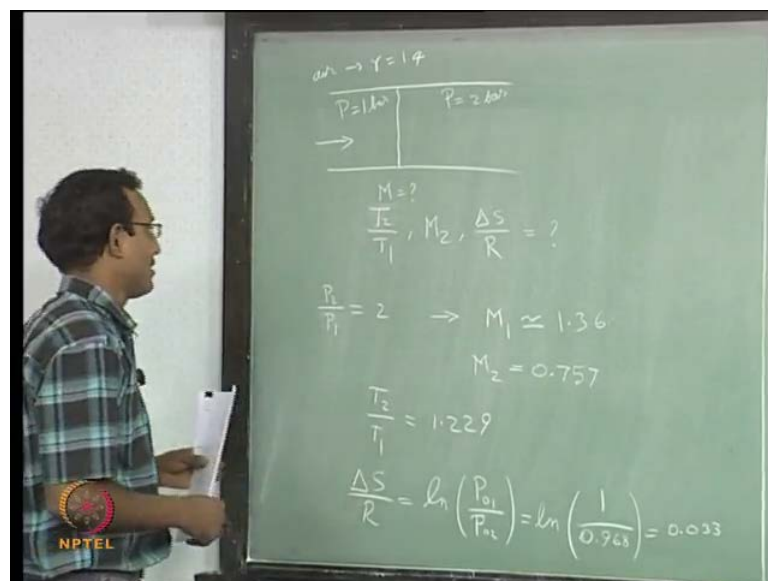


Gas Dynamics
Prof. T. M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 7
Lecture - 16
Solved Examples for Stationary and Moving Shocks

Hello everyone, welcome back. We started solving some numerical example and I picked up a problem that was more complex at the beginning itself. So, I thought about it again and I have prepared some more simpler problem with simple 1D flows and not with so much of area change. We will go and do that kind of problems again later, when we go to nozzle flows calculations, we will go and do it later. But currently we will just go for more simpler problems, so we will pick up some example.

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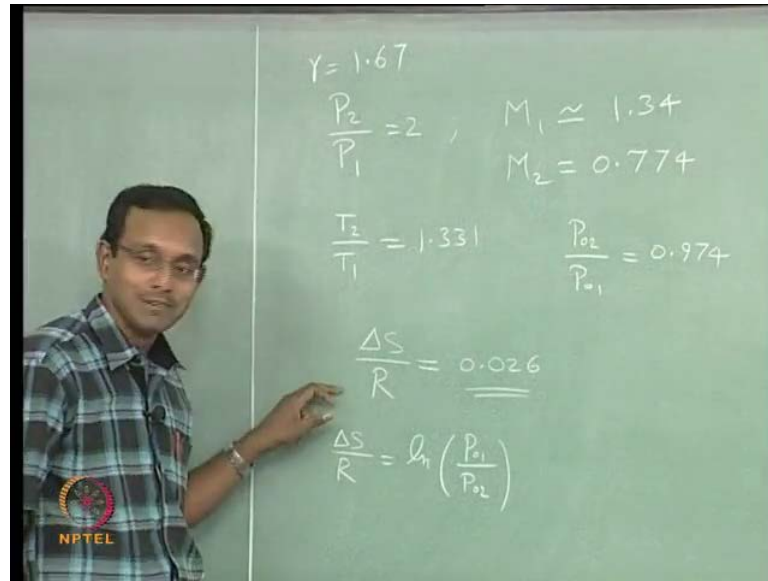
I am having a shock in straight area, constant area duct and we know that pressure here flows this way, pressure here is 1 bar and pressure here is 2 bar, this is the information I know. And now, I know that the gas is air, when I say this gamma equal to 1.4. Now, my idea is to find the shock Mach number, we know the pressure ratio, I want T 2 by T 1, I want the post shock Mach number and delta S by R. These are the things I want, all this we want to find, so how will I do this, straight forward question.

We know something about the shock that is, the pressure jump across the shock is given, so we already know that, normal shock tables will have P_2 by P_1 as one of the columns, so all I have to do is, find P_2 by P_1 , it happens to be 2 so now, I have to go to normal shock tables for γ equal to 1.4 and find, at what row it comes out to be P_2 by P_1 equal to 2. If I do that calculation, I am going to get M_1 approximately equal to 1.36 with a little bit of approximation, it number happens to be 1.996 or 997 something like this.

I have the tables, I can look at the number but anyway, I have done this work, just look at it. So now, we know that, this is the shock Mach number of course, we want other properties M_2 directly in the same row, I just have to read out the next column. M_2 is just another column there, that value happens to be 0.757 and if I read out T_2 by T_1 from there, T_2 by T_1 is 1.229 so I am getting answers directly that is why, I told it is a very silly problem, simple enough problem.

And then now I want ΔS by R , what is ΔS by R , is equal to \log of P_2 by P_1 , we proved this already. So now, I am going to pick this up and we want to find P_2 by P_1 , my gas tables whatever I have, the compressible flow tables whatever I have, gives P_2 by P_1 . So, I will write that as, \log of 1 by that ratio, that ratio for this particular Mach number for γ equal to 1.4, comes out to be 0.968. And so my ΔS by R 0.033 that is your answer, I can do this there other way also, I can say that ΔS by R equal to minus \log of P_1 by P_2 , I will get the same answer anyway. So, this is simple enough problem we solved this, now what if I say that, my γ is not 1.4, it is not air but it is argon.

