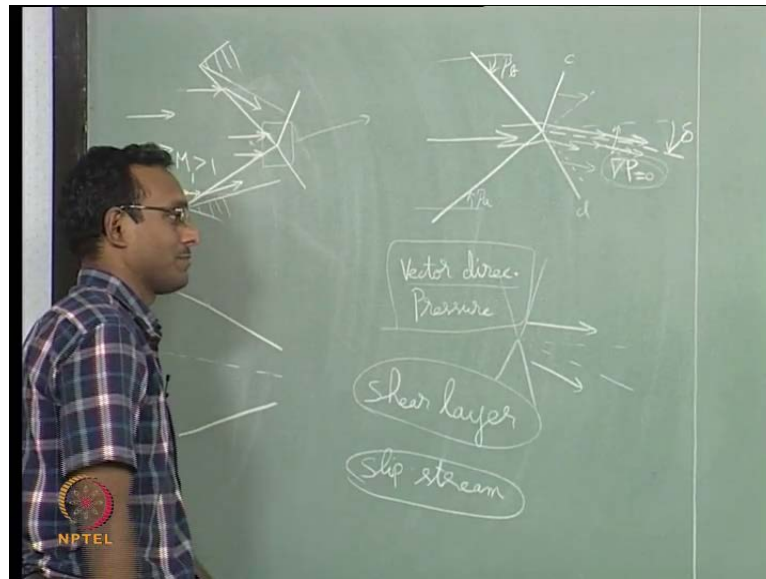


**Gas Dynamics.**  
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**Module - 11**  
**Lecture - 23**  
**Shock- Shock Interactions, 1 D Expansion Wave, Expansion Fans**

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Hello everyone welcome back last class we were looking at how to solve some problems with oblique shocks including cases where we had interaction in a wall I just realize that I forgot one more extra point I will start with that and then we will switch to expansions which will be interaction of one oblique shock with other oblique shock say in some flow field where flow is like this and mach number is supersonic I call it  $M_1$  is greater than one and the oblique shock is like this this is one oblique shock.

And I will have another oblique shock with another angle some other angle I will call this  $\beta_2$  one one will be confusing let us say I will use  $\alpha$  and this is  $\beta$  I have let us say I have two oblique shocks coming from say some model here say there is a wedge here and there is another wedge with the higher angle here let us say that is how this is created I am not worried about what is happening to the wedge later we are just interested in what happens to these two shocks when they meet typically when you go and read books, they will start talking about this one as a right running shock and this one as a left running shock to understand that you have to just be the fluid element. And just go along

this direction if I go along the fluid direction what will I see I am going to see that this shock is starting from if I am the fluid element standing here this fluid element is going to see and it is facing this way right it is facing the body here it is going to see that it is starting from the right and going towards the left this particular shock. So, it is called the left running shock while that one is the right running shock it will be it came from the idea of method of characteristics which we will introduce towards middle of the course,, but as of now we will just use the terminology it is useful for us now we want to know what will happen behind this for that I have to see what must be satisfied I will start I will remove these beta's.

So, that it is not confusing you know that it is beta a I am talking about that could be different from beta b the way I have drawn it beta b is higher than beta a which actually means that the deflection of the shock b is more than the deflection of shock a deflection by shock a this is what I am having if I draw the stream line close to the point of intersection this is going to go like this while this stream line this particular velocity vector is going to be turned like this which means these two flows are going to meet each other at that point now the way we think the we always think about this I am going to say that it is starting from supersonic it ends with supersonic we are still assuming it is all weak oblique shocks. So, it is mostly supersonic behind it. So, the supersonic flow coming here another supersonic flow coming from the top and they are colliding with each other and since the stream lines cannot not cross each other it. So, happens that there each of these stream lines are going to feel the other stream line as a body which means it cannot go just cross each other like this.

So, they will deflect in such a way that they will come to a new equilibrium they will come to a new equilibrium and I will draw it in a particular way we will explain it later oh I will correct it a little much something like this this is what will happen now let us zoom into this region and draw a separate picture here much more bigger picture and I am going to say that this is my beta a and this is my beta b and this is your right running shock and this is your left running shock now I will extend these shocks as if they are going straight without any interaction and then I will draw this lines with respect to that it will do something like this and we want to start with physical field right we always been doing that.

So, we will start with physical field what is the overall flow field for this flow here at this instant if I say this is extending forever if you just look at this duct which which the flow is seeing it is looking like this this is the duct it is seen this angle is less than this angle which means if I draw a centre line for this duct it is eventually going downward which means these two bodies together want the flow overall to go slightly downward. So, basically it is like we will go back to our piston analogy we always keep switching between piston analogy it is like I can imagine these two flows as if there is one piston coming from that side another piston coming from this side and they are both sending shocks as my piston cylinder is moving this way I do not have a flash animation to show this,, but it is going to do something like that if I do that then I am going to say this particular piston is moving faster than this one what is that mean.

