Gas Dynamics
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Module - 5
Lecture - 14
Normal Shock Relations Moving Shocks

Hello everyone welcome back. We were looking last time about variation of various properties across a normal shock. And we were looking at variation as a function of mock number.

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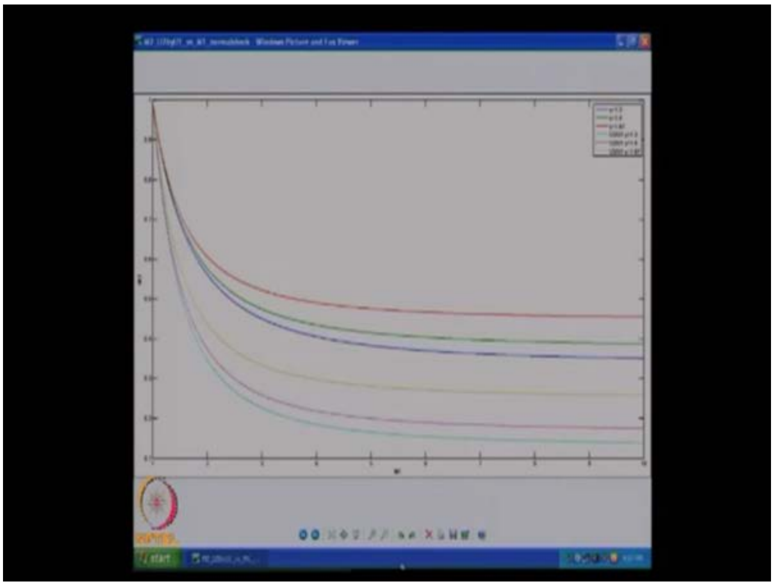

We will go to the screen, there where we saw that, if I look at downstream mock number as a function of upstream mock number for a static or normal shock that is stationary. Then as I increase my mock number as in the incoming flow mock number with respect to the shock, if that is increasing we find that the downstream mock number decreases from close to 1 to something very low. But, as we go more and more downstream, as we go to very high mock numbers, we are finding that the downstream mock number is tending to become a constant. It is becoming asymptotic value of some different values for different gamma values, which we have and if you look at u 2 by u 1, again that is also going to a constant in each of these cases. So, we can find the exact asymptote.

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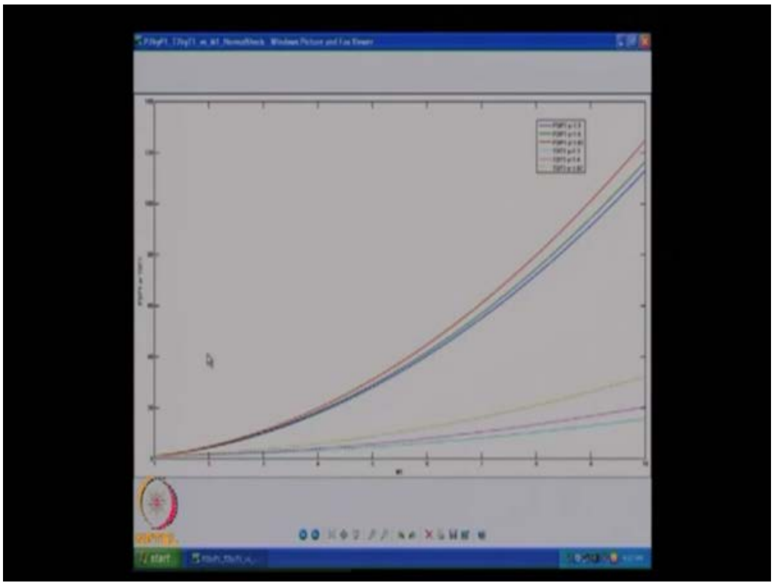

If we look at analysis and if I look at t 2 by p t 1 or p 2 by p 1, the pressure ratio or the temperature ratio, they are not going to any asymptote as we go to high enough mock numbers.

