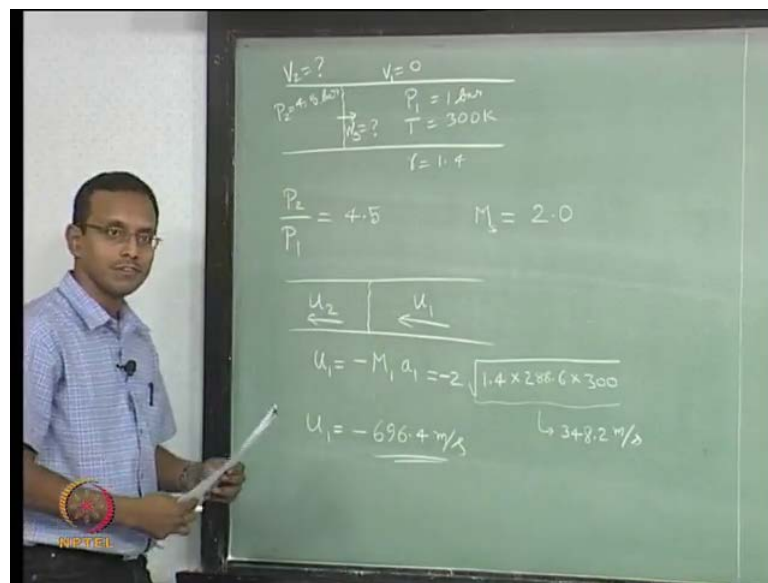


**Gas Dynamics**  
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**Module - 7**  
**Lecture - 17**  
**Solved Numerical Examples for Moving Shocks**

Hello everyone, welcome back. We are last time solving some problems from Gas Dynamics from the point of view of stationary shocks. This time we want to move on to moving shocks, we just want to solve problems with shocks that are moving. And that will give you a little more physical feel, which will directly lead us into going to oblique shocks; that is where, we are going to... So the first problem we want to solve is this.

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I am given that, I have a 1 d duct and it has stagnant air pressure 1 bar, temperature 300 Kelvin and I said air, so gamma equal to 1.4, this is stagnant air in this duct. And I am told that, there is a shock and it is moving, and behind the shock, pressure is 4.5 bar. Now, I am supposed to find the shock speed and the speed of the gas behind, everything with respect to the duct, the outside reference frame, so that is the basic problem.

So, to characterize the shock, what should I do? Basically I have to just start looking at the problem and immediately say the information given about the shock happens to be pressure P 2 divided by P 1; P 2 by P 1 is given for us across the shock, no other

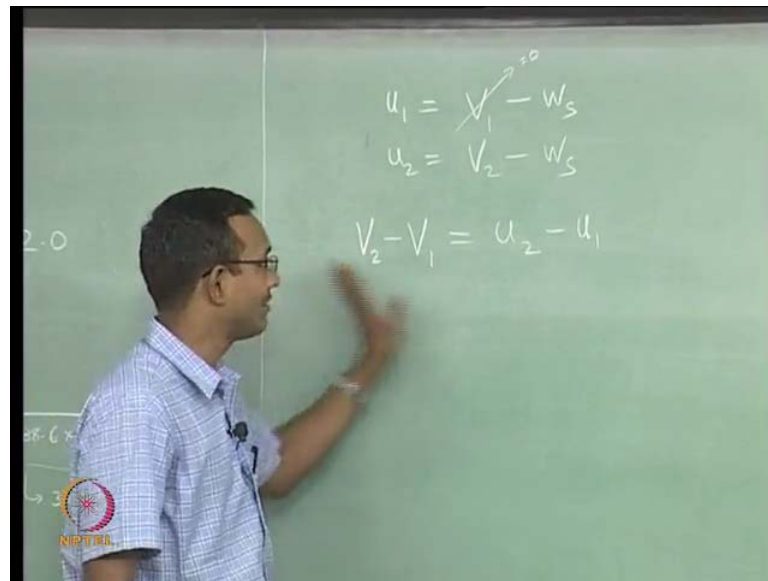
information is given across the shock, so the only information about the shock is  $P_2$  by  $P_1$  is given to be 4.5. Now, I will go to my normal shock tables and look at  $\gamma$  equal to 1.4 page and we will find that  $P_2$  by  $P_1$  equal to 4.5 occurs for mach number equal to 2.0 basically mach 2 shock that is what we are having.

So now, we have shock strength, once we have this information I do not want to really solve the problem in this coordinate system. We know that, this is  $V$  equal to 0,  $V_1$  equal to 0,  $V_2$  we are supposed to find out and shock speed  $W_S$  we are supposed to find out, that is the basic problem. So, I want to find in the other coordinate system, I will write here shock fixed coordinate system, I am going to have a  $u_1$  and a  $u_2$ . And now, I want to follow one convention for velocity vector direction, I want to follow the convention for original problem.

So, I am going to say, this  $u_1$ , this is supposed to be negative velocity, will keep it that way, we switched from that coordinate system to this coordinate system and I am going to say  $u_1$  is going to be negative. Because, it is going this way, I am saying  $V_2$  is positive,  $W_S$  is positive that is the way we are going to solve the problem. Keep one convention it will not confuse you, it will be very easy to solve, you will never get into trouble.

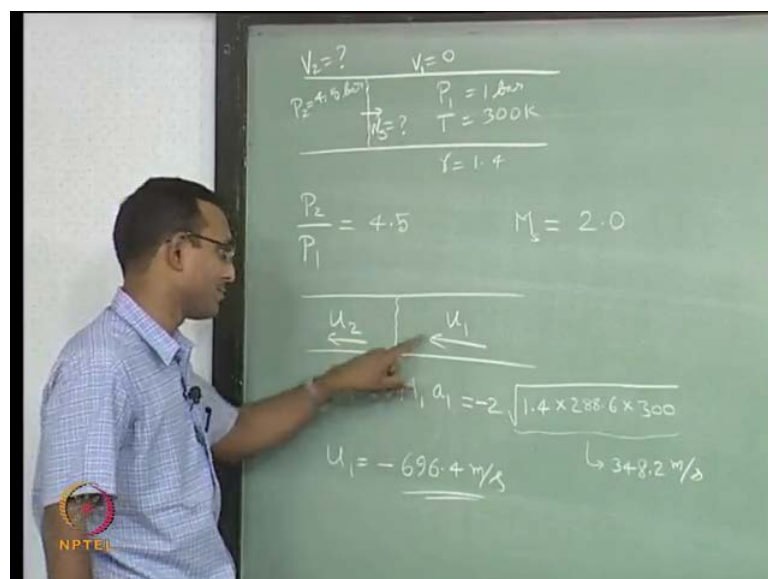
Now, I want to find  $u_1$ ,  $u_1$  is going to be minus of  $M_1 a_1$  because it is stagnant gas, it is like a bus travelling against somebody stationary, what is the speed of this person with respect to the bus, it is the same velocity the other direction. So, I am just directly minus  $M_1 a_1$ , this is going to be equal to minus 2 into square root of  $\gamma R T$ , 1.4 into 288.6 and 300. Just so we want to use this number again and again, I will give you value for  $a_1$ , it is 348.2 meter per second and so my  $u_1$  is minus 2 times that, which is 696.4 meter per second, this is my  $u_1$  that is given. Innovators given because  $P_2$  by  $P_1$  was given from there I found  $M_1$ , I do not want to call it  $M_1$  I will call it  $M$  of shock and. So, I am going to get to this point, we solved up to this point now, we will go and write the equation, which we already wrote before.