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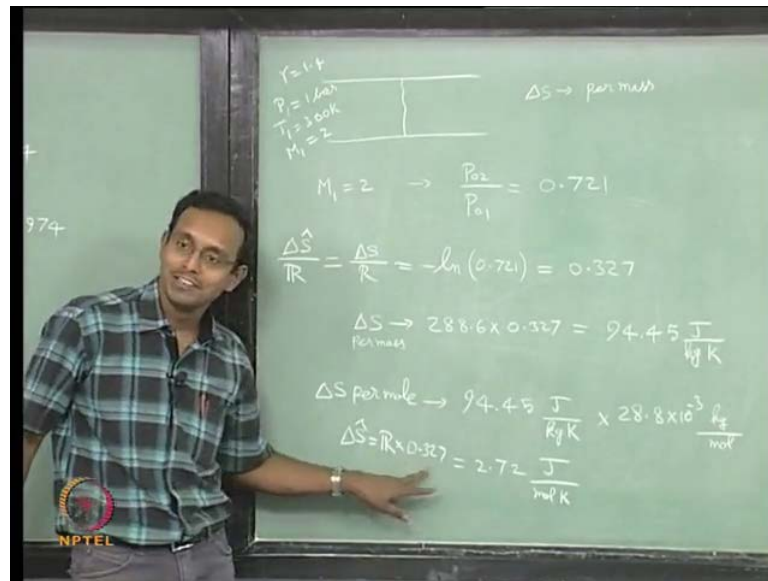


If it is argon, it is a noble gas gamma is 1.67 or 5 by 3, 1.66 or 1.67 whichever you pick, so I want to go and solve the problem with some other normal shock table. But, exact same procedure, P 2 by P 1 is still 2, so my M 1 I am going to find the closest match, that happens to be 1.34 for this particular gas. And so now I have to just read out the next value M 2 0.774, T 2 by T 1 again read out the value 1.331 now, the next one I have from my tables is P naught 2 by P naught 1, which happens to be 0.974.

So, I am going to delta S by R is minus of log of this number, which happens to be 0.026 what is the unit of this delta S by R, delta S by R is per kilogram, no it is not, what is the unit of this delta S by R, what is the unit of T 2 by T 1, dimensionless. What about this, what is this equal to, delta S by R is equal to log of P naught 1 by P naught 2, this side is dimensionless so this should also be dimensionless. If it is a good equation, if it is a correct equation, both sides should have the same dimensions.

So, this is also dimensionless which means, what is the dimension of my entropy, same as R, what is the unit of R now, joules per kilogram per Kelvin so this will have units of entropy per kilogram, remember that. The next problem I am going to give is based on that so we will go into that immediately. So, we will look at the next problem where, we are giving something different.

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I am given $P_1 = 1 \text{ bar}$, $T_1 = 300 \text{ Kelvin}$, $M_1 = 2$ and I am telling you that, there is a normal shock sitting inside my duct. If I have such a case, I am telling you this air $\gamma = 1.4$, I am solving this problem and I want you to find entropy change per unit mass for the air passing through this shock, that is what I want. I want ΔS in per mass basis, that is what I want to find, so how will I do this problem, I just have to go to $M_1 = 2$ for $\gamma = 1.4$ that tables, normal shock tables, $\gamma = 1.4$ and I will look at the row, which has $M_1 = 2$.

There, I will go and look for P_{02} / P_{01} that is what my tables have, there are tables which give you P_{01} / P_{02} , mine has P_{02} / P_{01} . So, this number for that case happens to be 0.721 so my $\Delta S / R$ happens to be equal to minus log of 0.721, which happens to be 0.327. We just now discussed this so ΔS is equal to R times this value where, R is my per kilogram basis gas constant. So, ΔS will be 288.6×0.327 , I am saying it is air so it is 288.6 so that is going to give me a number 94.45 joules per kilogram Kelvin, that is what I will get.

Or I can look at this as joules per kilogram is typically, units for entropy is joules per Kelvin is an entropy unit divided by kilogram, which is entropy per kilogram, entropy per mass that is what we are getting, that is another way of looking at it. Let us say, I do not want it in terms of per kilogram I want it per mole, what will it be, how will I find per mole basis entropy change. Now, I will write it, this is per mass now, I want ΔS per

mole basis, what will that come out to be, how will it, I am not hearing what you are saying.

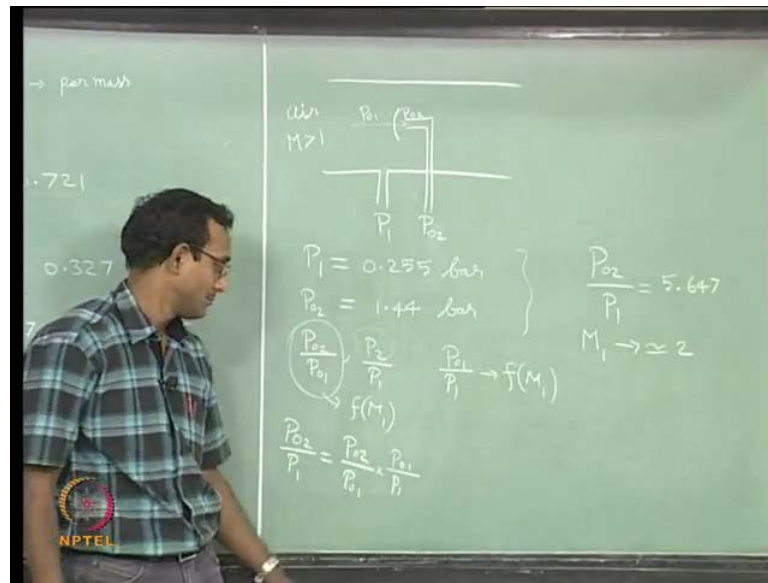
So, multiply this with molecular mass that is one way of solving the problem, I have already know the value 94.45 joules per kilogram Kelvin, this is per kilogram basis, per mass basis. Now, if I multiply this with mass by mole, I will get to correct units so I am going to multiply this with molecular mass of that particular gas then I will get the answer, multiplied by 28.8×10^{-3} kilogram per mole. Of course, you should know that, molecular weight of air is 28.8 grams per mole, I have rewritten this like this.