The shock that is coming here is going to tell oh run fast this way this is going to tell run fast this way,, but this fast is more than this because this is moving much more sharply into the flow which means say I am the person standing in the middle and the fluid elements going straight along the center one side the shock is coming, and telling run with two hundred meter per second this way other shock is telling go hundred fifty meters per second that way what will I do essentially I will go with fifty meter per second this way finally.

So, that is the overall effect if I go back to this picture since we said this angle is more this is the stronger shock the deflection more downward is what will eventually happen and I am going to draw a dashed line here these dotted lines are just for extension of the shock to tell that the shock moves more upstream in both the cases we will come back to it after some time, but this is how it is going to look I have drawn this line this is the final stream line direction after this interaction after this interaction all the flow will have that particular stream line direction I am just making sure that I am telling only direction and not the vector magnitude vector magnitude will be different for different cases we will go and solve such a numerical example later when we also introduce expansions fans then it is more fun, I mean then we will say one is an expansion one is a shock what will happen all kinds of combinations.

We can talk about we will deal with that after some time currently I will pick this particular case and what should be the criterion for finding out how much is the net deflection it is having initially the flow was coming straight horizontal now finally, it has

deflected down after these two shocks have interacted what will be that final deflection let us say I will call it  $\delta$  what will decide this is the main thing I am looking for if I am finding out what will decide this if I say  $\beta_a$  is equal to  $\beta_b$  the whole problem is very symmetric then my  $\delta$  will be zero if this is stronger then it will push the flow more down.

So,  $\delta$  will be more negative if if this  $\beta_a$  is stronger than  $\beta_b$  that is higher value than  $\beta_b$  then the  $\delta$  will be more backside and it will be positive angle that is what will happen finally, now how will I put it in mathematical terms what does the flow try and satisfy here I am saying the flow direction is like this what does that mean the stream line here which is processed by the top two shocks are going in a particular way while the bottom shock are going in the different they are experiencing different shocks they are very different shocks.

And finally, they are coming out to be here one thing it has to satisfy will be that the stream lines are parallel that is when the when the flow will be satisfying when will say that the stream lines are parallel when I when there is no perpendicular variation I will write it as  $\text{gradient } p$  is zero across that line which is the special line because it is a boundary streams line that are processed by the top set of shocks versus the bottom set of shocks that particular line is the special line we will give the name for it later,, but as of now I am just going to say that the streams line are supposed to satisfy this particular condition  $\delta \text{ del } p$  equal to zero that the gradient across that is zero what will happen if there is any pressure let us say this pressure is higher than this then what will any fluid do it will immediately want to go from high pressure to low pressure that is it will tilt down which means the stream line will shift till it comes to a point where it is all equal if it is more pressure on the bottom line and stream line will tilt more up.

So, it is going to a point where this is the steady condition we are talking only steady gas dynamics we are not talking unsteady gas we will talk about that towards the end of the course. So, that is one condition then what should be the other condition actually velocities are parallel that is one condition and pressures must be equal this is the other condition these are the two conditions I will look at I just gave both are almost the same looks like finally, the way to look at they are difference is if I consider the case were the top fluid which is processed by these shocks is going some other way and the bottom to fluid the bottom fluid process to the shocks going this way then I will be in trouble now

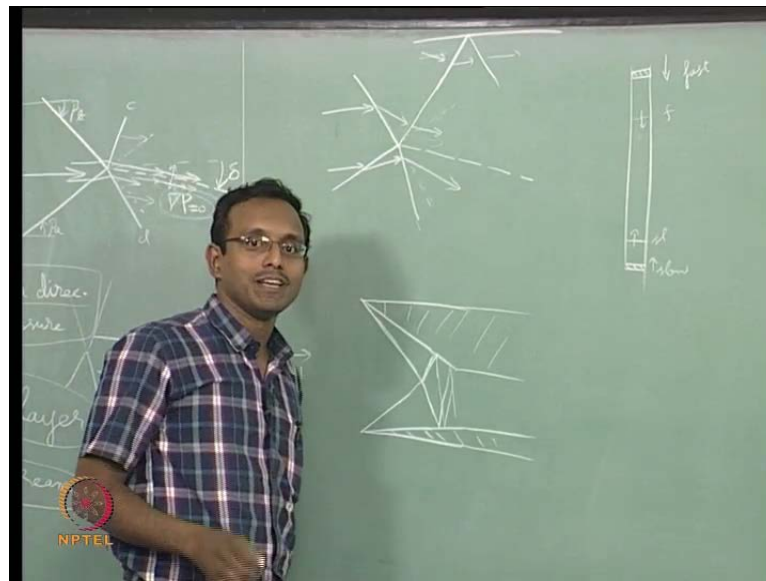
there will be a gap created here where there is no fluid passing that nature does not like ever if that is the case then fluid will immediately go and fill that region which will automatically adjust this shock everything will naturally take care of itself. So, I am going to say the conditions in this kind of problems will be pressures must match and velocity vector direction must therefore, that two things we will keep on coming back to these two conditions in typically in supersonic flows, we will always talk about vector direction vector direction and pressure these are very important in supersonic flow we will keep on coming to these two a lot these are predominantly applied as boundary conditions in your flow we do not care about magnitude will be the velocity I will make sure that the direction of itself magnitude will affect the energy in the flow,, but we are just talking about this currently. So, if I want to solve this problem to find what will be the angle of deflection this way for the new that are after deflections now I have to go and talk about satisfying these two conditions how many unknowns do I have in this problem I have beta c and beta d let us say this is my shock c and this is my shock d I have my beta c and beta d.

