**Gas Dynamics**

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**Module - 6**
**Lecture - 15**
**Moving Shocks, Solved Examples for Stationary Normal Shocks**

Hello everyone, welcome back, we stopped at a point, where we were trying to go for the Mach number of the flow behind a moving shockwave. We are trying to find that and we already did the expression for it and we said that it can go anywhere between 0 to some high value depending on the gamma, it is asymptotic to some saturation value. Can be 0 which is very low subsonic, ideally it could be 0 if it is a very weak compression wave, and if it is a strong enough wave then it goes to a maximum value, I will write that on board.

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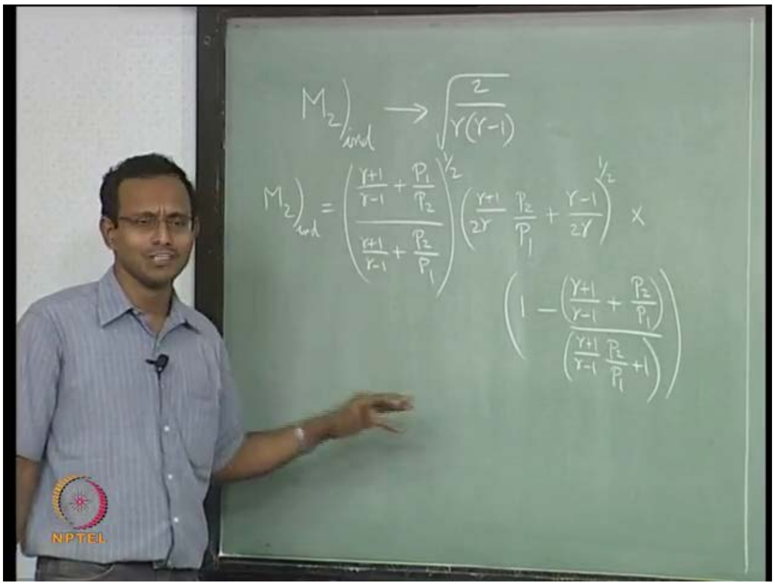

I will keep writing it as M 2 induced by the shock is given by 2 by gamma times gamma minus 1 we did this last class, this is I have to say tends to, it is not the only value, this is the highest value the asymptote that it will reach finally, that is what it will become. So, and we also saw that if my wave is really, really, really weak, then this number will go to 0. So, my actual induced mach number will go anywhere between 0 to this value; that is what we are seeing.

And we already saw that, if it is more compressible gas then the induced flow will be higher that is where, we were. It will respond better to the compression that is the way, it is supposed to be looked at, it will respond more if it is a compressible fluid. Now, we wrote a huge expression for this, I will write that one again, multiplied by I am folding the expression here, this is the expression. And now, I asked you this one nice question, what should be the p 2 by P 1 value for my incoming shock, the moving shock such that, the induced flow is just sonic.

That is, when it is going anywhere between subsonic to some supersonic value, we wanted to see of course, you should know that, this value is going to be more than 1. How do you know that, this is the number less than 1 the denominator, that will make this number more than 2 and this number is never more than 1.6. So, I can tell this whole number is always more than 1, square root of a number more than 1 will still be more than 1 that is why, this number is always supersonic.

So, I know that, if I make my P 2 by P 1 equal to 1 then M 2 individual becomes 0 and that is happening because of this term, we saw this already in last class. This will P 2 by P 1 will become 1, which will mean gamma plus 1 by gamma minus 1, plus 1 here again this becomes 1, gamma plus 1 by gamma minus 1 plus 1, this whole ratio becomes 1, 1 minus 1 is 0 so I am getting this to be 0. Now, I am saying that is, if P 2 by P 1 is 1 that is no pressure across, no pressure jump across my compression wave, a very weak compression wave.

If I have a very strong commission wave then we said that, this whole expression tends to this value, we saw that already. Now, we want to find when this M 2 induced goes past 1, what should be my minimum P 2 by P 1 such that, the flow goes to M equal to 1 behind the shock that is the way, we want to look at it. And to do this, I tried solving this expression, it was very complex, I found that finally, it is not easy to write an expression such that, I will invert this and say, P 2 by P 1 in terms M 2 induced, not easy to write. So, what will do is, will go to the graphical method, it is not very serious thing for our course, we do not need to solve algebraically. So, I will just go to a graphical method so we will go to the screen.

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We will first look at shock mach number that is, W S by a 1, which we looked at last time the mach number with which, that particular person wave, which is corresponding to this P 2 by P 1 is moving into as stagnant gas that is what, is given here. So, if I say my P 2 by P 1 is 1, I did not plot for less than 1, that does not make sense for us. So, P 2 by P 1 is 1, otherwise it is an expansion plan we do not want to talk about it, this compression goes from one all the way to have given up 100, P 2 by P 1 compression ratio.

What we are seeing is, as I increase my P 2 by P 1, my shock mach number increases this is what, we already saw last time. We said that, it is proportional to P 2 by P 1 square root. So, it will keep on going up and it looks like a square root curve obviously, it is just a square root of P 2 by P 1 multiplied by a constant that is what, it looks like. Now, what you are saying is, for different gamma values, gamma equal to 1.67 is the lowest shock mach number, it is the least compressible gas and gamma equal to 1.3 is the most compressible gas. So, it looks like compression wave travels faster in a compressible medium than in a non compressible medium or a less compressible medium that is what, it looks like. That is what, we are seeing here and of course, the remaining properties of, if my strength is higher it travels faster is also present here, I will go to the next one.

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We were looking for induced mach number is a last we will come back to it, induced mach number this is a induced mach number behind. That is my shock has gone past, the velocity is V 2 divided by the heated gas is now having a speed of a 2, V 2 by a 2 is the this is M 2 and with respect to my P 2 by P 1 across the shock. I am starting from one all the way to 100 I am going and in this side, what we are seeing is, for a large time it is just a say up to P 2 by P 1 of 4 or so all the gamma values are going to give you only subsonic solution.

My P 2 by P 1 is only around 4, that will only give subsonic solution behind the shock and you can also see that, it starts from 0 which is what we saw when P 2 by P 1 is 1, induced mach number is 0, we just told that today. And it does not add to a constant value for different gamma values is different because we saw that it is a function of gamma, square root of 2 divided by gamma times gamma minus 1. So, that is the function you will get finally, one more thing we want to show is, when I increase my P 2 by P 1, we saw that stock mach number increases.

