Gas Dynamics

Prof T.M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 9
Lecture - 20
Oblique shock relations

Hello everyone, welcome back. Last class we derived relation between before and after properties across an oblique shock, where we still had the variables theta, beta and m 1 not linked fully. We got a relation in terms of rho 2 by rho 1, and then we said if we know u 1 normal or m 1 normal, the normal Mach number with respect to that oblique shock normal to that if I can get that. Then I can go to normal shock tables for that particular gas, then I will just will get all the properties across that. We said that, and we tried to get that angle beta in terms of the deflection angle theta for the given Mach number; that is where we stopped. So, we will continue from that point.

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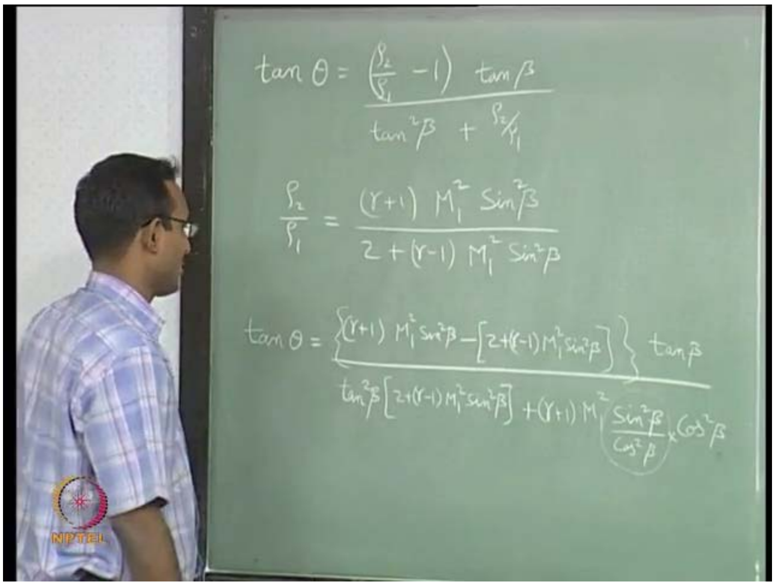

So, we derived this last time. We had this, and we said rho 2 by rho 1 can be obtained from normal shock tables or normal shock relations substituting it with M 1 normal instead of M 1. So, I will write my rho 2 by rho 1; from the previous thing you have one page of it. If you go back to this page you will see this formula; I am jut replacing M 1 with M 1 normal. I will replace M 1 normal with an expression for it which we derived last time which is M 1 sin beta.

So, it will be M 1 sin beta whole square which is M 1 square sin square beta. Again there will be a gamma minus 1 M 1 square. It will become M 1 square sin square beta. Now our job today is to substitute this inside this rho 2 by rho 1 and here, and simplify the expression so that we will have something very easy to work with. It may not be very easy, but it will be simple enough to work with; that is the goal, okay. So, let us start substituting inside that. I will directly start with rho 2 by rho 1 minus 1 where I will put minus 1 here, multiply this with that and ignore the denominator. I will do the same thing here; multiply this tan square beta with this denominator and then keep only the numerator here.

Both will have common denominator exactly the same. I will not write that expression, because they will get cancelled anyway. Now I will start writing. So, it will be rho 2 by rho 1s numerator sin square beta, that numerator minus the denominator, this minus the denominator; this whole thing multiplied by tan beta. That is the numerator of that function whole thing divided by; of course, we still have the denominator for this numerator. We would not write that because the same thing will get cancelled in the bottom, tan square beta times denominator of rho 2 by 1 that is this tan square beta into 2 plus gamma minus 1 M 1 square sin square; this plus numerator of rho 2 by rho 1.

Now this will be the simplified expression. Both the numerator and denominator will have this in their denominator if I write the original thing; I would not write it. I will just cancel those terms, fine. Now you want to simplify this expression overall. First simplification which I want to do is I want to take this tan beta to the denominator. So, I will simplify, wait wait wait let us not do it that way, because this does not have a tan beta in there. So, I will rewrite this as I want tan square beta taken common for the whole denominator. So, I will just rewrite this as cos square beta into cos square beta. If I do this now I have tan square beta in here. So, I have tan square beta common for the whole terms in the denominator. Now I have one tan beta in the numerator; they will get cancelled. So, now I will go write the next expression.

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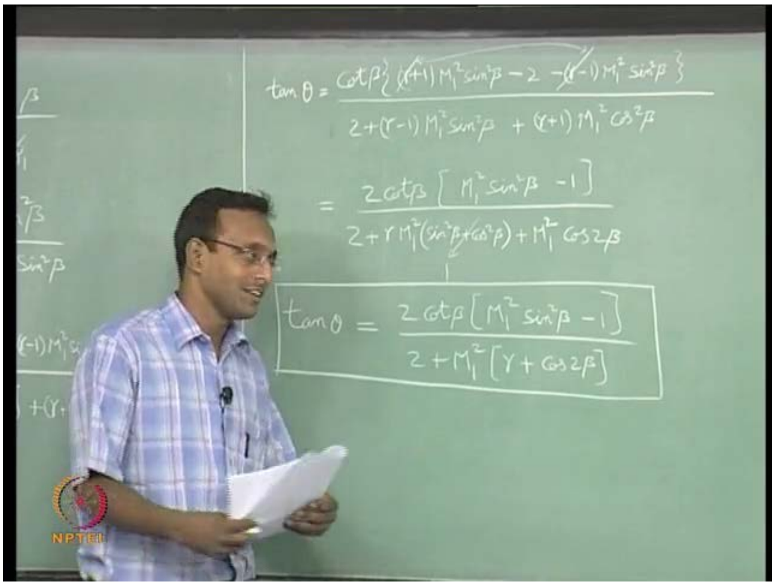

So, I do not have the tan beta anymore in there, okay. Instead I have cot beta which is coming from the denominator tan square beta multiplied by M 1 square sin square beta minus 2 minus gamma minus 1 M 1 square sin square beta. I have written the square bracket more open divided by bottom will be 2 plus gamma minus M 1 square sin square beta plus gamma plus 1 M 1 square cos square beta. Now this is what I have. I want to simplify this expression, expression inside this flower bracket, okay. If I look at this, there is a gamma M 1 square sin square beta, and here there is a minus gamma M 1 square sin square beta.