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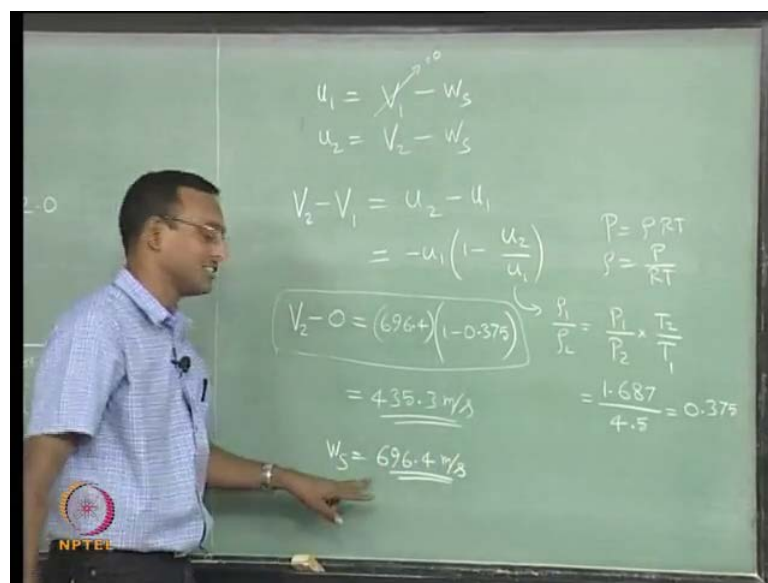
$u_1$  equal to  $V_1$  minus  $W_s$  and  $u_2$  equal to  $V_2$  minus  $W_s$  this is the transformation between the 2 coordinate systems, these are the expressions which are going to be used always in solving moving shock problems, very easy to solve this way. And we know, in our case,  $V_1$  equal to 0 and this information has already been used, we already found  $u_1$ . Now, all I have to do is substitute that inside here and then now I am going to have set of expressions to solve for it. Instead of that, I am doing it some other way, let us just go for this  $V_2 - V_1$  equal to  $u_2 - u_1$ . For me to write this, I should have one coordinate system direction for all the velocities.

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If I picked  $u_1$  to be positive in the other picture, let us go back here if I picked  $u_1$  to be positive here then  $u_1 - u_2$  will be positive because  $u_2$  should be less than  $u_1$  in the stationary shock condition. So,  $u_1 - u_2$  will be positive but in here,  $V_2$  is greater than  $V_1$  because the shock is moving that is what is telling, the gas to move this way. So, this will be positive velocity,  $V_2 - V_1$  is positive and here,  $u_2 - u_1$  is negative, this kind of problems will come up if you do not take care of the sign of velocity. So, always keep the sign information, I put an arrow this way, so I am going to call it negative velocity, keep it that way it will help you solve the problem easily.

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Now, I can rewrite this as minus  $u_1$  times  $1 - u_2$  by  $u_1$ ,  $u_2$  by  $u_1$  will not be effected why, both the velocities are negative, ratio will not get affected. Both the velocities are negative in shock fixed coordinate system so things will just work naturally, no problem for us. Now, we can just start solving the problem, how will I know  $u_2$  by  $u_1$ , I can get it from  $\rho_1$  by  $\rho_2$ ,  $u_2$  by  $u_1$  is equal to  $\rho_1$  by  $\rho_2$  from mass conservation across the shock.

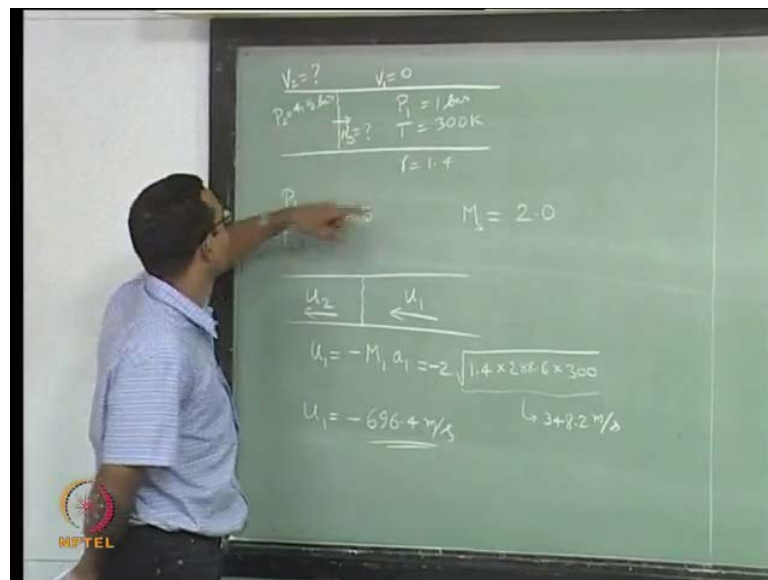
How will I get  $\rho_1$  by  $\rho_2$ ,  $P$  equal to  $\rho R T$  or  $\rho$  equal to  $P$  by  $R T$ , I will write it like this where, this  $R$  is specific gas constant of course, you do not need it in ratios but anyway I am telling you. So,  $\rho$  so I will write it as  $P_1$  by  $P_2$  into  $T_2$  by  $T_1$ , this is all I get it to be, most of the gas tables the compressible flow tables or gas tables, none of them will give you  $\rho_2$  by  $\rho_1$  across the shock. Because, they already have given

you P 2 by P 1 and T 2 by T 1, they do not need to give this separately one column, just waste of paper if they give it, they will never give it.