So, I can replace this axis with shock mach number if I want, by using the previous plot and what I see will be, as my shock mach number increases I find that, not much change happens if my shock mach number is high enough. I will go back to the previous plot, let us look at the number P 2 by P 1 of 60 and above, we will go back to the previous plot if

I have as 60 and above, that corresponds to mach number of 7 and above roughly, for all gamma values.

What we are seeing is, if my mach number is 7 and above, my induced M 2 is almost a fixed number irrespective of, what my P 2 by P 1 is, what my mach number is that is what, we are finally getting to. So, I am giving you a lot of hints towards hypersonic flow theory, in hypersonic flows typically, things do not depend on mach number, nothing depends on mach number of the flow. Finally, it will come down to that everything asymptotes to constant value, most of them that is, one more hint here given to you.

We would not deal with hypersonic flows much here, I will just tell you like this once in a while, so many places I have already told you, when mach number increases things become constant. So, again I want to see one more thing, if my gamma changes from 1.3 to 1.4 to 1.67, what we are finding is, the highest induced mach number happens for 1.3. Compressible gas responds with running faster behind the shock compared to a not so compressible gas that is what, you are seeing here. Not so compressible gas has a very low speed behind the shock, now I have some number calculated from this plot actually. I just wanted to put those numbers in we will go back to the board, if I find out at what point this M 2 induced becomes 1, for each of these gamma values.

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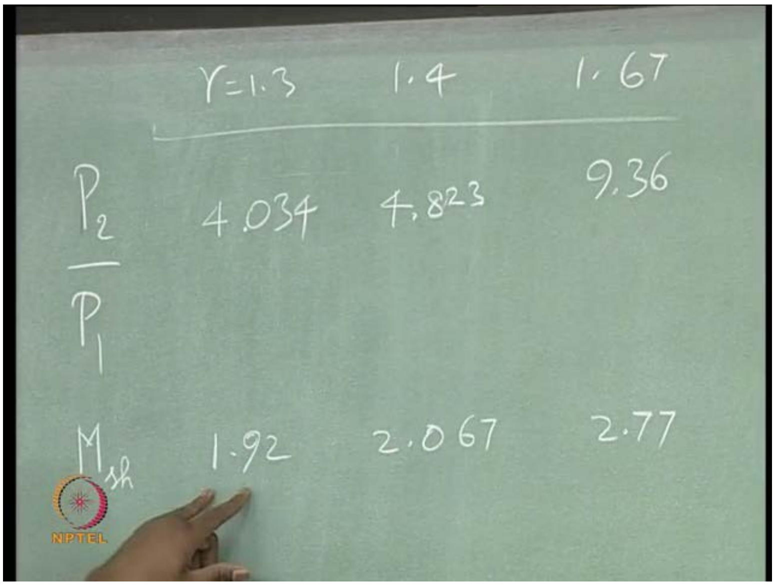

P 2 by P 1 I will put as a table, if I look at P 2 by P 1 value for gamma equal to 1.3 that is, 4.034, 4.823 and this is 9.36. And if I look at M of shock, that corresponds to 1.92 this is from the other plot using P 2 by P 1 in the other plot, 2.067, 2.77 this is what, we are seeing. If I want a supersonic flow behind my shock, for gamma is equal to 1.3 gas my P 2 by P 1 should be 4.03, which is what we roughly observed already in the plot, I said it should be above 4 for it to go at supersonic roughly that is what, you are seeing.

If it is 1.4, we are seeing it to be 4.8 if it is 1.67, unless I force it really, really hard, it does not want to go supersonic. Unless I go more than 9.4, it does not want to go supersonic, unless I compress the gas really hard, it does not want to move, does not want to move supersonic, does not want to go high speed. I need to force it more, if it is a less compressible gas that is what, I am seeing here. Overall, I am telling you only one statement last few classes, when I compare across gamma I am telling, the gas responds better to waves, if it is a more compressible gas.

If it is less compressible gas, it does not respond to the waves better that is what, you are seeing overall. If my shock mach number is 1.2 itself, that is enough to create supersonic flow behind in 1.3 if it is 1.67, I need to have 2.7 mach number that is the way, it goes. And only when I compress it with this 9.36 for this less compressible gas, it goes at least to 2.7 mach number but I compress it by half, it goes close to 2. Shock can move faster relatively in a more compressible gas, again that is coming from these numbers, I am comparing like this.

Now, I want to go and look at the actual velocity values, actual velocity values induced behind the flow, that is the next thing I want to see. Before that, I wanted one more observation, when I think about most of these plots I showed that, if my P 2 by P 1 is not strong enough, it still sends out a wave that is supersonic travelling we found that, shock mach number is always more than 1, for any P 2 by P 1 more than 1. But, it induces a flow less than 1, mach number will be very small behind the flow example, clap my hand you hear that sound, that is a sharp sound, that is a shock moving past your ear.

People have done shear and experiments of clapping and they found that, there will be shock waves going from your hand. How is a shockwave created, I am compressing the gas in between my hands and I am making it go to very high pressures locally and it escapes through a corner, it finds a hole and escapes. When it escapes, there is a interface between the high pressure gas and the outside low pressure gas so that causes this P 2 by P 1 the energy is transferred from this high pressure gas to the low pressure gas and that is going to send out a compression wave.

That gas is going and telling information to the outside world that, this region has high pressure that is the overall thing, that is happening that shock is going that way. And the flow behind it, we known that it is not supersonic flow, when I do this the paper itself does not want to move so much. But, I still create supersonic flow, I create a shockwave it is not going very seriously fast behind, flow behind is not very seriously fast. So, that being said, when I think about any unsteady change in the flow and again inducing you to think more towards unsteady direction.

Because, that is the current day requirement for gas dynamics people, that is what is missing in gas dynamics world. People do not know how to think unsteady, they are all so much trained and steady gas dynamics. So, I want to induce more and more of unsteady. So, when I think about I want to make any change in gas, I send out a pressure pulse, if it is a compression wave, if there is any compression required P 2 by P 1 is atleast more than 1 even a little bit, there is a shock going.