So, now I have to find those betas I have two unknown and I have these two condition. So, I will solve for try these two values and count to a point where it satisfy this two condition velocity directions must be equal pressures must be equal across this special line that is how we will solve it we will go and solve it later,, but I just want to give this introduce this phenomena here itself. So, that when you go there it will sink a little more inside your brain that is the idea here now I will give a one more information about this particular line which is dashed line there the velocities across here and the top and bottom these two need not be equal velocity values need not be equal in our inviscid gas dynamics world that is we are assuming there is no viscosity if that is the case then the velocity magnitudes can be different.

We still have to obey this vector direction must be same,, but the magnitudes can be different we will keep that currently if the magnitudes are different what is that special layer that particular layer were above is moving faster below is moving slower if that is the case then that particular fluid is called the shear layer because there is a sphere across it there are that particular has been sheared, but initially since we did all inviscid gas dynamics they gave a name which was slip stream that is a stream line across which the fluid elements are slipping that fluid element let us say is going fast and this is going

slow and this stream line are going slipping on each other on that boundary that is why it is called the strip plane, but if I include viscosity then immediately that changes the name to sphere layer because there it will grow another sphere like the jet flow there will be a sphere layer along the or in the boundary layer there will be a boundary layer which is the sphere layer itself it will become a sphere layer once I put in viscosity you should get used to this name slip stream also ok.

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So, if I draw flow without all these arrow I will go and draw it again this is what it will be looking like the flow comes here it turns down and then it turns a little bit this shock is not very strong comes here turns up and then it turns down very sharp ok So, depending on how much the turning should be this shock string will change,, but I will say you the quick way to remember this I can easily tell that if I extend the shock on this dotted line here and the dotted line there I know that the reflected shocks will be ahead more upstream of that dotted line a simple heuristic argument or hand waving argument will be that I will take this piston analogy. And I will say when this shock is coming through that piston down straight like this I will draw the picture once again I have this column that is one piston it is another piston this is going up slowly it is coming down fast if that is the case then there will be a shock coming this will be fast and this will be going up slowly this is what will happen now till these two shocks touch each other they come to the same point the gas that is going to go through will be that stationary gas in this world which will be one initial temperature, but let us consider this particular

solution just after it crosses that shock it will still keep going it it sees more fluid it will keep on going and telling all the way till end now that is going crossing that shock and going passed it is going to see the new gas which is heated by the shock.

So, it is going to travel faster in the perpendicular direction right it is going faster that way while this one is also going to travel faster this way,, but not as far as that change now imagine this whole thing happening here while I move this cylinder this way that becomes our piston analogy from two d to one d right. So, if I do that as I move then I will go to this particular case were the strong stronger shock gets to go passed this weaker shock heated gas; that means, it will not be very strong after the shock interaction. So, it will just go a little forward well the other one the weak shock is going to go after this point into a strong shock heated gas. So, it will deflect more that way the idea of these shocks are this weak shock your here left running shock while the stronger oblique shock is your right running shock ok.

So, the right running shock will just go straight and cross that point and just keep going way while the left running shock will go by this if in case if I have a wall somewhere here and the shock hits there. Now whatever we did in the last class coming right the whole thing of that c is the flow its going in a particular direction and then it is turning and then it has to turn to that particular wall all that will happen out there, but we are currently looking at only this region only the local region ideally in gas dynamics what we will do is we will just break down the problem into small small bits we already experienced and then we will put them all together, if I currently pick a very small complex let us say I have a body like this in supersonic flow and ideally I should not be drawing this point beyond this point because I haven't thought you enough to go pass this point ,, but if this is the case when the flow comes in let us say it is a two d object like a wedge then it is going to see that very small angle like this other side let us say is parallel to the flow direction.

So, there is nothing there same thing happens here and now when the shocks come in whatever we did here will happen all that happen after that now I have to think about interact of this shock with this wall and come back this will go bounce off and come back now this will keep on oscillating like this several times this is what they start calling shock strains inside we will go deal with this kind of problems later currently we will jump to some other problem after this another problem I can think about this what if

this reflection across here for this particular angle change happens to be a detached shock that is the next case.