So, all I have to do is, just go to that particular condition, which we know it is not very difficult, we know that it is mach 2 now. I have to go to that particular row, which has P 2 by P 1 equal to 44.5 or M 1 equal to 2, that condition. I will find T 2 by T 1 value, that happens to be 1.687 and divided by P 2 by P 1 value, which we know it is given in the problem 4.5 and this number comes out to be 0.375. So, basically it is coming down to 1 minus 0.375 now, I know minus u 1, we just calculated u 1 to be 696.4.

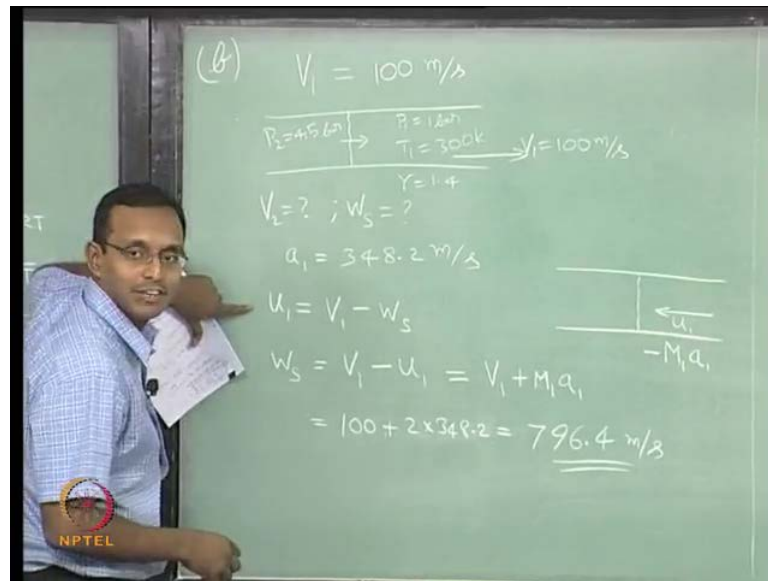
So, V 2 minus 0 equal to, I put 0 of V 1 is 0 for us, 696.4 into 1 minus 0.375, this becomes your expressions. Now, you can just directly solve, V 2 comes out to be 435.3 meter per second now, have I found the shock speed, yes because from this expression, minus u 1 happens to be the W S. Minus u 1, we already put a number here, shock speed happens to be 696.4 meter per second, it is positive which says it is moving towards the right, from left to right, V 2 is positive, velocity is from left to right.

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So, if I go back to the picture and say what is happening, I am going to say the shock is moving at 696 meter per second and the flow behind it, is going at 435 meter per second. That is what, is the final outcome of this whole problem, we will go and look at another problem with a little more modification.

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I called this one problem a, I want to refer to this problem again so we will call this one problem b where, we want to say that,  $V_1$  is non zero, it is given to be 100 meter per second, exactly same problem.  $V_1$  equal to 1 bar,  $T_1$  equal to 300 Kelvin, gamma equal to 1.4 it is air so  $R$  is also 288.6, there is a shock moving and  $P_2$  is given to be 4.5 bar, same problem as before. Except for  $V_1$  is given to be 100 meter per second from left to right that is, there is a velocity here already now we are supposed to find the same thing, what is the value of  $V_2$ ? And what is the value of shock speed.

These are the 2 things we want to find, just an extension to the previous problem now, we have one variable non zero, that is all. I picked the same number so that, I can go a little faster, I can use the same  $a_1$  value so I will just put a 1 otherwise, you just have to calculate, square root of gamma  $R T$  348.2 meter per second. Now, I will go and write  $u_1$  to  $V_1$  minus  $W_s$ , I told you this is the expression we keep on using, we will use this. Now, we have a  $V_1$  that is non zero. Previously it was 0 in the previous problem now. I have to go and find out  $W_s$ ,  $W_s$  will come out to be, I will take it the other way  $V_1$  minus  $u_1$  and now, this is going to be  $V_1$ .

Now, you have to be little careful, in shock fixed coordinates, I will draw another picture here, this is  $u_1$  and it is having a value  $M_1 a_1$  in shock fixed coordinates. Now, I want to make sure that, when I use this problem, I am not going to be messed up I want to use it correctly. One way of doing it like this, after that I will tell you another method both

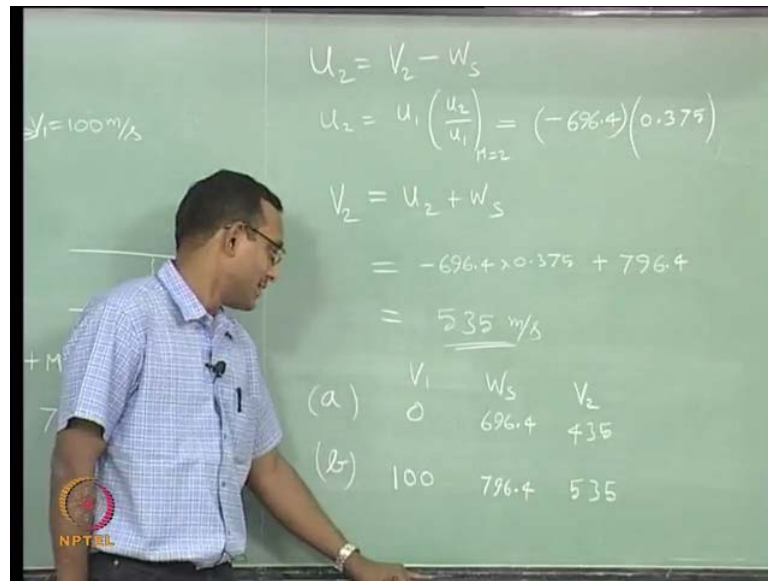
should match exactly so I am looking at this problem and I am going to say, I have to put  $u_1$  as a minus of  $M_1 a_1$ .

Why am I putting this minus sign, I already did this in the last problem also but I did not specifically tell this clearly there. But here, I have to tell it, it is going from right to left that velocity is negative, if it is left to right that is positive for us. So, I have to be very careful,  $M_1 a_1$  is the magnitude of the velocity, it does not tell you the direction so I have to be very careful and put a direction minus  $M_1 a_1$ . Now, I will substitute that so it just becomes  $M_1 a_1$ , if this value comes out to be positive then the shock is moving to the right, from left to right, that is the way we are going to do it.