So, if I do this multiplication, I will get to 9 times 0.2 or something, which will be roughly 1 by 5 around 2 I am getting the answer, 2.72 joules per mole Kelvin. But, this is because I already have this ΔS per mass I got this way, what if I did not have this already, how will I solve this problem, will I go and solve this, and then solve this, unnecessary. Look at this, I can rewrite this as, this is also equal to ΔS cap by universal gas constant, is not it right.

I am multiplying and dividing by molecular mass, that is all I have done from here to here, that is also true where, S cap is entropy per mole basis. So, that is what I wanted to find so I can write it as ΔS cap is equal to universal gas constant times that minus $\log P_2$ by P_1 , which is 0.327, 8.314 multiplied by 0.327 will also give you this number, it is all the same. If you look at the actual math that is happening, it is all exactly the same, you will get to the same spot.

Another way of looking at it, this R can be rewritten as, this R divided by molecular weight so I will write this as universal gas constant divided by molecular weight. Molecular weight will go to the top and that will exactly be looking like this finally, multiplied by molecular weight. You will get to the same spot, finally answer will be the same, various ways of looking at it final answer is this, will go to some other question.

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Now, I am going to go to more practical problem, I am having a wind tunnel supersonic wind tunnel, I would not tell you why the static pressure is measured one particular way, stagnation pressure is measured one particular way. Let us assume those are all correct, I am going to tell I have a side port on the wall and it is measuring pressure and I call this static pressure. Why am I calling this static pressure, just believe me right now, it is correct it is called static pressure and it is static pressure.

And I am having a pit out probe, we discussed pit out probe already, the flow is coming here and it is slowing down to come to stop inside the tube and so I am going to say the flow is forced to come to rest inside this duct. So, I am going to call it close to isentropic and so it isentropic pressure finally at stagnation condition. I am going to call this actually, I should call it P_{naught} only, I do not want to call it $P_{\text{naught } 2}$ already, I will just call it P_{naught} .

Now, I am going to say, I am having air flowing supersonic let us say I know this already, it is supersonic flow and I know it. If I know it, I want to find the Mach number here, given these two pressure values that is the goal. One thing about supersonic flows, if it is stopped by anything, there will be a shock in front of it, in this particular case the shock looks like this, we will go and deal with it later after we go to 2D problems. But, as of now, we will just assume I get a shock like this like 2, 3 weeks later, we will get into this problem as of now, we will just continue like this.

And I am going to say, flow through the centre is, what is going through the tube and because of that, it is seeing almost a normal shock in front of it and if it is seeing a normal shock in front of it, the P_0 is going to be decreasing across the normal shock. So, it would not be P_0 , but it will become P_0 on the other side so whatever I am measuring here will become P_0 , it is not stagnation point upstream of the shock, stagnation pressure downstream of the shock.

So, these are the numbers which I will practically measure, if I put some pressure measurement device here let us say, I am doing that in my lab and I am getting these numbers, P_1 as 0.255 bar. I am giving you numbers that are close to what can be achieved in our lab, P_0 comes out to be 1.44 bar now, we want to find Mach number from here. It so happens that, I have a good compressible flow tables where, there is one extra column given P_0 by P_1 , this is given in my normal shock tables.

If I have this column given, very easy for me to solve this problem, all I have to do is to take this ratio, that ratio happens to be 5.647. Now, I will just go to that particular column in gamma equal to 1.4, I said it is air gamma equal to 1.4 and I want to look for this number in that column, wherever it happens I will just read out my M_1 value and that value is approximately equal to 2. It actually comes out to be 1.999 and will just keep it to, that is what it comes out to be but let us say, I do not have that good gas dynamics tables, how will I solve this problem that is the next question.

If I do not have that and have only these columns, P_0 by P_1 and P_2 by P_1 , these are the tables that I have, let us say and I still want to find this. How will I find it, P_2 by P_1 does not help because I do not have P_2 value it so happens that, I do not have P_0 also but I have P_1 . So now, I have to do something else, I have to link P_0 and P_1 , I do not know P_2 will eliminate this of course, I could link P_0 and P_2 also but we want upstream condition M_1 to be figured out.

