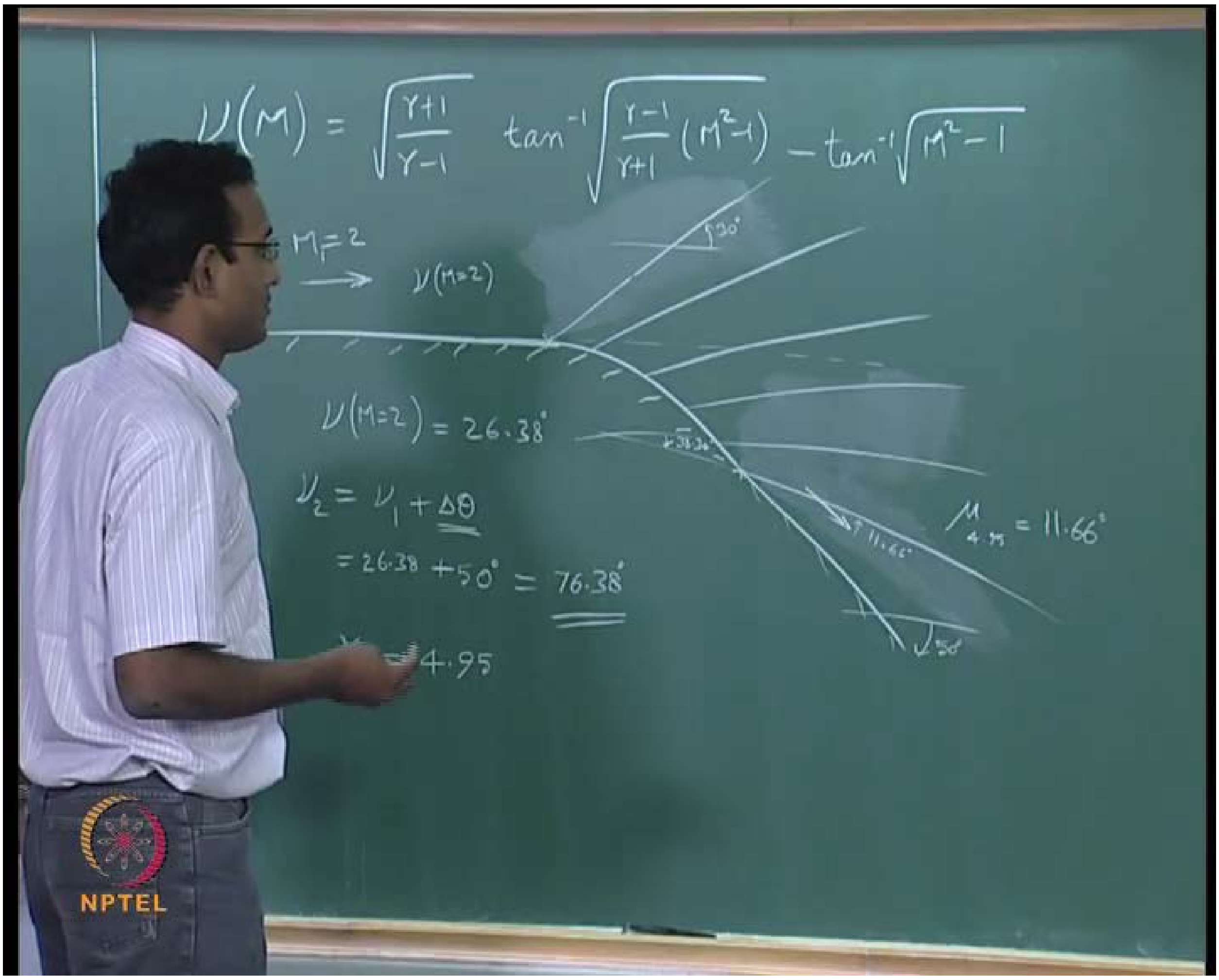
Gas Dynamics
Prof. T. M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module No - 12
Lecture No - 25

Prandtl-Meyer Function, Numerical Examples, Shock-Expansion Theory.

(Refer Slide Time: 00:22)



Hello everyone, welcome back, last class we stopped with the functional form of prandtl meyer function. I just write it once more, this was the function we had last time and we said that typically we work with angle and degrees. So, this tan inverse should be converted to degrees units that is all it is nothing right. Now, we will look at this in functional form on the screen there.

(Refer Slide Time: 01:06)

What I have plotted here is mach number on the x axis all the way up to 100, and on the y axis it is angle and degrees starting from M equal to 1 course you should know that below M equal to 1 this function is not valid. It is not suppose to have any function value and you know that it will go complex numbers anyways.

So, from M equal to 1 onwards I have plotted for three different gamma values, what we are saying here is specifically. Let us say we will pick say gamma equal to 1.4 the green one. We are seeing that it starts here they are all almost the same for very small mach numbers up to two or something there is not much variation on the angle after that, when I go they split apart when you go to higher and higher mach numbers.

They split apart reasonably right, but if I look at only gamma equal to 1.4 they tend to become one particular value beyond some point. It tends to become an asymptote and we gave the asymptotic value last time we get just to be 130.5 for gamma equal to 1.4 and similarly, we can find such angles for the other cases also, and I believe I gave that also the last class anyways. So, now, I want to see the effect of gamma in here as usual we always want to discuss compressibility.

We said that gamma changing the gamma is one way of changing the compressibility effects that is not the only way to change compressibility. It is just one of the ways of changing compressibility and I am going to say that gamma equal to 1.67 is least compressible, and as I go more and more it is becoming more and more compressible that is, what I am having here when I have gamma equal to 1.64 to 1.4 my gamma decreases compressibility increases, what we are seeing is the prandtl meyer function drops the functional value drops that is the change in the function need not be. So, high to change the mach number mach number still changes.

Say, I want to go from mach number say 5 to 10, if I have to do that in gamma equal to 1.3, I have to go from 60 degrees to something of the order of 73 degrees or. So, rough estimate I am just picking up numbers from the plot the same thing, if I have to do from 1.67 and say I am going from 5 to 10, it has to start somewhere around say 95 degrees all the way up to 125 degrees rough estimate. What we are seeing is I have to turn the flow a lot for me to accelerate the flow, if I have less compressible gas gamma equal to be very high value, when I have a easily compressible gas, when I turn it when I change the flow direction that is I am sending an expansion fan, when I send it I need to turn it by a small angle to increase mach number very high, that is the special effect of compressibility input, this is what we have to get out of these plots.

So, now, we will keep these in mind we will typically have only problems of gamma equal to 1.4 in our simple gas dynamics world and with that we will get some numerical problems and work on them today. Lets go to the board again, if I pick a case of we did this a little bit last time, but I will just give some numerical examples. Let us say I have a smooth corner smooth expansion corner, and I am going to say I have mach number equal to M 1 equal to 2, and I am going to call this situation M 2 and.

I am given this angle to be 50 degrees, the net turn I am giving for this flow is 50 degrees. Let us say, if I give this and I am telling you that this is air gamma equal to 1.4, we want to find what is the final mach number. I am going to achieve in this we gave a formula wise. Now, we just have to work through with the same formula, which we discussed list time the way to think about this is I have to find the nu of M equal to 2 here, which is what I will call as nu 1, I have to find nu 1 from this function above, but instead of this function we will go and use tables.