I can just cancel them directly, and I will put this arrow as usual to link that this is how I cancelled it, okay. Now the next term M 1 square sin square beta with a positive sign; here again it is positive M 1 sin square beta. So, it will be 2 M 1 square sin square beta. The only thing remaining is minus 2. So, I can take a two out common, and I will write this as 2 cot beta multiplied by M 1 square sin square beta minus 1; that is our numerator. Now in the denominator we want to again to do the same thing, gamma M 1 square sin square beta gamma M 1 square cos square beta. They are not the same, but you know that sin square beta plus cos square beta is 1, okay.

So, I can write this as 2 plus gamma M 1 square with sin square beta plus cos square beta. And of course, we know this is equal to 1. Remaining terms are minus M 1 square sin square beta plus M 1 square cos square beta. I will write it as plus M 1 square cos 2 beta. Hopefully you remember this formula from trigonometry. It will become cos 2 beta, sos 2 beta is equal to cos square beta minus sin square beta. So, I will get to that form from here. So, I have my final expression. This is the final form cannot simplify it any further.

So, I will have tan theta equal to 2 cot beta multiplied by M 1 square sin square beta minus 1 divided by 2 plus M 1 square times gamma plus cos 2 beta. This is the expression I have, okay, and very very essential expression for oblique shocks problems calculations, very important. This is what is linking the deflection angle, shock angle and the incoming Mach number. They are related by this expression. Now let us say I have a problem where I know that the shock is having a particular inclination to the incoming flow, and it is having incoming Mach number some value, than I can calculate the deflection angle directly from this formula.

But in most of our engineering application we have will have the opposite. We will have a case where I will be given the theta; I will be knowing the Mach number; that is I know my body shape. I know that the body is tilted to the flow with some angle. So, now I have to find what is the shock angle there? Because only after if I find the beta I can go and calculate any property, right. I need to know M 1 sin beta; that is a normal Mach number. Once I know that I can go to normal shock tables to find out properties across the shock. So, I need to know the beta; that is what will typically be the problem when we are solving in engineering. So, given a theta and M 1 I need to find a beta. Now I have to invert this relation; it is not very easy. So, we will go use the age-old method of using a plot using graphs. So, let us go to that screen.

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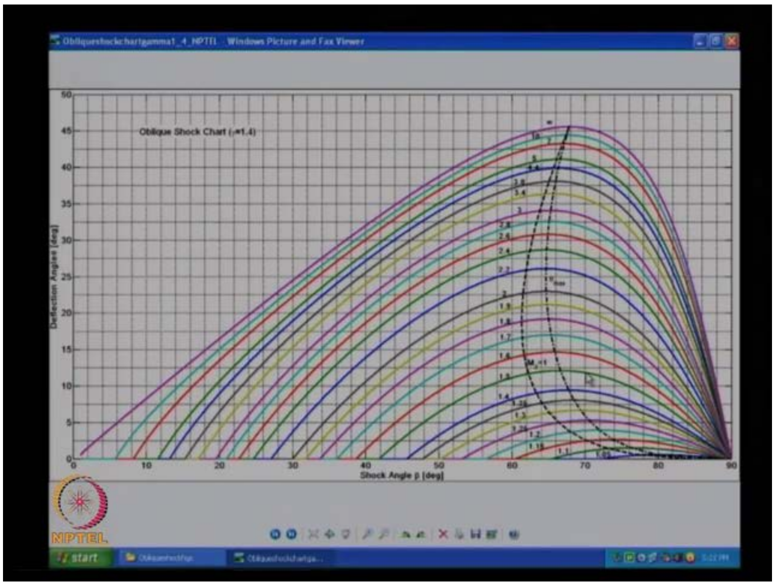

So, what we are seeing, I do not see the mouse first, okay. What we are seeing here is shock angle beta going from 0 degrees to 90 degrees and the deflection angle theta in degrees again, okay. I am plotting it for oblique shock gamma equal to 1.4; of course, I could get the whole plot for some other gamma value. We are using this, because it is the most common thing. The plot looks like this. It seems like for every deflection angle there is double value for any given Mach number. These value is written here 1.1, 1.2, 1.4, all this, up to infinity Mach number infinity, everything. It seems to be an inverted U-shaped curve which is for every deflection angle let us say I pick 10 degrees and I go for same Mach 2, I could have an angle of the order of 39 degrees or I could have an angle of the order of, wait I have the order of 84 degrees or so.

I could have two different values like this. So, which means I am going to have two different shocks. I could have a shock that is having a low shock angle or a very high shock angle; first observation we are making. We will make more observations, and I just wanted to plot this. I do not want to use this plot to explain things; I will go and draw this on board and explain things, okay. One more thing to note; it has a peak value for every Mach number, and above that we cannot get a solution for beta; that is what we are having. And beta does not have a value below a particular mach angle below a particular angle.

