Gas Dynamics
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Module - 9
Lecture - 19
Oblique Shock

Hello everyone, welcome back. We were looking last class at trying to look at the 2-d problem as an analogy of a 1-d problem, like we were trying to link 1-d x versus t as looking the same as x y is unsteady 1-d problem is equivalent to a 2-d steady problem. That is what we were trying to look at.

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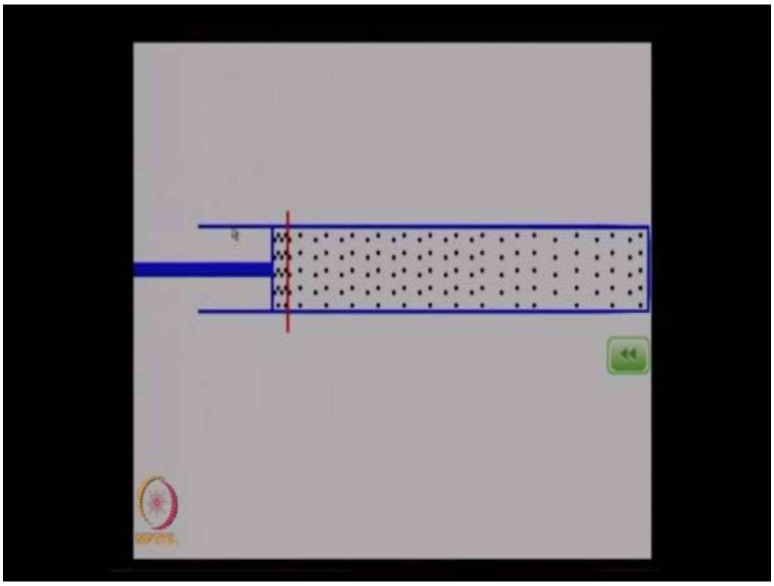

Today we have an animation to do that. Let us go to the screen. What we are having here is a piston cylinder arrangement where there is a duct, and the piston is moving and as it moves, these dots are supposed to be representing gas and the gas is getting compressed, that is the dots are getting closer to each other.

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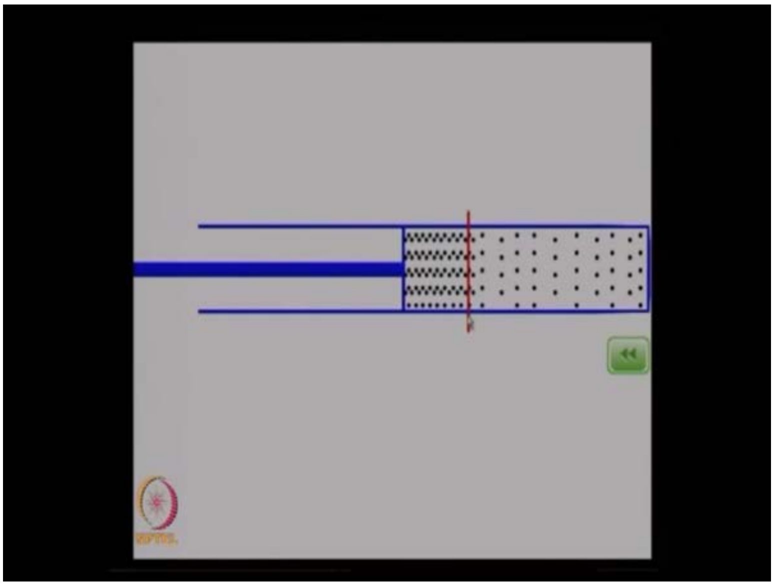

And this is the interface line. The red line which is supposed to be are shock, it is shown as if it is outside, but it is really supposed to be only inside. Can you see that, this is physically possible right such a thing is possible to happen. So, this is our we already solved a problem with this we said that if a piston moves at twenty meter per second, and the shock will be going much faster than the piston itself. We solve such a problem, this is supposed to be some such analogy.

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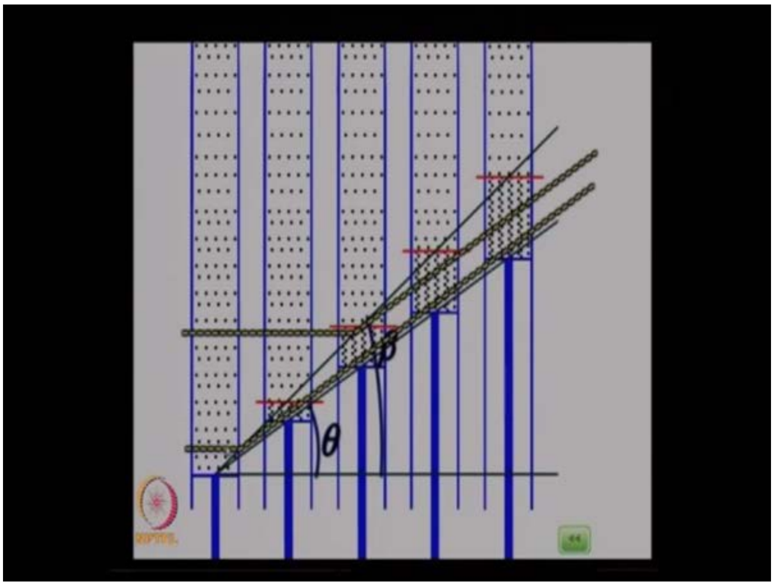

Now, we will go to the next one, which is supposed to be giving you the feel for what is happening. Here, we are moving the piston parallel to itself in the direction in this direction while the piston is moving the other direction. What we are seeing is we are drawing the trajectory of the piston along with this line and this is the trajectory of the shock. What I am doing here is x versus t whatever I was plotting before it is a same thing I am getting here, a horizontal line is roughly x versus t and the vertical line is here position horizontal line is a time axis, vertical line is a position axis.