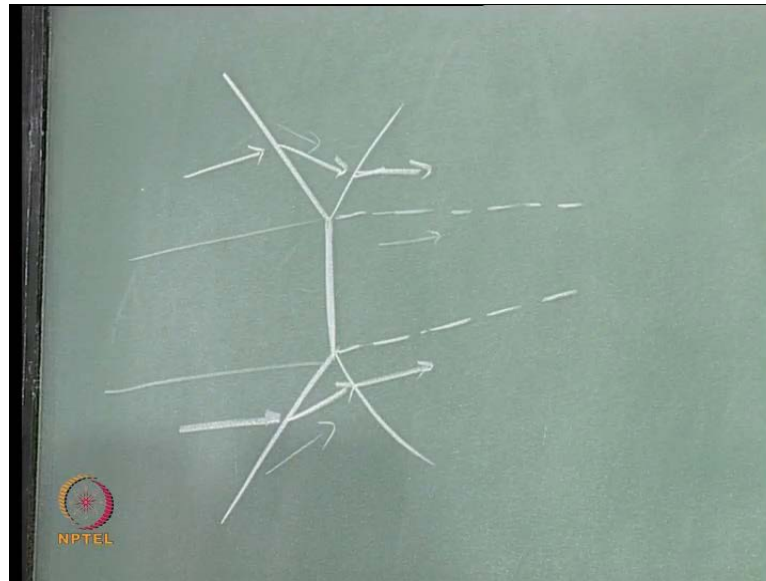
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I am going to a case where I am saying this is the shock deflection from the original shock I have another shock this is the deflection when they come and meet they have to let us assume that we have symmetric shocks easier to work with and then I am saying it has to turn back,, but the mach numbers have decreased after the first shock when that happens it is going to go to a point where this particular theta is crossing the therachange in angle how do we look at this as an oblique shock problem. I will block this particular part like what we did last class the flow is coming flow is supposed to come to the parallel to the stream line like what we did last class right I will draw a line like this the flow is coming along this paper and then turning to that direction if that is the case then there will be an oblique shock sitting somewhere here,, but if we find that that deflection angle is more than the critical angle for this approaching mach number then there should be a detached shock sitting here, but there is nothing to detach from there is no real solid object there and the same since I said it is a symmetric problem the same thing is happening here now what did we do last time we erase the shock a little future and then put a normal shock I will do the same thing here.



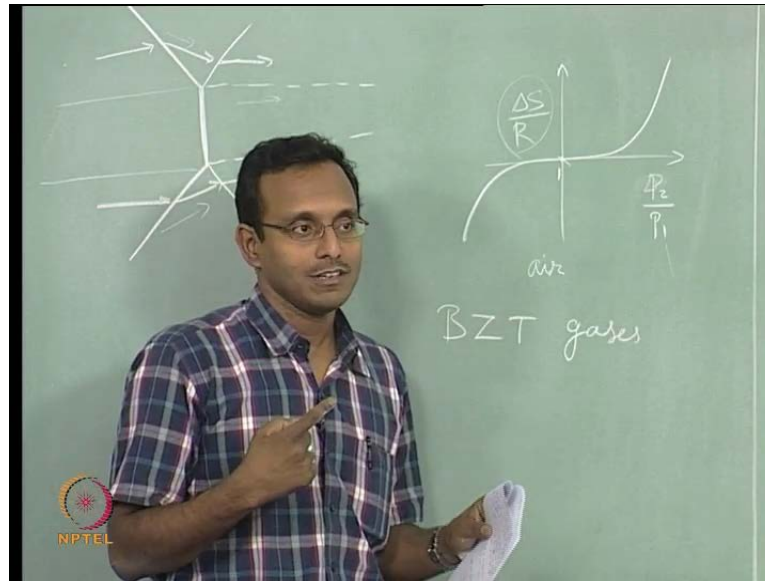
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So, it will do look something like this I will draw a better picture it will look something like this shock and this shock if they have their turn back straight here if they find that this mark number is not very strong say these mark numbers are not very any one of them will be strong it will end up with this condition after that it will have the normal shock at the center flow that is coming inside this region will just face a normal shock and it will go this way and there will be two slip streams this time two slip streams and they need not go parallel to each other I just drawn it such that it is converging most likely it will converging, we will keep it like this for now a flows come here turns down and then this is going to turn it out to parallel to slip stream there.

And here the flow comes here turn and locally it is going to parallel to that slip stream this is what will happen in the this is just a more complex example of what two shocks can do with this we will stop oblique shocks for now later we will go on start solving flow field in different flow situations individual problems we will take and solve,, but it currently we will switch over to expansion waves in compressible flow when we go to expansion waves in compressible flow it is not exactly a straight opposite equivalent of the shock shock causes compression expansion in the straight opposite I cannot tell that it is a very thin strong discontinuity in the expansion why we already proved this in when you are deriving normal shocks we said that if I have a strong discontinuity if I have  $p_2$  by  $p_1$  more than one then my  $\Delta s$  is greater than zero.