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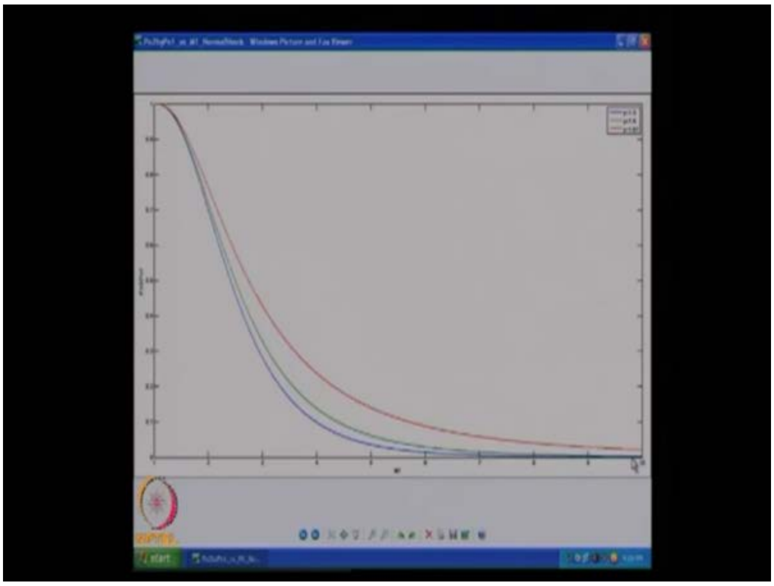

And of course, p naught 2 by p naught on it looks like it is going to asymptote. If I go and look at it in long scale, it looks like it is continuously dropping. It is not going to an asymptote.

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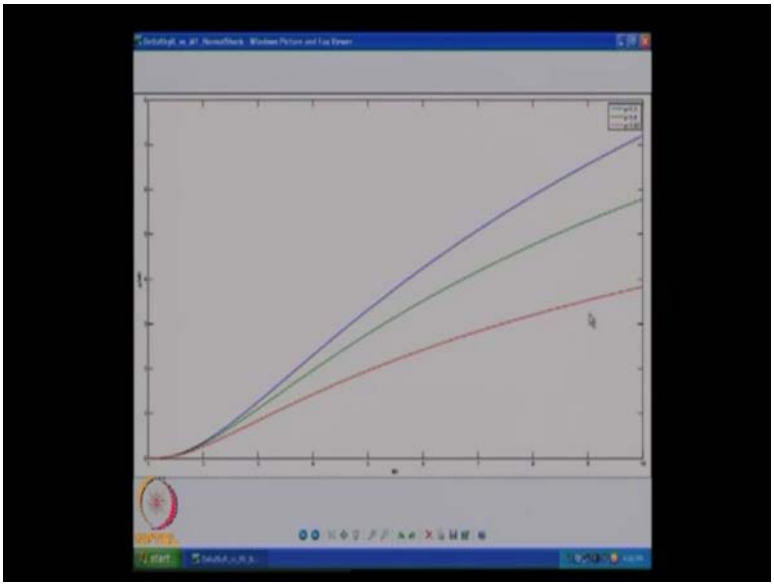

Again that will be very clear if, I go take delta s by or which is just log of p naught 2 by p naught 1. If you look at that it is continuously increasing.

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So, if I go back and look at the previous one, where it is p naught 2 by p naught 1 it is dropping continuously. If I take log of these it will be some negative value, because it is less than 1 in the axis. So, we have delta s by r is negative of this, log of p naught 2 by p naught 1 is delta s by r.

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So, you are getting and you are seeing that it is continuously increasing. It is not going to any asymptotic value.

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Now, will go back to the very first one M 2 alone it seems that it is going to a constant. If I look at mock numbers if I go above 5 or so the downstream mock number across a normal shock is almost the same value always irrespective of what my incoming mock flow is. This is the resume where we call hypersonic, were things will go almost the same. If I go above mock 7 that is more valid, it is better if it is mock 7 and above, you want you can call it above 5 itself that it is doing that, and u 2 by u 2 is almost a constant after that point. They can actually find the value for high mock number approximation which is what we are going to start with today.

So, now we will go to the board and find the asymptotic values for this.