That is not easy function to work with we just use tables nu of M equal to 2 comes out to be 26.38 degrees for gamma equal to 1.4 of course, now you know you have seen the plots, you have to look at for a specific gamma values for this gamma equal to 1.4 this is the value we are getting now, with this you remember the formula nu 2 equal to nu 1 plus delta theta, where theta is the angle expansion, how much is there delta theta is the net expansion angle, if it is compressing then it will have a minus sign automatically.

It will get a negative delta now, I will just put numbers here 26.38 plus the net change in stream line direction is 50 degrees. So, I will add 50 degrees here. So, my final prandtl meyer angle happens to be 76.38 degrees. Now, I have I know that this flow will reach a mach number, such that it will have 76.38 degrees as the prandtl meyer function. So, what I have to do is go back to that plot and read out 76.38.

What is the mach number and instead of doing that from the plot, I am doing it from my tables in compressible flow tables book, and I am getting that mach number to be 4.95 what has essentially happened is my mach two flow, when it expands over this corner has gone to a point where it is having mach number of 4.95 here, that is what has happened. I am assuming that after this is going just straight. It is not changing anywhere now, this being said I have to inculcate the habit of drawing expansion fan the correct way.

So, let us say till here the line is all straight and this is the first point of change and let, us say this is the last point of change and since it is a smooth corner. There is expansion fan will be spread over this whole region this is, what we discussed last time now, I have to say that the very first wave will be at an angle with respect to that local stream line direction, such that this will be equal to mu locally why the expansion fan each individual wave travels at speed of sound locally. So, with respect to that local velocity vector, it is going to have that mu value remember that it is always local velocity vector local mach number. So, for M equal to 2 this whole region is uniform flow up to this first line.

So, M equal to 2 mu angle will be sin inverse 1 by 2, which is 30 degrees. So, it should be drawn such that it is exactly 30 degrees. Since, it is not very nice I will draw it again we will make it look like it is close to correct. So, it is going to more like this for this particular problem. It should actually be probably slightly lesser, but anyways this is your 30 degrees and it is going to keep on going there.

What should be the final mach final not mach number final angle of my expansion wave, if I look at it every expansion wave is like a sound wave. So, for that local mach number, which happens to be 4.95. I have to find the mu for that particular mach number and find the local velocity direction, which will be parallel to the wall here.

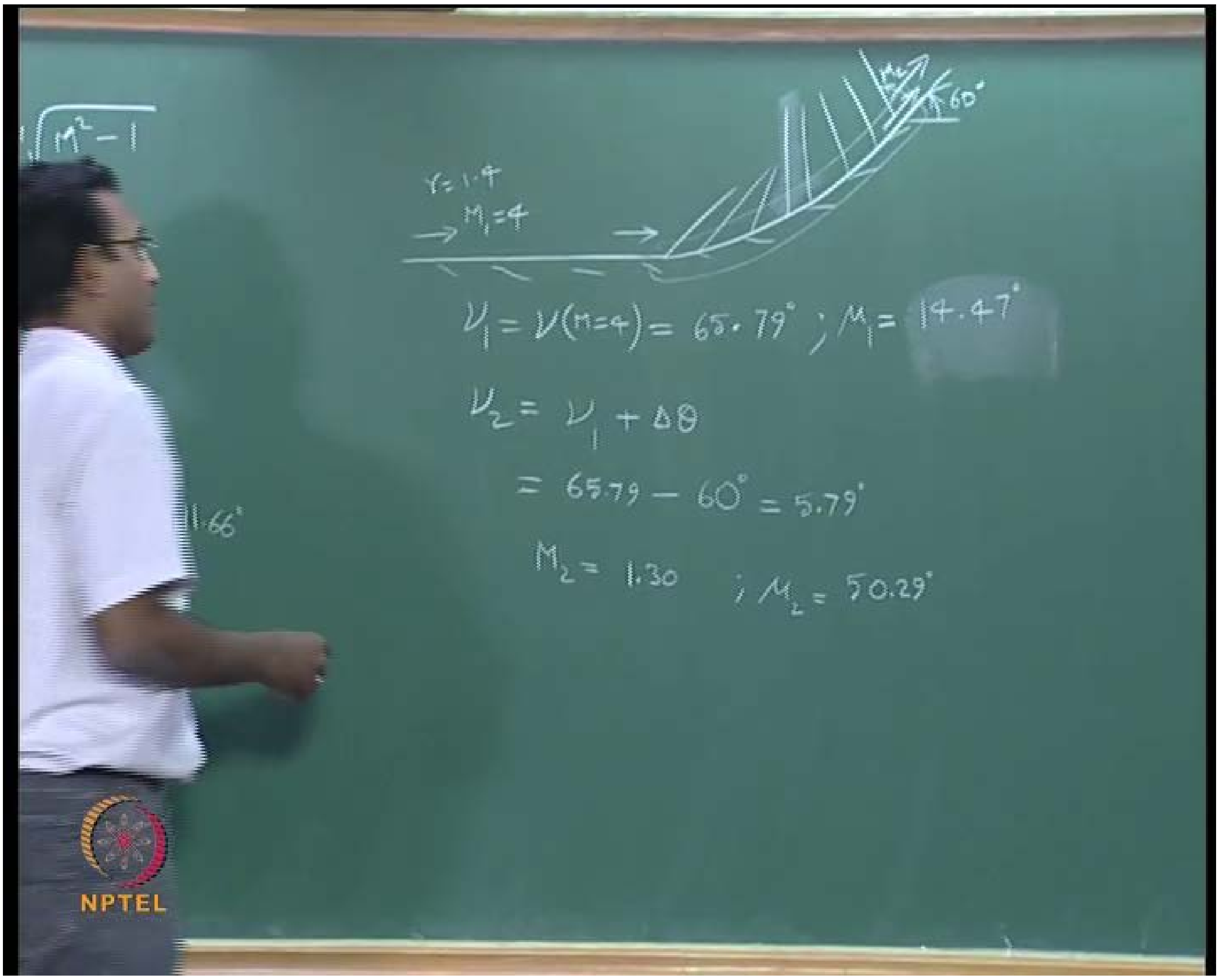
So, I will draw this local direction of velocity vector and I have to find the local mach angle for that mach number, which happens to be mu of 4.95, happens to be 11.66 degrees. I will use this number this is sin inverse 1 by 4.95 that happens to be this. So, with respect to this flow velocity direction. I am going to have 11 degrees from here, which is a very small angle like this. So, from the last point of change it is going to have an angle like this, now I will erase whatever is unnecessary.

So, that it is not confusing I draw this again and this actually, I will remove this also now you know it is 50 degrees, if you want we will mark it here this is 50 degrees with respect to horizontal line that is this wall angle. Now, my initial expansion wave line goes like this final one goes like this this is the correct way to draw it actually crosses the horizontal and goes below also for this particular case it does what matters is with respect to the local velocity vector direction in there, I am going to have that 11.66 degrees this is the last wave.

So, overall this final angle with respect to my horizontal will be what 50 minus 11.66 right what is that 40 30 930 8.34 38.3 degrees that is what will be the angle below horizontal that should be the final angle here think about it that way this. Now, let us say I want to draw intermediate expansion fans a few of them there are. So, many ways I said its infinite number of expansion waves here in, which case I can draw few more when I draw others. I have to draw it, such that there is a slope change in a linear fashion to some extent.

I am assuming that this change is linear it need not be, but to some extent this angle and this angle must be around this middle angle or the middle one should be having an angle in between these two angles. It cannot be changing just think about it that way every angle should be like that it is a smooth change in the slope of this expansion this is what you have to think about this is one particular case of smooth expansion lets go for smooth compression ok.