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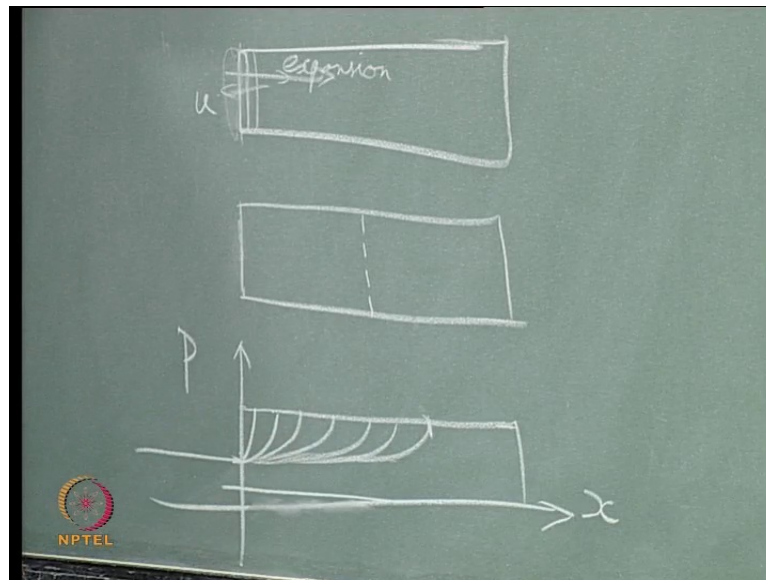
I will just remind you of that case  $\Delta s$  by  $r$  versus it was  $p_2$  by  $p_1$  I think if I have something like this this is your one and this is less than one this is more than one if I have this case you had a curve that was like this I showed this plot sometime back maybe we will think about it in the video I have this kind of plot this particular plot if I look at  $p_2$  by  $p_1$  more than one  $\Delta s$  is positive which means it will occur when  $p_2$  by  $p_1$  is less than one which is our expansion it. So, happens at  $\Delta s$  is negative when that happens then that is not naturally possible this is what we said that time itself and we said only compression shocks are possible ok.

So, I am going to use that statement compression shocks are the only ones that are possible,, but I also gave a caution statement there saying this is for our kind of gas and most of the common gases will follow this this type of a curve there are special gases I gave the name on that time also b z t gases the special gas which will have this curve inverted that is it will go the other way were only expansion shocks are possible compression shocks are not possible,, but most common gases do not have this property. So, we would not talk about it and simple gas dynamics only that caution being given now I know one extra statement from before which is expansion shocks are not possible now I have to tell you one more thing which I will introduce later,, but I will it is the right time I induce right here, and then talk about it in detail later when I say there is a compression shock I am going to call it as it is a whole bunch of compression waves

come together and form a very thin layer they are all one on top of each other when it is expansion wave that is not the case expansions.

If I say there is a expansion shock possible even that will be doing exactly the same thing,, but it. So, happens? That expansion waves cannot accelerate to go and sit on top of each other we will see why after we do some expansion analyses. So, now, the way to think about it I will have the set of wave let us say in my flow it will not cove lest to form a expansion shock, but if I send the whole bunch of compression waves into my flow they will all cove lest together to form one thin layer called compression shock which is what you have been doing all this time normal shock. So, the way to look at it I am going to consider expansion as happening over a big region in the flow what is the typical example for an expansion.

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I have a box and I am going to let us say open this portion suddenly this is having high pressure outside is let us say very very low pressure what is going to happen our stadium example. There are. So, many people inside the stadium the gate is open and there is empty space here what will happen people will run out they do not want to be sitting in the crowd. So, when they come out the people sitting next will realize there is more empty space. So, not moving this way everybody will start moving this way while the information that the gate is open goes that way this is your expansion wave expansion goes that way while the velocity goes this way mass goes this way while the expansion

wave goes that way that is what I have to remember previously, we said compression wave were as more people dumped into the stadium and closed then they have to readjust they have to compression wave all the people will go that way along with the compression wave expansion. If the wave goes to the right the velocity induced by the wave is to the left right you already did this I just want to reiterate it several times. So, that is easier to you to remember all this now let us go look at one particular wave lengths this actually I wanted to tell you one more point here, if I open up like this the very first time the stream the particles that are sitting on the edge will see that there is a huge pressure space it will send a very strong expression wave strong in the sense it will tell that the pressure jump is very high that is all it will tell information is going in tell you that it will accelerate very fast this way, but when it goes to the next fluid element here this fluid element is already moved out, because there is velocity now this region is not as lower pressure as before it is slightly higher.

So, what will happen the next fluid element that is coming here and seeing this is going to tell the pressure is not as low as the previous one telling,, but it is still less. So, the very first wave will tell that the pressure difference is very high and after that every wave is going to tell that the pressure is slightly lesser slightly lesser and there will be a whole bunch of such waves going into this flow. So, if I draw like this here then I am going to tell if I open this edge and wait for say a few seconds then I will tell that the first wave will be here and the last wave will be the current wave will be created whole region there are.