It will be a very weak shock that P 2 by P 1 may be very, very small and the flow behind need not be supersonic, it will be just a very small value, it could be even 1 centimeter per second. Because, we know if P 2 by P 1 is 1, it is 0 value, 0 meter per second so 1 centimeter per second is more than 0, I can have such a flow field also. So, any unsteady phenomenon, if I have a pressure increase as there is a compression wave moving into some flow field.

Then, I am reading flow behind it in the direction of the wave and it could be any value between 0 to some high asymptote value, which is equal to square root of 2 divided by gamma times gamma minus 1 this is what, we saw already today. All these you just need to remember, I am just going to reiterate this several times finally when you are going to unsteady gas dynamics towards the end of this course, you should be ready and you just say that is what, we have been talking all this time and I will just accept whatever I am telling that is what, it should become later.

So, I am just training you for that now, we will go and look at, the velocity induced is not just related to what is happening in the shock and the gamma value and the P 2 by P 1, it also depends on the speed of sound in the medium in which it is moving, there is also one more factor. So, I calculated the actual V 2 value, the velocity behind the shock and if you go look at the formula, it has a 1 in it, V 2 is equal to a 1 times a big function of a gamma and P 2 by P 1, you will get such a big function we wrote it last class. So, you will get such a big function there now, we want to go look at that V 2, I called that induced velocity here, let us go to that picture, it is here.

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And I had chosen different gases and T 1 equal to 300 Kelvin, I will have to use the mouse for people on video so T 1 equal to 300 Kelvin, that is the gas still having in 300 Kelvin temperature. And I have chosen different gases, I said gamma equal to 1.3 for H 2 and Co 2 it is roughly 2, one is 1.29, other is 1.308 something it is all, almost there that kind of numbers, we will just use 1.3 for both. And they have different molecular weights one is 44 and other 18, very different numbers and because of that, you are getting speed of sound to be very different, even though T 1 is the same.

I am giving speed of sound just next to it, 424.4 meter per second, Co 2 is having 271 meter per second, air I am having 348 meter per second. If I have gamma equal to 1.6, I have picked 2 noble gases helium and argon, very different in molecular weight one is 4 other is 40. And again you are seeing, helium being very light is having very high speed of sound because R will have denominator molecular weight and so it is going to be very

very high-value. And argon is heavy so it is going to be having lower speed of sound that is what, you are having finally.

Now, will go look at let us say, we will pick gamma equal to 1.3, the 2 cases blue and green alone, blue is here green is here, both are same gamma. Now, it is not effect of compressibility, you are just looking at effect of molecular weight. When the gas is heavier, it is not moving faster, I would call it inertia in my opinion, I am pumping in energy through the shock and it does not want to go fast, it wants to go slower, it is heavier mass it is moving slower, simple way to look at it that is the easy answer.

If you want to go give a complicated answer, a 1 is lesser, a 1 is lesser does not give me any physical field but when the heavier gas gives me a lot of physical field, heavier gas is more difficult to push. Even though if I spent the same energy, it does not want to go with the same velocity, it is lesser velocity. Kinetic energy of the flow may be the same but it is having lesser velocity, density is higher for Co 2 of course.

So, kinetic energy may be the same in both cases I did not plot that, may be I should plot that next time and show you that is what it looks like, may be you can plot it by yourself, it is not that difficult to do this. Now, if I look at gamma equal to 1.67 those two cases helium and argon, this is your light blue color and the other one I do not know what color it is, some color I think it is this one. So, I am picking this case and this bottom most case, again speed of sound is making a huge difference but same observation, helium is a very light can and it is moving extremely fast for the same P 2 by P 1.

It is moving extremely fast, I cannot tell it is a gamma effect as of now, I am not looking at gamma right now, it is the same gamma doing this. Now, we will go and look at mixed gamma I know that, argon is 40 and Co 2 is 44, they are close we will pick that particular case, it is almost same molecular weight, very different gamma values. One Co 2 is more compressible than argon so green versus the last one, they are seemingly here. There is a effect of gamma here a little bit but predominantly it is having the effect of molecular weight because Co 2 is having lesser speed of sound than this and that is having some effect.

If it is exactly same molecular weight, probably this will be a little higher, Co 2 is 44 argon is 40 if both were 40, this will be a little higher. And then you just see the effect of gamma, which is more compressible gas will be pushed faster that is, you are seeing here also, it is too close that is all. Now in the middle, I have air, I do not have any other comparison for air, I could have put in ethylene I thought about it only now, C 2 H four has the same molecular weight almost 28 it will have.

I did not plot that but if I put C 2 H 4, the gamma value will not be 1.4, it will go to some 1.26 so I did not want to think about it too much, there is no good companion for air to compare. So, we just left it as is, I could have used oxygen it will be almost the same, gamma is 1.4, 28.8 will become 32 not much of a difference. So, it will look like this kind of curve, even closer so I did not want to plot it. But, the two observations we have from this, one is if it is a more compressible gas, it is going to be moving faster for the same P 2 by P 1 and T 1.

If it is lighter molecular weight, it will travel faster, those are the two observations I am getting from this. Now, I want to go look at other properties, which we did not look at till now, other properties I want to go to the board and derive the formula, before I go and use it. So, I will go here, I want to look at P naught 2 by P naught 1 across a shock, P naught 2 by P naught 1 is less than 1 for a shock fixed coordinate system, which is what you have been seeing. P naught decreases across a shock we use to say now, I am going to say the opposite, we will see why after this.