(Refer Slide Time: 12:34)



We will keep this just go for a smooth compression case, where I am going to have a wall and it is going to turn to a 60 degree angle, 60 degree angle and its turning into the flow, what does that mean it is going to block the flow it is going to decrease the mach number.

So, it is a compression wave sitting there, and I am going to say the incoming mach number M 1 equal to 4 gamma equal to 1.4. We have not currently worried about finding the pressure and temperature, if needed we can do it we will do it in some other example for lines how will I work on this problem exactly the same way nu 1 will be nu at M equal to 4 this value, you get from tables that value happens to be 65.79 degrees nu 2 equal to nu 1 plus delta theta, but by our convention delta theta is the amount of expansion in this case it is compressing.

So, I have to give a negative value for this 65.79 minus that 60 degrees we said, which happens to be 5.79 degrees this is what I am going to get 5.79 degrees. Now, if I go back and look at my table such that nu of that particular M 2 let us call this as my M 2 nu of that particular M 2 happens to be 5.79 that M 2 value will be 1.3 mach number. So, essentially if there is.

A smooth compression of my flow from mach 4 up to 60 degrees, then I will end up with the flow 1.3 mach number this is, what I am saying again I have to go back and think about how to draw these compression waves it is weak compression waves. So, I can still think about it as isentropic and. So, I can say that the angle at, which it is going to go with respect to the flow will be mach angle again, if I say it is mach angle. Now, I have to find the nu value for this, I will put mu 1 for M equal to 4 is sin inverse 1 by 4 that is 75.21 degrees, and for this case mu 2 happens to be 50.29 degrees.

I think there is a problem somewhere I made a mistake somewhere, I think it cannot be that you guys have a calculator you can check whether this 75.21 is right. I think there is some mistake there anyways I know that, when mach number increases the mu angle should decrease. I am seeing that there is a mistake there this value sin inverse 1 by 4 is 14.47 degrees. We will use that number and tell me the same for 1.3, what is this value this is correct good now, I am more comfortable with this.

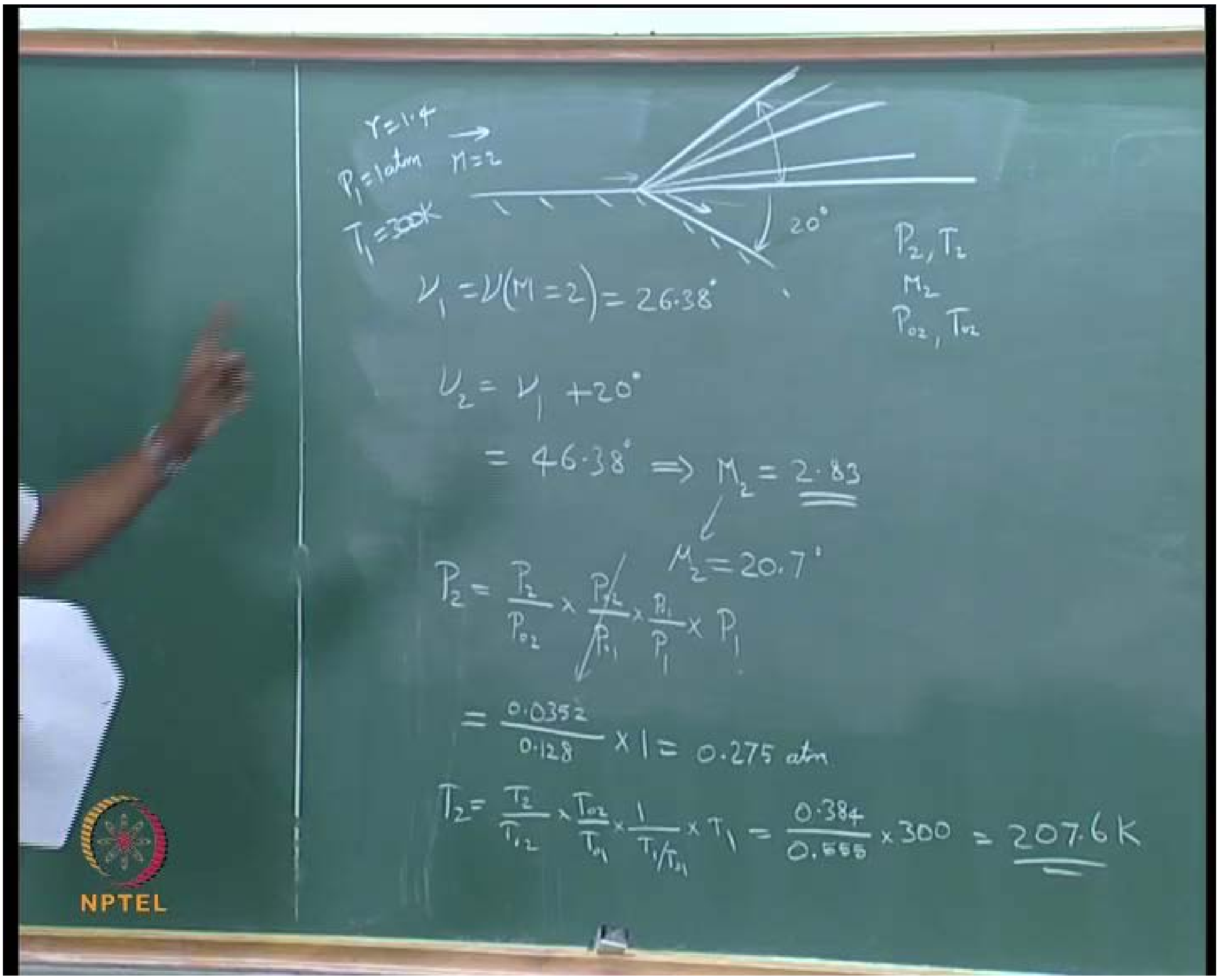
So, what I am seeing is locally with respect to this flow vector mach 4 flow has mach angle 14.47, which is a very small angle subtending an angle very small with respect to this, I am actually drawing it slightly big. So, that it is easy to visualize, and then as it goes mach number decreases, which means my mu angle should increase. So, when I am drawing my local velocity vector like this with respect to that my mu would have increased. I am drawing too high probably here and I do not want to draw it this high, because the final angle is 60 degrees plus 50 degrees.

So, it is going to go somewhere like this and with respect to this final angle. I am going to say my mu is 50 degrees with respect to that, which means with respect to this, I have to draw a 50 degree thing there, which means with respect to horizontal it has crossed 90, and it is coming inward going to go like this this will be your situation.

If I think about a case it is more like this it is going to look something like this. We would not worry about how they interact right. Now, this is a very complicated case, when such a thing happens there is probably a normal shock sitting in front of it. Typically, we do not see compression was running up stream we will currently keep it like this, we are interested in.

Studying what is happening only very close to the wall this region alone. Nowhere, else just this region were whatever theory. I have said will be applicable, because it is all very small change at any time very small changes close to isentropic. So, this will work when I go more out these compression waves will come together, and they will form a strong compression wave, which will become a shock eventually when that point happens it is non isentropic. I cannot use any of these relations things will go wrong there. So, this will be the case here hopefully. You will get comfortable with drawing these mach angle that is the idea now we will go for another example, which is all sharp corners.

(Refer Slide Time: 18:53)



This is most common example in expansion fan in gas dynamics. I am having a sharp corner change, and it is 20 degrees and the incoming mach number happens to be 2 it is here.