What we are seeing is if there is something some particle that is coming here only. When it sees the shock from that point on it will start moving that is what is represented by this yellow line here and it becomes your trajectory we looked that it on the board, last time just wanted to put a animation in here and if I pick another case here, if we follow any point on this line up to this point till the shock touches. It is going to be standing in the same spot after that it starts moving up that is what you are seeing here, that is the idea. Now, if we look at the angles we are going to label them the same way what we are going to use in the class. The angle of the line representing the piston part is denoted by theta and the angle representing the line denoted for the shock is given by beta is our shock angle if it is x y coordinate system currently we are having x versus t coordinate system and if it becomes x y coordinate system theta is the angle of deflection of this incoming line.

The particle that comes here is deflected by the shock. Now, I am talking 2-d flow when I am having a wall like this along theta direction and incoming flow is straight parallel to the incoming, this yellow line then there will be a shock formed at angle beta such that, the fluid particle that comes straight when it touches that shock immediately, after that it will be turned to go along this way, that is the idea and same thing happens in this particular line also, that is what you are seeing and you can say that all these lines any lines in between these two those particles will be going in between these two, which means my overall fluid element that was here which was having a big volume.

Now, going to be compressed to have a small volume, this is where we stopped last class we ended there and now I am just going to show you the animation explanation for the same thing. Now, we do that is all we want to show in animation, let us go to the board what we want to do is we will start with how to look at this oblique shock problem, somehow we have to look at this public shock problem such that it is easy to solve analytically. So, happens that people have already figured out how to look at it, is there is a very nice way of looking at it which is again using a moving reference frame, we are already comfortable with moving reference frame from one thing to the other. Now, we are going to pick a special reference frame movement.

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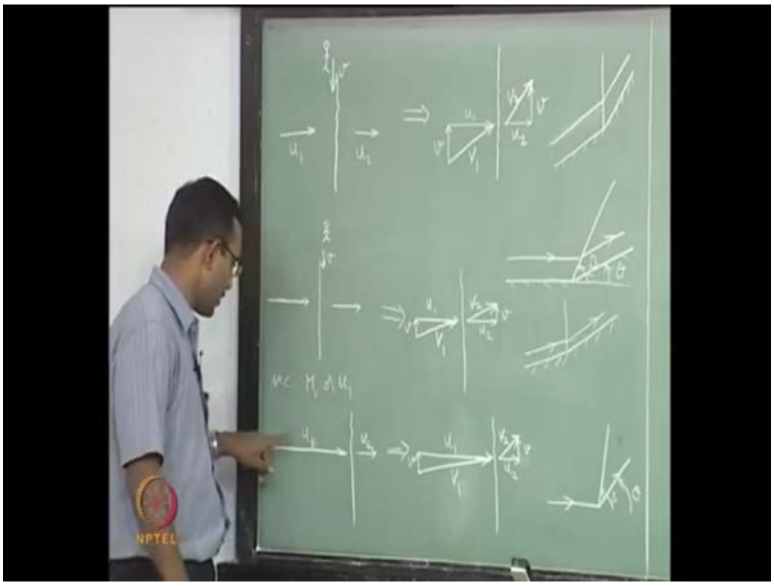

Let us start with stationary normal shock from now on when I say normal shock, it is stationary normal shock. We will go for moving only later after that, I am having some velocity u 1 coming in and some smaller velocity u 2 going out. Now, I want to have my observer, this particular person is going to be moving straight along this normal shock with a velocity V. We are going to pick such a case, if that is a case what will be the flow field as seen by that moving observer that is what we want to see. So, we are going to transform to the coordinate system as viewed by the flow field as viewed by this person, this particular observer. So, I have this original velocity vector u 1. Now, I have to add this relative velocity because the flow is going to be going this way with respect to that person I add that also.

Let us say, this is my V vector, I will call this capital V 1 this will be the velocity of the incoming fluid as seen by the observer and then there is normal shock but it is still going to be here. Normal shock does not move really, it is in fact moving along this line. We would not see it. Now, the next thing is u 2, we said it has some u 2 velocity and the same V vector should be added to it and this becomes the final net velocity. I have to mark all the velocities u 2 is the same as that plus this relative velocity V should be added the net velocity is V 2 capital V 2 there. So, overall what is happening is if I draw a particular streamline it comes at this angle and then there is a shock and then this velocity here. It is supposed to be a straight line, just assume it is a straight line that is fine. So, if I pick another streamline. Now, I have to be a very careful drawing a parallel line that is going to do something like that.