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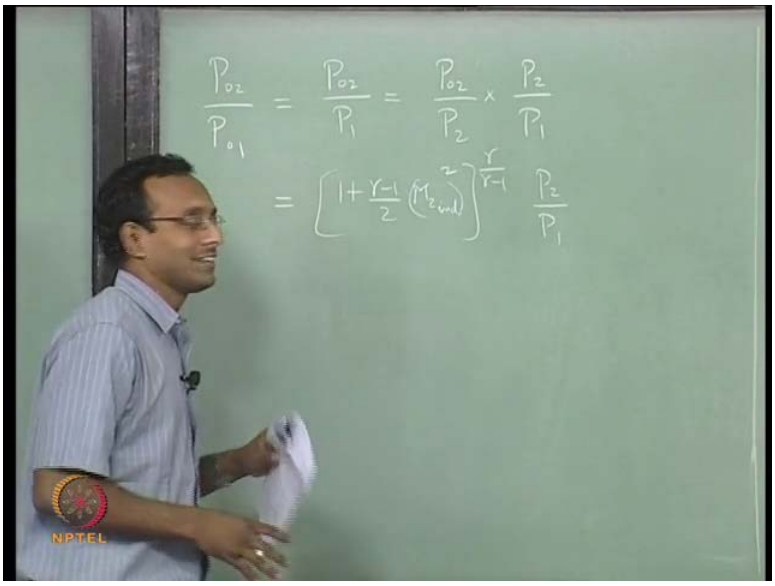


I am deriving the formula for it P naught 2 by P naught 1, I can write this as P naught 2 by P 1 why, my mach number for my gas 1 that is the incoming gas is 0. So, P naught 1 by P 1 is 1, 1 plus gamma minus 1 this is stagnant gas, so P 1 is same as P naught 1, it is stagnant gas, stagnation pressure is same as static pressure, so I am having this. Now, I want to write this the favorite way of gas dynamics people cycling ratios, most of the books will use this cycling ratios, very nice way to express things.

Now, we know this because I know induced mach number M 2 so I will write this as 1 plus gamma minus 1 by 2 M 2 induced square to the power gamma by gamma minus 1, this is the same formula nothing changes, multiplied by P 2 by P 1. Now of course, M 2 induced we wrote a huge formula just sometime back in terms of P 2 by P 1. So, the whole expression in terms of P 2 by P 1 I can get, now you want to go look at, what this plot looks like then we will talk more.

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This is my P naught 2 by P naught 1, I did not put it very clearly near 0 and surely, let us use the mouse here, if I zoom in, it starts from 1 on the x axis, it starts from one on the y axis. This is the last point I have, it starts from 1 on the x axis, it starts from 1 on the y axis since I have number like 1000, that 1 is diminishing to look like 0, it is almost 0 in this scale, that is all is happening anyways. So, if I look at it, if my P 2 by P 1 is 1, there is no change in the flow which means, P naught 1 is same as P naught 2 that is true.

But after that, any higher P 2 by P 1 that is, if I send a reasonably compressing wave then we find that, P naught 2 is higher than P naught 1 why, that is the question we want to answer next. Why is it higher typically, it is supposed to be lower when it is processed by a normal shock, this is higher why is that. All this time in normal shock, we have been discussing stuff and we said P naught 2 is less than P naught 1 always now, we have the opposite case, why is this happening. It is of course, related to reference frame shift, something more is happening, I think physically it is easier to answer than going for mathematical expressions.

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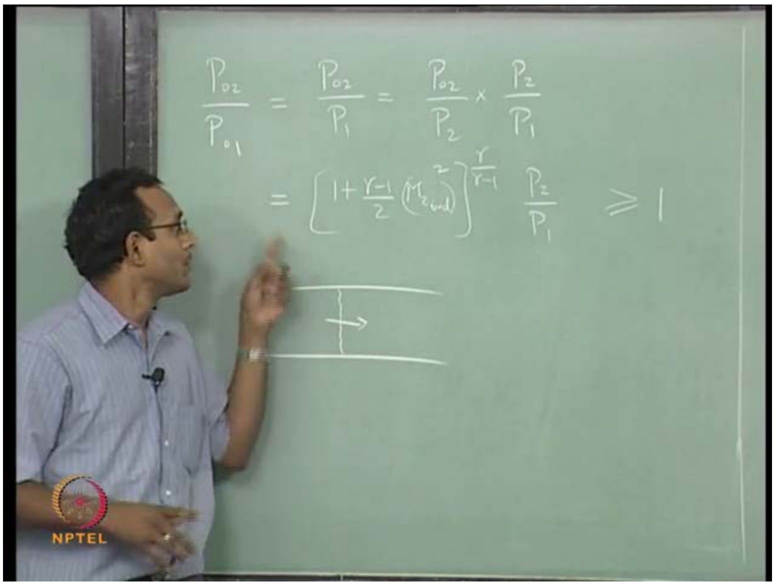


I just plotted this, I know M 2 induced in terms of P 2 by P 1, I plotted this as a function of P 2 by P 1 that is all I did where, I know M 2 induced in terms of P 2 by p 1. I used that calculated P naught 2 by P naught 1 for a whole bunch of P 2 by P one and plotted this curve. Now, we are finding that, this is always greater than 1, I will say greater than equal to 1 now, we want to explain this, that a goal for us finally.

So, how will I explain this so what we are seeing, if I have a stream tube and I have a moving shock in it, the main difference is the shock is moving, the gas here is stationary. And here, the gas is moving which means, first thing velocity is higher, second thing pressure is also higher. Basically, I took a gas which has only pressure energy no kinetic energy and gave it pressure energy and kinetic energy.

When I do that, if I bring this gas to rest through a imaginary isentropic process, the way we discussed stagnation pressure. When I do that all the extra kinetic energy, which I gave to the gas should also be converted to pressure energy and imaginary isentropic process making the gas to stagnate which means, kinetic energy is converted to pressure energy. When I do that, I find that already pressure is high, I added all kinetic energy in terms of pressure energy so it will be much higher that is the reason, this should be more than 1 if it is a moving shock based flow.

P naught 2 by P naught 1 should be greater than 1 for this flow, this is something very different from ordinary gas dynamics, normal shock shock fixed coordinate system where, we will tell that the shock absorbs some energy and sends the gas with lesser energy. Pressure energy is lesser, P naught 2 is less than P naught 1, in a moving shock system P naught 2 is greater than P naught 1 that is what, it will come down to. That being said, you want to look at T naught 2 by T naught 1, stagnation temperature ratio.