So, gamma equal to 1.4 P 1 equal to 1 atmosphere t 1 equal to 300 kelvin. We have this whole thing now, we are told to find what will be the final pressure temperature mach number standard things every property that is needed P naught 2 T naught 2 everything we want to find all the properties.

So, how will I do this it is exactly same procedure as what we did just before this board, which was find the local prandtl meyer angle nu at M equal to 2 this is my nu 1, which from tables will be 26.38 degrees and nu 2 equal to nu 1 this is expansion angle. So, it is plus 20 degrees. So, the final value is 46.38 degrees this, if I go and use my prandtl meyer function tables from compressible data books. I will get M 2 equal to 2.83 this is the first phase.

The next thing I want to do is draw that shape their expansion fan shape exact shape, how will I do that what is the first angle going to be 30 degrees sin n was 1 by M 1 sin n was 1 by 2. So, with respect to my flow vector going to be a 30 degree line that is my first it is really not 30.

Assume, it is close to 30 this is a first line now, the next one for this I have to find mu 2.83 that comes out to be 20.7 degrees. So, the final stream line angle is going to be parallel to this wall, which is what is the use of this expansion fan and. So, now, with respect to that I have to draw a 20 degree line it. So, happens that this change itself is 20 degrees. So, I know what my 20 degree looks like 20.7 will be slightly above this 20 degrees.

So, my final expansion line happens to be only straight above horizontal, since I am exaggerating it looks like too big it should not be. It should be almost that horizontal line may be, I will draw one more line very close. So, this is my set of expansion fans this whole region is my expansion fan. This is the way to draw for this particular picture that is the easy part now, you have to go and find the pressures and temperatures, how will I do that very simple expansion is all isentropic

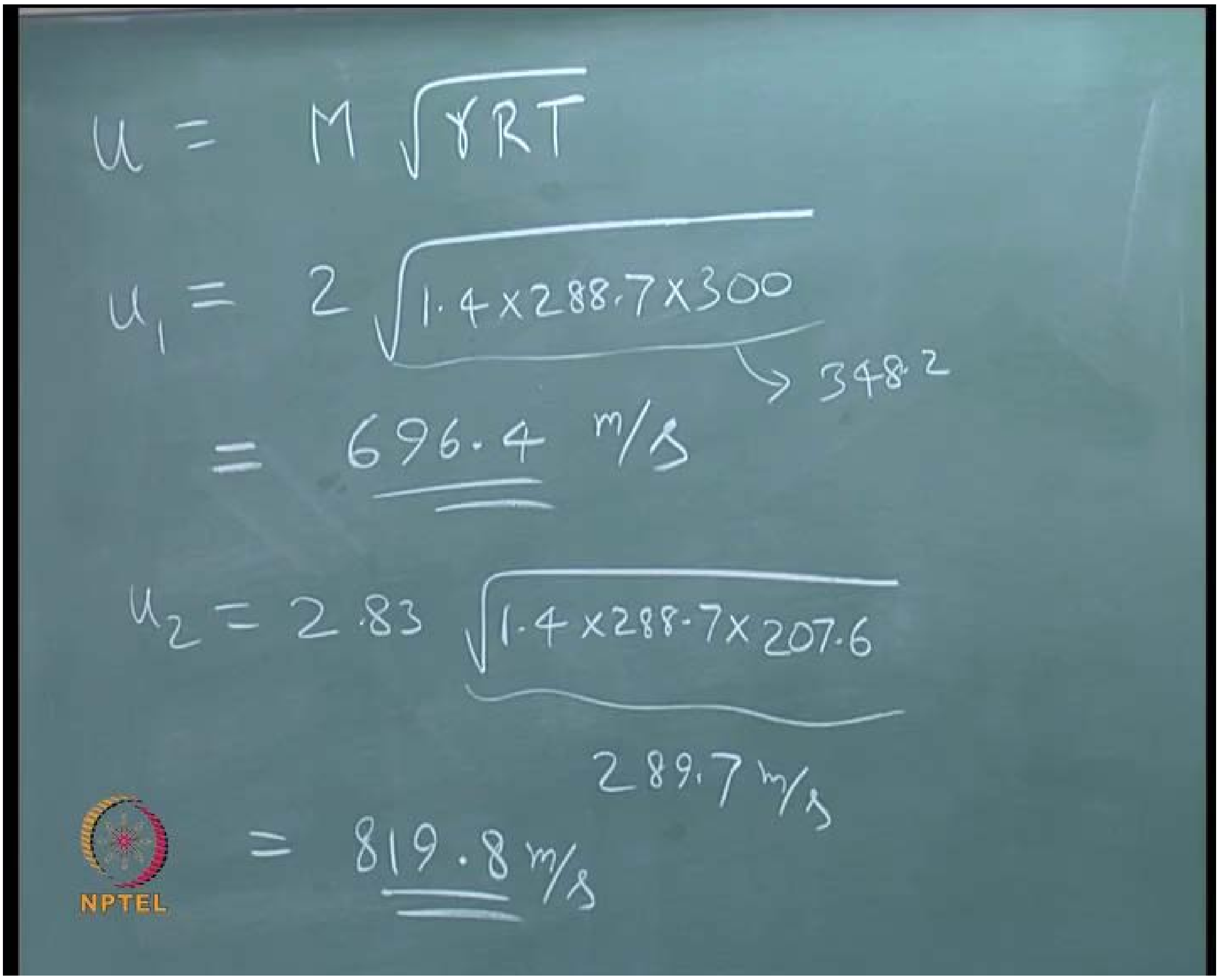
So, I know my p naught and t naught remains the same. So, I have to find P 2 equal to this is the gas dynamics way of writing this term. So, get use to this P naught 2 by P naught 1 into P naught 1 by P 1 into P 1 it is the nice way of writing pressure equal to something pressure multiplied by a set of ratios. Now, we know that there is 1 atmosphere remaining numbers we have to find, since it is an isentropic expansion I can say that p naught does not change we did this long back.

It is also T naught does not change anyway here I will make this 1 P naught does not change. So, this P naught 2 is equal to P naught 1 reaming things is just numbers. I have to find from isentropic flow tables for M 2 P by P naught is given in my tables, and I used that that is 0.0352 divided by my tables has P by P naught.

So, I will do divide it by the other value for M 1 that is 0.128 multiply it by pressure P 1 for us it is one atmosphere. So, I will just multiply by one final answer is 0.275 atmospheres this is what I have now the same thing. I have to do for temperature of course, I can of course, just say to you that T 2 instead of all P's replace it with T's that is all you need to do really. Let us say I will just go and do that T 2 will be I will just do it once more T naught 2 into T naught 2 by T naught 1 into 1 by T 1 by T naught 1 into T 1 this is a nice way of writing it for me and.

I am going to find this to be 0.384 divided by 0.555 multiplied by 300 kelvin and that answer happens to be 207.6 kelvin that is what I have there. Now, the remaining thing is finding T naught 2 and P naught 2 that is what is left not very difficult. It should be same as t naught 1 and P naught 1, we have numbers for all the ratios. I have T 2 by T naught 2 or P 2 by P 2 by P naught 2 or the same thing for case one anyone of them I have to use.

(Refer Slide Time: 25:01)



Let us say I use P 1. So, P naught 1 is equal to P 1 divided by P 1 by P 01 1 divided by that number was 0.128, that was to be 7.81 atmospheres and similar thing T naught 1 T 1 divided by that ratio is 0.555.