It so happens that it will become mach angle; we are going to look at that now, okay. And I have also drawn one more line here; we will come back to this line later. On the left of this line it is supersonic, and to the right of it, it is subsonic; we will get back to that later. Now we will go back to the board and look at this same expression again. I will go back to this itself; it is enough if I look at this. This expression denominator cos of 2 beta where beta is going to be anywhere between 0 and 90 degrees, alright, so think about it. Two beta will be anywhere between 0 to 180, okay. So, there will be some values for which this will become minus 1, some values for which it will become plus 1.

But gamma is always more than 1.1 or more than 1. So, this whole function will never ever become negative. This is always positive, and one square is positive, two positive, everything is positive. Now I want to find values where the curve hits 0 where theta will become 0 no deflection angle; that is what we are looking for, we will see why. So, only the numerator can go to 0 now, denominator is not going to go to infinity really. If at all Mach number going to infinity we will come to that special situation later, because if it is going to infinity numerator also has M 1 square it would not really go to 0, okay. So, now I have to think about other reasons why it will go to 0, either this square bracket can go to 0 or cot of beta can go to 0.

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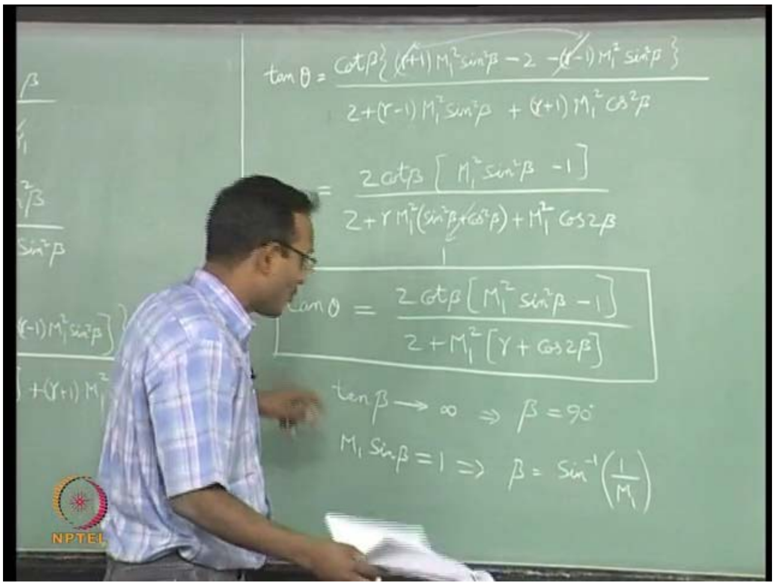

If cot beta is 0 tan beta turns to infinity which means beta is equal to 90 degrees not turns to, it is equal to 90 degrees; that is the way we write it. And the other expression will tell me M 1 square sin square beta equal to 0 as the minus 1 equal to 0 is M 1 square sin square beta minus is 1. So, I will take a square root of that; it will still be 1, plus minus 1, minus 1 is not possible. So, we will keep it as plus 1; minus 1 just simply means that the flow is from right to left instead of left to right. Mach number negative does not make sense for us; it is just absolute value of velocity for us.

So, I will just have one solution for it M1 sin beta equal to 1, minus 1 just simply means that I have either a negative angle beta or I have a negative mach number which means flows from right to left. Negative angle beta is the shock instead of like this it will be like this; that is all; that is nothing great. So, we will come to that situation later. Then we would not call it negative beta anyway; we will keep it like this. So, this implies that beta is sin inverse of 1 by M 1 which is your mach angle, okay. So, we are finding that we have solutions for beta anywhere between this to this.

What if I just jumped ahead of myself? What if my beta is less than this value? If my beta is less than this value then I am going to have sin square will become lesser than that value, right; sin is an increasing function from 0 to 90. It will become less than that value which means M 1 square sin square beta minus 1 will be less than 0. It will become negative; that does not mean anything here, theta will become negative. We cannot tell anything special here. Let us go one board back. We have this expression here. No no I will write a better expression P 2 by P 1. Let us go to a fresh page.

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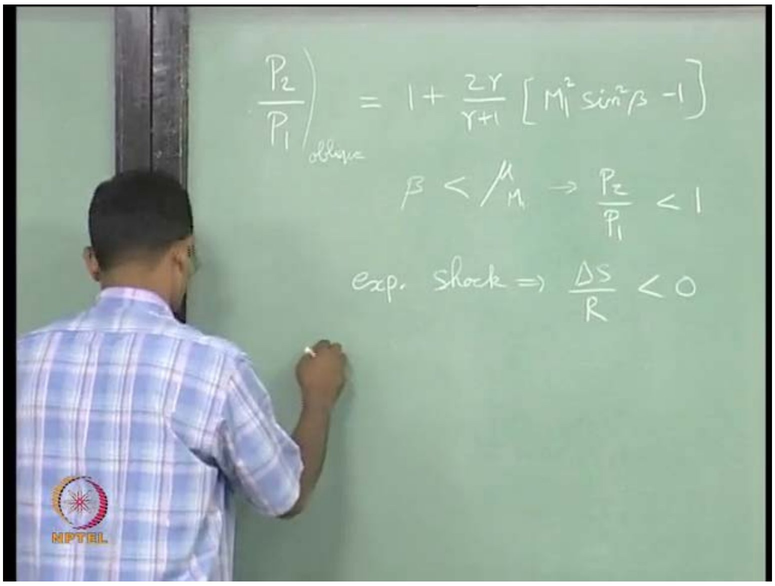

We will write P 2 by P 1 for oblique shocks; of course, it is going to be written in terms of M 1 normal. I will write M 1 normal equal to M 1 sin beta. We already have this kind of expression written before. So, I will just write it directly. Here it will be I will get this. It used to be M square minus 1; it just comes out to be this, okay. So, now I will go look at that and say if my beta is less than mu of M 1, Mu of M 1 is your sin inverse 1 by M, alright. So, if I have such a situation then this whole square bracket will become negative. If this becomes negative I am going to have a condition where 1 minus 2 gamma by gamma plus 1 which means multiplied by some other number; that is fine which is 1 minus something which basically means that P 2 by P 1 will be less than 1, which means it is becoming.