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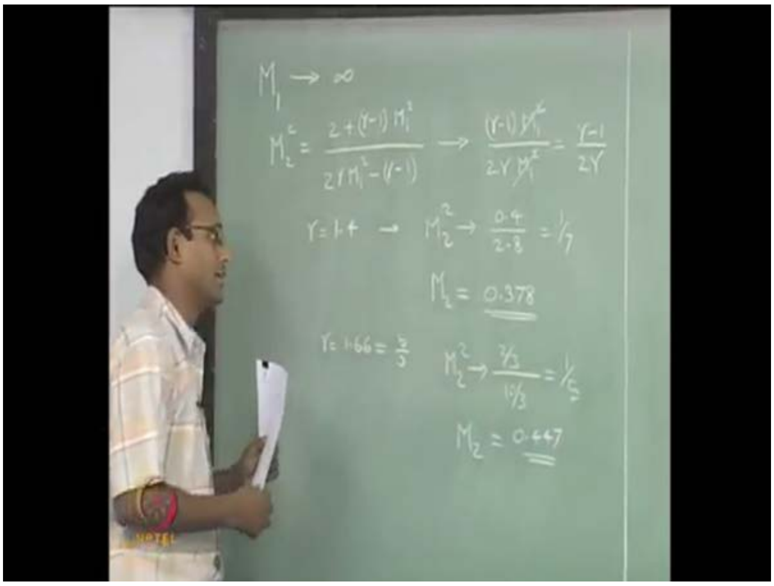

So, we are thinking about incoming mock number tending to infinity that is what we are looking for. And we have this expression, M 2 square we have derived all this normal shock relations, I am just writing, we are having this. Now you want to say if I set M1 tending to infinity. M1 is being very large, then this will tend to I can neglect this 2 with respect to infinity, I will just ignore this 2. So, it will tend to something like gamma minus 1 M1 square divided by the denominator again this M 1 square is very high compare this gamma minus 1, which is almost like 0.4 0.3 that kind of numbers. I can neglect this compare to M1 square. So, it will become 2 gammas M 1 square and we find that this is independent of mock number that is why you are going to constant value of course, this is what you are getting.

Now, if I want to find the asymptotic value for various gamma values, let us say I put gamma equal to 1.4. This is actually remember it is M 2 square, M 2 square tends to 0.4 by 2.8, which is 4 by 28, 1 by 7. So, I pick find my M 2, it is 1 by root 7, which is 0.378, that is the value which our plot would have asymptotic to, if you go back and look at the plot it will look like that but, it is not go to the plot anyway you know it is true.

If I look at gamma equal to 1.66, which is for more atomic gases, I can say this is exactly equal to 5 by 3, it is not really 1.66, it is actually 1.66666 bar, if you want to think about it right irrational numbers anyways. We will just keep it as 5 by 3, it is easier to work with. Then I am going to say my M 2 square will become 2 by 3 divided by 10 by 3 which is 2 by 10, 1 by 5. So, you are going to get **1** by root 5 as your mock number for that case again. So, M 2 becomes 0.447, this is what you are going to get. If I look at gamma equal to 1.4, to gamma equal to 1.66, gamma equal to 1.4 gas will have lesser speed of sound, which I said, in a way is related to more compressible gas. If I think about it that way more compressible gas will go to much lower mock number than a less compressible gas.

Just a feeling you are supposed to get developed out of this course. If it is more compressible, it will be more useful if you are going for serious calculations with gamma changing during your flow. But, we are not going to deal with that in our whole course, I will just show you what happens if I change gamma that is all and we will let go here if you go to in any case, if there is some course if you want to have a course called high temperature gas dynamics, there we will think about variation of gamma which temperature. If that is the case, then I will have to worry about what happens to the compressibility as the gamma changes. This is just give you a little feel for it. If you ever have a course on high temperature gas dynamics, that will give you a lot more feel for it. I will just leave it there if you want you can go read up more on high-temperature gas dynamics. What will happen to pressure ratio?