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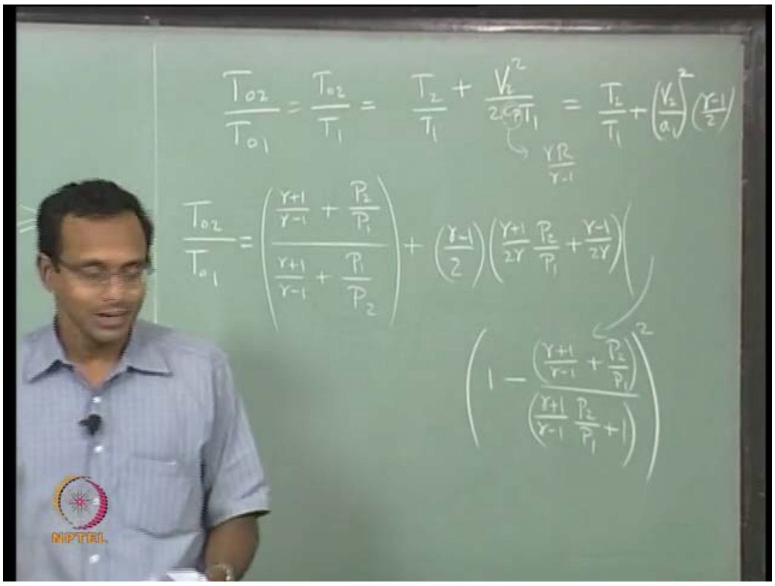


And of course, stagnant gas so I can rewrite this as T naught 2 by T 1 and I will expand this T naught 2 for a c p equal to calorically perfect gas, T naught 2 will become T 2 plus 2 square by 2 c p, this whole thing divided by T one this is what, I will have. Now, I can rewrite this c p, which we have been doing all this time anyway, it will become gamma R by gamma minus 1 where I will pull this gamma R T 1 as a 1 square. And we already wrote an expression for, I will write this once more T 2 by T1e plus V 2 by a 1 square multiplied by gamma minus 1 by 2 this is what, I will have.

And if you go back to last class, you will find that V 2 by a 1 we wrote an expression for this, we wrote V 2 in terms of a 1 and P 2 by P 1. So now, if I put all this together, I am going to write that expression anyway today T naught 2 by T naught 1, it is again a big expression. T 2 by T 1 we know this already from normal shock itself, plus gamma minus 1 by 2 times V 2 by a 1 square. So, that is going to be gamma plus 1 by 2 gamma P 2 by P 1 plus gamma minus 1 by 2 gamma.

It is used to have a square root on the top now, I have removed that because there is a square, multiplied by I will write the next term below so I will say that term is here, 1 minus gamma plus 1 by gamma minus 1 plus P 2 by P 1 this is a numerator, divided by gamma plus 1 by gamma minus 1 times P 2 by P 1 plus 1. This did not have a square in V 2 by a 1, so now I will put a square on top of this term. So, it is T naught 2 by T naught 1 is T 2 by T 1 multiplied by gamma minus 1 by 2 multiplied by, this term is your W S by a 1 by the way.

W S by a 1 square that is this multiplied by, this is your u 2 by u 1, 1 minus u 2 by u 1 square that is what, you are getting finally, that is your whole expression. You go back, remember the derivation you will see that, this is 1 minus u 2 by u 1, this is your W S by a 1 square and this is your gamma minus 1 by 2 is coming from here, this is your T 2 by T 1 coming from normal shock tables itself, normal shock relations itself. What do we expect in this case T naught 2 by T naught 1, will it be less than 1 or more than 1 or equal to 1, that is the next question.

Physically speaking what should it be, physical feel is more important for us, physically speaking T naught 2 by T naught 1, what should it be less than 1, more than 1, equal to 1. More than 1 why, P naught 2 is greater, do not link that, it is non isentropic process. We cannot say, P naught increase so T naught increases you cannot say that, it is non isentropic process. Only for isentropic process, if pressure increases temperature increases, I cannot say that here.

So, you cannot gives that answer but your answer is correct give me some other answer, velocity is higher, V 2 is more than V 1, V 1 was 0 V 2 is some nonzero value and T 2 is more than T 1, shock has heated the gas. So, if I look at going to stagnation through a imaginary process taking the gas to stagnation condition, enthalpy plus kinetic energy now, I am saying enthalpy increased kinetic energy increased so the net stagnation temperature increased.

So, that is the way we have to look at it, I am energizing the gas so T naught 2 will be more than T naught 1. If it is a shock fixed coordinates, T naught 2 by T naught 1 will become 1 that is what, it will be. This is not shock fixed coordinates so you are getting all these kind of results, that is the difference. If it is shock fixed coordinates, you will suddenly get that this will become W S by a 1, I cannot tell that directly from here, we just leave it like this. You can go think about, how to make this go to shock fixed coordinate, is not easy I have to subtract another term. When I subtract that term, this whole thing will become 1 let us not worry about that here. I thought, I can do that simply, it is not possible.

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Now, we will go look at the plot of T naught 2 by T naught 1 for this, which one is it, it is here what we are seeing in this case, again I am plotting P 2 by P 1 going from 1 all the way to 100 and T naught 2 by T naught 1 of course, is starting at 1 like every other plot. If P 2 by P 1 is 1, there is no change in T naught 1, T naught 1 is equal to T naught 2 because it is a very weak wave no change in the gas. Now, as I increase my P 2 by P 1 you are finding that, T naught 2 by T naught 1 is increasing. But, look at the numbers, this is like going from 1 to 40 or 1 to 45, while P naught 2 by p naught 1 went all the way up to 1000.

So, again we can say that, enthalpy energy added is not as high as the pressure energy added, it is a compression wave, of course. The pressure energy added should be more that is what you are seeing and one more way to look at it, supersonic flow response very fast by changing its pressure than changing it is temperature, which you have been seeing all through, even from isentropic flow onwards. You always saw that, pressure was changing faster than temperature and density will always be somewhere in the middle.

I did not plot rho 2 by rho 1, if you want you can plot that also, it will be somewhere in the middle but anyways this is the idea here, we are seeing something opposite. Gamma is equal to 1.67 is the top most, gamma equal to 1.3 is the least, all this time we said let us go look at the previous curve P naught 2 by P naught 1, where is P naught 2 by P naught 1.