Another way to look at the same problem, I will get the same answer from another method, I will tell you that method also. Imagine this, this 100 meter per second flow is like a river flowing and the shock is moving on the river or it is like a boat going on the river. If I think about it that way, there is a boat moving with respect to the water in the river and the water itself is moving. So, the actual velocity of the shock from outside, actual velocity of the boat for somebody was standing outside, will be velocity of the river plus the velocity of boat with respect to the river.

So, it will come out to be that  $M_1 a_1$  plus the velocity of the river, that happens to be this 100, that also will give you the same answer  $V_1$  plus  $M_1 a_1$ . You have to just a crosscheck, I do not need to do it to 2 different methods, one is enough, if you just follow the math, it will come out to be right as long as you do not make any mistakes. So now, we just find this number, it is we know that  $M_1$  is 2 because  $P_2$  by  $P_1$  is given to be 4.5, it is the same value. So, I am going to say it is 100 plus 2 into 348.2, the answer comes out to be it is almost the same answer, 696.4 was the previous answer plus 100, 796.4 meter per second this is the answer you are getting here. Now, I want to go and solve  $V_2$ , now similar to this formula we have a  $V_2$  formula  $u_2$  equal to  $V_2$  minus  $W$  S.

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So, how will I find my  $u_2$ ,  $u_1$  into  $u_2$  by  $u_1$  at  $M$  equal to 2, easy way to do it, gas dynamics people like this a lot, canceling ratios very nice way of doing things. It works great for us because in all compressible flows tables, in all gas tables whatever you pick, they all give you only ratios so I have to multiply all the ratios to get to whatever number I get.

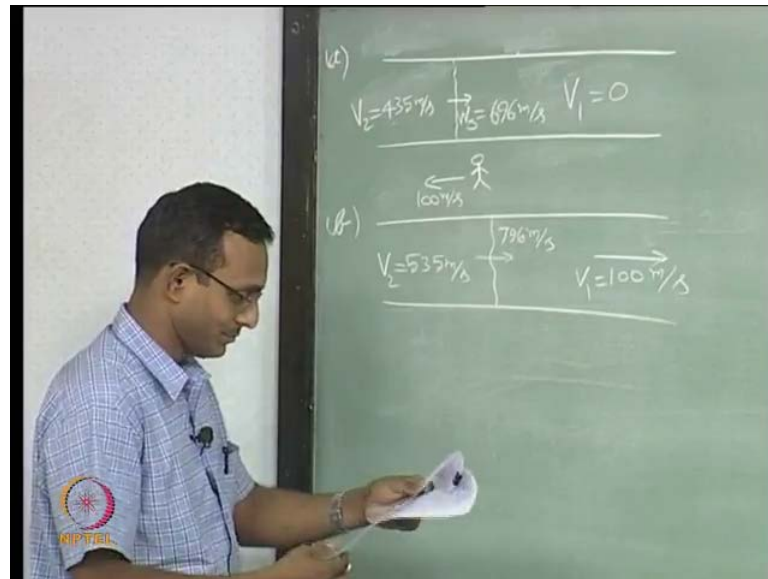
So, I know this  $u_1$  value, it happens to be minus 696.4, this multiplied by  $u_2$  by  $u_1$  I want to find out,  $u_2$  by  $u_1$  we already found out from the previous problem it is the same value, it has to be  $T_2$  by  $T_1$  divided by  $P_2$  by  $P_1$ , that we found out to be 0.375. If I do this, this is just  $u_2$  now, I have to find  $V_2$  so I will write it  $V_2$  equal to  $u_2$  plus  $W_s$  remember the minus sign, if you do not use the minus sign, things will go wrong. Minus 696.4 into 0.375 plus 796.4, this is what it comes out to be and the answer happens to be 535 meter per second, this is what you are getting.

Now, I want you to compare problem a, and problem b, I will make a small table  $V_1$  for problem a was 0,  $V_1$  for problem b was 100,  $W_s$  696.4 and here, it is 796.4,  $V_2$  435 here, it is 535, these are the numbers we are getting. If you go look at  $u_1$ ,  $u_2$  those numbers will be different, other numbers if you look at it, the same to be matching very nicely like this in fact, every number will have this 100, it so happens that it will be there.



Now is this logical, is this going to be just this, if I have a  $V_1$  some value, I just added to all the values and it will come out to be this answer always, that is the question we want to ask. It so happens that, for this type of a problem, yes it is, why that is the question we want to ask. Now, we will go to that side and start drawing pictures, more pictures so I have all the pictures I need it looks like, I have all the pictures I need.

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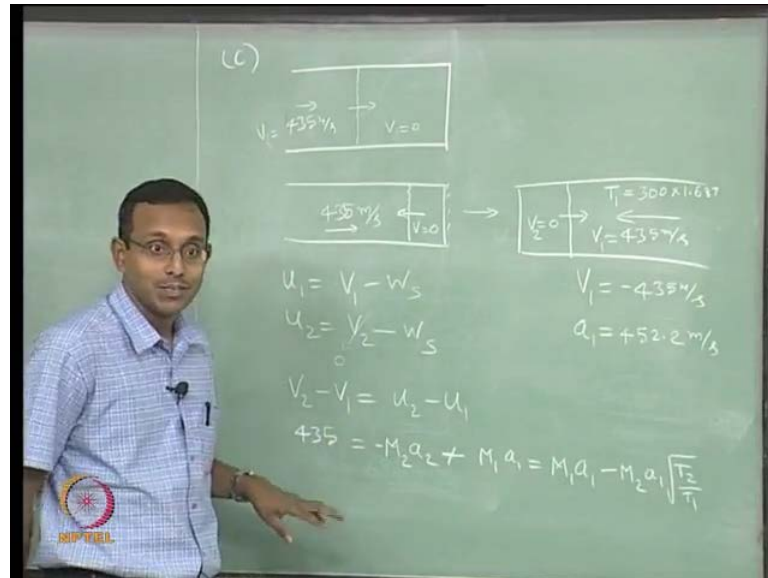


This was problem a and problem b picture, I have to change, I have to draw new pictures, problem b I am putting 696 it was 696.4, that does not matter so much for us. If I draw this picture, it becomes easier for us to understand the problem, if I look at the picture from problem a, I have this case and if I say, I have an observer standing here and he is observing this particular problem. Now, if I say, he is not just standing there but he is moving this way with 100 meter per second, he is on a bus if he is moving like this, what will he see as this velocity 100 meter per second.