Now, in our flow we have assumed that, it is invoice id which means I can call any streamline as a wall there is no affect of boundary layer anywhere. I can call any streamline as a wall because no flow can go past that streamline. So, I will transform this to a case which I like. Now, I want to look at this problem like this till my head and look at it. Now, it will look like I have just rotated the problem slightly to the clockwise direction, this is what I will get. Now, basically if I look at the normal shock problem with an observer moving along the shock direction with the given velocity, then I am going to get some such case.

Now, I transformed a coordinate such that it is as observed by the particular observer who is moving with respect to the stationary normal shock, this is what that particular person will see it is a easy way of looking at the problem, even though I gave the analogy like the piston cylinder arrangement, this is the best way of to transform. The thing, the piston analogy which we saw in the animation will have a little bit of a error, because there we assumed that the horizontal velocities a constant before and after shock it is assumed to be constant which is not the case in real life but this particular analogy it will work perfectly, because we are moving along the line which separates the before and after shock we are moving along the shock basically. Now, we will look at what happens of course,, we have them. Now, mark those angles this is our theta and this is our shock angle beta will keep those.

Now, what if he is moving with a lesser velocity? I will draw that case again I have to draw the same kind of lengths for velocity vector. So that it is comparable. Now, he is moving with a lesser velocity V. Let us say only this length, previously it was long. Now, it is lesser V is lesser this is the movement of that particular observer. Now, this I want to transform and that is going to go to shock the same velocity here but a smaller relative velocity which will give me some other V 1, same u 1, some other V. I will get some other V 1 on the other side same u 2 a lesser V.

So, I will get some other V 2 if I look at this problem. The same way as before I transformed this to a flow field, it is going to look like this is how the streamlines will go which is equivalent to saying. There is a small angle of change here, there is a small change here, theta is lesser in this case what is happening is my theta has decreased, that is what you are noticing what happens to beta would have increased a little. Those are the things that you will see from here, if I change the relative velocity for the observer, then it is going to change the theta and beta for my problem is there any other variable in my problem of course, the strength of the normal shock itself which was our original variable in our normal shock problem. So, I can change that also, if I say I increase. Let us say I will keep the same V as this 1 but I will increase my actually, I will keep the same V as this previous case.

And I am increasing my m 1 or u 1 in our case. It has to be u 1 will just keep it as u 1, which is equivalent, if I say t 1 is the same for all the cases, it is a same thing. So, which means my velocity vector upstream is going to be longer and it is a same height. I have to draw the other case first and if this is my u 2 sorry, u 1 sorry, u 2 will be much smaller right. We know that already if we have high u 1, u 2 will be lesser, because my mark number increases. It is much stronger it will be stopping the flow much more drastically. So, it will be a smaller u 2. Now, I will transform this same way as before. Now, it is going to become I need more space. It looks like the vector itself is too long and the same V I have to add and this will be my V 1, this is my u 1, this is my V and that is my V 1 and if I pick the same U 2 from here across here. It is a small value, I have drawn it too long and I will add this V, I am going to get some particular V 2, if I turn my head slightly.

So that, this is horizontal in my eye, then I will get a particular picture, that is looking like it is turning that nicely and the shock of angle is something like this is what I will get, I just took a shortcut I should have gone and drawn this kind of picture and then came back to this picture. I just took a shortcut I am just telling you that the gap between beta and theta will become lesser if I increase my u 1. The gap between beta and theta will be higher, if I increase my V, if I decrease my V, that is what we have done till now basically, I am telling I need these two as my variables for my fixing my problem, if I want to say, I want to choose a particular oblique shock problem I have to pick a particular u 1 and a particular velocity with which my observer is moving along the normal shock.

So, if I pick these 2, then I will get to a nice condition which will match my flow problem of course,, will this work we are just going through some particular analytical approach will it work, if it is not working, I would not be teaching you this currently. That is the only proof we know. It works because gas dynamics people having using this for a long time why do we need two variables. Now, previously it was just one variable normal shock, I want to give strength I needed only one variable. That is the strength of the shock p 2 by p 1 or m 1 a mark numbers of the shock anyone variable given. I can give every other property for the shock. Now, we need two variables why is that. It is 2-d one more dimensions extra. So, I have to give some information about the second dimension. So, I need one more dimension. So, I need one more data point on that.

That is the reason, why we need two variables. Now, of course,, it is not very nice to look for this V and u 1 as the two variables instead of that we have to come up with better variables. We will go and look at that soon beta and theta could be nice variables but they are related probably I should pick m 1 and theta or m 1 and be tight will work. We will go and look at that as time goes.

Now, we will start solving the problem as if it is a fresh problem, we would not assume that, I can solve the problem by using normal shock analogy with relative velocity with respect to observer added and then it will work, because it is a very complex thing to find that V value to fit my problem. Instead, we will just go and pick, this is just to give you a feel for what is actually happening.

