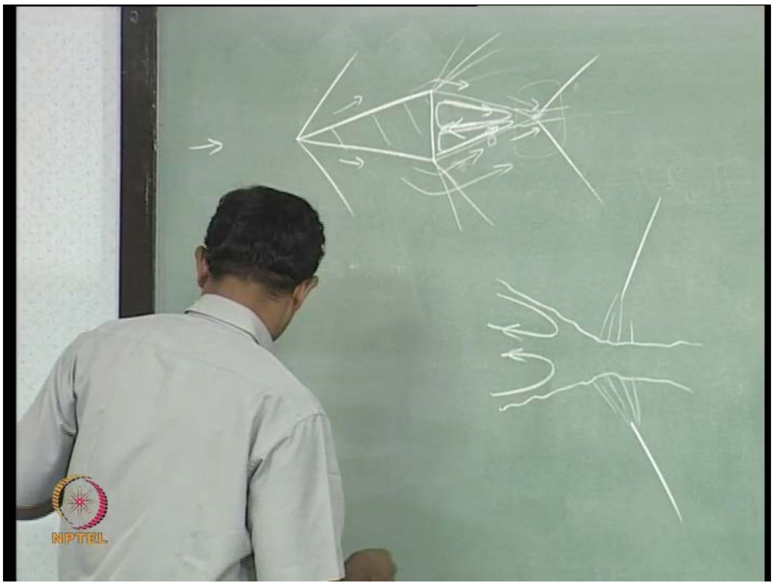

This new concept I have to introduce here. I am having a wedge which is terminated like this; this is my wedge. I am having a new case, and I am again having Mach number high enough that I would not have detached shock. I am going to have this very easy to think about; flow is going like this. Now what will happen at that corner? If the flow starts turning like this, what will happen? Then I will have an expansion fan like this, but the same thing happens from the bottom. This flow will also turns like this. Now what will happen? These two flows are coming head on against each other. They are going straight against each other.

It is coming vertically down; this is going vertically up. Typically such a flow is not possible, okay. If you try to turn this flow to any angle, it is not possible to turn at this line center line. So, what the fluid does is in the transient process this all thing will happen. They will go collide head-on, okay. We will probably go to this problem if I get permission to use some book on video directly, but as of now I am going to say that the flow will come hit head on and then figure out what to do.

What really happens there is initially they will come and hit, and then they realize that there is no space. So, there will be extra shock wave coming here which will come and neutralize this expansion wave, and so the expansion wave decreases in strength. What does that mean? It cannot turn as much. If it does not turn as much all this I am talking about is all unsteady word. All this is happening for the very first time in the flow start over this body. Now I will draw the steady state picture. The expansion wave has become weak which means it is going to turn at some angle. If it turns like this, the fluid inside will just not do anything. This flow will go like this and the same thing on the top.

I am saying it is a symmetric problem. So, it is going to turn like this, and they will both go and meet somewhere here as if there is the body that is shaped like a diamond wedge, and then there will be a shock here. What is going to happen inside? If it is a viscous fluid, then this fluid element will pull the next fluid element along because of shear, and so this flow will go like; this flow will go like this, and there cannot be empty space here. So, flow in the center should go like this to fill that empty space which is created by that pulled out.

So, your whole flow field will look like this. There will be a recirculation zone, double recirculation zone sitting inside that. This is a kind of flow field you will have at the end of this, okay. Now of course, after this the flow is just going to go straight, because I am assuming symmetric here; in this case they are symmetric shocks, everything is symmetric. If I have the viscosity this problem is not as simple.

There is going to be a wake created, not just recirculation zone, but there will be a wake, which means a shock will not be a clean shock. But the shock I will expand this small region alone. This is your shear layer, and it will just form a wake like this. So, you are going to have some unsteady smooth compression. So, you are going to have a bunch of waves together compression waves coming together to form that shock note. This is what will happen there, but this final angle will be same as what you have drawn with your simple case. The final oblique shock angle will be the same. This is the kind of flow field you will get, and here is your recirculation zone; this is what you will get.

This for all the bodies; any body that has this kind of sharp termination sudden termination, every bodies that has this sudden termination will have flow separating behind it. This is not because I have reached my m equal to infinity here; that is not the case here. Even at m equal to infinity it will leave the body, but that is not the case here. I think I have to give you one more and then I will go for shock interaction and expansion interaction. I will just give you this because it is useful for people who are working in compressible flows.