So, many waves here that is what will happen if I draw let us say I will draw this is my location and this is my pressure if I plot along inside this box then initially at time  $t$  equal to zero it has all constant all constant pressure,, but in my axis it should be all straight now because I dropped this pressure to some very low value let us say this value the outside world is this value.

So, it is going to go for something like this with time it is going to do something like this it will do something like this with time region were the pressure is constant will keep on decreasing very few first change the pressure is dropping very fast after that it is dropping slower and slower and slower and at this point it is dropping very, very, very little the very first way it was very strong after that it is all lesser and lesser less strength waves  $p_2$  by  $p_1$  across the wave length the  $\Delta p$  will be slightly less that is a best

way to put that is what you will see that is the physical feel for this things now we will just go and put this in equations form just one simple expression.

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mass:

$$\rho_1 u_1 = (\rho_1 + d\rho)(u_1 + du)$$

$$0 = \rho_1 du + u_1 d\rho$$

$$\frac{d\rho}{\rho} = -\frac{du}{u_1}$$

$$d\rho < 0 \Rightarrow du > 0$$

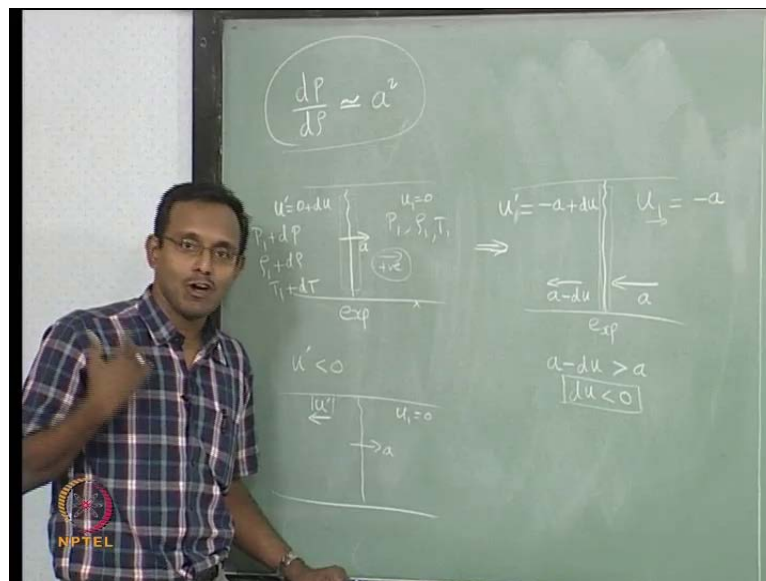
I want to get and then we will talk a little more let us say I go to the condition were I have only one expansion wave and it is wave fixed constant. So, this is a stationery wave sitting here I am going to say there is the velocity  $u_1$  into wet this wave with pressure  $p_1$  one density  $\rho_1$  and temperature  $t_1$  on the other side I am going to say since it is a very thin wave there will be a small change this just put arbitrarily  $d\rho$  plus  $du$  if  $du$  is positive then velocity increases if  $du$  is negative then velocity decreases we will just keep it as some variable and then this will be  $p_1 + dp$   $\rho_1 + d\rho$  and  $t_1 + dt$  all this will happen now let us do this mass conservation across this mass conservation is I am going to pick a control volume across this wave this way across I will make it equal area constant area control volume.

So, I will just have  $\rho_1 u_1$  and equal to this density into this velocity now of course,, I can expand this  $\rho_1 u_1$  will cancel with this  $\rho_1 u_1$  and I will neglect the  $du d\rho$  term because it is two very small numbers multiplied by each other. So, the remaining things will be zero equal to  $\rho_1 du + u_1 d\rho$  this is all I will have or I will rearrange this expression  $d\rho$  by  $\rho$  equal to minus  $du$  by  $u_1$  similar thing we derived when we are doing the sound derivation I am just telling you were it is coming now one of the important things I have to note here is I am going to have a condition if

my density increases my velocity will increase that is if  $d\rho$  is less than zero then this will be more than zero  $du$  will be more than zero and vice versa after that which means if I have an expansion which is  $d\rho$  expansion is density decreasing. So,  $d\rho$  is less than one which is the condition we are looking at  $d\rho$  is the lower if it is expansion then density decreases if that is the case  $du$  is positive.

So, I will go back and look at this now I am going to say if this is an expansion wave I did not put a expansion wave if this is an expansion wave then  $du$  is positive  $d\rho$  is negative that is what we are seeing which means my expansion wave accelerates the flow if I am sitting on the wave itself if I am on shock fixed coordinates in steady whirl then I am going to have velocity increase this wave I will keep on telling this because we will go and do the other reference frame were I am outside reference frame lab reference frame or the board reference frame if you want and then we will study if the wave is also travelling what will happen that is the more complex form we will do not deal with fully we will just use this piston argument or stadium argument if you want that is the easier thing to use now the next thing I do not want to go through the same derivation as we did in the as we did in the speed of sound argument speed of sound derivation,, but it is going to be almost the same. So, if I use my momentum equation along with mass equation I will end up with the case.