So, I have to multiply this with 300. So, 300 by 0.555 that number is 540.5 kelvin and of course, P naught 2 is equal to P naught 1 T naught 2 equal to T naught 1. So, now, I have solved this full problem of expansion any other property. I want I can find what are the properties important in this problem anything else that, we have not talked about till now only one more happens to be velocity. So, I will just find the velocity also, why do I have to talk about this just in case, if somebody has a doubt that mach number increases.

I cannot just say velocity increases, because temperature decreases. I have u equal to M into square root of gamma RT. We know that T decreases M increasing what happens to velocity it can go the other way, but we will just check it has to accelerate, then only you can call that the flow is accelerating, when it is expanding we did this long time back, when we were discussing stagnation points and stagnation enthalpy there.

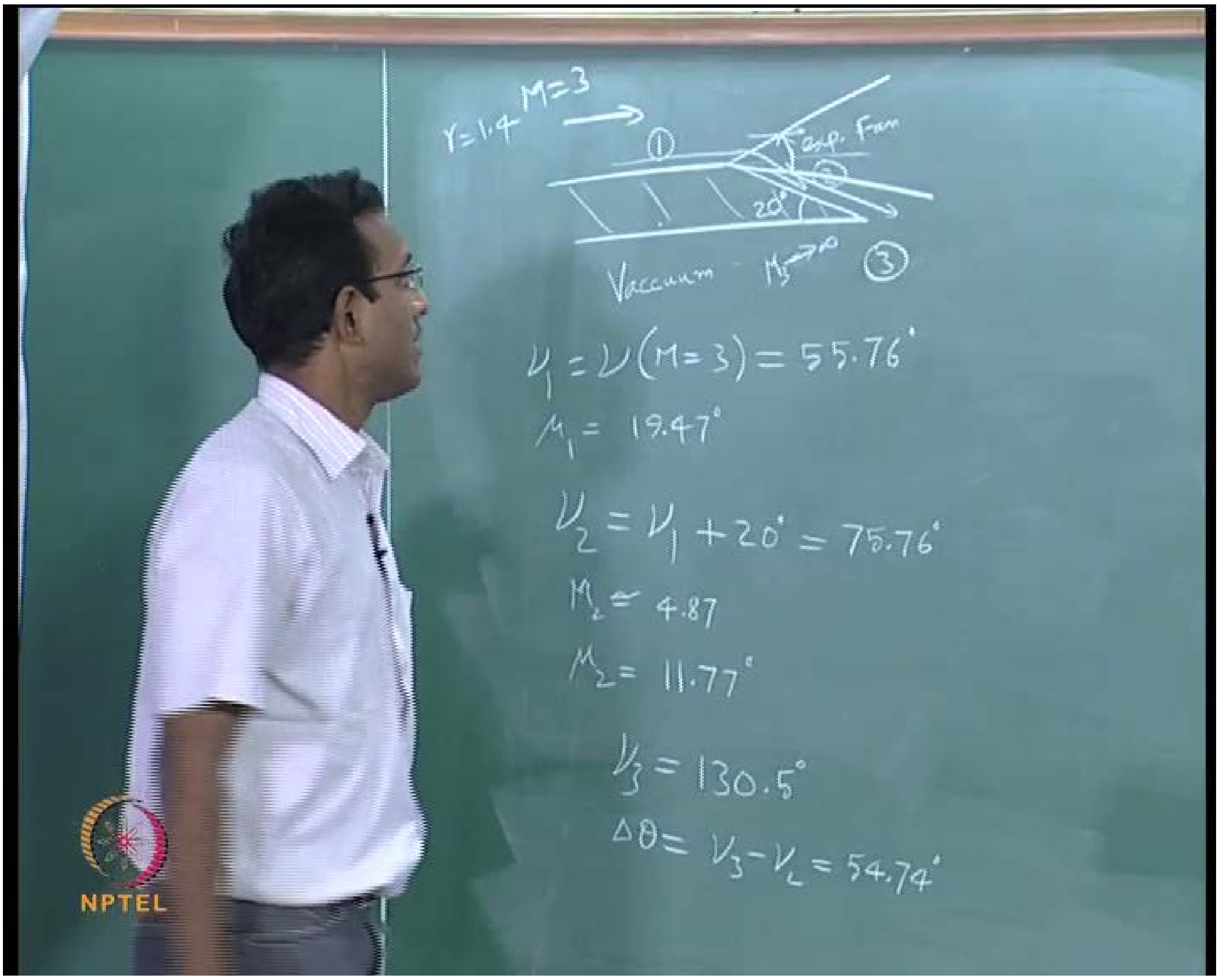
We said if my temperature drops my velocity increases that should be satisfied even here. So, we will just check it, u 1 I want to calculate. So, I will put M 1 was 2 for our problem into 1.4 into 288.7 into temperature was 300 kelvin and this answer happens to be 696.4 meter per second this exact number we have been using a lot 696.4, because we typically it started mach two problem in supersonic.

So, we will always get that u 2 we found that the mach number was 2.83 into square root of 1.4 288.72 into temperature we found was 207.6. So, if I find this number it is 289.7 meter per second this multiplied by 2.83 is my final answer, which is 819 .8 meter per second. If I go look at this number here, if I just see the speed of sound here, what will be the speed of sound I will divide this by 2 it will be a 348.2. So, my speed of sound has gone down yes, because my temperature is lesser it is colder.

So, it has gone down, but still a mach number effect is higher. So, we are finding that the velocity is increasing. Even though, I am getting colder my velocity is still increasing eventually we will go to a point, where if I keep on increasing if I want to increase velocity I have to change my mach number by a huge amount, because temperatures will become close to 0, then you have to change your mach number by a huge amount there will be a point where you cannot change anymore.

What is that point called the v max? right We did this v max, when we introduced T naught T naught equal to T plus u square by 2 c p we said and, if I put T equal to 0 you will get T naught equal to v max square by 2 c p that v max what you will reach eventually it will go a steady state. We have seen plots of this also right. So, this is one problem which we solved like this. I will give you another case a little more complex version of this and that will be the last numerical example, I think I will solve in my expansion alone world.

(Refer Slide Time: 29:45)



Let us, say somehow I have a flow that is like this and here mach number is 3 gamma equal to 1.4 of course, I could have done problems with gamma equal to 1.67 or 1.3. It is just a change in one number it does not change much. So, much for us now I am telling you that this angle is 20 degrees, 20 degrees is the tramp angle, and I am telling that this is parallel to this.

So, it is a straight section and outside is vacuum that is the setting we have I want to find what is the flow field here the absolute flow field let us say this is my nozzle exist from my rocket and it is sitting in space currently the flow coming out is M equal to 1 M equal to 3 at gamma equal to 1.4 now, I want to see what is the flow field here that is the idea, if I think about it that way what should I do I will go just faster this time mu of M equal to 3 I will mark this region 1, this region as 2 and somewhere outside is 3 we mark this something like this.

So, I will just go nu 1 is nu at M equal to 3 happens to be 55.76 degrees and mu 1 happens to be 19.47 degrees. Now, nu 2 must be nu 1 plus 20 degrees, because it is expanding its plus 20 degrees which is 75.76 degrees. So, from here I will get a M 2, which happens to be approximately equal to 4.87 I do not have clean resolution at this high mach numbers in my tables.