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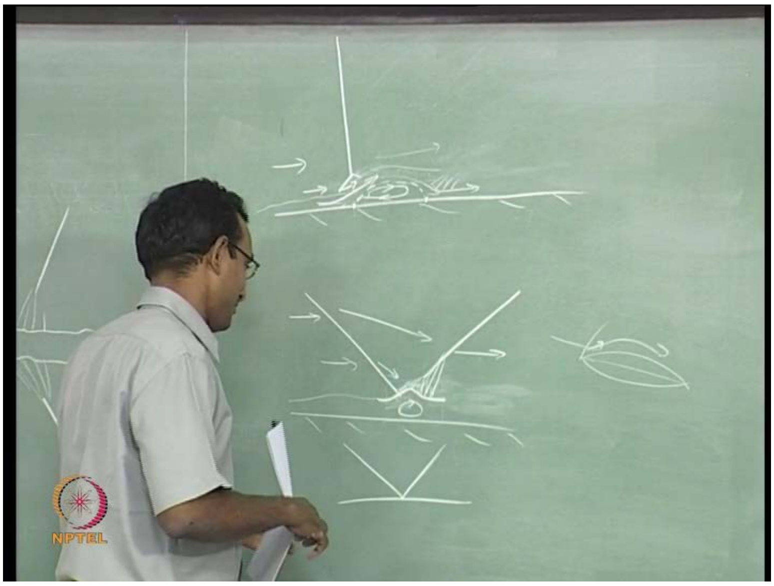

That is I am having a wall, and let us say there is boundary layer on it. And I am saying there is normal shock suppose to stand there for whatever reason. I do not know the flow field; for whatever reason there is a normal shock supposed to stand there. And ideally it should be this in an inviscid flow where boundary layer; there is no subsonic region. So, shock was all the way to the end. But if it is a viscous flow that will be region where there cannot be a shock because it is a subsonic flow inside. When the shock is coming and sitting here you have to know more about boundary layers; I am just giving to you some information from boundary layer course.

When there is pressure increasing along the flow direction, boundary layer will suddenly separate from the wall, and the flow inside will just want to go up away from the wall; that will start happening. I am just giving you a very special flow field. And when this happens we will just assume that this happens. This is called boundary layer tripping or separation of boundary layer from the wall. We will figure out what happens after that, but when this happens, the flow that is supposed to go straight will now go and crash into the next flow in the stream line, right. This stream line is coming up, and this stream line is going straight. So, there is the problem there.

What will happen? There should be an oblique shock to adjust this flow to go like this. So, this normal shock is no more a normal shock. I am talking again unsteady problem. I am giving you how a normal shock boundary layer interaction happens. Now it is going to go somewhere like this. There is normal shock from the centerline, and this shock is going to be ahead of this. If it is ahead of this, what happens? The flow comes like this and it turns here itself, which means now the boundary layer tripping is going to happen earlier. Previously it was at the line where the normal shock touches the wall, now it is going to be earlier.

So, it is going to go to a point where the flow will start going like this, but after sometime the flow is going to be very low velocity, slow speed flow. And it so happens that the boundary layer will reattach after sometime, and then it will again go like this. It is low velocity; it is decreasing velocity, it is just going to go like this. So, now what happens is there will be a recirculation zone sitting here. There will be what they call a separation bubble formed here; that dotted line is the one stream line which is separating the fluid that is circulating and the fluid that is moving. That is your separate stagnation point here and here, separation stagnation points.

That is too much of aerodynamics probably for you right now, but what happens at this corner is there will be a small shock let here; we would not worry about why it has formed right now, alright. And after that the flow field is again supersonic flow or subsonic flow depending on the conditions here. If it is supersonic flow then there will be expansions. If there is the expansion and then again the flow has to turn back there will be compression waves which will again form a shock. This is the case not for a normal shock problem but an oblique shock problem.

For a normal shock problem there will be no shocks form downstream, but if it is transonic zone if the Mach number is close to one, then you will still see the expansions and compressions if you go look at some particular density map or something. You will see that the density decreases and increase again; all that you will see. I will pick another case where it is oblique shock reflection on the wall. If I consider something like this, and I am going to have a boundary layer. I am drawing like this, but I am going to change this picture immediately after this. What it is going to look like is when this shock comes and interacts to this; boundary layer is going to do this.

The streamline in the boundary layer will suddenly jump up. I will erase this local region, going to come here and suddenly this stream line will jump up and then go back down. When something like this happens, I have drawn too big; I will draw it smaller, and it will look more like this. If something like this happens, then this flow that is coming here is going to see this. If it is far away what is it going to do? It is going to go like this, it is going to go like this, and it is going to go parallel. This we have seen already when we were thinking about two shock reflecting across, shock reflecting across a wall.