What will he see this problem as 696 plus 100, what will he see this plus 100, that happens to be this problem exactly. It is just a shift in coordinate system, person who is watching is moving, that is the only change if you look at it, moving shock problems mostly it will come out to be something like this, it is easier to work with. For a given strength, there is a particular  $\Delta u$  you will get, if the change just this variable, all I have to do is add that number everywhere and the problem is solved, we do not need to resolve the problem. That is, assuming we already solved the problem once for one

particular V 1, if the change just V 1, very easy to solve just add it, that is all it suppose to be. Now, we will go to next problem, continuing the numbering I will call it problem c.

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We will continue the same problem in a way, I am going to say the shock is going like this and it reach the wall, there is a wall at the end, it should be a straight line, there is a wall at the end. So, what is happening now, it is a shock it is going like this and there is a flow coming here, what was the velocity there, we will pick the problem a, 435 meter per second, this is V 1 equal to 0, V 2 is this and this value was 696 meter per second whatever.

Now, this problem does not change till the shock goes and hits the wall, after that the shock does not have to tell any more gas. What was the job of the shock, it has to go and tell everybody move that way and it has told all the gas that is present in the tube, it is job is done, it vanishes. But now, there is a problem, what is the problem, will go to the case where shock is reached the end so I will draw one more picture. There is no V 1 equal to 0, every condition is all this value, all the gas is moving with 435 meter per second into a wall.

Now, there is a problem, things cannot go just through a wall, wall stops all the flow which means, now there should be a new shock created here. And that has to be moving against this flow, this is going like this such that, the velocity behind it is supposed to

come to 0, that is the job of this shock. This shock is going and telling this, what they call the reflected shock, this shock's job is to go and tell every fluid that is coming this way that, there is a wall you cannot move this way.

So what will happen, all the people thought, all the molecules actually, we will go back to molecules, all the molecules thought there is a lot of space they are all coming this way that is what, this shock told them. When the shock went here, till that point there was no information about this wall to this fluid so they all kept coming. When the first layer of fluid that touched this, when the shock that touched happen immediately after that, that fluid knows that there is a wall there.

So, it tells every other fluid this way coming in, that do not come in this way there is a problem here, that information is going this way. What will happen, this inducing a velocity this way, in a way it is slowing down this fluid coming to the wall that is what, is happening here, this is the physical feel. Once you have the physical feel, problem is easier to solve and it has to come to  $V$  equal to 0 that is what, we are trying to solve. Now, instead of looking at the problem with shock going right to left with all velocities negative, I will just go look from the behind of the board.

What will it look like, I will just transform this problem to I turn the board the other way and look at it from the other side. Then, it will look like, this is the problem we are going to solve, here I am given  $V_1$  and  $V_2$  both the values are given, I am renumbering things it is no more connected with problem a. I am renumbering it here  $V_1$  and  $V_2$ , this old numbering I will remove, would not get confused with these numbers. This is the problem we want to solve now, we are supposed to find what is the speed, at which the shock goes.

So, let us go and find out how to do this so again we are going to write the same set of equations, we will say  $u_1$  equal to  $V_1$  minus  $W_S$ ,  $u_2$  equal to  $V_2$  minus  $W_S$  remember, there is no compressible flow tables for moving shocks. So, we have to do this transformation of coordinates everytime to go to stationary shock case and then use the tables come back to moving shock problem, transform to one side come back transform again.

We are now given  $V_2$ , this happen to be 0 and  $V_1$  has a value 435 meter per second, that is given to us. So, I will again go and write  $V_2$  minus  $V_1$  equal to  $u_2$  minus  $u_1$

remember, to keep the velocity directions important. If it goes negative if it goes leftwards, it is minus sign, will keep that into account. So,  $V_1$  minus 435 meter per second, I will write it here I have put an arrow on 435 so that, we do not want to touch that, I will change it only here.

$V_1$  equal to minus 435 meter per second which means, it is going right to left, will keep it that way and  $V_2$  is zero. So, I am finally getting it to be this side 0 minus, minus 435 will be 435 equal to now, I do not know  $u_2$ , I do not know  $u_1$  and I am supposed to guess mach number of the shock or  $P_2$  by  $P_1$  of the shock. So, we have to work through this problem only, how will I do it, let us say, I will write  $u_2$  as minus  $M_2 a_2$  remember, minus is coming because in my shock, fixed coordinate it is going from right to left.

Similarly, I will do for  $u_1$ , it will become a plus  $M_1 a_1$ , do I know  $a_1$ , yes I know  $a_1$  because I found the temperature here.  $T_2$  by  $T_1$  was known, for the problem a,  $T_2$  by  $T_1$  was I think 1.68 something like that, I did not give you that part of the problem anyway, I will just do it right now, 300 into 1.687. We wrote this  $T_2$  by  $T_1$  value before in fact, in this page in fact, it is slightly shown here, anyway. So from there, I will get a 1 value I have to find square root of  $\gamma R T$  and that comes out to be 452.2 meter per second.

You get it to this value, you know up to here, I do not know  $M_1$  but if I know  $M_1$ , I will know  $M_2$  now a 2, can I write in terms of a 1. Yes I can, will write it that way  $M_1 a_1$  minus  $M_2 a_1$  into square root of  $T_2$  by  $T_1$ , can I write it like this. Yes because a 2 by a 1 is just square root of  $\gamma R T_2$  divided by square root of  $\gamma R T_1$  so I can simplify to this. So, what I did is, a 2 is equal to a 2 by a 1 times a 1, again the favorite multiplication of ratios, gas dynamics deal with.

So now, I have this, I do not know  $M_1$ , I know  $a_1$ , I know  $a_1$ , I do not know  $M_2$ , I do not know  $T_2$  by  $T_1$  but if I know  $M_1$ , I will know  $M_2$  from normal shock tables, I know  $T_2$  by  $T_1$  from the same row normal shock tables. So, if I know  $M_1$ , I know this whole expression and then now I have to see, whether 435 is matched so it is now a problem of iteration. I have to guess an  $M_1$  and see, if 435 is matched by this side and I keep on doing this, till I solve the problem.

I will start guessing  $M_1$  value, next value, next value till it solves, instead of writing it like this, we will rearrange this and write it a little better, will go to the next section.