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Were I will get to  $d p$  by  $d \rho$  is equal to a square,, but I cannot say this to be equal to a square directly it is just suppose to be equal to the wave speed that is all I can tell from the similar arguments has what I did for the speed of sound that is some I believe it was class eight or class nine somewhere there anyways.

So, if I do that I have to be proving that my changes are very small we gave the argument in speed of sound derivation that my wave does not change any anything in the flow it does not change pressure it does not change density it does not change velocity it does not change anything then it is my sound wave and that is the particular wave which will travel with speed of sound,, but this is not really a wave which does not change anything it actually changes the velocity pressure density temperature everything if it is changing can I still use that argument we are going to use that argument right what is the justification I am giving for it I am going to say since it is one single wave out of a whole bunch of waves I am going to say this particular wave is going to change only by a small bit and there are enormous wave this we are we will typically represent it by five or six lines as a wave,, but a bunch of waves, but actually this infinite number of waves inside that band that is what it is really.

So, each wave individually will change it by a small amount overall affect will be the integral effect which will be a big number that is what it is I am going to use that argument and say the change caused by this change is almost zero the  $d u$   $d \rho$   $d t$   $d p$  whatever we have in the last pole everything is almost zero that is what I am going to say if it is almost zero then I am going to say this is  $d p$  by  $d \rho$  is almost equal to speed of sound because the expansion wave moves or behaves as if it is a sound wave almost the sound wave.

So, it will travel with almost the same velocity that is the argument I am going to use and which means it is almost isentropic we are going to say that it is almost behaving like sound wave and it is almost isentropic,, but when in using expressions we will directly go and say it is isentropic it is not really isentropic in reality a slight single compression wave will travel slightly faster than speed of sound and a single expansion wave will slightly travel less than speed of sound,, but it is very difficult for us to find the difference between these two values. So, we will just use this number for engineering it will work good unless there is a very serious research problem where you need to have that kind of resolution it should not affects typically it will not affect us for any of the

problem now let us look at it from the point of view of the shock the wave that is moving it is not the shock it is actually just an expansion this came from speed of sound derivation I do not want to derive the same thing again lets go back to the same lecture now I will go to another reference frame were I have a wave that is moving a wave that is moving and velocity is zero what will happen here this will be  $u' = 0$  plus  $\frac{dp}{\rho} = u$  I will put zero one I will put here  $\rho$  one  $t$  one  $p$  one plus  $\frac{dp}{\rho}$  one plus  $\frac{d\rho}{\rho}$  one plus  $\frac{dt}{t}$  same thing expect this now I am going to call this the expansion wave the single expansion wave not all bunch of them just one wave now I want to look at in this coordinate system even in this coordinate system we should have,, but it is very difficult to use mass conservation here because my control volume should be moving this wave.

So, to analyze this problem I am going to transform this to my wave fixed coordinate system I will transform it and come back to it just a minute I will come back to this again I will transform this. So, how will I transform this this is my velocity let us say is  $a$  it will be exactly  $a$  if  $u$  one is zero. So, if this is  $a$  then I will impose a velocity to the left everywhere in all the velocity fields. So, my  $u$  one will be equal to minus  $a$  and here  $u$  one prime actually not  $u$  one prime  $u$  prime equal to minus  $a$  plus  $\frac{du}{u}$  and all the pressure density temperature will just be varying by  $\frac{dp}{\rho} \frac{d\rho}{\rho} \frac{dt}{t}$  that does not change anything only the velocity field changes with change in coordinate system.

So, if this is the case now we already solved this problem I have put a control volume across this and I am going to say since it is minus  $a$  in this direction it is equivalent to having  $a$  in this direction and it is going with a minus  $\frac{du}{u}$  this way this is what I have,, but we already have this kind of result and I know that this is an expansion wave in an expansion wave if a flow crosses it its velocity increases that is what I know. So, I know that  $a$  minus  $\frac{du}{u}$  is greater than  $a$  which actually means that  $\frac{du}{u}$  is less than zero right just immediate results from there now I will switch back from this transformation back to the original coordinate system where I am saying I am fixed to the wall here outside reference frame,, but I have this value  $\frac{du}{u}$  difference in velocity does not depend on reference frame switch.