It is still a shock in our case, but it is a shock which has P 2 less than P 1, which means it is an expansion shock which we already disproved a long time back in normal shock situations where we said that delta S by R for those cases will be less than 0. So, I can say that for beta less than this I will have P 2 by P 1 less than 1 which means it is expansion shock, expansion shock which will have delta S by R less than 0, not possible. So, I will never have a situation where beta is less than my mach angle for that particular incoming Mach number M 1, right. So, now my solution exists somewhere between mu of M 1 to 90 degrees which is what we were seeing in the plot anyway. So, now we will go draw the plot once more. I will draw it on a fresh page; I want to draw it big.

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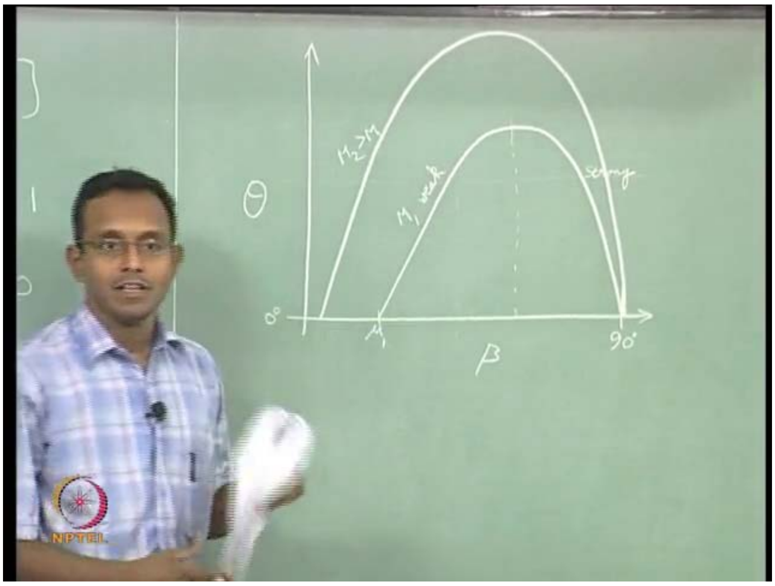

This is our beta, this is my 90 degrees, and here this is 0 degrees, and this is our theta. And for a particular mach number it is going to do something like this. For another Mach number higher mach number it will do something like this. It is supposed to reach the same point. It is M 1, M 2 greater than M 1; this is what it will do, okay. So, now we can already say we already proved that for a given mach number it goes from mu 1 which is our sin inverse 1 by M for that M 1 to 90 degrees; that is the only place where we will get any solution. Now we will say one more. For any particular deflection I get two values. Now I have to label them separately. So, I will call this the weak solution not for no reason, but I will tell you the reason after writing this, and this side is the strong solution.

Before the peak it is the weak solution; after the peak it is the strong solution, we will keep it like this. Why is it called strong and weak? We will go back to our P 2 by P 1 here. We find that beta is anywhere between mu 1 to 90 degrees. In this place sin beta is an increasing function continuously which means if I have a higher beta, I am going to have a higher P 2 by P 1, which means it is more compression. That is the reason we are going to call lower beta as less compression shock, higher beta as higher compression shock. So, I am going to have strong shock and a weak shock; two solutions are possible.

So, I am going to this plot, and I am going to say this is my weak shock solution. This is the strong shock solution; two solutions are possible. One is a weak shock, other is the strong shock. If you look at the curve it is not like a simple symmetric parabola. It is shifted skewed more to the right; that is what it will look like. Now if I want to find the maximum value maximum beta for which theta will be maximum. If I want to find that, of course, I can go and derive it. It is just derivative equal to 0 you have to set; let us say we would not do that. I will give you the expression here.

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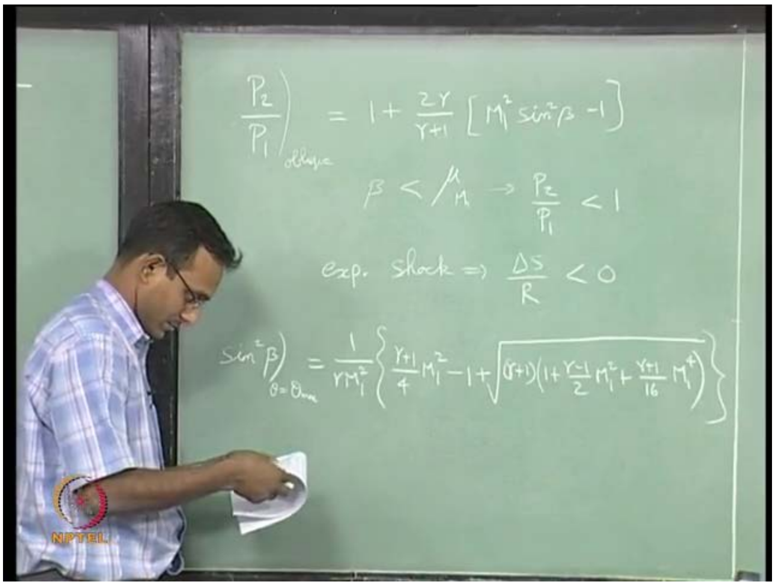

I can find that beta at which you will have maximum for any given mach number and that is given sin square beta at theta equal to theta max is given to be. I derived it; if you want you can go derive it. I do not think it is needed to be derived in our class time, and it is not very difficult to derive also. It can be derived; I have to close bracket, okay. So, this is the expression we have. So, this is only a function of gamma and M 1, nothing else, gamma and M 1 it is going to give me a beta. Now I have to go use this beta in our tan theta formula to get the maximum theta; I would not do that. You guys can do it; I will just leave it for you guys to do it. So, now the next thing I want to talk about is the other line which we looked at in the plot.