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$M_1 - M_2 \sqrt{\frac{T_2}{T_1}} - 0.962 = 0$

$M_1$	2	3	1.5	1.70	1.72	1.74
$M_2$	0.577	0.475	0.701	0.641	0.635	0.631
$T_2/T_1$	1.687	2.679	1.32	1.458	1.473	1.487
LHS	0.285	1.26	-0.267	-0.036	-0.013	0.008

$M_1 = 1.74$

$W_s = M_1 a_1 + V_1 = 351.8 \text{ m/s}$

$\frac{P_2}{P_1} = 3.336$

It so happens that, for most of the problems it will be this,  $\Delta V$  by a 1 equal to  $M_1$  minus  $M_2$  square root of  $T_2$  by  $T_1$ . I will keep getting this always in most of the moving shock problems where, I do not know the shock strength. The previous problem section a and b when be solved, we knew the  $P_2$  by  $P_1$  for the shock. So, it is very easy to solve. Here, we need iteration but this is the general form where,  $\Delta V$  is  $V_2$  minus  $V_1$  that is what, you will have.

Now, will get to numbers so I will take this also to that side, I will write a function that looks like  $M_1$  minus  $M_2$  square root of  $T_2$  by  $T_1$  minus, I know  $\Delta V$  was 435 and I know  $a_1$  is 452.2 I will get that number to be 0.962 equal to 0. So, I have to form a table for iteration  $M_1$ ,  $M_2$ ,  $T_2$  by  $T_1$  and the whole thing, I will call it left hand side. So, we have to start guessing, each column will be one guess, I will start guessing with mach number 2.

And I go to normal shock tables,  $\gamma$  equal to 1.4 and I look for  $M_2$  that is, 0.577,  $T_2$  by  $T_1$  happens to be, we already know this number 1.687. Now, if I do this calculation on the left and side, that number comes out to be 0.285, I do not know how to guess the next number. I will generally guess 3, I could be having to guess less than 2 or more than 2 I do not know, I say I choose 3, will see what happens. This number is 4.475, again

normal shock tables gamma equal to 1.4 for mach 3, this number is 2.679 and this is 1.26 so it looks like I went the wrong direction.

I guessed wrong, I should not be guessing from 2, I should have guessed lesser, I guess the more, the number went the same direction. Most of the functions in gas dynamics are monotonic, it is a good thing for us that is what, is used here. Now, I will go guess something lesser, this is very close to 0 and when I go from one number increase, it is too high. So, I will guess something very close to this but lesser, something nice I will start with 1.5, that number is 0.701, 1.32.

And now, my left and side comes out to be minus 0.267 now, I know the answer is between this and this, I just have to guess a number in between. It so happens that, in my gas tables there is no 1.75, my gas tables has 1.7, 1.72, 1.74, 1.76. So, I thought I will pick 1.7, I am still guessing I am not interpolating, I could have interpolated and got to the answer faster. I got this answer which says, I should go more towards 2 even now, I could interpolate but it is so close that, we will just try the next number, in the next row in my gas tables, that is easier.

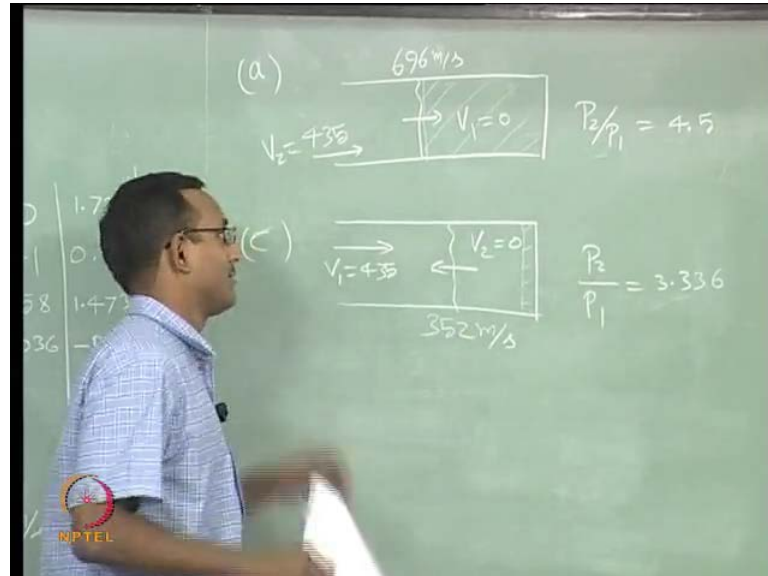
So, I will go 1.72, this is 1.70, the next one is 1.72 and that number is 0.635, 1.473 and this number comes out to be minus 0.013 it is even close but still the same side. So, if I took a interpolation probably, I would have been a closer match, I do not know I am guessing so it comes out to be this, I could have picked 1.74 instead of, 1.75 I picked 1.7. Now, we will look at the next one 1.74, I am moving the correct direction, 0.631, 1.487 and this number is 0.008, this is the closest I could get and I have crossed.

If I want at this point, I could interpolate between these two to get to a closer answer, which will probably be 1.735, which if I approximate to second decimal the answer is 1.74 so this happens to be my mach number. Typically, in gas dynamics, if you guess right at the initial conditions, you will solve in the 6 th or 7 th iteration, most of the problems. Unless you start with say, mach 20 and slowly decrease every 2 then you will take some so many iterations to get to the answer.

So, once I know  $M_1$  equal to 1.74, I will go and find  $W/S$ , we had the formula on the board before so I will just write it. This you know, how to derive it from two different methods, this number happens to be 351.8 meter per second, this is the speed with which the shock moves with respect to outside observer with respect to the duct. And  $P_2$  by  $P_1$

for this shock is 3.336, now we will again observe stuffs, I have numbers anyway, I will erase and do it.

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Problem a, we had a case where, a shock is moving this way  $P_1$  equal to 0, this was 435 and this was 696 and now we are saying, it went and it found a wall and so that becomes your what was this problem, problem c. Problem b I said, I did not erase b probably here, it is not problem b we are working on problem c I think. So, I just did not erase that part, anyway problem a set this, problem c it says, now this is the velocity here now, I am thinking about shock going this way and the flow incoming is 435 meter per second,  $V_2$  is 0.

And when the shock goes this way, it is going with velocity we just found 351 meter per second, it is close to 352 meter per second. We are not interested in the actual number I just want to tell you the trend, here  $P_2$  by  $P_1$  was 4.5 and here,  $P_2$  by  $P_1$  is 3.336 some such number, I could have written as 3.34 that is enough. So, what we are finding is, when my shock goes first, it is having some particular pressure ratio and then it is created a supersonic flow.