So, I just gave roughly 4.87 I did not interpolate very well, and this will give you mu 2 value of 11.77 degrees. Once, I know this I should be able to draw the expansion fan there. So, I am having 19 degrees roughly 20 degrees is the angle. We know that this angle we said as 20 degrees.

So, it should be roughly that that is the first expansion line with respect to the incoming vector it is 19.5 degrees above the last one going to be 11 degrees from this wall, which means it should be roughly midpoint is 10 degrees. So, it should be slightly above that. So, its somewhere here this whole region is my expansion fan that whole region is expansion fan. So, the flow comes here turns and then goes out straight like this that is what is happening this is the first case of course, now you can go and find p 2 t 2 every other property you want u 2 whatever right.

Now, we will go out of that I just I believe you can do remaining part, now the next thing is we have not given any angle change, but you have given pressure outside. We said in gas dynamics, there are two types of boundary condition velocity direction and pressure. We have been using velocity direction till now, suddenly we are giving pressure boundary condition, if I gave pressure boundary condition here what should it be I have to just go and find what is the pressure here it. So, happens that p is 0, if it is 0 what should be my mach number keeping same P naught isentropic should be M tends to infinity right.

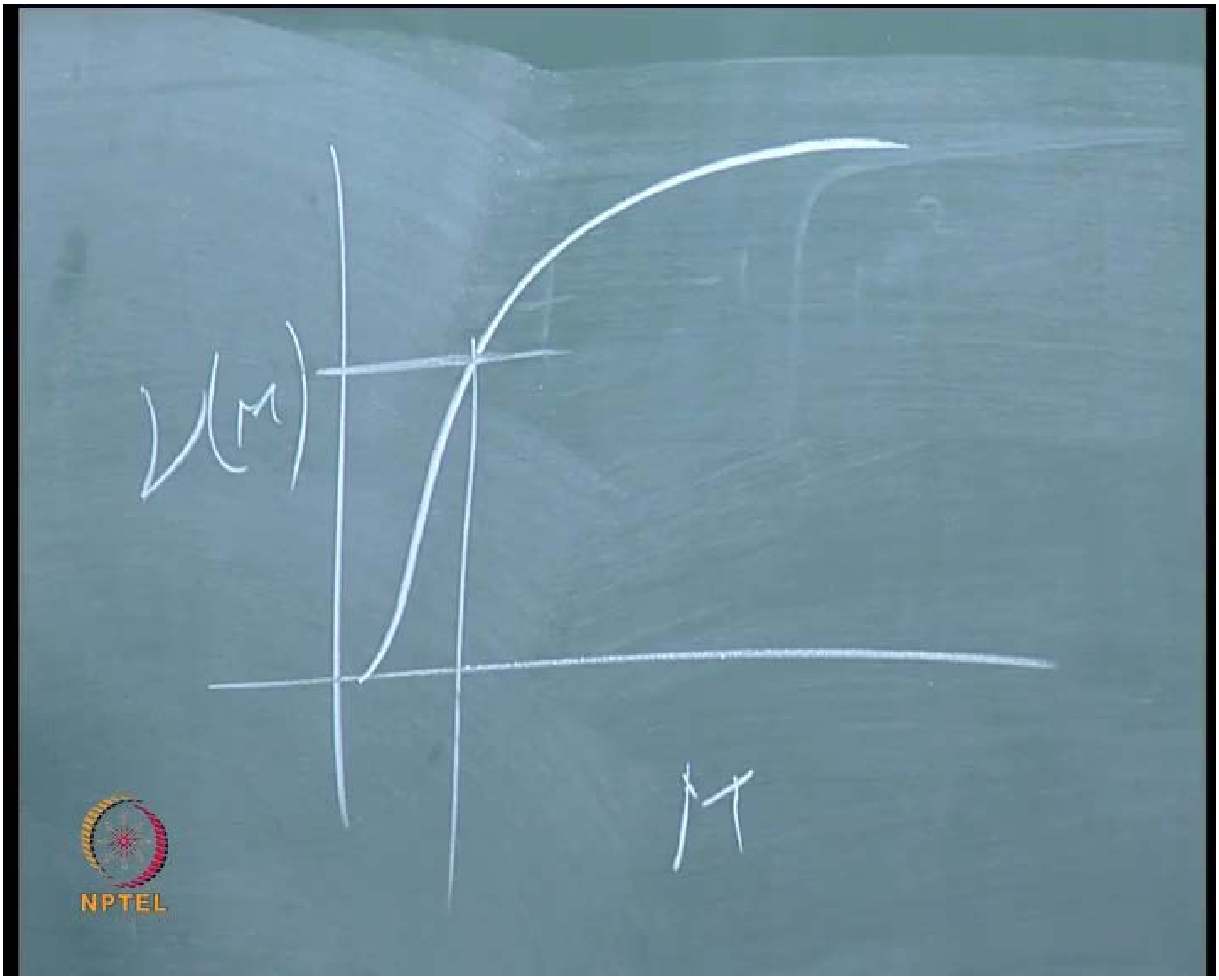
So, I know that my final mach number M 3 actually, I will just put tends to infinity not equal to it tends to infinity, which means now I know my M 3, I know my M 2, I just have to draw the expansion fan problem is different. So, I will find my nu 3 what is nu 3 for gamma equal to 1.4 this number. We discussed last class that number happens to be 130.5 degrees and nu 2 we calculated just now 75.76. So, my delta theta the change in angle of the stream line comes out to be nu 3 minus nu 2 happens to be 54.74 degrees, which means the flow is already turned 20 degrees from horizontal, it can turn only 54 degrees more.

So, I will erase these things for clarity it can turn only 54 degrees more from here, this is my 90 degrees its 60 degrees. So, it can turn only 54 degrees. So, the final velocity vector direction is this. It cannot go anymore turning around the body it can turn only till here and this change is 54.74 degrees that is this change. Now, I just have to draw the expansion fan angle my first wave here, what will be the angle for that my first wave of the expansion fan what should be the angle for that the same thing here 11.77, but we already have an expansion there. I do not need to go and look at it anymore, I know that the mach number here is all the same velocity vector direction is all the same.

So, I just have to draw a parallel line to that that is my first expansion wave this and this are parallel right the same condition and the last expansion wave. What is the mu value for that sin inverse 1 by infinity sin inverse 0 is 0. So, you do not have a mu angle 0 with respect to the flow velocity vector direction my last expansion wave is going almost on it. So, it comes to a point where it is not physical for us. We already told you that M tends to infinity is not a real situation by then my continuum assumption breaks down.

So, I cannot be using this analysis here by here by here pressure is 0 here and non 0 here that situation will never arise and at this particular situation, where pressure is close to 0 I cannot apply continuum any more it is no more continuum there is no molecule it cannot be continuum anymore. So, it is just a hypothetical analysis, but one thing it tells is, if I have a case like this the flow will just go straight like this it will never try and turn around the body that is the very important thing. You should know if, I start with high mach number it cannot be it turned too much already, because it is already turned a lot ok.