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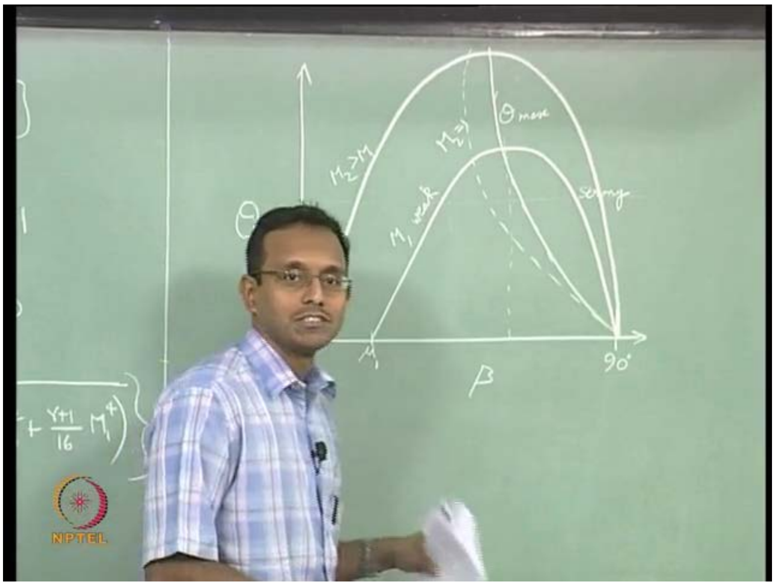

We had one more line which was going like this while the other one was your theta max line that was going like this. This is your theta max line. It goes through for all mach numbers the peak value; it goes through all of them, that is one line. The other line is to the left of it; remember that it is at a lower beta value, and that is your M 2 equal to 1. Actually I did the calculation for all the mach numbers, for all betas and found M 2 for all of them and then put a contour of that along here. Of course, you can find an expression for when M 2 becomes 1; it is not simple enough. So, I just did the graphical method; it is easier for me. I just did this.

What you have to note is if I think about a strong solution strong oblique shock, then I am going to have only subsonic condition here behind the shock. If it is weak for low values of my theta, I will have supersonic flow behind the weak solution behind the weak shock solution if you want to think about. Why do I call it solution suddenly? It is a solution to our equations; now it gives two answers. So, I call one of them weak solution, other as strong solution. So, I am basically having a weak solution; for low thetas it will have supersonic flow behind; for thetas very close to theta max typically that is the case. When theta is very close to theta max you may have a subsonic solution. You have to be now careful in looking at this that region. It may have a subsonic solution behind the shock, okay. You just have to be careful about it a little bit about it. Now I want to look at why can there be two solutions for the same deflection? So, we will go to that side and we will have to start drawing pictures.

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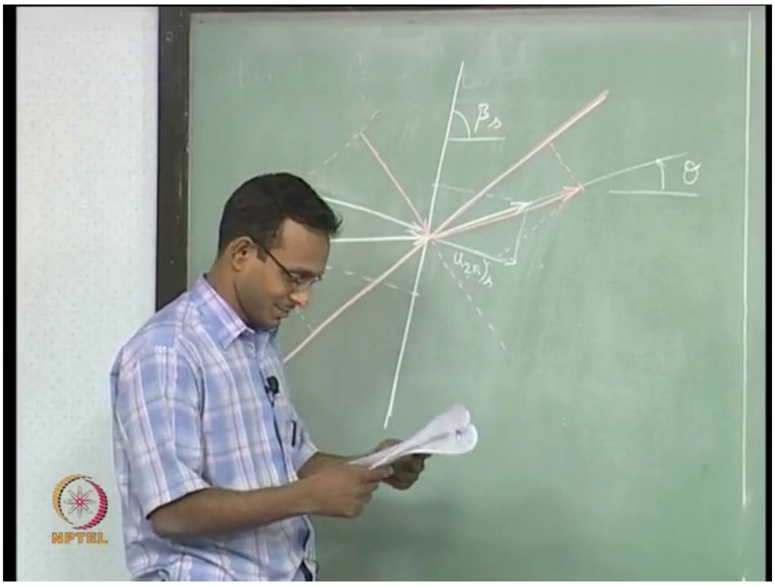

So, let us say we have a very long vector that is my incoming velocity vector, and that is parallel to our horizontal. And now I am going to say I have one solution that is at this angle; first watch then draw, it is easier. I will tell you how to draw it, so that it looks like you can always draw it correctly, okay. But I will also tell you what goes wrong if you draw it too many times. I will draw one more shock angle that is lesser than this angle; this let us call it the strong solution. Now I will draw another beta weak solution. I have two solutions. Oh, I have one more color. I will use this color; hopefully, it can be erased later. The pink one is our weak solution, the white is our strong solution.

So, let us look at only the strong solutions first. So, I have to figure out the normal component. So, I will have to drop a normal for this vector drawn. I am drawing in a particular way so that I will get to the point where it looks like the downstream velocity vector can be drawn. So, now I have to find that the U tangential should be translated just across that side; that will go to somewhere here on that shock, and now I will draw again normal direction. I am going to say my velocity vector; the final angle is going to be along this line. This is my theta; this is my theta direction. I draw that line also already. Now I will say this long vector which is my u 1 normal for strong shock solution is going to be brought down; u 2 by u 1 will be less than one for a shock normal shock.