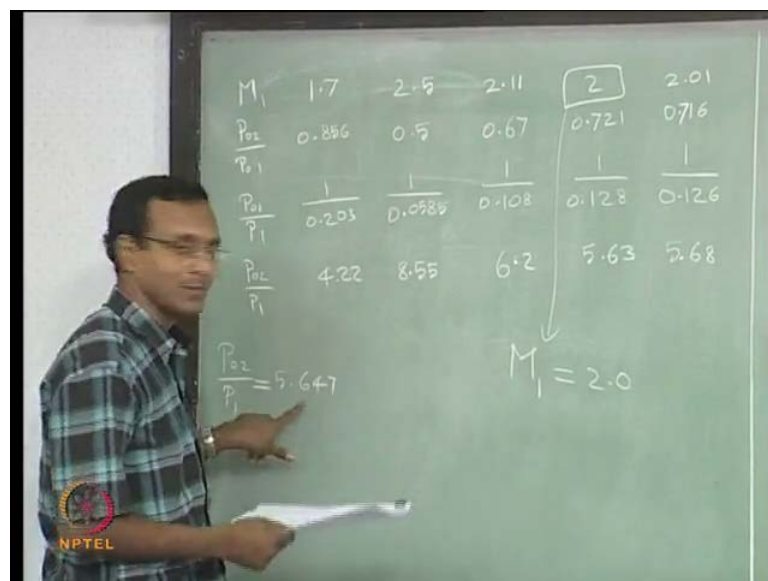
So, we would not to link these two, we want to link P_0 and P_1 , how will link that, P_0 and P_1 is a function of M_1 why, it is isentropic flow. If I pick your flow, here it is having this static pressure and it is having some stagnation pressure. If I take this flow is entropically through an imaginary process and make it stagnation, it will

become $P_2/P_1 = 1$. So, I will take it that way and find the Mach number upstream, that will be a function of M_1 .

We know this formula in fact, it is $1 + \frac{\gamma - 1}{2} M^2$ the whole to the power $\frac{\gamma}{\gamma - 1}$, that something you should just remember, should just come into your mind from now on. Of course, this is given in tables, you do not need to remember the formula really if you have tables in hand. Anyways, I know this and I know this, this is also a function of Mach number, this is coming from normal shock tables, this is coming from isentropic flow tables.

Now, all I have to do is start guessing, guess a Mach number, find I want P_2/P_1 by P_1 , this is equal to P_2/P_1 by P_1 into P_1/P_1 . Now, if I guess the Mach number, I can get this number and this number, this is coming from normal shock tables, this is coming from isentropic flow tables. Now, my job is simple, I just have to form a table and start guessing numbers, I will do that in the other board and I have to reach the final P_2/P_1 , you know the answer now. So, you know how to guess correct answer but let us say, I do not know we will assume, I do not know that M_1 is going to be 2. Because, that is what you will be ending up when you are solving the problems in exams so we will assume some case like that.

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I am going to guess a Mach number, I will find normal shock tables value, I will find isentropic tables value and then I will multiply these two to get this number. And I want

to see, whether this is matching my I am looking for $P_{naught 2}$ by P_1 equal to 5.647, this is what I want finally. So, let us see if it is possible so initially, I do not know what to guess of course, typically I will guess 2 first. Since I know the answer is 2, I do not want to guess 2; first, because I want to go through the iterative process.

So, I am guessing 1.7 nice number to guess, always remember guess 2 initially, in exams typically the number will be Mach 2. So, if you guess 2, you will be close to the answer already, easy to solve iteration will be less time. If it is 1.7 and I am going to get 0.856 and in my tables it happens to be P_1 by P_{naught} that is what is given for me, P by P_{naught} is what is given in my tables. So, I just write numbers like this, I just write it inverted now, I have to multiply these two to get $P_{naught 2}$ by P_1 .

So, that number multiplied comes out to be 4.22 this is one case now, I will guess something higher, I do not know whether I should guess higher or lower right now, I have one number, I know that this is less than this. $P_{naught 2}$ by P_1 is not easy to judge, if I increase Mach number, will this increase or decrease currently, I cannot tell very clearly. Of course, once you solve this problem, you know which way to go, right now I do not know so what will I do, I will just guess randomly some number.

I am picking 2.5, I could have guessed 1.5 also then I will find that this will be lower and then I have to go higher and I will go to 2.5, that is fine. I am guessing 2.5 right now and this number is 0.5, this number is 1 by 0.0585, I will write it clear 585. And so this total comes out to be, multiplication comes out to be 8.55, that is this so this is higher, this is lower, my answer is somewhere in the middle. So, I have to get something in between these two, I can guess or I can use interpolation between these two.

Of course, we know the function is not linear, but I can interpolate between these two and get to some other Mach number that will be closer. I can do that, let us say I do not do that, I just guess something, it looks like the answer should be closer to this number than to this number. Because, we want 5.65 this is 8, it is more closely this number than that number in a way but somewhere close to the middle. So, it should be somewhere in the middle but more close to this so let us say, I will guess 2.1 in here, I have guessed 2.11 I do not know why, to be correct I will keep it 2.11, I have interpolated also.

So, anyways we will do it is this way, I am picking this number that comes out to be 0.67 and this one will be 1 by 0.108 and typical mistake in exams will be this, the tables will

be giving P by P naught and you will write P naught by P as that value, 0.108 instead. So, be safe about that, should always be clear about what you are writing, which number you are getting from the tables and what you want. And that comes out to be 0.108 and this product comes out be 6.2 it looks like we are closer but I am still not close to the answer but it is getting closer, it is pretty close to the answer compared to this one.

So, I will go lesser so the next guess I will go 2, it so happens that I know the answer 2 because I had other tables. And if I do not know, typically I would have guessed 2 directly here, next number but I just wanted to avoid 2 for some time so that, you know the iteration process. So, if I picked this, that number comes out to be 0.721, 1 by 0.128 and 5.63 comes out there, 5.63 but we want 5.647, not exact answer so answer is somewhere in between these two but very, very close to this.