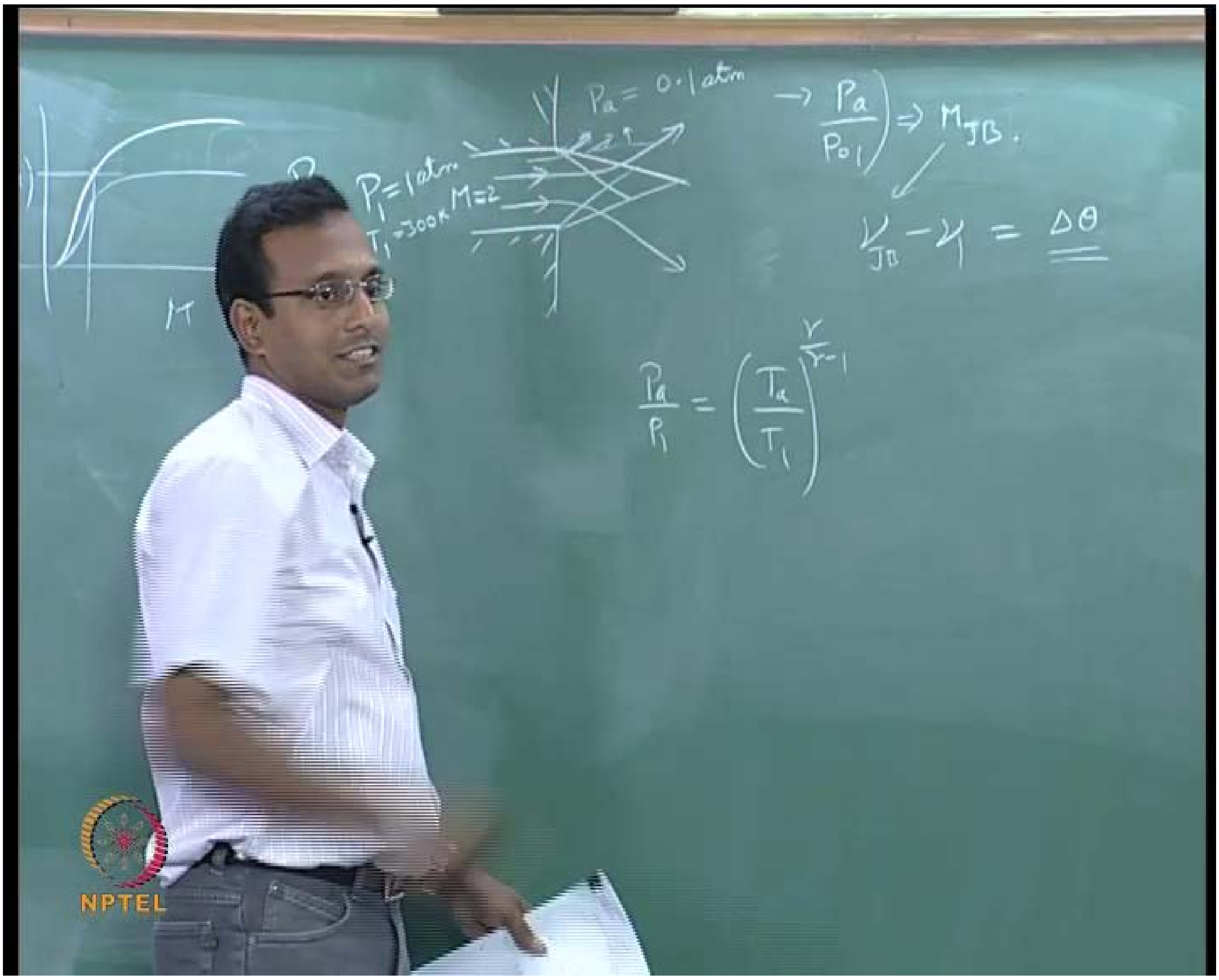
(Refer Slide Time: 37:10)



That is the way you have to think of and you should of course, remember the plot, which we just saw nu of M versus M it looks something like this, which means if I start somewhere at reasonably high mach number say five after that I have I can turn only. So, much especially if it is a very compressible gas I can turn only lesser.

So, if it is a less if it is a more compressible gas. It will turn faster the curve will become flat very fast this kind of effect will happen, which means I want to turn my flow more it is probably not possible if it is very compressible, but still my mach number can go to infinity that kind of situation can arise. So, the next thing we can do is once you understand these it is just the simple extension of this ok.

(Refer Slide Time: 38:05)



I will consider a first flow problem, which is I have some jet coming out with a non mach number and velocity direction. Let, us say M equal to 2 and I have to tell you P naught value let us say I will give you P 1 equal to 1 atmosphere, if such a situation is coming out and let us say I do not have P exist is equal to let us say 0.1 atmosphere, if I have such a situation, and let us say I will give a T 1 also 300 kelvin.

If I have something like this, I will just go for how will I calculate this problem remaining thing is just going through the tables that much of use for us. We will just let us assume that you can do it. I will give you exercise later on the website. Now, we know that mach number is pressure is 1 atmosphere and it has to go decrease to 0.1 atmosphere, which means the flow has to expand more if it has to expand more, because this is the new boundary condition.

Then it has to expansion fans the very first fan will be with respect to this angle 30 degrees something like this this angle is 30 degrees that is the first fan angle last one will be decided by what is the final mach number? Let us say you can calculate all that, I will just draw some angle roughly currently let us assume this is correct for now, but how will I calculate this final mach number.

The flow is turning like this right you can imagine that flow is going like this and it is turning out, if I think about the same thing on the bottom it is going to go like this, and its going to turn down like this, this is what should happen let us start getting use to this, because we are going to draw more and more of this as the classes come up next week or the weak after next. We will draw more of this a lot more of this and how will I find it, I know the pressure here and I am going to assume this isentropic. So, I know P naught value is same as P naught 1. So, I will just go and use P e by P naught 1 this will give me my M at the exit M at the outside boundary.