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Will just go start with solving as if it is some new problem in gas dynamics. I am going to say it is there is a shock which has angle beta with respect to my reference line which could be my horizontal axis and there is an incoming velocity u 1. This u 1 is different from the u 1, we have been discussing in the previous page of the board alright this u 1 is now in 2-d plane it is parallel to my reference line, that is my u 1. Now, we already know that we want to look at it somewhat related to normal shock. So, I have to decompose this into normal complement and tangential component along the shock, we want to get to these two components when I do that naturally this angle will become beta, because I am looking for tangential and I am going to label this as my u tangential ideally. I should call it u tangential one and then this my u 1, normal ideally it should be u 1 t. You will see that u 1 t and u 2 t are the same.

Now, I will draw the other picture from u 1 normal I know that there will be a small u 2 normal, because it is a normal shock based thing and we said that, this tangential velocity is like velocity along the shockwave, which is related to my the small v in the previous discussion. It is related to my moment of my observer my reference frames, if you want to think about. So, this cannot change across the shock.

So, I will just keep the same value here that is why I did not put it as u t 1, I will just remove it. Now, it is just u t u tangential will keep, just this is my u 2 normal and u tangential is going to be the same I think this is the same and this is your final velocity vector u 2 well this is u 2 u t. I have drawn it, such that this line goes straight through this. So, it looks as if the velocity vector has shifted ideally I have to move this triangle such that, this vortex comes and touches this point and move this triangle such that, the vector touches this point then only it is exactly correct but it is just to give a feel for what is happening, this is the same thing happening at every point along this shock. So, I could say that, it is the same. Now, what is my theta angle? It is the angle with respect to my original reference line in my 2-d problem made.

By the final velocity vector after the shock, if I extend this line that u 2 line if I extend that is the theta I am talking about. So, what can I tell about the relation between u 1 n and u 1 that is just direct trigonometry here. It will be u 1 sin beta is my u 1, normal u 1, sin beta and u t equal to you 1 cos beta. Now, we want to write something more of course,, I can also say that m 1 normal equal to u 1 normal by a 1, which is equal to u 1 by a 1 which is m 1, m 1 sin beta. It just comes out to be that we will just keep it there we would not.

Use this relation immediately we will use it after sometime. Now, I want to look at this triangle again in little more detail I will take this triangle draw it a little bigger I change the angles. Let us say we would not worry about this change in angle from the actually this is correct. Now, it may be close and we know that. This is our 90 degree angle because this is normal and this is tangential this is your 90 degree angle.

Now, I want to relate this u 2 with this u 2 normal, which means I should know either this angle or this angle, what I currently know is this angle. This angle is known to be theta, that is what I know and I also know that this u tangential is parallel to my shock which means with respect to my reference line. This is having an angle beta these are the things I know already. Now, I need to find, let us say I want to find this angle. So that, I will be get a sin theta relation. So, I want to find this angle, how I will find it? This is some 8th standard trigonometry I think.

So, I am drawing parallel line to this original reference line. Now, you look at a slant line with parallel line you know this angle will be beta right included opposite angle, I think that is what it is called. So, this whole thing is beta and now I know there is one more parallel line here, which means this angle is theta, this triple thing angle is theta from here to here while the whole thing is beta which means, this particular angle which I am talking about this particular angle. I am talking about will become beta minus theta, that is what it should come out to be this particular angle between u 2 vector and u t vector, that is what I should get once, I get this triangle is very simple to solve. I can relate any two vectors with the third one, because I know trigonometry.

Now, I can go underwrite u 2 normal is equal to u 2 times sin of beta minus theta I wanted sin. So, I took this angle, if I wanted cos I would have used this angle which will be 90 minus beta minus theta anyways. So, it is going to be u 2 times sin of this angle will be this which is from trigonometry simple steps. Now, u t I have another expression u 2 cos beta minus theta remember, this will be used after sometime u t has 2 values but I know u t is the same across both.

So, I can link this u 1 cos beta and u 2 cos of beta minus theta later we will do it, that is one relation between beta and theta. So, if I want to find m 2 normal all. I have to do is divide by a two similar to that, I will get it to be m 2 cos m 2 sin m 2 sin beta minus theta this is what I will get. Now, of course,, it is so tempting to just go and start using normal shock tables with m 1 normal, because we know it already because we said that only thing that matters across this u 1 normal and u 1, u 2 normal with respect to my observer. It is just a normal shock flow which means u 1 normal and u 2 normal are related by my normal shock. Let us say I do not want to do that I want to analysis from scratch as if I do not know normal shock relations and then after sometime will say yes, I know normal shock relations, if I want to do that I have to start with mass conservation momentum conservation energy conservation across the shock for a 2-d problem.