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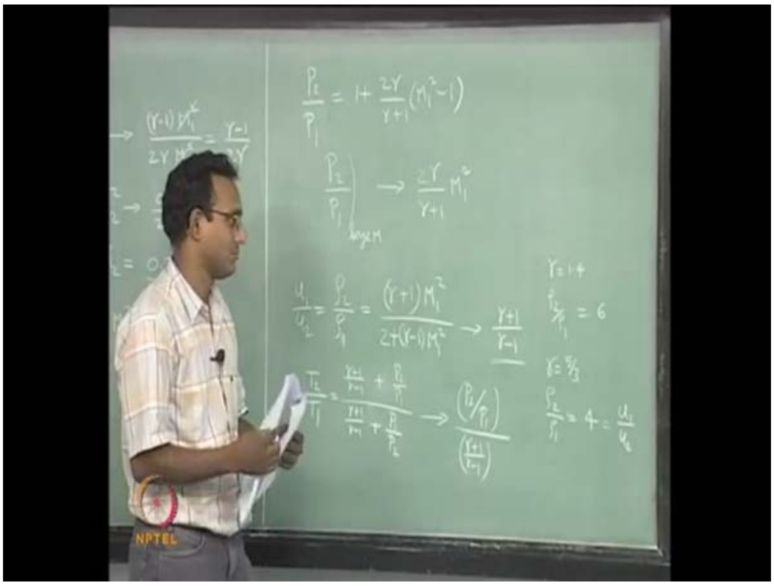

We had this formula very easy to remember one. So, we will just keep it this way, these is one of the forms of P 2 by P 1 we had. If I, look at this, if I say M 1 is very big number I am trying to go to asymptote. So, P 2 by P 1 for large, M1 will tend to I will neglect this 1 with respect to M square, I will neglect this 1 also with respect to M square. So, it is just going to become 2 gamma by gamma plus 1 M 1 square, if M 1 is very high P 2 by P 1 is also very high. It is 1 M not independent of mock number, it increases if I keep on increasing mock number. So, it is not going to a constant that is all. So, it is not going to asymptote which is what we saw in our plot also. Now, I will go rho 2 by rho 1 density ratio rho 2 by rho 1, density ratio. We had one more nice form easy to earth quiet in this case of course I have all ready tabulated all the various formulae which you can us. I am just picking one of them which will be very comfortable for this analysis.

So, I am going to take this 1, and I am going to neglect this 2 with respect to this M 1 square of course, product multiplying by gamma minus 1 should keep we would not remote that. So, this comes down too. I will just neglect this 2 and if I neglected this 2, this looks like M 1 square will get cancelled and I will just have gamma plus 1 by gamma minus 1. So, this tends to gamma plus 1 by gamma minus 1, this is all you are getting. Now will go and look at this is again, this says that this is independent of mock number by rho 2 by rho 1 goes to a particular asymptotic value. We did not have a plot for rho 2 by rho 1 specifically in the beginning of the class. But, if you look at this rho 2 by rho 1 is same as u 1 by u 2, if rho 2 by rho 1, goes to a constant u 1 by u 2 goes to a constant, which means u 2 by u 1 also goes to a constant, which is what we plotted and we showed that u 2 by u 1 goes to a constant, anyways.

Now, we will go look at what this number looks like for different gamma values. If I put gamma equal to 1.4, 2.4 by 4, that is going to be 6, if I go to gamma equal to 5 by 3 which is officially 1.6666 and we will just keep it that way to rho 2 by rho 1 will be 8 by 3 divided by 2 by 3 that will be 4. What are we seeing here? I already told you when gamma decreases it is more compressible gas. So, what we are seeing is, if I have an infinite strength shock then I can compress the gas to 6 times its original density, While if it is a less compressible gas, it is going to go lesser compression ratio, cannot go to very high compression ratio, this is why I am, saying gamma is in a way measure of compressibility. In a way, I will not tell it is exact as of now, it is slightly in a way it is correct, it is not always true.

There is also molecular factor inside, if I want to think about speed of sound. But, anyways gamma is in a way related to how compressible the gas is to some extent draft measure. It works reasonably well if this is a case, what will happen to my velocity this is going to be equal to u 1 by u 2 velocity goes to 1 4^{th} , here velocity goes to 1 6^{th} here that is what is happening here.

So, we have already looked at I will just write here, u 1 by u 2, then I will say I have also looked at velocity ratio. I do not need to deal with it separately, we will pick a particular relation. Actually I will, we can even pick, I am not going by the nodes. Now I am, just going to go free-form writing 1 plus, this is going to be more complex. So, I have, to go by the nodes that all we will go back by the nodes because, I will get a m 2 square here, then it will be m 2 and m 1 I have to link. Let us not do it that way, I will just go gamma plus 1 by gamma minus 1 plus P 2 by P 1 divided by gamma plus 1 by gamma minus 1 times P 2 by P 1 plus 1 we had such a relation. No, I made a mistake here, this is not right this happens to be density ratio, whatever I wrote now was density ratio. Pressure temperature ratio is P 1 by P 2 with a plus sign in front of it. This is correct this is t 2 by t 1 and this formula already made a mistake and I was in a hurry.