So, I have to draw something small enough such that it will go and meet along that line. I am drawing in a particular way so that it will go along tan theta. So, I will say this is my u 2 normal for strong shock, and this is my other tangential velocity. So, now my final vector happens to be this. This is one possibility. I am saying that incoming velocity is now turned by theta degrees with this beta. I will call this beta strong; with that beta I am going to get to this; that is one angle. Now we will go to the weak solution with the other color. I want to do the same thing for this, but the angle is different. So, I will have to drop the normal again.

I will erase that u 1 normal value, less confusing. It is roughly that; I will just extend that normal here. And now I have to say it has to reach that line. So, I will draw a line parallel to it, and I have to tell that this is my normal u 1 normal sitting here. U 1 normal is sitting here; I have to take this for length of u tangential across there. That will be roughly somewhere here. Wherever it meets there that is your final point; I will just extend this across here. So, now my I will just draw the vector slightly shifted so that you know where the other one is, but actually it is supposed to be on the same thing. So, you are finding that I can draw two solutions which will exactly match.

Now you may ask if I follow the same procedure I can draw how many ever shock angles I want with the same angle finally, and if I follow the procedure I can draw more the same process if I follow, if I already fixed the theta and choose beta and draw all this. I can do that, but you should know that I do not have the freedom of choosing u 2 normal u 1 normal to u 2 normal ratio; that is decided by normal shock relations. It so happens that the relations will permit only two such values, not any angle you choose; that is what your equations are showing; that is what your final plot is showing. It just looks like we kept it as a free variable; we did not choose this from normal shock. So, I could choose how many ever currently if I want. If I go and tell normal shock it can only be two values; that is how it will be. This is just for people who want to ask different kinds of questions; I just want to tell that also extra.

Now I will tell you one more thing. It so happens that we have two solutions, but in real life we only see mostly only weak shocks. If I have a steady problem I see only weak shocks. Till now I have not seen a steady stable strong shock solution, I have not seen anywhere in literature also stable strong shock solution. That being said that does not mean people have not observed strong shock solutions. They have observed it but in unsteady problems; that is the idea of stability here. It so happens that it has now been recently proved recently in the sense that it is last year or this year it was proved that the strong shock solution is an unstable solution while the weak shock solution is a stable solution.

Because of that if there is ever formation of a strong shock solution any small disturbance, noise, any small disturbance in the flow will throw it to a weak shock solution. That is why when we wait for long enough to form steady stable solution steady state and when looking at the problem, we will always see only weak solutions; by the way stability can also go to the other side of the mountain also, right. If you think about stability as a ball sitting on a mountain I can push it that way or the other way. If I push it the other way it becomes a normal shock or a bow shock. This is just extra information; you do not need to know it right now, but I am just telling you anyway so that you are ready for it. Now I just want to put some observations in a table form for strong shock and weak shock. I have told you most of them; anyway, I will just put them in an order.

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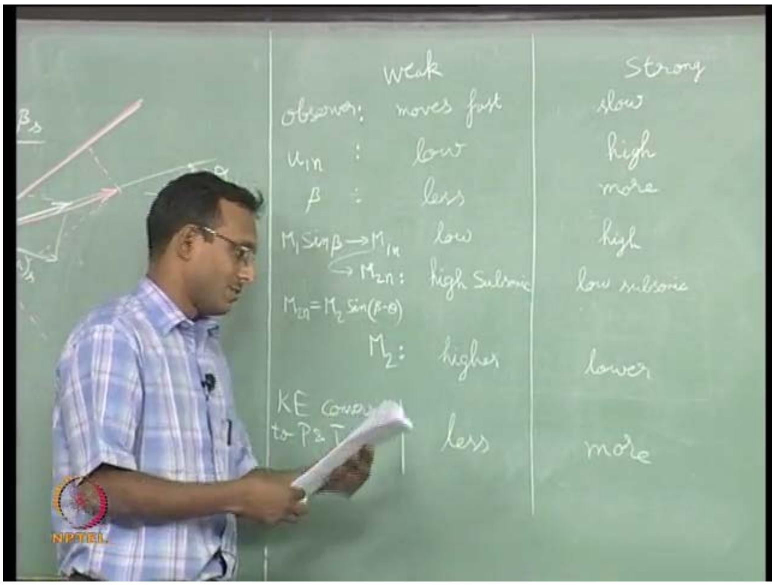

And now I want to look at it from the point of view of the tangential velocity which is caused by our observer moving along the shock this side, okay; if I think about it that way I want to find observer. It so happens that the observer moves fast in a weak shock and in a strong shock slow, okay; that is, I am having a higher tangential component in a weak shock, lower tangential component in a strong shock which we just now saw in the plot. It is anyway there. The next thing we want to look at is normal component u 1 normal let us say. I am looking at u 1 normal; u 1 normal is low for a weak shock, high for a strong shock. Of course, now you can easily tell beta we already defined everything based on beta, here it is less more. This is how we defined; actually probably this line should be at the top anyways.

So, now the next thing we want to see M 1 sin beta which is your M 1 normal, right; of course, you can tell that from U 1 normal I can directly tell that that will be low, but that is not what I am interested in. I am interested in; I will plug this into normal shock which will give me M 2 normal. I am going to look at only M 2 normal; M 1 normal you know what it will be. It will be low and high; I do not need to write it separately. I am more interested in this. This you have to remember from normal shock solutions. In normal shock if my mach number is low, the M 2 normal will be closer to 1. We know it should be subsonic, but it will be high subsonic. M 2 normal will be high subsonic while here it will be low subsonic.