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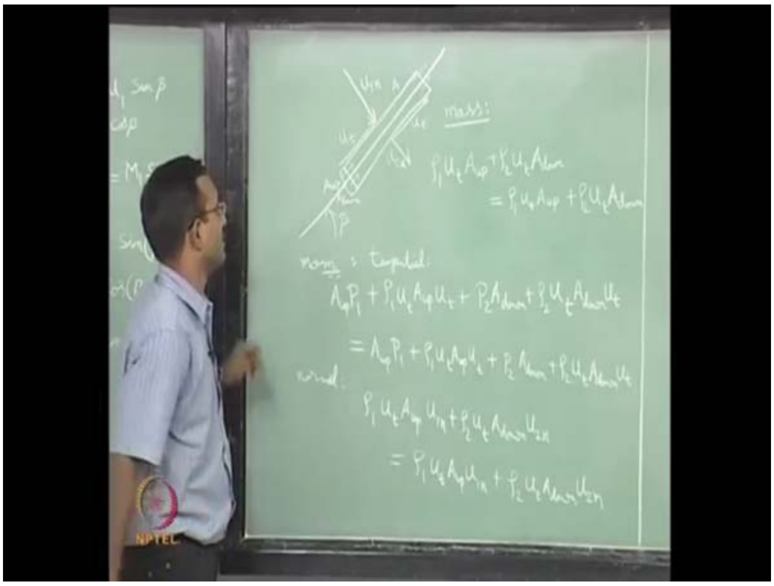

So, I will go start again with a shock at angle beta. Now, I am going to pick a control volume like, this is my control volume, I am picking assume it is a rectangle there seems to be a bend there and. I am saying there is a velocity incoming, it is u 1 normal and it also has a tangential velocity incoming u t on the other side, there is u 2 normal outgoing, it is also has a tangential velocity u t outgoing, I am drawing it differently. Now, previously I wanted to add vector. Now, I want to show that at this point this is the vector velocity I want to solve this problem by shifting my coordinate system to parallel and perpendicular to my shock.

So that, it is easy to solve, because I already know the velocity vectors in these dimensions, it is easier to solve that. Let us say the area on this line for my control volume is some area for the flow for the normal flow alone the other region. Let us say I have a upstream and a downstream and I am picking a rectangle such that, a up the same here and here and a down is same here and here, that is nice to look that particular case. So that, your equation will becomes simpler.

Now, first thing I want to do is conservation of mass momentum energy along the shock direction. First we will pick, that is along the tangential direction. So, let us pick mass, if I pick mass conservation, if I pick along this direction as it is, if it is a 1-d problem. So, I can start using my 1-d relations rho, u is constant for my control volume. It is a big control volume I am going to say rho u is constant. So, I have to find out how much is the mass entering through this section, how much is leaving this section? This section, I have to link all of them together. So, my mass expression will be rho 1 time u tangential times a upstream plus rho 2 u tangential, a downstream these are the masses. That are entering this way which is equal to rho 1, u tangential a upstream again there plus rho 2 u tangential a downstream. So, happens that this is automatically satisfied always for any value of u t it is a silly equation it looks like it is identically satisfied for any value of u t that is all we see. So, u t cannot be governed by this equation really it is automatically.

Satisfied for any value of course,, this is happening because I have pictured my control volume such that a upstream is the same for both the sections here and here it is same as down. It is same here and here if I picked something else, then I have to take into account that there is a change and then it will become more complex problems. Let us not worry about that. Now, I want to do momentum equation momentum conservation. Now, momentum conservation I have to be worried about it is a 2-d problem. So, I have to worry about two different momentums going along this way of course,, when it is going along the shock, it can be my along the shock momentum that is going along the shock or the normal momentum being carried by the mass flow, this way along the shock both are possible. Let us say, we will pick the tangential momentum, first if I pick the tangential momentum.

That is u t being carried by the mass flow, this is way, that will be rho 1 u t A up times u t, this is my mass flow rate times momentum times velocity, because my momentum rate of course,, I have to have this is normal component no. So, I should have a pressure also along with this P 1. I will call this plus on the other section P 1 A up sorry, A up is also there with this, I am looking at only this section and plus this downstream section P 2 a down plus rho 2 u t A down u t this is all the momentum entering this way. Now, this is to be equal to the momentum leaving on the other side, which is equal to if I do the same thing here. I will get exactly the same relations a up P 1, because it is still on the same side of the shock plus rho 1 u t A up is my mass flow rate multiplied by u t again plus, the other section exactly the same thing will come up again P 2 A down plus rho 2 u t A down times u t it.

So, happens that, this expression is also identically satisfied, tangential momentum equation is identically satisfied. Let us look at normal momentum, this the u 1 normal being carried by the mass flow going this way, that is also happening if I do that since it is this way momentum, the pressure term will not be there in that, it will just be without pressure normal momentum equation. There will be no P 1 related and there is no shear stress we assumed it to be 0.

So, there is no other stress on this area, it will just be rho 1u t A up times u 1 normal plus rho 2 u t A down times u 2 normal, these are the net to momentum of normal momentum being carried by this mass in this way, This is equal to on the other end, again you will get the same thing A up times u 1 normal plus rho 2 u t a down times u 2 normal again it comes up that, it looks like some expression equal to the same expression, which means it is again identically satisfied. So, we are coming up with some set of expressions which looks as if, the tangential components will work irrespective of whatever u t I pick. Now, let us I do not want to do the energy again you will again see that m dot times h if I put h plus u square.