All this time we have been doing, what happens when mock number turns to infinity suddenly I am writing an expression in terms of P 2 by P 1, and P 1 by P 2, why because I already know from here, if mock number turns to infinity P 2 by P 1 turns to infinity.

So, I will take the shortcut instead of substituting all the expressions here it will be very complex.

So, I am, just taking a shortcut saying P 2 by P 1 goes to infinity, if I say that is very high compared to this number. So, I will neglect this. So, this will tend to P 2 by P 1 divided by, now this is, there and this P 1 by P 2, this is 1 by infinity. So, this is very small number compared to this. So, I will keep this 1 this is what I have. But, there is not a lot P 2 by P 1 is going to turn to infinity. So, T 2 by T 1 will also attend to infinity.

But, not as fast as the other one because, this number is always more than 1, we just did some calculations roughly. We found that, that is 6 for gamma equal to 1.4. So, this will be going up slower, temperature goes up slower across the shock compared to pressure goes up faster. That is what you are seeing here anyway which is what we saw in the plots alright. So, that brings us to the end of discussion on normal shocks that are stationary. Ideally I have to go at this point and start solving numerical examples, which I will do after we go to moving shocks after we finish moving shocks.

I want to give you a little more feel for moving shocks, then what is given in most of the books and after that we will go and start doing numerical examples, that is a little better that gives you a different feel for what can be done. But, in case I want to solve a problem. Typically what will be given will be there will be compressible flows tables which we already said there are tables like this available, we said that we will use isotropic low tables that is what we did before. Now I am, saying there are also normal shock tables for compressible flows already available, they deal with only stationary normal shocks, not moving normal shocks. Stationary normal shocks tables are available.

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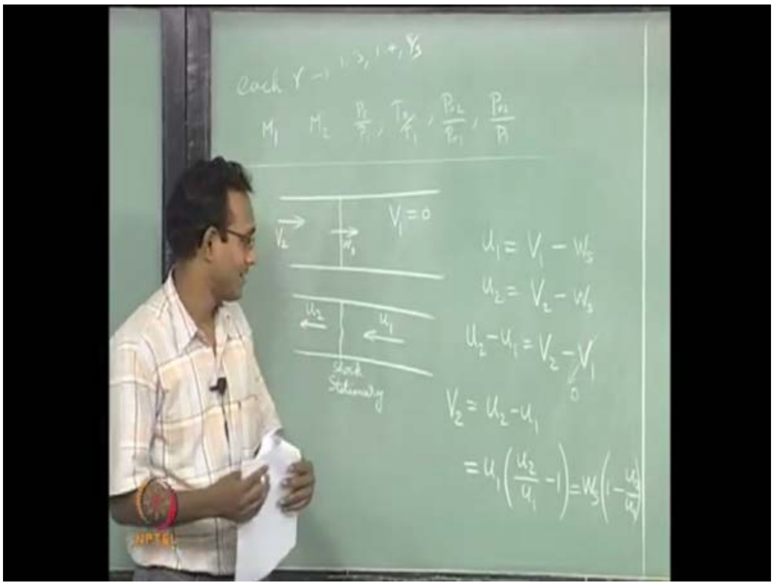

So, what they give will be, for each gamma typically they will give 3 gamma’s 1.3, 1.4 and 5 by 3, 1.666. These are the 3 values for which, they will give data typically. What they will have be M 1 as the input variable and they will give you M 2, P 2 by P 1, T 2 by T 1, P naught 2 by P naught 1, P naught 2 by P 1, these are the things they will typically give you. There are some more things where they will give you rho 2 by rho 1. But, I think it is unnecessary if I know P 2 by P 1 and P 2 by T 1. I can find rho 2 by rho 1, as long as my gas does not change its molecular weight during the shock that is not the case in our ordinary gas dynamics. So, we can very well say that there is no reaction during the shock. All right in that case I can say that I can get rho 2 by rho 1 directly. So, I do not need to have a table which has rho 2 by rho 1 also because P equal to rho r T is valid before and after.