So, I will try one more 2.01 and see, what that looks like, if I picked 2.01 this is 0.716 and 1 by 0.126 and the number comes out to be 5.68 I want an answer close to 5.65 so it is more close to this than to this. So, I can go and interpolate between this and say, the answer should be 2.007 or something like that, 2.003 or something or I will just leave it as 2. So, we will currently do that and say, answer is just 2 so my M 1 or the Mach number in my test section happens to be 2.0.

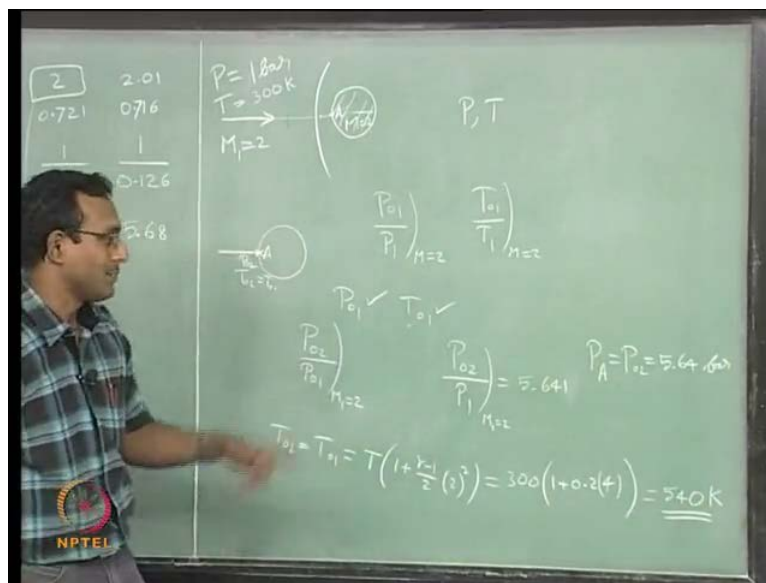
If you want you can be more clear and taken interpolation between this and it will be what, 2 out of 5 in that gap, it is 40 percent from here to there, it will be 2.004 if you want or I will just give it as 2. If I am accurate enough up to second decimal place that is enough, that will be 2.00 third decimal is 4 so I will ignore that so I am getting number Mach 1 as 2, this is one way of doing it. Typically, when you are solving such a problem, I would start guessing with 2, guess nice numbers like 1.5 to 2.5 that kind of numbers, nice way to guess.

Once you guess that, you will finally go to a point where, the answer is between two of them, you could directly interpolate but I would not. I will go pick a number, that is nice something in between and I will get much closer to the answer like this. I can interpolate at this point but I know the answer is very close to this so I will guess something in between this but very close to this. I am very, very close to the answer, at this point I can interpolate, this way you will solve the problem faster than if I start interpolating here, it will go somewhat close to the answer, I will tell you what it will be also.

If I interpolate across here to get to this number, I will get 2.2 so then I will find that the answer is between 2.2 and 1.7. If I interpolate between those two, I will get 1.91 so it is answer is between 2.2 and 1.91. If I interpolate between those two, I will get 2.11 that is how, I have this number in my chart, I did the full interpolation method 2.11 you will get. Now, I know the answer is somewhere between 1.91 and 2.11 now, I will go and guess 2, that gives me this number, I find that it is close but slightly less.

So, I will guess something higher, I will get to this answer ideally, I will guess guess guess then interpolate, guess 3 or 4 times then start interpolating, that is easier that way you will solve the problem faster, you would not waste time in exams. Go to the next one another problem now, it is more of a practical problems, I am having a cylinder moving in still air at Mach 2 speed.

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I am having a cylinder, we are looking at a 2D problem, this dimension is infinite, minus infinity to plus infinity. And this is moving at Mach 2 what does that mean, if I shift my reference frame to this on the cylinder then I will remove this arrow here and I will say, my flow is coming in with free stream Mach number of 2 this is what, we are having. And as I already told you, if I put any object in flow in supersonic flow, there will be a shock in front of it.

Now, we are interested in finding the leading edge, I will call let the point A, I am interested in finding pressure and temperature at that point A, P A and T A that is what

we are interested. If I am trying to find that, it is a simple enough problem of course, I have to first cross the shock, the streamline that is going straight there will be the streamline that is going to cause the pressure there. So, across the shock of course, P_{naught} drops but T_{naught} stays constant, Mach number drops.

After that, the flow is subsonic now, if I look at flow around a cylinder subsonic flow, it will achieve stagnation pressure at the tip at the front most point, which is what we are interested in this problem. So, I am going to say, it will reach the stagnation condition $P_{\text{naught } 2}$ for this shock and $T_{\text{naught } 2}$, which is equal to $T_{\text{naught } 1}$ for any normal shock. So, I will end up like this of course remember, it should be stagnant shock then only, $T_{\text{naught } 2}$ equal to $T_{\text{naught } 1}$.

We just did moving shocks and we saw that, $T_{\text{naught } 2}$ by $T_{\text{naught } 1}$ not 1, it is more than 1 always that is why, we shifted coordinate systems. Now, the shock is stationary in front of this cylinder and so I will have $T_{\text{naught } 2}$ equal to $T_{\text{naught } 1}$. So, I just have to solve this problem for so I need to know $P_{\text{naught } 1}$ and $T_{\text{naught } 1}$, that number that information is given in the question itself. P equal to 1 bar, T equal to 300 Kelvin or somewhat standard condition I am getting this numbers, these are static conditions not stagnation.