That will again give exactly the same kind of result. So, I do not want to do it. Let us say that tangential component transfer or transport along tangential component, it is not very useful for us. It is not giving me any governing equation really, it is just telling me a equal to a or b equal to b something like that not very useful. Let us go to the normal direction. The same picture, I am going to solve for normal direction across area A.

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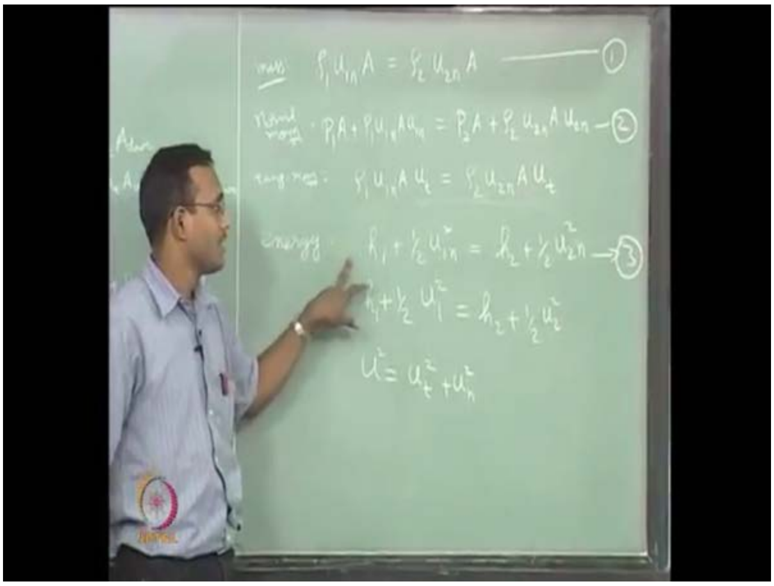

Now, I will get rho 1 u 1 normal times A equal to, this is my mass conservation rho 2 u 2 normal times A. This is one expression yes; this is very different it is not A equal to A kind of relation. So, we need to keep it, next one normal momentum component. It is going to have P 1 times a plus rho 1 u 1 normal A times u 1 normal equal to again I am using 1-d relations, because I am having flow, only along I am considering only that side. We have decoupled the problem P 2 A plus rho 2 u 2 normal A times u 2 normal. This is what, I have here does not seem like anything will get simplified, we will keep it as is tangential momentum being transported along the normal direction, that is going to look like rho 1 u 1 normal A times, the tangential velocity this is going to be equal to rho 2 u 2 normal A times the tangential velocity.

Now, in here I could have assumed that u t 1 is different from u t 2, if I assumed that then all these expressions none of them will go to 0 actually I should go back here. So, if I had gone and said that u t 1 is different from u t 2, if I ever said that then I will have a situation where none of these expressions will become identically 0, actually even then it will all become identically 0. So, I do not need to worry about this whole set of expressions tangential transport is always 0. But, even if I said u t 1 is different from u t 2.

I will go here now tangential momentum where, I will get u t 1 here u t 2 here. Now, if I go and use the mass relation inside here, we have put rho 2 u 2 normal A is equal to rho 1, u 1 normal A. Then I will get rho 1 u 1 normal A times u t 2, you will get from here that u t 1 equal to u t 2 that is what you all get finally, but if I do not do that, if I already know that u t is constant, because I am saying it is the velocity with which my observer is moving along my normal shock. Once I say that u t is the same, that is what I have used and that is the understanding.

I used to solve this if I did not use that then I will get u t 1 equal to u t 2 at this point. Now, since I used it this equation, just becomes mass equation. It is not anything new, it is a dependent equation, it is in fact the same equation multiplied by u t, it is not giving anything new will go to energy equation h 1 plus half u 1 normal square 2 plus half u 2 normal square. Now, is this correct or not really, because kinetic energy when I call it kinetic energy it does not know direction. So, I cannot just write you 1 normal square or u 2 normal squares alone. I should also take into account u t square that is what, it should be really.

So, this is not really true but wait a minute will keep this expression but I will write the more correct expression which is half u 1 square, half u 2 square I will write it this way, this is the perfect equation, this is correct as long as there is no heat transfer across there is no work done by the fluid and those kind of other expectations. This is correct, this is your basic h naught 1 equals to h naught 2 relations which we had in normal shock now, since I know from before that u t 1 equal to u t 2, when I decompose this expression u 1 square u, u will become u square will become u t square plus u normal square of course,, I can put subscript 1 or 2 for both, it will work that is why I did not put 1 or 2, if I do that and I say that u t 1 equal to u t 2. This expression reduces to this expression.

This is the more correct expression h naught 1 equal to h naught 2, this is the more correct expression it. So, it happens that, this becomes this, If I use u t 1 equal to u t 2. So, we will keep this expression also. Now, we want to say something special, I want to say that, if I pick equation is 1, 2 and 3, if I pick these three equations, they look exactly same as what we had for stationary normal shock some 8 or 9 classes before, when we derived the normal shock, we had this exact same set of equations without the subscript n, that is the only change, that is why I wanted to write this expression here.