So, P 2 by P 1 divided by T 2 by T 1 will be my rho 2 by rho 1 and there is one special thing P naught 2 by P 1, It will be very helpful in solving problems will see that if I want. When will I use this P naught 2 by P naught 1 to find entropy jump. delta S is minus log of this value. You have to of course, know in your tables whether they are giving it as P naught 2 by P naught 1 or P naught 1 by P naught 2, they will give you different books will give you different things you have to know what is written there on the top.

And you have to of course, get used to picking the correct gamma page, each gamma will be one page typically. So, you have to look the correct page otherwise you will make a lot of mistakes. This is what happens in most of the exams people will just take the wrong tables and because of that, they will get wrong results. Anyways we will go to numerical solutions later. Currently we will switch to moving shocks it gives you a little more feel for waves in gas.

So, I am going to start with moving shocks and I am going to assume that there is a shock moving and it is moving into still air or still gas, no movement in the gas. I am going to assume the gas is stationary till the shock comes in. So, I am going to draw, still we are in 1 d world I am going to draw 1 d duct, where there is a shock moving with some wave velocity. I am going to call it W s velocity of the shock wave velocity, and I am going to say, I have to be careful about using variables. I want two different reference frames. Because I have to compare, I have to go from 1 reference frame to the other. Till now we have been solving problems with reference frame fixed on the shock that is, I am sitting on the shock. I see that the incoming flow is coming with some mock number and it is going out with a lesser mock number that is what we have been seeing till now.

I am going to say I am sitting on the outside as in say on the duct on the tube or on the ground and there is a shock moving stationary gas, shock is moving and this is a set of compression waves that is what we said a shock is. So, what does that mean it is a strong compression wave coming in, what will happen really behind it flow will follow right it is like a snake example, there is a snake and everybody run. And that is the information that is going they are all running along this way, there is too much crowd there is less crowd here everybody wants to go this way that is the feel right.

So, I know that the velocity behind is going to be this way and here I am going to say, now I am being careful using correct variables. I am using V 1 here, we will stick to U as velocity in the reference frame, where shock is stationary, we will keep that as the reference. I am, going to say shock stationary. In this reference frame, I will call the velocity is U, in the other reference frame I will call the velocity is V. This is the reason why we chose volume to be V cross and here I am going to have a velocity V 2, some velocity V 2.

And just because I switch my reference frame, the P 2's and T 2's will not be any different. Shock is just a shock. It does not care who is looking at it whether somebody sitting on the shock is looking at the shock or somebody standing on the ground is looking at the shock does not care, shock is just a shock.

So, the pressure ratios, density ratios, velocity ratios they are all going to be the same as in this except for velocity I should not have told. Pressure ratio, temperature ratio, density ratio those will be the same. Velocity depends on reference frame that is a very important thing in engineering. Velocity depends on reference frame. Now, I am choosing one reference frame here and this is another reference frame, where I am saying I am stationary with respect to this incoming gas. I am saying there is still air in the room and there is a shock passing by, that is the condition. So, that is this case, I am keeping V 1, V 2 as velocities in that reference frame, where it is with respect to lab our ground reference frame and this one is with respect to the shock. Maybe the shock itself is moving currently.

All I know is I am sitting on the shock. I am sitting on the shock and because of that I am going to see some velocity incoming this way. I call that U 1 and we know from all the analysis last few weeks we are going to get this to be U 2 and it is less than that value. We know all this already and T 2 by T 1, P 2 by P 1, rho 2 by rho 1, those ratios of thermo dynamics properties across the shock does not change with reference frame. So, I will just use the same ratios that are very important statement I am making here.

Now, I want to go from one reference frame to the other. How will I do it? I want to write U 1 in terms of V 1, easy right, change in reference frame is just because of this reference point is now moving with respect to my original reference. So, I am going to say there if I add a negative W S to this whole system then the shock will become stationary, W S to the left will make the shock stationary. So, I will add that to all the velocities in this whole frame anywhere, any analysis I do all the velocities I will add this. So, I am going to say U 1 here is equivalent to V 1 minus W S and similarly, U 2 is equal to V 2 minus W S.