So, I have to find the stagnation conditions for M_1 equal to 2 so if I do that, I have to find $P_{\text{naught } 1}$ by P for M equal to 2 and $T_{\text{naught } 1}$ by T_1 for M equal to 2, I can find these numbers. And once you know this, all I have to do is I will find $P_{\text{naught } 1}$ from this, $P_{\text{naught } 1}$ is known, $T_{\text{naught } 1}$ is known. If I know these two numbers, all I have to do is, $T_{\text{naught } 2}$ is same as $T_{\text{naught } 1}$, this is directly your answer, pressure and temperature at that point, temperature answer is directly there.

If I want to find $P_{\text{naught } 2}$, I have to go to my tables and look for normal shock tables M_1 equal to 2 at particular condition, $P_{\text{naught } 2}$ by $P_{\text{naught } 1}$. I can do all this or I will take a shortcut, it so happens that my tables has I can do this, this is one way of solving but I do not have numbers for this in my page. I have in my tables $P_{\text{naught } 2}$ by P_1 , which is very, very useful, I am given P_1 I want $P_{\text{naught } 2}$, I have already gone through that discussion.

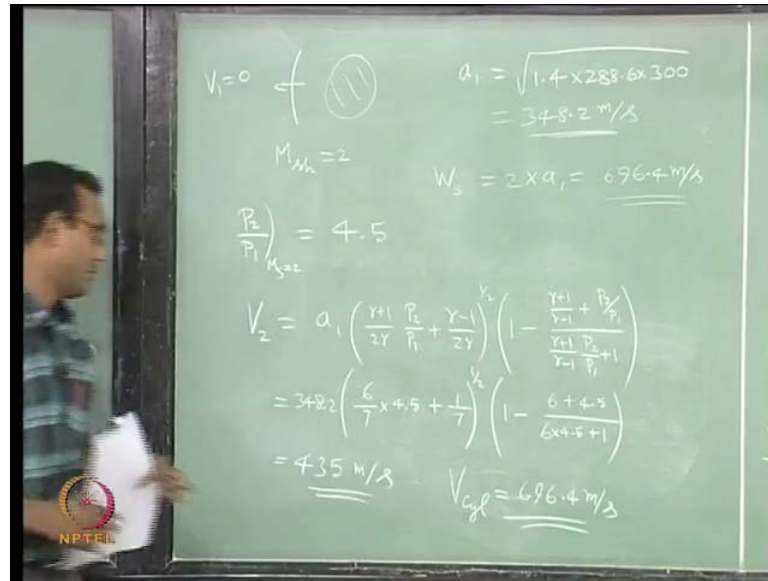
So, all I have to know is this number for M_1 equal to 2, if I have this then of course, it is M equal to 2, we just solve the problem it is the same number 5.647, 5.641 apparently for

M equal to 2. So, I am getting this number here, now from here, P 1 value is given to be 1 bar so I will directly get this number to be 5.64 bar. P at A equal to P 0 2 equal to 5.64 bar that is what, you are getting. What about T naught, let us find it T naught 2 equal to T naught 1 is equal to T into 1 plus gamma minus 1 by 2 into 2 square that is all.

It is 300 into 1 plus 0.2 into 4 that is, 1.8 times 300, 1.8 times 300 is 540, 540 Kelvin is the temperature experienced and the pressure experience happens to be 5.6 times atmosphere. These are the rough numbers, which the cylinder faces at the front this is why you have to worry about high speed flows. Everything will experience much higher pressure than what is really happening there because it is moving faster, your stagnation conditions are very, very high.

If I move at Mach 10 temperature will go crazily high, if I use gamma equals to constant and find the temperature, it will come to 10000 Kelvin or something very, very high number. Let us just look at that number here roughly, if I put 10 here it will be 100, 100 times 0.2 is 20, 21 times 300, it is not very, very high, it is 6300 Kelvin but I already told you, more than 2000 Kelvin, gamma will start changing for air. So, that number is not correct, I have assumed gamma equal to 1.4 here, that number will not be right, number will come down a little bit, it will come to probably 4800 or 5000 Kelvin. But, we do not want to worry about that for this moment, we will say in our world, it is all nice. Now, I want to look at the same problem in a different form now, we are going to look at moving shocks, it is the same problem cylinder.

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Now, I am going to say, that the shock is having the same strength, P_2 by P_1 is the same as this case and so it is having a same strength M shock equal to 2. We already solve the problem partly, I am going to look at it from moving shock prospective. And because of that, if I will look at it from outside reference frame, it will look like this Mach 2 shock is moving into still air, V_1 equal to 0, Mach 2 shock is going this way. I want to solve this problem, first thing I want to do is find the shock speed, shock speed is nothing great, I know V_1 is 0, I want to find a 1.

Cylinder is still here, we will keep the cylinder also, a 1 we said it is 300 Kelvin so it is 300 Kelvin and air, 288.6 into 300, this happens to be 348.2 meter per second, you can call it 348 meter per second if you want, that is fine. So, my shock is moving at Mach 2 so it will be double this number, 2 into a 1 which is 696.4 meter per second, this is my W_s , I used to call it W_s . So, we will keep it W_s , speed of the shock with respect to my outside reference frame, this is my moving shock.