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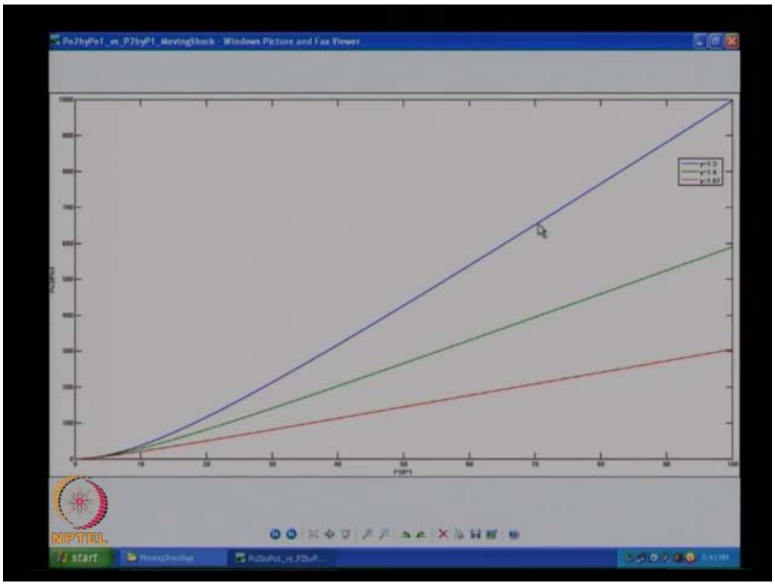


Here, if I look at P naught 2 by P naught 1 also I say that, most compressible gas will get most compressed or it will be more energized.

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But now, when I look at T naught 2 by T naught 1 it is giving the opposite result, this is the only change. If I have to explain this, I have to go into how molecule uses the energy inside, it is something to do with internal energy. It is related to c p that is how, it is related to internal energy, has to do something with the c p and that is something to do with gamma so you are having this effect. It so happens that, the way by which gamma is increasing, is by decreasing the number of internal energy modes.

That is, molecule can use the energy to just move or it can rotate about it is centre of mass or it can vibrate about it is centre of mass or electrons can move to higher energy levels, there are so many things that are possible. How does the gas used it is energy, will decide it is c p value and that is decided by actual energy present in the gas itself. There is too much energy it will give it to all the possible energy modes, if there is not enough energy it will not give to all the energy modes.

So, when the temperature is high you can expect c p to change, when the temperature is low not other modes are available, it will be just one fixed value. That is what, is most commonly used in our gas dynamics where we say, gamma is constant, we used calorific perfect gas, you are going to say gamma is constant there. Here we are seeing that, when there is more energy modes the temperature drops, that does not mean the energy is dropped really, but I cannot prove it here for you.

If I look at enthalpy H naught 2 by H naught 1, you will find that it will be following our regular expression, the c p is higher and so it looks like it is dropped that is what, is happened here. If I look at H naught 2 by H naught 1 and plot that, blue will be the top most, green will be in the middle, red will be the lowest. Then it will explain everything for us then it will look like more compressible gas, will be more energized, that is logical for us.

This looks like it is opposite simply, because more compressible gas has higher c p and I have divided by the c p for the higher condition and it has dropped the other way. That is what we are seeing currently, may be I should have shown you also H naught 2 by H naught 1 but you have the expression, you just going and plot it, it is very easy to plot. So, I will just leave it like that for you of course, you have to know that, when it is higher temperature the c p value changes.

If I use constant c p there is no difference, H naught 2 by H naught 1 is same as T naught 2 by T naught one but I cannot explain it in any other way, as of now. We will just leave it like this, this is the only one curve, which does not explain our statement, it does not go along with our statement of a more compressible gas will be more energized by a moving shock, it is the only thing that is going against. We will look at only thing left, only variable left in our flow field, entropy change.

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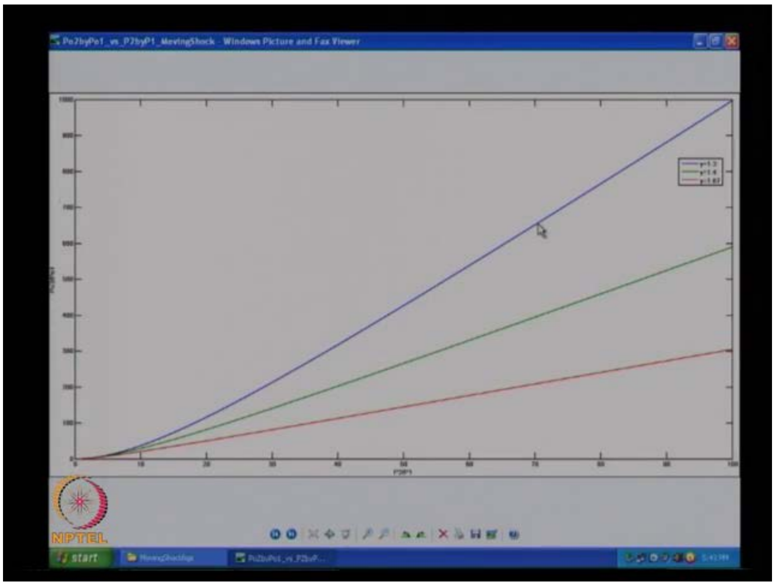

Here, we have the delta S by R curve for various P 2 by P 1 for the moving shock, for different gammas as usual. If we look at the curve, it is like almost going flat for some time for low P 2 by P 1 and then it is going up and straitening out, as we go to very high P 2 by P 1 values. What we are seeing is, if we just look at gamma value high, which is least compressible gas, it does not take a lot of entropy and when it is most compressible gas, it takes a lot of entropy that is like the bribe, which we will talk about later.

The function how do we calculate this, we remember we derived sometime back delta S by R for normal shocks to be equal to log of T naught 2 by T naught 1 to the power gamma by gamma minus 1 multiplied by P naught 1 by P naught 2, the whole inside the log., that function is what we will use. A long time back, we made the T naught 2 by T naught 1, 1 and then we got the expression for normal shock delta S by R. Now, we cannot make T naught 2 by T naught 1 equal to 1 because we just saw the previous curve T naught 2 by T naught 1 will be changing with various moving shocks, that is the main modification. Now, we will go to the next plot, which is zoomed in version of this one.

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I just wanted to show this little detail here where, we are seeing this for small changes in P 2 by P 1 that is weak moving shocks, we find that entropy is not at all increasing, it is very, very negligible. And this is the regime where acoustics people work, this is basically the regime where we say, it is almost isentropic compression wave and this is that region. And beyond that point, it is not true say, if my P 2 by P 1 is something like

1.4 or above, that will not be the case and that is the main change I wanted to talk about here.