Actually, it was not supersonic, was it  $M_2$  we did not calculate, I think I calculated I did not do that part for you people,  $M_2$  was 0.577 it was not supersonic really, the temperature is also high, velocity is high and temperature is high, it is still subsonic flow. Because, the speed of sound for this case, a 1 we picked was 450 or something like that,

something higher. So, it is still subsonic case somewhere here, anyways it is subsonic flow created, it is going like this and it is suddenly meeting a wall and that information has to be transferred to the whole fluid by the second shock.

It is not moving very fast as fast as this one, it is moving slower but with respect to the fluid, it is actually moving very fast. With respect to the fluid if I think about it, it is going extremely fast. The fluid is going this way but still the wave is going this way, it is like a treadmill you know, it is like a treadmill what is the thing the belt, the belt is going this way with 435 meter per second and the person running on it is running this way for 352 with respect outsider.

Which means, is actually running the total velocity added these two, which is 787 meter per second, the shock is actually travelling faster in the gas that is heated, the main thing is the gas is hot, it has been heated by the first shock. So, it is traveling very fast but when it reflects, it does not need such a high strong shock for it to tell the fluid to stop, that is the main thing here. Previously, it was going with 4.5 now, it is going with 3.336, that is enough.

With that, it can say stop there is no more movement needed, that is the information taken by this shock, that is all it is carrying. Overall problem if you look at it, I had a duct, I had zero stagnant gas initially, there was a shock sent in P 2 by P 1 4.5, it created a flow of 435 meter per second this way and then it stopped and that information is carried by this. And then every gas that is processed by this second shock will come to stop, it will not move any more.

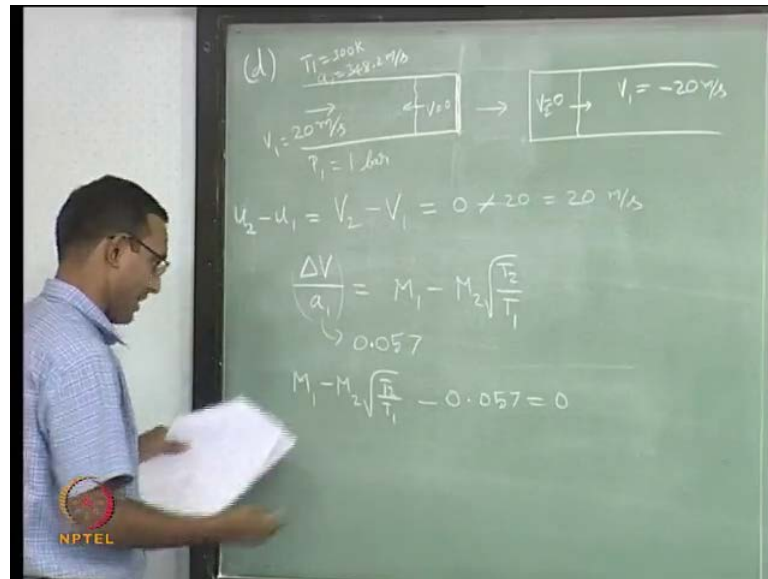
What happened to the overall gas that was initially sitting in this whole region now, it is compressed and it is sitting in a very small region here, that is the other thing that you have to think about. If I think about let us say, I will pick this whole volume, that whole volume is probably now sitting in this small area here, that small volume here why, that gas was compressed twice once by this, once by this. So, overall if I want to call it P 3 by P 2 then I can say the net final pressure is 4.5 times 3 of the order of 14, of the order of 14 times compressed.

So, the volume will be 1 by 14 of that roughly, it need not be that because temperature is also changing, it is roughly that. So, all that volume is now only here, all that gas which was initially there let us say, if I had this gas or some color gas, all that color will be only



inside this small region, what is here then the gas that was in this volume here, like that it gets compressed on the other side, you have compressed it twice with two different shocks. I guess I will not be able to complete problem d fully, we will see how much ever we can go, if needed we will stop with the iteration part. We will go to problem d, I can take a shortcut a little bit with using the expressions from forth, we will see.

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It is almost problem c except for I am picking a particular velocity, this is a case where, I am having a flow through a duct 20 meter per second. I am having a simple flow, flow through a pipe 20 meter per second and suddenly, somebody puts a lid on it, they closed the flow. I want to find what happens inside the duct now, this is the unsteady problem, we are trying to solve an unsteady problem already. What is going to happen, all this flow has to be stopped because there is a lid that is closed and says no more flow through it.

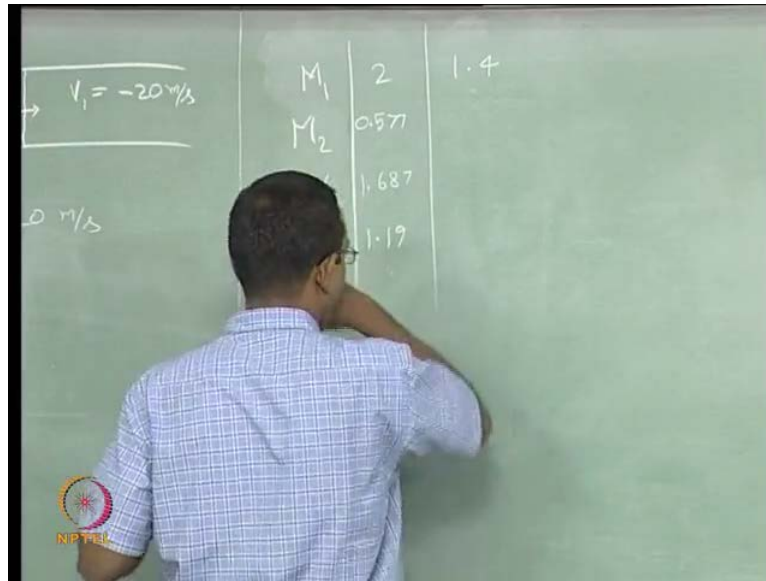
So, there has to be a shock that is going this way and  $V$  equal to 0 should be enforced there by that shock that is the goal that is what, the goal of this particular shock, it is going to do that job. Let us say, we know this  $T_1$  equal to 300 Kelvin temperature here,  $V_1$  equal to this and this means that,  $a_1$  equal to 348.2 meter per second, I am using same number so that, it is easier. Now, we do not like solving the problem from right to left, we are always using left to right.