Now that being said I am actually interested in M 2, actual M 2 value. M 2 normal is equal to M 2 sin beta minus theta; this is what we know already. This is our formula for it, right. So, if this is the case now we said that beta is low theta is the same, and M 2 normal is higher we said for weak shock; beta is low, M 2 normal is higher which means M 2 must be higher; that is what I am interested in. M 2 here it will be higher, here it will be lower. How will I know it is lower for strong? Let us say compared to weak, beta is higher. If beta is higher and this is lower, then to compensate for this increasing and the overall this is decreasing, this has to decrease a lot more, right. So, I can tell for sure that strong shock Mach number downstream will be very very low. This Mach number is not normal Mach number; it is a full mach number including the tangential component everything together, the actual flow velocity Mach number.

Now I just want give you one more field for things. So, I am going to look at kinetic energy conversion; that is the next thing. Kinetic energy conversion what is it getting converted to? It is getting converted to internal energy which is expressed in the form of pressure and temperature. I will say two pressure and temperature here. If you are more into molecular gas dynamics I can tell that it is going into internal energy in the form of translation energy, rotation, vibration, etcetera, and the pressure is one way of expressing translation energy; that is what it will come out with. So, pressure and temperature are related through translational mode. That is too much information for our simple gas dynamics course, but anyways that is sitting here.

So, basically I am taking the kinetic energy in the flow and putting it into the internal energy in the molecules. Now I am thinking molecules while this is I am thinking fluid element. If I take it and put it inside the molecules inside the fluid element, then I am finding that the conversion will be less for this case and more for that case. How can I say that? I can directly say that u 1 normal is higher for strong shock which is here; u 1 normal is higher for strong shock, but the outcome is very small. I am putting it as normal in Mach number terms; of course, you should know the temperature is also going to be higher because M 1 normal maybe I should write M 1 normal. M 1 normal low high; if M 1 normal is high T 2 by T 1 is higher.

So, temperature of the flow behind the shock is higher, but Mach number is lower which means the velocity is really really low, okay. Velocity we will be really low which means I start with very high normal component velocity. I am ending with very low normal component velocity. What happened to all of that energy which is in the form of bulk movement of fluid, the kinetic energy? They all went into internal energy in raising the temperature and pressure of that fluid. That is why you are having higher temperature and higher pressure downstream compared to weak solutions. So, that is why I am saying that is less and this is more. This is one set of things we wanted to say. Now I want to look at one thing which is very tricky in solving problems. What happens to P 0 2 by P 0 1? This is just across oblique shock. I am not any more thinking about just strong shock and weak shock. This is for any oblique shock, could be strong or weak. So, I am thinking strong or weak oblique shock.

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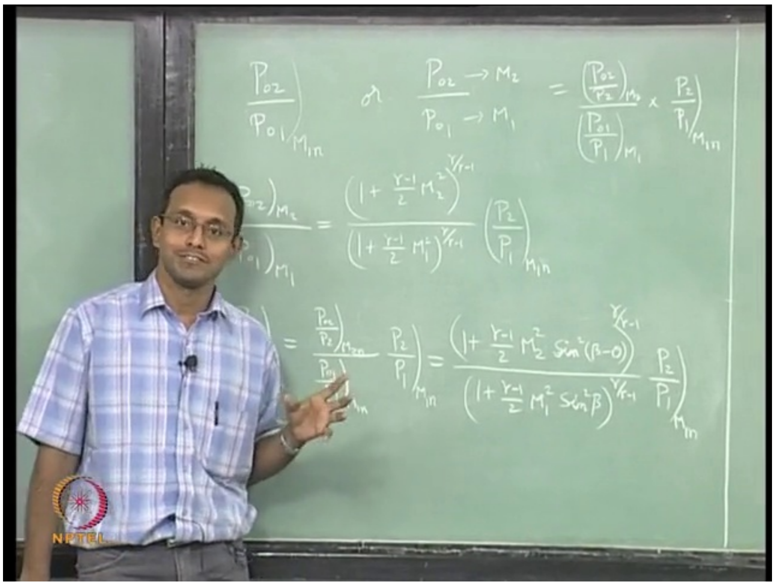

I am interested in finding P 0 2 by P 0 1; which way should I find it? Should I find it from M 1 normal or should I find this from M 2 and this from M 1; which way should I find it? That is the question we want to ask, okay. It looks like if I write this expression this is going to be P naught 2 by P 2 divided by which is based on M 2 P naught 1 by P 1 based on M 1 multiplied by P 2 by P 1 which is of course, it does not matter whether it is M 1 or M 1 normal; we already showed that last time. So, that is here, because movement of the observer does not change the actual pressure jump across the shock; only the velocity components have changed. So, it is going to remain the same; that is what we said. Okay. So, that is that case.

So, now we want to write expressions for these. Let us say since I started that I will write that, P 0 2 from M 2 divided by P 0 1 from M 1. If I expand those terms it is going to look like M 2 square to the power gamma by gamma minus 1 divided by 1 plus gamma minus 1 by 2 M 1 square to the power gamma by gamma minus 1 multiplied by. I will just leave it as P 2 by P 1 from M 1 normal, because that is going to be the same for both. I do not want to write the expression; I will just leave it like this. This is from one form. The other form P naught 2 by P naught 1 from M 1 normal is going to be expanded the similar way. I will write it as P naught 2 by P 2 from M 1 normal; that is the difference between the previous one and this one divided by P 0 1 by P 1 from M 1 normal. Oh, that should be M 2 normal by the way.