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And again we see that, if you go back to the previous curve I am going to say that, delta S by R is not going to be a constant, it is going to be increasing continuously. And that is related to, we just saw that T naught 2 continuously increases as we go to higher speeds of the wave. And again I am saying that, more compressible gas draws more energy into entropy which means more compressible gas will waste more energy from my shock. The shock will be dissipated faster in more compressible gas, even though it is getting energized fast, it will get dissipated faster also.

Because, entropy is one form of waste of energy in my way or I want to call it entropy as the bribe, we have to give to nature for the gas to do what we want it to do, that is the way we want to look at entropy. Any system let say, I take refrigerator common example for entropy in thermodynamics, for me to take energy from a system that is already cold and send it out to a hotter section. It is against nature but we want it to happen so for this to happen, I have to supply energy.

Where is all this energy going, it is actually net energy balance across, whatever energy is taken from the cold section is going to the hot section. But, I am pumping energy extra every time, why am I wasting electricity, that extra energy is directly going to satisfying nature, that is the bribe I am giving nature continuously. So that, it keep on cooling that gas, keep on cooling in this case. the ice box or whatever inside the fridge.

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The same thing here, in this case I want this compression wave to compress the gas and the gas should move away from the high pressure section, go towards the low pressure section, that was the use of my compression wave anyway. It was going and telling, there is high pressure here get away from here, that was the information the shock is taking away. So, when it is taking that information, for the gas to follow this, somebody has to give bribe to this nature, that is done by this entropy.

We are finding that, if the gas is more compressible, I have to give a little more bribe that is what it says, I called that bribe here because very immediately easy to understand for us. You will never forget it if this is the case, nature wants bribe in the form of entropy increase, any process that is naturally occurring is occurring because there is some entropy increase. Somebody is paying some energy to nature so that, it is happening, I take this chalk and drop it on the ground entropy increases, it produces sound when it hits the table, entropy increases.

Everywhere I am wasting energy how, I wasted I added energy by increasing it is potential energy and letting go that is how, I started. If you go and do the thermodynamic analysis of this, you will find that there is some energy imbalance, the collision that happens on the table is not energy conserving collision. That extra energy, part of it goes to entropy increase, part of it goes to creating the shock, you are going to hear that sharp sound when the chalk goes and hits the desk.

when that sharp sound comes, that is creating a strong compression wave and behind that, there is flow. If I drop a book on the table, there is a reasonable sound I have actually seen shear end pictures of dropping a book on the ground, it produces nice shockwaves on the sides. You can go find it on the web, it is available, go find such things it is nice, e fluids I think is where I saw, I am not sure there are. So, many websites like this, e fluids is one such nice website where, you can see a lot of gallery of fluid mechanics, fun stuff. There are other websites also, I remember this one first, how stuff works is another where, they give a lot of stuff like this. There are so many other websites, I should not be advertising websites on NPTEL probably, but anyways we would not worry about that so much.

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So, that is the way we have to look at entropy change, the next thing we have to do, actually I am going to close moving shock discussion here except for, I will start solving problems actually, numerical examples. I think I can finish one numerical example today so we will push it, we will try and solve one problem today, that is just it says 3 minute I will probably take 4 to 5 minutes and finish it off. I am going to pick a simple case, we did not solve any problems using normal shock also. So, we will first go and do problems with stationary shockwave, we would not to moving shock today, we will do it next class.

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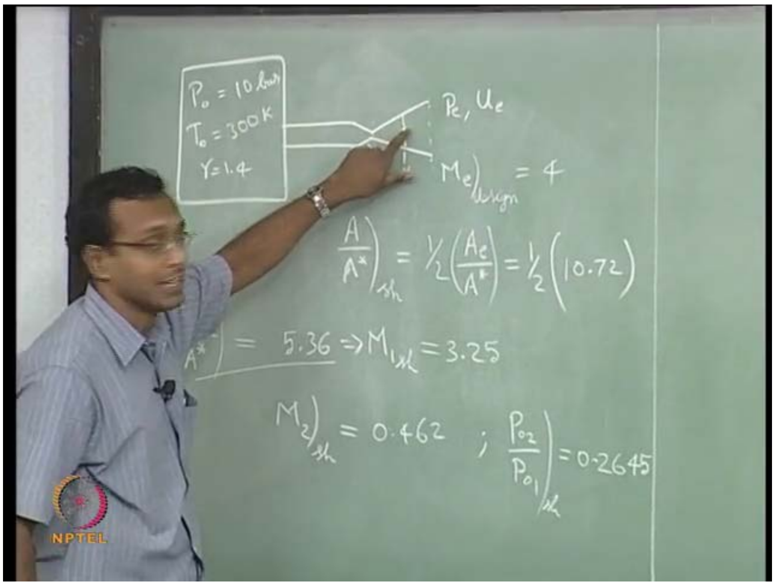


So, I am starting with a big chamber with pressure of 10 bar and temperature of 300 Kelvin, this is the condition which we have in our gas dynamics lab, I am just using that condition directly here. And I put P naught and T naught, while I told you pressure and temperature why, I am going to say the velocity is inside this tank, the tank is so big and the mass flow rate out of this is very small but the velocities are almost close to 0 and so I will call this my stagnation condition.

And I am saying, there is a small pipe and there is a nozzle, conversion diversion nozzle sitting at it and there is a shock sitting at, I know the location and I am going to tell that, it is sitting at half the area ratio of the exit area. The nozzle is designed for M exit designed equal to 4 and I am going to tell the shock is sitting at A by A star of the shock, is half of A exit by A star. These are the information given now, I want you to find the exit pressure and the exit velocity this is what, you need to find.

So, we have to know that, if there is a shock here and there is stagnation here, mach number goes from 0 to sonic to supersonic and then there is a shock. So, I know that, the nozzle here is choked, I am introducing a word which you should not know as of now, all I know is the mach number is 1 at the throat that is what we know, as of now. So, if I know that mach number here is 1 now, you want to find out, what is the mach number at this location, which is my shock location.

This is my star condition because I know mach number is 1 somewhere there so it is a star condition. So, I can say of course, we will go and solve again c d nozzle problems later, I just wanted to pick this example and solve it. Now, I want to find the conditions here, I want to find the mach number just ahead of the shock, it is a very thin layer so I will use the same A by A star for the shock. A by A star, I will go look at my compressible flow tables, I want a supersonic mach number for it, I can get a subsonic or supersonic mach number for A by A star.