So, I will transform this to a problem of this type and I am going to say there is a shock that is moving left to right and it is job is to make  $V_2$  equal to 0 while  $V_1$  was having a minus 20 meter per second velocity. Now, I am putting correct direction minus 20 because it is moving the other way, so  $V_2 - V_1$  equal to 0 minus of minus 20, 20 meter per second, this is also equal to  $u_2 - u_1$ . So, we want to solve this, we also know this, we wrote the expressions in different form now, I am writing another form that is all.

Actually, I do not need to do all these, I will just we do not want to derive the whole thing, I will take the expression from below. From the previous time, we wrote this know  $\Delta V$  by a 1 is equal to  $M_1 - M_2 \sqrt{T_2 / T_1}$ , I can use this directly, I do not want to go derive this whole thing for you. It is same problem as before except for we gave a new dimension to the problem, we said it is an unsteady problem suddenly, there is a wall that appeared in the duct after that, the problem is exactly the same, that way I will cut short and finish it off in this 2 minutes left.

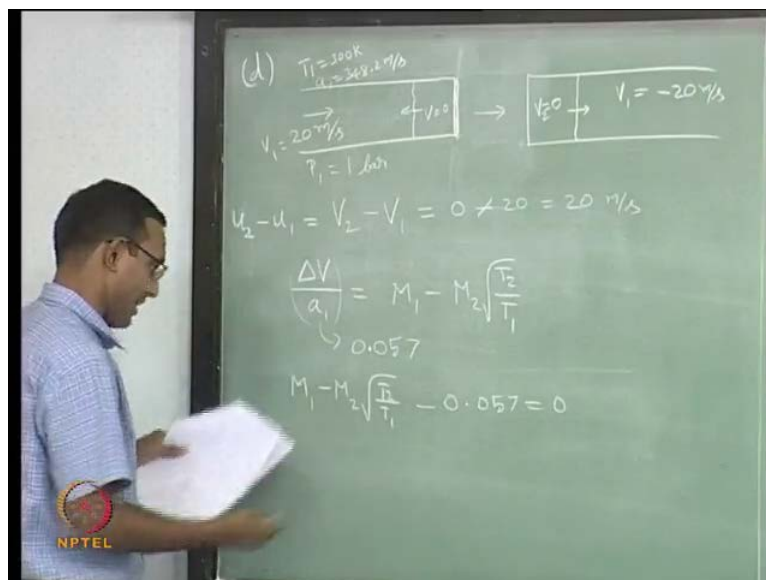
So, what do we do exactly same procedure,  $\Delta V$  is given to be 20 meter per second, a 1 is given to be 348.2 so I will get a number for this, that number is 0.057. So, I just have to solve this problem with this expression  $M_1 - M_2 \sqrt{T_2 / T_1} - 0.057$  equal to 0, this is my left hand side and I just want to solve this problem. Again I have to go on guess that is the same thing, I will go through the same guessing process, will again go form the same table I do not want to write below so I will go to next page.

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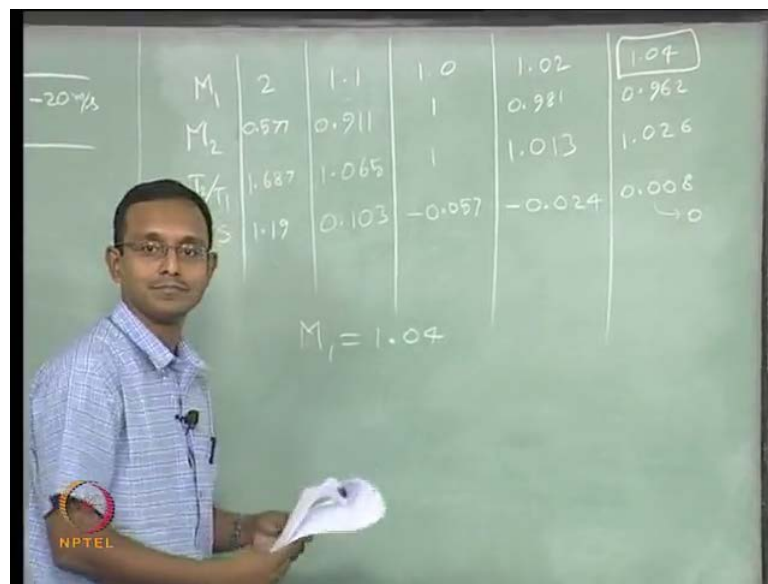
Again, I always like starting with  $M_1$  equal to 2 so we will start with that in fact, I remember this number 0.577 also, this gives you left hand side of 1.19, while it is supposed to be 0. So, I am guessing too bad, now I have experience from the previous problems I know, if I increase mach numbers left hand side increases so I will decrease immediately. Let us say, I will guess something low 1.4 because it is too high I am going more, I cannot guess 1 if I guess 1, things will go wrong, it would not go wrong very much.

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Go back here, if I guess 1, what will happen what will be  $M_2$  value, it will also be 1, what will be the value of  $T_2$  by  $T_1$ , no change it is a mach wave  $T_2$  by  $T_1$  is also 1 so  $1 - 1$  will be 0 and it is minus 0.057. Now, you look at it, it is very close to 0 so I should not be guessing 1.4 really, I should be guessing something close to 1 so that, I will get to the answer. We will do that instead of 1.4, I was not thinking like this when I was doing the problem in one paper.

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So, I will start guessing 1.1 here, I skip one column in my guess 0.911, 1.065 and this comes out to be 0.103 this is what, we have here. And I know if it is 1.0, we know the values it is we just did this know, 0.057 you know this also so the answer is somewhere in between this, probably very close to 1. Now, I just have to guess let us say, in my tables I have only a set of numbers I will get 1.02, that is the closest I can get to 1.

I have 0.981, 1.013 that gives minus 0.024, I want to go even closer so I have to increase from here a little bit, go 1.04, 0.962, 1.026 and this number comes out to be 0.008, which we will say is approximately 0. So, I am getting a mach number to be this, for if I stop at 20 meter per second flow in a duct with a sudden lid completely closed no mass flow through it then I will produce a 1.04 mach number flow, mach number shock going against the flow that is what, we know as of now.

We will go and find the implications of it next class, because I have one more full page of implications, because of this. We cannot go through that, now we will stop here, we

will just stop with  $M_1$  equal to 1.04, will come and continue the same problem next class at the beginning. After that, we will do one more problem and then we will go to 2D problems, any questions you have. So, easy way of solving, if you just remember that one formula and just iterate, see you people next class.