A quick check, if I change my reference frame, any difference in a particular reference frame should not change, difference between this to this should be the same as difference between this to that you will get directly. Here I want to write it in a particular form. So, I will put it is U 2 minus U 1 equal to V 2 minus V 1 because I want to proceed from this point. I am going to say yes, this is true, a quick check. Now, I want to proceed from here and I want to find the value of V 2 given V 1 is 0. So, I am going to set V 1 equal to 0. I will just make it 0. So, my V 2 has a relation already U 2 minus U 1 in shock fixed coordinates that will be my V 2.

That is what I am getting. So, I am going to have P 2 equal to U 2 minus U1. I want to write it in terms of mock number, how will I do it? I want to write it as U 2 by U 1. Then I have expressions for U 2 by U 1, U1 by U2 everything we have expressions. So, I will pull out a U 1 and I will have U 2 by U 1 minus 1. Now, I want to find the value of U 1 what will it be. From here I am shifting to here. Simple thing I can just go to math and say V 1 was said to be 0.

Then I will get U 1 is W minus of W S minus is important here, minus W S or I can say this is stationary, this is moving, Now I am switching reference frame to this. So, this will be moving this way with the same velocity and this is stationary. So, it is minus, it is just coming out directly. So I will rewrite this as W S times. I will take the minus sign in to the bracket it will become 1 minus U 2 by U 1 that will become I expression. Where U 2 by U 1 is shock fixed coordinates.

We know everything about shock fixed coordinate properties. We have already thrashed it out like last for classes 5 classes. Now, all we have to do is just start substituting things inside. Only one question W S is the velocity of the shock, do we know the value, in a way we know the value because, I have been dealing with U 1 all this times. How will I write U 1 in terms of say mock number, U 1 can be written as M 1a 1. Now, I know U 1 is equal to minus W S and I have already used up the minus sign. So, it should just become M 1 a 1, I have already used up the minus sign and the minus sign went into this to flip the terms the other way. So, I am going to say this is equal to M 1 a 1. Now, I will go and write a better expression.

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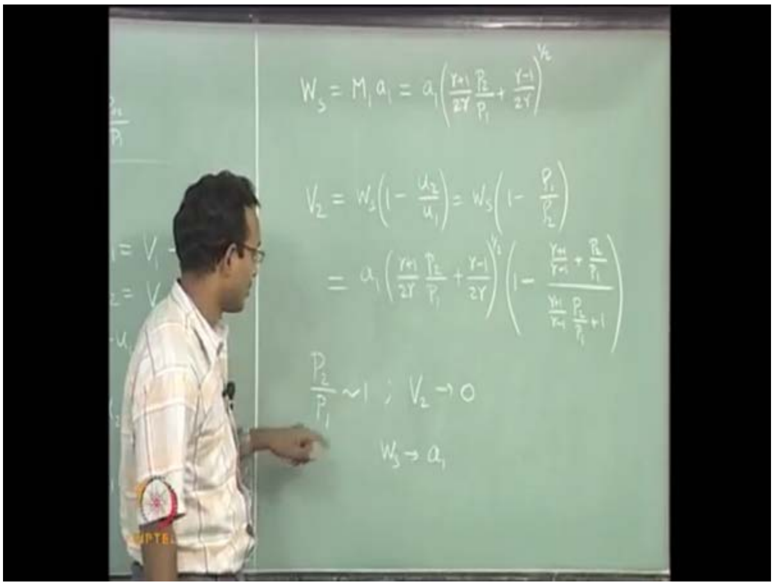

I am picking up one of the relations which we wrote before, it should be 2 gamma, this expression I never wrote separately. But this comes from whatever we wrote today for P 2 by P 1 when we did the asymptotic analysis. 1plus 2 gamma by gamma, plus 1 times M square minus 1 will give you this form. I just rearranged that term to get to this, nothing great you can get to this form it is not very difficult to get.

So, I have rewritten M 1, M1 square I have rewritten and then put a half power on top of it. So, I got this expression. Why did I go to p 2 by p 1? In reality when we want to create a moving shock, the way to create it, is to create a pressure differential, if the pressure differential is strong enough then I am going to form a moving compression wave which I am going past myself, I have to wait for this to happen shock tubes case. But anyway I will go ahead and tell you that if there is a whole bunch of compression waves come together, they form a strong thin-layer called the shock.