I will get the supersonic value and that comes out to be 5.36, this happens to be your shock mach number or mach number just ahead of the shock, this is equal to M 1 for shock. From here, I will go to normal shock tables, this information I got from isentropic flow table now, I will go to normal shock tables, I will find M 2 for shock. m 2 for shock I will put in this number, I am assuming it is air, gamma equal to 1.4. I have to look at tables corresponding to gamma equal to 1.4, I am going to get 0.462 and since we are interested in pressure at the exit, I also want to carry over the other information.

P naught 2 by P naught 1 across that shock, that information is, I made a mistake I did not give you the mach number itself, I will take a step back I made one mistake. When this is the case, it is half of 10.72, this is the first point where I have to start, A by A star for M equal to 4 corresponds to 10.72. From isentropic flow tables, half of that is 5.36 this is equal to and this if I go back and look at isentropic tables, this is going to give you a mach number of 3.25, this is my shock mach number, I made a mistake already everything else is correct.

If I go look at normal shock tables for this mach number, I am going to get this and P naught 2 by p naught 1 across the shock is going to be 0.2645, I am getting these numbers. Once I know this, the remaining stuff is downstream of this shock is subsonic flow and areas varying in a subsonic flow, that is all I need to look at now. Subsonic flow and it is areas varying in a subsonic flow, so when I look at it is the same area there.

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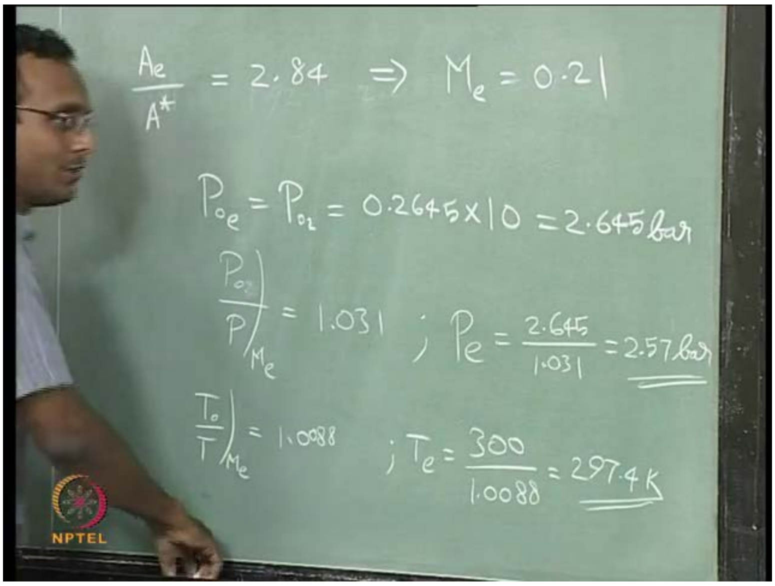


But, my mach number changed, I will go and find, for this mach number what my A by A star is A by A star for M equal to 0.462, that number comes out to be 1.42 which means, my A star across the shock has changed. We will just keep it like that, we will go talk about it later, this will tell me that A exit by A star is supposed to be twice that value because shock is sitting at half the area, it is going to be 2.84. If I go and look at isentropic tables again, that is going to give me exit mach number, it is supposed to be subsonic flow, it will become more subsonic than this and that value comes out to be 0.21.

Now, I am almost done, I want pressure at the exit, I know P naught at the exit is equal to P naught 2 because the remaining problem is isentropic flow, only at the shock it is not isentropic. We know the P naught 2 value because we know P naught 2 by P naught 1 0.2645 multiplied by P naught 1, which is 10 bar, so it is 2.645 bar. So now, I will go and look at P naught by P for mach number at exit, which is 0.21 if I look at that number that is, 1.031 I know P naught is this value, I want P.

So, it will be P naught divided by 1.031 will be my P value therefore, P exit equal to 2.645 divided by 1.031, which is 2.57 bar, this is my exit pressure. Only one more thing exit temperature, T naught by T for this mach number, exit mach number of 0.21 is 1.0088. And this tells me that, T e is, I am using 300 Kelvin because there is no change across T naught, change across a shock for T naught 2, T naught 1 and T naught 2 are same because we are using shock fixed coordinates. And I am having this, divided by this value 1.0088 that is going to give me 297.4 Kelvin from here, I have to find speed of sound and then multiply with mach number and I am getting velocity.

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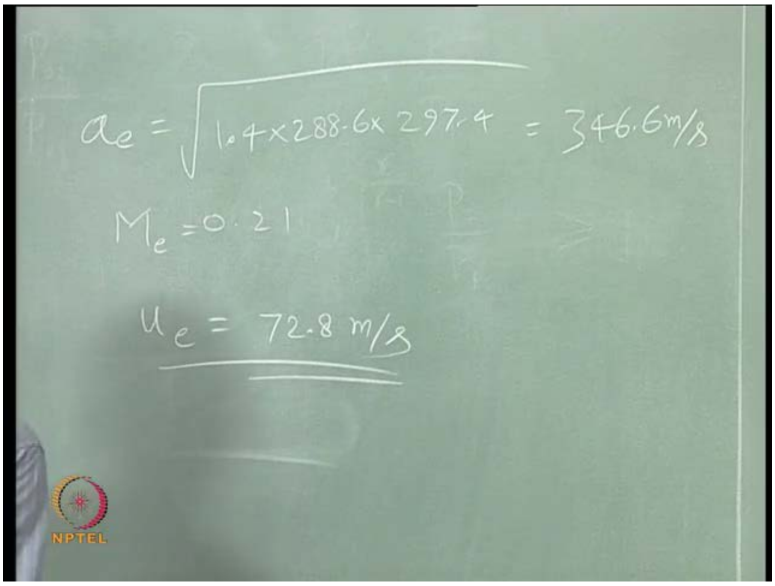


The a at exit is square root of 1.4 into 288.6 into 297.4, that is giving me 346.6 meter per second. I know my M exit is 0.21 so my u exit is going to be a product of these two, that is going to be 72.8 meter per second. So, I just used both isentropic flows and normal shock tables to get to pressure and velocity at the exit. So, we will solve more problems next class, we will stop here for today.