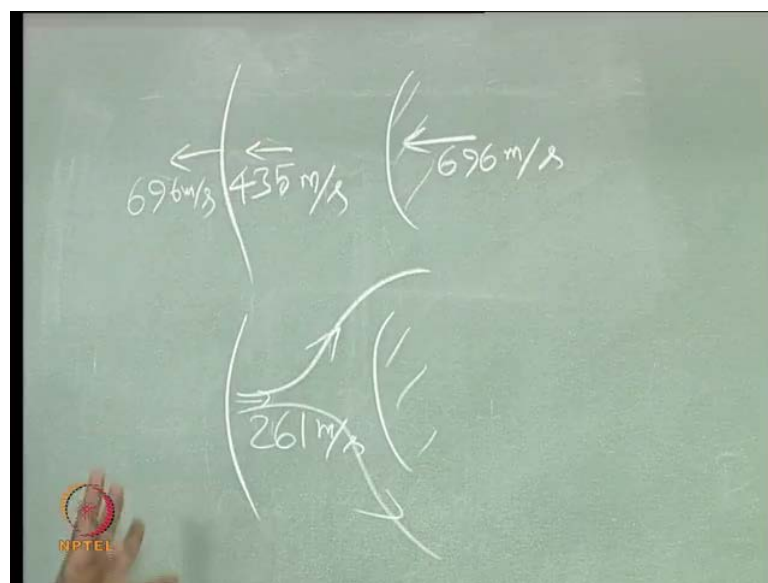
Shock is moving into still air, now I want to find the velocity behind the shock caused by this shock. So, we have directly one formula we will just use it, I am going to say P_2 by P_1 is what is given in the formula, in all our formulas we had P_2 by P_1 . So, I will find P_2 by P_1 for M shock equal to 2, it happens to be 4.5. Now, I will use this number in my V_2 formulae in my shock calculations.

If I use that, it is a $1 + \frac{1}{2} \gamma P^2$ by P^1 plus $\gamma - 1$ by $\frac{1}{2} \gamma$ to the power half into $1 - \gamma$ plus 1 by $\gamma - 1$ plus $\frac{1}{2} \gamma$ by P^1 $\gamma + 1$ by $\gamma - 1$. I think that is multiplied by, multiplied by $\frac{1}{2} \gamma$ by P^1 plus 1 , this formula we already wrote I think two classes back. They are having this now, all I have to do is, substitute numbers for this, by the way you should get used to $\gamma + 1$ by $\gamma - 1$ for γ equal to 1.4.

We already did the number, what is that number 2.4 by 0.46 , it will be 6 plus 4.5 divided by 6 into 4.5 plus 1 exactly will get. And here it is $\gamma + 1$ by $\frac{1}{2} \gamma$, we will write the numbers anyway 348.2 into 2.4 by 2.8 , 24 by 28 . So, that is 24 by 28 is 6 by 7 into 4.5 plus, this is 0.4 by 2.8 , 1 by 7 . You have to get used to this kind of numbers, it is not very difficult, to the power half into $1 - \gamma$ we already know this number is 6 , 6 plus 4.5 divided by 6 into 4.5 plus 1 .

You have to just calculate this, nothing more to simplify here and that number happens to be 435 meter per second. Now, my critical question, what is the velocity of the cylinder with respect to my outside reference frame. We said, that was moving at Mach 2 which means, that will be equal to 2 times a 1 , this is not equal to 2 times a 1 , I will just write velocity of cylinder. V_{cylinder} is equal to 2 times a 1 as 696 , V^2 happens to be this value so what is happening, this is your physical intuition part.

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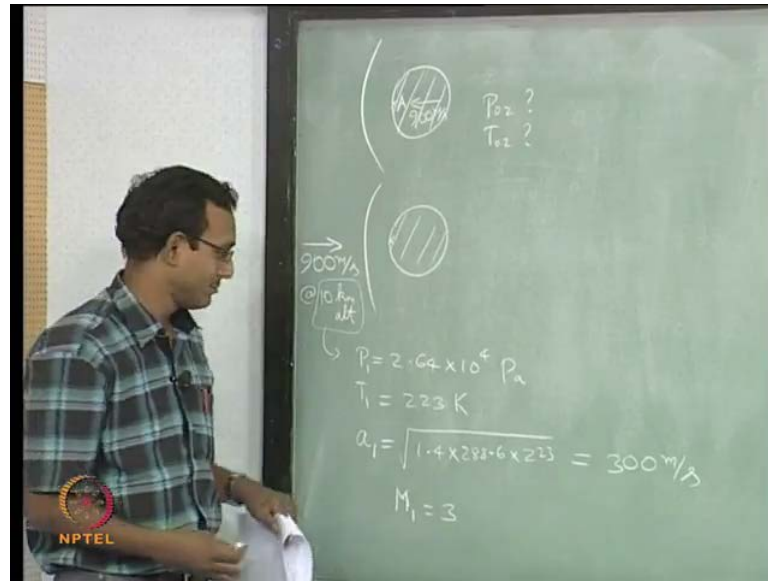
So, now, I am going to say there was a shock, after the shock I have velocity 435 meter per second, it is not this way it is the other way, arrow is other way. Shock is going at 696 meter per second and gas is going with 435 meter per second and there is a cylinder, which is also going at 696 meter per second. So, if I suddenly switch to the other reference frame what will happen, shock is stationary with respect to this cylinder.