Now when it is near the wall when it is viscous flow and there is some boundary layer tripping happening, then you will see that the flow comes like this, and then suddenly it is going to see a bump like this. So, there is going to be a set of compression waves; they are all going to mix together, and there will be expansion waves after sometime, because it is a smooth curve similar to the airfoil we had before. It is an angle of attack let us say; this airfoil at an angle of attack. Flow is going to come here, and then it is going to see all kinds of thing after this; it is going to expand after that.

Similar thing will start happening around this bubble, and at this bottom corner it is going to turn again the flow field, because the recirculation bubble is like this. Flow is going to turn back to straight; there will be compression waves here. All these waves interacting together will finally go and form that final shock which we have drawn already; it will finally form that wave, because some of these compression waves will be negative by the expansion waves immediately after. But after that the remaining things will just interact to form a nice clean shock.

If I zoom into any boundary layer very close to the wall these kind of thing will happen; if I zoom out and go far away and see what happens, then I will just see a wall shock just bounces off the wall depending on the scale of the problem. This whole thing may be happening inside this small zone here, depending on the scale of the problem I have to think about it. Typically in supersonic intakes this is a serious problem, because now this recirculation can cost Eddy shedding and unsteady flows. This will start going increase and decrease in size and that happens. This shock is going to oscillate up and down, because the size of this body is changing.

So, this is going to respond to it unsteadily and that happens. They are going to have unsteadiness in your flow field itself; that is going to cause a lot of trouble. This is one reason for having intake buzz the way they call it in supersonic combustion ramjets; one reason for it, not all the reason. This is just one of the reasons possible. But there is also a part of gas dynamics. I have just introduced it, but it is viscous gas dynamics; you should remember that. It is not unsteady gas; it is not inviscid gas analysis.

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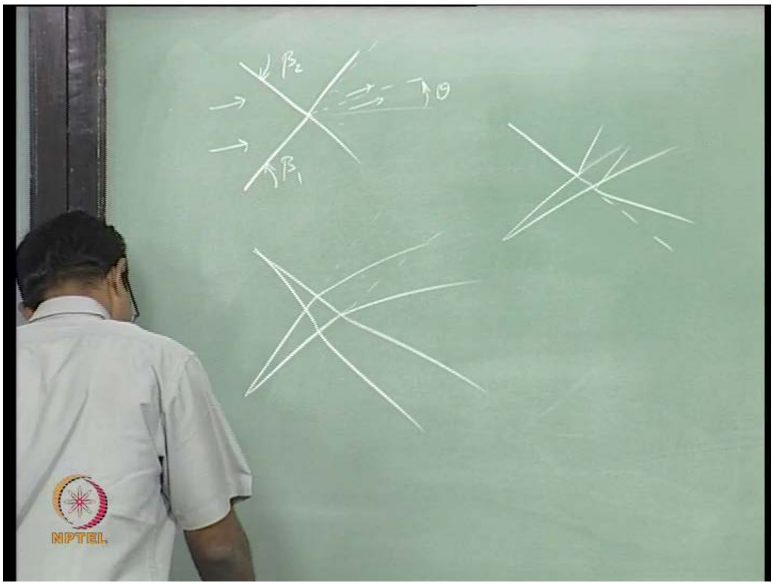

Before you come to next class probably you can think about these problems; I will just stop after this. I am having two shocks. I am having some Mach number; let us say this is beta 1, this is beta 2. I want to find let us say this is some theta, the final stream line direction and the shock angles after they reflect. How you will solve this problem? I have given you enough already to solve this problem. What are the conditions you need to use? Same thing, pressures must be same and vector directions must be same. That is both will go parallel that is theta; both should satisfy that theta. That is the same thing.

You can solve this problem for a given condition let say you picked numbers for these; you can solve this problem. Similarly, I can give you an expansion fan, another expansion fan; they can interact and they will do this. We will talk about this next class, but they can do this. That is another way of thinking about it. If I draw this expansion fan straight this is just the dotted line it has gone more downstream; we can give reasons for that later. Where if I draw the dotted line here it has gone more upstream which I have already given you reasons in some class before; similar reasoning will work for this.

These are easy stuff. Now I can think about expansion interacting with the shock, more complicated case. Now we will find that this will go up like this while the shock will go downstream like this; that is what will happen. And all kinds of interaction like this can be handled after this. Once you know this, now I can go to any jet problem and start solving the jet problem. We will go there slowly; after this we will go pick up variable area of flow; that is flow through nozzles. We have done external flows currently, flow around bodies you have done today, shock expansion theory. Next we will go pick up internal flows, flow through ducts with one d assumption. So, it is it is just going to be flow with varying areas; that is what we will solve next, okay. So, we will meet next class.