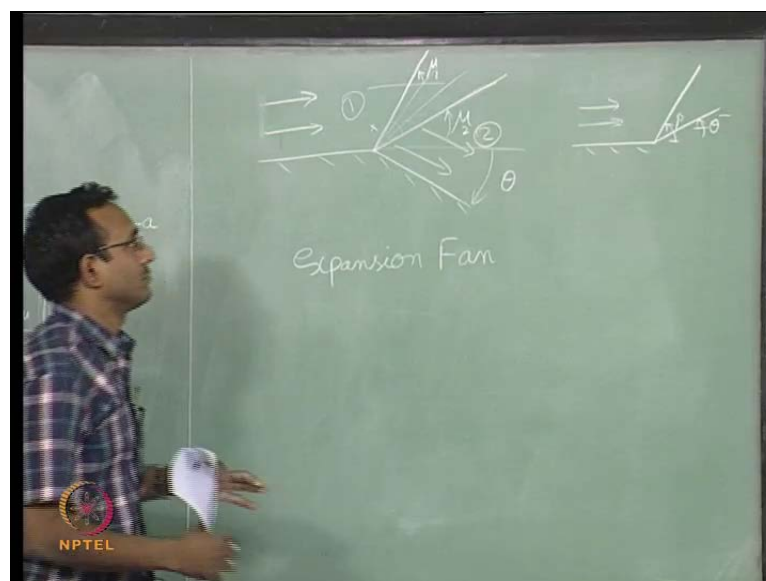
So, I will just put the same  $\frac{du}{u}$  here  $\frac{du}{u}$  is less than zero which means now initially I am I am having a convention that velocity this way is positive right if  $\frac{du}{u}$  is less than zero I will put that in here  $u'$  is less than zero which means I having a flow actually with  $u$



one equal to zero I am having a wave travelling this way with a what I am finding is there is a velocity induced this way finally, because  $u'$  is less than zero now I am using that convection and now I will just put the magnitude here the direction is already taking care of this minus less than zero.

So, this is what we were talking about at the beginning when we introduce the expansion wave right s when there is one wave going this way it is telling that there is lesser pressure here move this side and that is the induced velocity that is what you are saying I can do the whole expression with  $u'$  one naught equal to zero if it is not equal to zero then I have to find the relative velocity between this and then use that to move from across this reference frame which I can believe you can do by yourself you can find that the velocity induced by the expansion wave will be to the back backward direction now I will just think of expansion waves as some person who is running then it is going to tell the fluid which it is just now contacting that go to where I am coming from that is the information I am giving while in contrast the shock is going to tell the opposite come along with me is what it will tell they are going to tell the opposite direction if you think about this it becomes easier later when the problem becomes more complex now typically people encounter these expansions in two d flow than much simple one d flow most of the flows are two d anyway,, but we will consider two d problems because...

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We will come back to the one d expression later if there is a supersonic flow going like this previously when we did shocks we remembered this basic flow shape last class I told you right this is your reflection of the wall is into the field flow is coming like this the wall moves into the flow the flow and this becomes the compression wave whole bunch of compression waves come together to form and contrast.

Now I am going to remember that other flow field other basic shape a theta deflection away from the flow direction if the flow was just going straight it will see there is the huge gap here this is why it wants to expand think about it that way now the expansion set of expansion waves must start from the corner why this is the change were occurs it is as if there is a whole bunch of people walking straight like this and suddenly the road becomes wider what will they do they will slowly start filling the space who will see it first the person in the corner will see it first they will walk like this first and they still want to travel in this way and they travel little faster this way and once this last person moves the next guy sees that there is more gap next to him.

So, he will travel faster the whole sequence goes, and they will have slight delay the person here will see this change only after he has travelled some more the person next to him had moved down if I imagine that analogy the region were change start happen will be along some like this and there will be one line where it will tell this is the final change there is no more change possible that line will be somewhere here there will be two lines start of expansion end of expansion.

Now after this whole thing the flow turns this way and since we did this is one d that after the expansion is experienced the flow accelerates I can naturally put that this velocity vectors are shorter than this velocity vectors I can give that already here we will keep that way now ideally what we have to do is find expressions for for a given theta what will be the change in mach number what will be the change in pressure what will be change in temperature density everything across this expansion wave set of expansion waves now you give a nice name for this since it look likes the fan the fan which opens up like this that kind of fan they call this expansion fan this is the expansion fan if you want the japanese paper fan it is something like that that is why it is called the expansion fan we will go deal with more of this,, but I will do one more thing since we have only few seconds the very first wave that is going like this the speed at which it will go in will be almost equal to sonic line right every wave is going to do that exact thing the very

first wave is going to move with speed of sound of this gas the gas that is approaching that is what will happen which means I am going to have this line having an angle to mach angle of the incoming gas I will put  $\mu_1$  and by similar arguments I can tell that the last wave if I call this as region two and the region one the last wave will have with respect to stream line this angle  $\mu_2$  and this is very important point it is with respect to stream line always knew, we will go try and find expressions for the final angle final mach number with respect to theta we will try and get it we will start with that deviation next class and we will probably going to explaining more things after that that is probably one more class of expansion fan then we will move on to more practical flows then we will see you people next class.