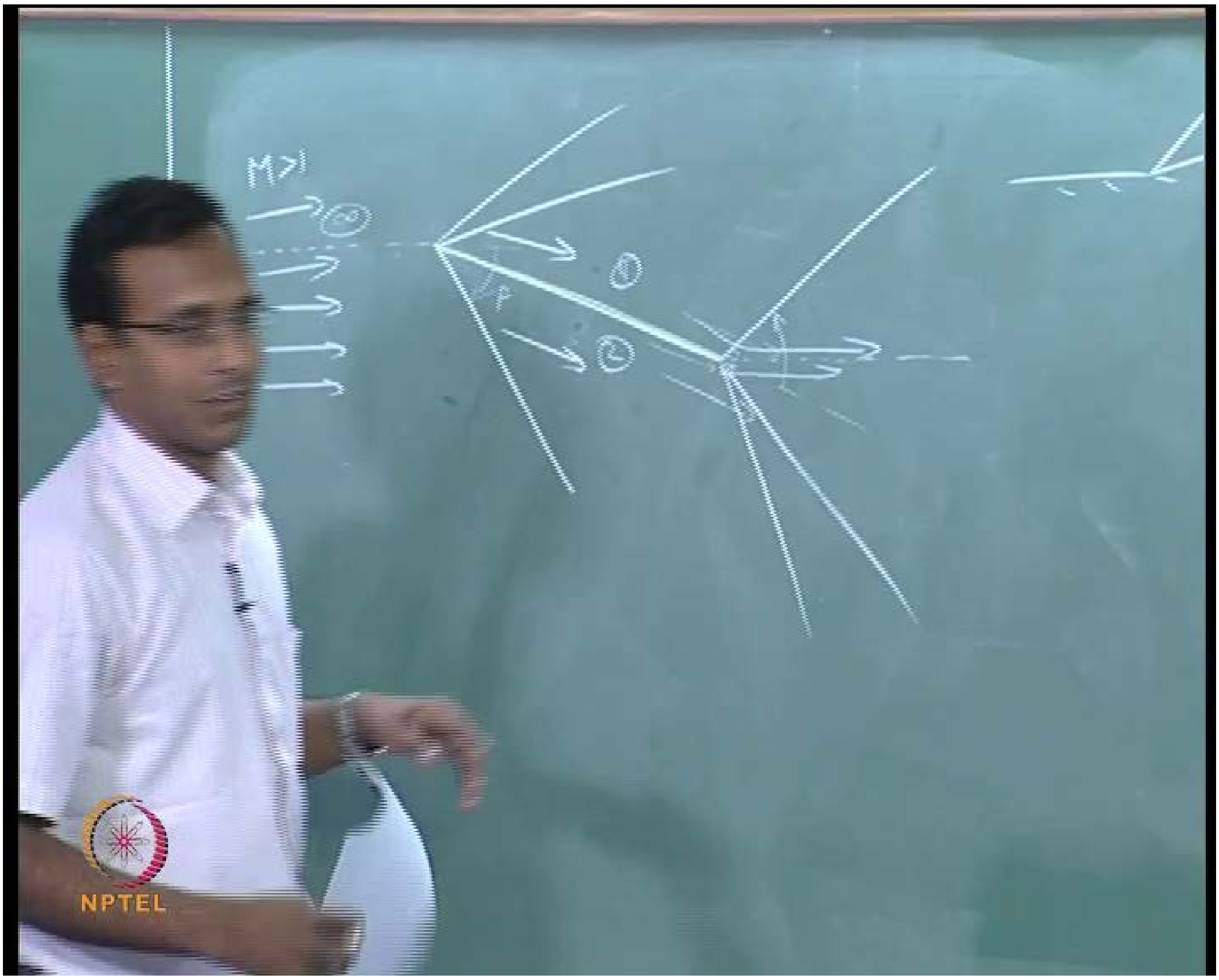
Let us call this ambient pressure I do not want to call it exist it will confuse you later, and jet boundary mach number at the jet boundary the final line will be the jet boundary to find the mach number. We will go and deal with jet separately later, I am just giving you this one example, I will find the mach number how will I find this mach number. I am having P a by P naught that from isentropic tables for gamma equal to 1.4, it is going to give me some mach number. Once, I know the mach number I know the delta nu right. So, I know how much it will turn by this will give me a nu at jet boundary. I know that nu 1 oh sorry nu minus nu 1 is my delta theta. So, I have to find the delta theta now, I will tell that at the end of it.

It has turned by so, much angle that is what it will come out to be it will turn by whatever angle it is I have not calculated this, but you can calculate it is not it is the same procedure. I have to go follow it again and again and I will go to a point, where I know this once I know mach number I can find temperature also even, otherwise you can find temperature how we said it is isentropic.

So, I know my P a by P 1 is equal to T a by T 1 to the power gamma by gamma minus 1, I can use this also to get my temperature at the exit at the ambient condition. That is also a possibility two ways of doing this, this is the way we will approach this problem why did I pick this problem separately all this time. We have been considering boundary conditions as velocity vector direction. This is one problem were pressure at the exit is given pressure is the boundary condition given for the flow direction two boundary conditions are typically used in gas dynamics world.

Once, this is done the next step is just go and use this expansion fan concept along with all the previous compression wave concepts. You have all that mix together in flow situation applications, that is the only thing left and the way to start is by picking the simplest problem even possible in supersonic flow.

(Refer Slide Time: 42:49)



A flat plate 2 d flat plate at an angle of attack, let us say this angle is some alpha as in aerospace terms and I am having uniform supersonic flow coming in, and I want to tell what will happen here? It is of course, considered to be inviscid they are still in inviscid world.

So, I am going to draw this one line that is going to touch this now, this is going to help us to understand the problem the way to approach this problem is to say I want to draw what is the wave nature at the front, if I look at this dotted line and above then I will just block this region with paper. It looks like this flow with a sudden expansion corner stream line can be equivalent to a wall in inviscid flow we are in inviscid gas dynamics. So, all I have to do is, it will have an expansion fan like this flow will come like this, and turn around like this that is the easy part now a little more difficult part. I will block the other side of the dotted line.

I am having only this dotted line and flow below, what is this saying there is a wall, which is coming into the flow this is like a compression and it is a sudden change. So, it is going to have an oblique shock, which is some few classes before I have to go find the angle. We hope that currently it is not a detached shock, it has a shock the case is a complicated case for this problem. We would not deal with it right now, let us say it is attached shock and I can find the beta value from there, this is my beta value. I can find it let us hope we can just go find the mach number and gamma, I can get a beta value.

So, I have drawn this let us just change the color a little bit for the plate. Let us say that green thing is my plate this is my shock this is my expansion. I have drawn this my flow field here is also parallel like this. Now the next thing to do I have to find what will happen on the back this flow field is going to go like this. This flow field is going to go like this, if there is no problem they will just keep going straight like this, but that is not going to be the case why, because the pressure here is going to be different from the pressure here. How do we know that this below this dotted line region is going to go through a oblique shock, which is a compression it is going to increase pressure here. Let us mach region let us call this infinity region this is called 1, this is called 2.

We can tell now that P 1 less than P infinity and P 2 is greater than P infinity. I can tell that for these two stream lines. Now, because of this I am going to have a case were this pressure is higher than this. When these two packets of stream lines packets of fluids come along the closest to the wall, when they touch what they are going to see immediately is there is a pressure jump, what will happen flow will want to go from high pressure to low pressure. Now, once that is said I will erase these things.

Once, that is said we can also tell that the stream line wants to turn that way, because that region is lower pressure the same reason. I can tell that the stream line has to go away from high pressure region for the low pressure stream line that will also turn, if I say this when will they stop turning it will go to a point, where the pressures will become equal consider a case where it is turning too much, what will happen then now that side will be finding that oh these guys are pushing too much without any force a better way of explaining this is after I put the waves here right, if I think about what will happen here. I will bock the top side I am having only the bottom side. I am having flow along this wall and it is now turning I do not know by how much let us just say it is turning, because it is turning away from the flow direction, which means it is an expansion corner.

So, it has gone to be an expansion wave here expansion fan here this region is my expansion fan. let us say if I look at the top section alone flow is coming like this and wall is moving in, why do I call this a wall it is actually the stream line center line will be as, if it is a wall in a inviscid flow stream liens are equivalent to a wall in inviscid flow. So, I can just say that the wall is moved in this is equality to our problem, which we did some days back this problem. So, it is going to have a shock in this region with respect to this line. It is going to have a beta value there will be a shock there now, because of this what is going to happen this shock will cause the pressure, which is low to become higher while this pressure, which is high will decrease now across this expansion fan to become lower.

So, ideally these two waves should be such that they will come to an optimum point, where they will reach the same pressure is that the only condition. There is one more condition needed I told you already in gas dynamics always. There are two conditions what are the two conditions one is the velocity vector direction, other is the pressure. We keep on repeating this all through we will see how it goes you have to make sure that these two vector directions are the same at the same time. These two pressures are the same two conditions two constants.

I have to get what are those what is the full delta theta here, which is same as delta nu I have to find this how much is the change and the second thing is this beta angle two variables two conditions. I can definitely solve this problem. Now, it takes a lot more effort to solve this problem we would not solve it now we will solve it after some time it needs iterative procedure and we do not have enough time anyway. So, this is a simplest problem I have done now. I will do one more and then we will switch on to some other case. we will go and solve more problems after that see you people next class .

.

.