If I say the shock is stationary with respect to the cylinder, there is a velocity of this fluid with respect to the cylinder. How much is that number, actually it is 261, 261 meter per second, this is what is the final flow around the cylinder? This is what, is going to go like this that is why, you have to get used to the problem from both points of view, whichever reference frame I am in, I should be able to solve the problem.

My shock is such that, this flow around the problem will give you a steady state it so happens that, shock will stay stationary at some particular distance from the body, there is something called a shock stand off distance. We would not deal with it now, will go deal with it when we have bow shock, we have not gone to bow shock, we are still in normal shock and moving normal shock, that is what we have done till now.

And shock stand off distance is decided by, how much mass flow rate has to go through and turn around the cylinder, is it having enough space and all. I think I have time only for one more problem and with that will stop, I have lot more questions that is possible. I will just modify this question to as spherical bullet, I am shooting a bullet at supersonic speeds into still air.

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Spherical bullet, for our plot it is still looking the same and of course, it is going to have shock in front of it and we are interested in the same front point a, we want to find pressure temperature there. The question is asked differently here but it is still the same thing, what is the pressure and temperature at the leading edge. But of course, from our discussion we know that, we are looking for P naught 2 and T naught 2, this is what we are asking essentially.

And of course, you have to be given all the information, I am going to tell that, the bullet is moving at 900 meter per second. So, it is if I go to other reference frame where, flow is coming and this bullet is stationary, this is coming at 900 meter per second and it is given that, it is at 10 kilometer altitude. So, I am at 10 kilometer altitude and that is like typically, the attitude of aircraft flying long distances.

At 10 kilometer altitude, I am shooting a bullet at 900 meter per second, I want to find the pressure and temperature experienced by the front end of the bullet. So of course, you need to know information about the atmosphere, several tables exists typically, if you take separate booklets, which are gas tables booklets like so many authors are there, I do not want to name any one particular author. So, if you pick any of that then you will also get standard atmosphere tables, we will use standard atmosphere.

I will give you the values P_1 is equal to 2.64×10^4 Pascal and T_1 is equal to 223 Kelvin, standard atmosphere tables are also available separately. It may not be

available in typical compressible flows books but if you have separate gas tables or compressible flow tables as a book, there you will have standard atmosphere tables as the very first table, typically, in most of the books, you will have this also. So, once I have this, I have P 1 and I have to find the bullet speed, 223 Kelvin if I pick, a 1 is 1.4 into 288.6 into 223 Kelvin.

It comes out to be 300.6 meter per second, we will make it 300 meter per second some such number, it is almost 300 meter per second. This says that, my M 1 happens to be 3 because it is 900 meter per second, M1 happens to be 3. Now, the problem is simple, I just need to find P naught 2 by P naught 1, so I will get the answer directly P naught 2, because I do not have P naught 1 yet I have P 1, we have to find P naught 1. But, it is similar of course, I have my special tables where, P naught 2 by P 1 is given, I can just go multiply the P 1 and I will get answer let us say, I do not go that path this time.

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$$\frac{P_2}{P_1} \Big|_{M=3} = (1 + 0.2 \times 9)^{3.5} = 36.73$$

$$P_{01} = 36.73 \times 2.64 \times 10^4 \text{ Pa} = 9.7 \times 10^5 \text{ Pa}$$

$$P_{02} = P_{01} \times \left(\frac{P_{02}}{P_{01}} \right) \Big|_{M=3} = 0.3283 \times 9.7 \times 10^5 = 3.18 \times 10^5 \text{ Pa}$$

$$\frac{T_0}{T_1} = 1 + 0.2(9) = 2.8$$

$$T_{02} = T_{01} = 2.8 \times T_1 = 624.4 \text{ K}$$

I want to find P naught 2 by P 1 for M equal to 3, how will I do it, if I do it by formula it is going to be 1 plus 0.2 into 9 to the power, gamma by gamma minus 1 is 3.54, gamma equal to 1.4. I just have to find this number, it is what, 1.8 2.8 to the power 3.5, that number happens to be 36.73. If I go look at the tables also, I will get the same number, isentropic tables for M equal to 3 will give you this number, if you go to gamma equal to 1.4 tables.

So, I have to find P_{naught} , which is 36.73 multiplied by P_1 given there, $P_{naught 1}$ is equal to 36.73 multiplied by 2.64 into 10 power 4 Pascal, which is 9.7 into 10 power 5 Pascal. It is roughly 9.7 atmospheres that is what we are getting, this is $P_{naught 1}$ so $P_{naught 2}$ is equal to $P_{naught 1}$ into $P_{naught 2}$ by $P_{naught 1}$, for M equal to 3 from normal shock tables. So, that is 0.3283 multiplied by 9.7 into 10 power 5, answer is 3.18 into 10 power 5 Pascal, this is the pressure experienced by the leading edge of my bullet.

What about temperature, T_{naught} by T for M equal to 3, which will be 1 plus 0.2, which is γ by γ minus 1 by 2 actually, multiplied by 9 which is, 2.8. So, T_{naught} is equal to T times 2.8, $T_{naught 2}$ equal to $T_{naught 1}$ for normal shock, for stagnant normal shock equal to stationary normal shock better word, which is equal to 2.8 into T_1 , that number happens to be 624.4 Kelvin. These are the various ways of solving simple problems, next class we will go and solve more of moving shocks. This we had one problem solving moving shocks but we directly used the formula but this formula will not be easy to handle. It is more difficult, will just start using intuition and normal shock tables from next time. That is a better way of solving, you do not need to remember such formulae any more, we will see how to solve that next class.