P naught 2 by P 2 is from M 2 normal, and P 01 by P 1 is from M 1 normal multiplied by P 2 by P 1 from M 1 normal. If I look at this expression it is going to look slightly different. I will write it in terms of that M 2 so that I can compare the expression M 2 square sin square of beta minus theta. We did this M 2 to M 2 normal last time whole to the power gamma by gamma minus 1 divided by 1 plus gamma minus 1 by 2 M 1 square sin square beta to the power gamma by gamma minus 1 multiplied by P 2 by P 1 at M 1 normal; this is what I have.

If I look at the expressions there is no connection between these really, okay. They should be very different, right; that is what we see. So, in my opinion you should not be using this but wait. We said P naught 2 by P naught 1 is related to entropy jumps across shock, right. We had this formula; I will go back and write that again. I wanted to confuse you a little bit. So, I am taking you this particular path so that you will be more clear at the end of it.

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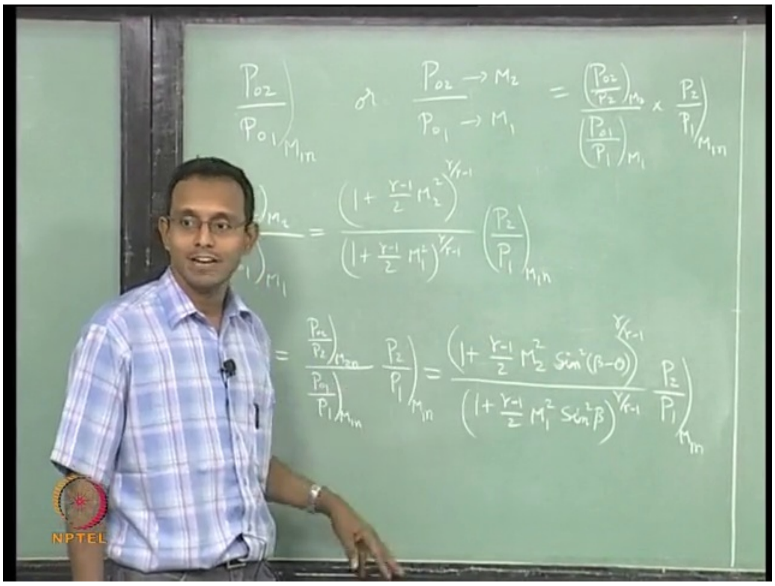

We had this formula delta S. We had this formula which we said for normal shocks T naught equal to T naught 1, and so it became log of P naught 1 by P naught 2 for delta S by R. And we said in moving shocks this is not equal to 1, and so we have to keep the whole term. And now in our oblique shocks we said we are again in shock fixed coordinates, and the motion of the observer is perpendicular to the fluid motion in that particular reference frame. So, it does not change my T naught value. The movement of the observer the V value does not change the T naught; it is going to be the same across both sides.

So, we said T naught 2 equal to T naught 1 there also which means my expression comes out to be P naught 1 by P naught 2. Now that being said there is one oblique shock; whichever way I calculate there is only one entropy change. That being said I can say one of them is correct, the other one is wrong; that is also there. But I am going to say one more thing. The change is because of some observer observing it differently; that does not change the fluid's state. Observer is moving along the shock looking at it once; other case he is just standing still and looking at it only in the normal component. Those are the two ways of looking things which means the entropy jump across should be exactly the same whichever P naught 1 by P naught 2 I use.

That being said P naught 2 by P naught 1 which we derived here, whichever method I use I should get the same expression. We will go see a numerical example next class, but that happens to be true. That being said I will still keep the caution that expressions look very different in my gas tables. I told you already that I have one more P naught 2 by P 1 column. Do not use this column to find the P naught 2, because the P naught 2 from this expression is not the same P naught 2 from this expression. It so happens that the P naught 1 is not same for both also, but the ratios are the same. If I go and use P naught 2 by P 1 I will make mistakes. In my calculations we used P naught 2 by P 1 for normal shock examples we did this already. And I told I have extra column in my gas tables I used this, and we did it without that also. We did it in two methods.

So, I could do that, but in oblique shocks do not use this ever, unless you want to calculate it by normal this method M 1 normal method. If you are using this and finding P naught 2 and finding P naught 1 based on M 1 normal using this kind of method, then you can use this column. Otherwise, you will make mistakes if you ever take this P naught 2 and say that it is equal to P naught 2 for M 2; that will be wrong. P naught 2 for M 2 will be different from P naught 2 for M 2 normal; that is what I wanted to say here. Once I say that then everything else is clear; you cannot use this for my actual calculations. It so happens that the ratio is matching nicely.

If I start from here and use the expressions for M 2 in terms of M 1 I can go to this form finally; it is a long procedure I do not want go through it. If you want you can go prove it to yourself that this is equal to this, or if you want a simpler way to do it take ratio of this divided by this and prove that this equal to 1. It can be proved; it is just a very lengthy procedure I do not want to do it. Actually it is not very difficult; if I just take ratio of this, this and this gets directly cancelled. Everything has to the power of gamma by gamma minus 1 I will remove it. The remaining thing must be equal to 1; I have to prove that. Not very difficult, but it is very lengthy.

If you finally go and see it will just come out to be one of identical normal shock relations which you will anyway have; so, you will say yes its identity. So, it is going to be equal to one finally; that is what you will end up with. So, now next class we will go and look at what happens with around theta max; that is the next thing we want to see. What if my theta is more than theta max, and from there we will go into why is there a theta max, that kind of stuff. That we will do in next class, and then probably some example problems of how to solve oblique shock problems in the next class. See you people next time.