Ideally it should be written this way and with this, it should become this, I wanted to write it like this. So that, I can just say 1, 2 and 3 directly, since I am getting exactly the same differential equations the solution should be exactly the same. It is not differential equation sorry; differential equation can have different solutions based on boundary. This is algebraic expression will give exactly the same answer, which means I do not need to go and solve this whole thing and spend two more extra lectures for getting to you, m 2 to m 1 relation it is going to be the exactly same derivation.

I will just skip that part and say that, if I put you 1 normal alone in my normal shock calculations, I will get that particular set of solutions directly from there. That is the important aspect we have to just convert to normal component from original coordinates and then we will start solving the whole problem. If I go to that picture, it is easier to look at. Not this picture. I think I will take this picture, if I go to this picture, if I am given this u 1 and this beta. Let us say, and then if I want to find P 2 by p 1 across the shock, all I have to do is go find the u 1 normal after that, if I go to the equations it will look like normal shock equations. So, I will get my solution same as normal shock tables solutions. I will go find the m 1 normal from there, I will go find the m 2 normal actually I do not need m 2 normal currently, we wanted only P 2 by P 1.

So, from m 1 normal in my normal shocks stationary normal shock tables, I will get P 2 by P 1 across and that is my solution for the problem. If I wanted the velocity direction, then I have to go find u 2 normal and find this u t. Put this triangle complete it and get the V 2 or of course, you can cheat and say I will go use this function. I will go use this function and I will get my u 2 from beta and theta I can get the relations anyway I want I can do. It is all the same it is all trigonometrics.

So, we have different ways of looking at this problem and we found that t naught 2 by t naught 1 is equal to 1 even for an oblique shock even if there is extra velocity there on the tangential component, why is that ? It is because the tangential velocity is the same before and after, that is the special reason P naught 2 by P naught 1 will be less than 1 which you will see, that is similar to normal shock P naught 2 by P naught 1, that is what you will get. Now, you want to start solving for beta, theta in terms of each other. If I ever go and write an expression for P 2 by P 1 for an oblique shock, how will I write? It is not very difficult to write, it is going to be the same expression as in normal shock.

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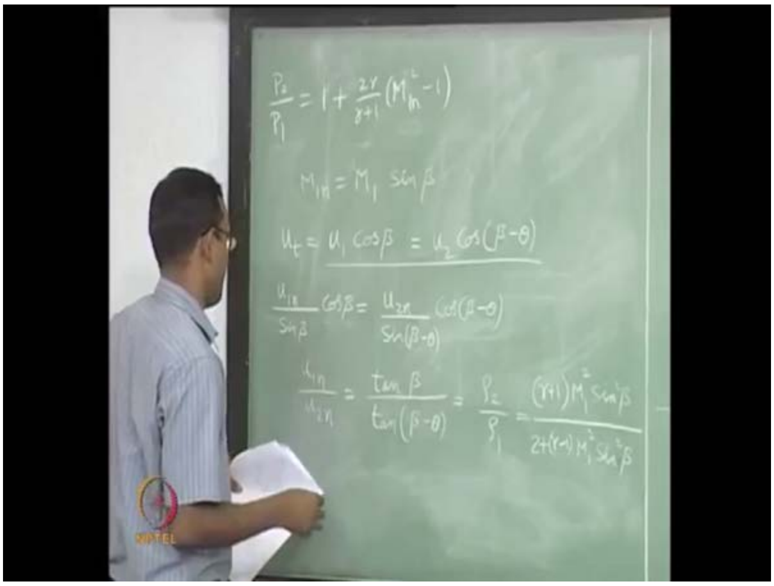

So, P 2 by P 1 equal to 1 plus 2 gamma by gamma plus 1 times m 1 square minus 1 this was one relation just check whether this relation is correct. I think it is correct. I do not have that expression here. So, here instead of it being m 1 all I have to do is make it m 1 normal, then suddenly I solve this problem to get P 2 by P 1 all I have to do is this but will I know my m 1 normal already that is related to my beta.

We said that m 1 normal is equal to m 1 sin beta. So, as long as I do not know my beta I cannot tell my P 2 by P 1 if beta is 90 degrees of course,, I will get the same thing then the shock becomes normal shock. It is perpendicular to the flow direction and u tangential becomes 0 naturally. So, to solve these, I need to basically it is coming down to I need to give you m 1 and beta for me to solve the problem, which is what we said sometime back, I need 2 variables to define this problem fully. Now, once I know this I can find theta ideally. I should be able to find theta how will I do it? Let us say we will go back to this board.

If I am given m 1 and beta and of course, I am given all the properties in state 1 then I can say that, I can find m 1 and beta from u 1 and beta. Once I know m 1 and beta, I can find u 1 normal or m 1 normal from there, I will get m 2 normal. I already know u tangential, because I am given beta u 1 cos beta will be my u tangential. It is the same value here, which means I know u t here, once I know u t here and u 2 normal.

Here, I will know the final vector u 2 and I have u 2 normal already, if I have these two I will go to this expression u 2 normal equal to u 2 times sin of beta minus theta in this. I know u 2 normal u 2. I know beta only thing I do not know is theta. So, I can get theta from here but this is such a big process to get to theta. So, instead we want to write an expression for directly theta equal to some big expression in terms of m 1 and beta that is the goal for us. The next few moments, we are trying to get to that point.

So, the way to do it we already said that u t is equal to u 1 cos beta from the downstream section. It is equal to u 2 cos of beta minus theta, we know this already. Now, I am going to use the other expression from the next page again which was u 1 in terms of u 1 normal. Now, I am going to use just these 2 as my expression u 1 in terms of u 1 normal will be u 1 normal divided by sin beta multiplied by this cos beta equal to again u 2 normal u 2 in terms of u 2 normal will be u 2 normal by sin beta minus theta multiplied by this cos beta cos beta minus theta. I have this expression from here I can write u 1 normal by u 2 normal equal to this tan beta. This will become tan beta on the denominator which I will take it to that site become tan beta by tan of beta minus theta, this is one relation between u 1 normal and u 2 normal.

We know one more, what is that in terms of m 1 normal sorry, the normal shock relation but let us say we would not write the full expression. Now, I will just call it this is equal to rho 2 by rho 1 which is true from mass equation in the normal direction. It will be true you are going to get to this form. Of course, I can write in terms of m 1 which is our old formula gamma plus 1 times. It should be m 1 square but in our case it is m 1 square sin square beta. I have used m 1 normal square and I have substituted in terms of m 1. This divided by 2 plus gamma minus 1 times m 1 square again I am going to put m 1 sin beta square m 1 square sin square beta, this is what I get.

Now, what I have to do is take this equal to this and manipulate it such that, I will get tan theta equal to something I have to expand this and make it tan theta equal to something instead of directly starting with this. I want to write this expression for so many times. I will just put rho 2 by rho 1 for a few seconds and then we will go on substitute this to get to some big expressions. I would not finish this today I will go to a point where rho 2 by rho 1, if I substitute I will get a good answer I would not substitute rho 2 by rho 1 today.

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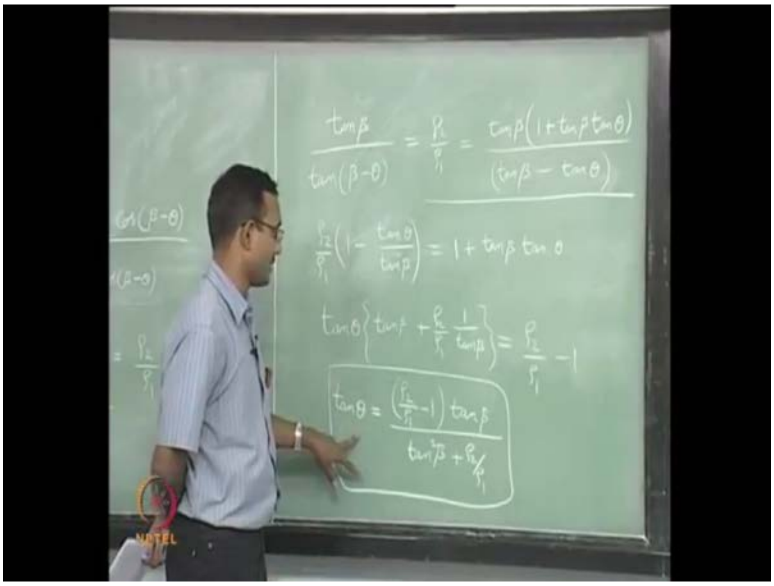

So, I have tan beta by tan of beta minus theta equal to rho 2 by row 1. Now, you should know my expansion for tan of beta minus theta is equal to tan beta minus tan theta divided by 1 plus tan beta tan theta. You should know this if I use that expansion, then I am going to get to a form where this is equal to tan beta multiplied by 1 plus tan beta tan theta divided by tan beta minus tan theta.

Now, what I want to do is club this denominator with this tan beta take it to my left-hand side of this expression just this equal to this expression and I do that I will end up with rho 2 by rho 1 multiplied by 1 minus tan theta by tan beta and this is equal to 1 plus tan beta tan theta, this is what I will have. Now, I want it tan theta equal to something I have to group all the tan theta terms together. So, I will take this tan theta term to the other side and I will write tan theta terms on left of my new equation tan theta multiplied by it is going to be tan beta in here, tan beta this minus sign when it goes there.

It will become plus. So, plus rho 2 by rho 1 times 1 by tan beta, this is 1 term. Now, I will take this 1 to the other side, this is equal to minus 1 here and rho 2 by rho 1 here, rho 2 by rho 1 minus 1, this will become i expression. So, I will end up with tan theta equal to rho 2 by rho 1 minus 1 times tan beta times tan beta divided by tan square beta plus rho 2 by row 1. Now, I have an expression for tan theta which is from which, if I invert tan, I will get theta in terms of rho 2 by rho 1 and beta where. Now, my rho 2 by rho 1 depends on my m 1 and beta m 1 sin beta. It is related to m 1 normal. So, next class we will substitute that m 1 normal in here and simplify this expression everything in terms of m 1 which is the original mark number and beta. So, we will continue that part and it will give you a very nice expression will do that next class. See you people next class.