We will go into details of this again later, I will just tell you now that you whole bunch of compression waves together is what forms this thin-layer called shock. And now if I know my P 2 by P 1 that is caused because of the shock, then I can tell what will be the speed at which the shock will move given a 1 and the gas in which it is moving. I should know what is the temperature of the whatever stationary gas and it the stationary gas is gamma value, I need to know these 2 and the P 2 by P 1 which is what is typically referred to as a strength of the shock. How much compression can the shock give, that is called the strength of the shock so, if I give the strength of the shock and in what medium is it travelling, we can tell at what speed it will travel, that is the idea that is why we write it like this.

So, most of the moving shock expressions I am going to show you are all going to be in P 2 by P 1, very useful in P 2 by P 1 form. So, the next one, we already left it half done V 2 is W s times 1 minus u 2 by u 1 which can be written as W s times 1 minus rho 1 by rho 2. We already wrote an expression for rho 2 by rho 1, in terms of P 2 by P 1. Now, I just go and invert that expression and I am going to write it. So, my expression W s, I am going to substitute the top 1, now we want to go again to these two extremes of the analysis, 2 extremes of the formulae. I am going to say what if my shock is very, very weak, it is going to cause a very small pressure jump, a weak compression wave, extremely weak compression wave. If I pick that particular case, if it is a weak compression wave I can say that my P 2 by P 1 is approximately 1.

So, let us say we will set the ultimate weak shock will be P 2 by P 1 equal to 1 or tending to 1 is what I have to do really. Let us say we will make it equal to 1. I want to see what happens to my W s? Actually, we will look at V 2 first because the formula is just right above V 2 will tend to, what will happen that will become 1, this will become 1. It so happens that P 2 by P 1 will become 1, gamma plus 1 by gamma minus 1 plus 1 numerator also gamma plus 1 by gamma minus 1 plus 1.

So, these 2 will just become 1 overall, 1 minus 1 becomes 0. So, my V 2 tends to 0 that means, if it is a very weak compression wave there is no flow behind it. Now, what is the speed at which this way will travel that is the next question W s. I want to find what it tends to, I am going to look at this top expression and again I am going to set P 2 by P 1 equal to 1. What is going to happen, P 2 by P 1 becomes 1, gamma plus 1 both the denominator are the same 2 gamma. So, it will be just add the numerators gamma plus 1, plus gamma minus 1 minus 1 plus 1 cancels 2 gamma by 2 gamma becomes 1, square root of 1. So, it becomes a 1.

So, What happens is if I have a very weak compression wave, it is going to travel at speed of sound in that medium and it may not change any flow as in no pressure jump, no velocity jump that is what you are going to get if it is very, very weak, if it is anything more than P 2 by P 1 equal to 1, then velocity will not be 0 and I will have slightly stronger than speed of sound as the shock strength. So, if I have already told you this is ahead once I will tell you again.

If I have, a case where 2 3 shock waves, 2 3 compression waves, weak compression waves come together, then they will get together I will not tell you how currently it gets together, we will just keep it like that. We will go and do this analysis again. When we go and deal with shock tubes, when the shocks come together, when the compression waves come together, they will tend to overlap 1 on top of the other and become 1 strong wave, there are some 10 waves, they all will coalesce to form 1 strong wave. It will still be that thin shockwave as far as our gas dynamics is constant assumption. It will be reasonably thin. If that happens, then my P 2 by P 1 is the overall, it should actually be P 2 by P 1 multiplied by P 3 by P 2, multiplied by P 4 by P 3 like that, overall it will become P 4 by P 1 or something or P n by P 1, if there are n waves whatever.

So, overall that P 2 by P 1 across that whole set of waves together will not be 1, it will be strong, when that happens this is not really true, it will be moving with some reasonably high-speed. At what speed, we will go look at it, if P 2 by P 1 is anything more than 1, what will happen? This number will not be 1 but, higher. If this number is higher, then my speed of the shock is going to be more than a 1, it is going to travel supersonic. Any compression wave, if it has any reasonable compression anything more than 1, P 2 by P 1 will move supersonic, that is what we are seeing here, opposite is true for expansion waves we would not deal with it right now. But, we will go to expansion when we have to, first we will deal with all compressions then we will go deal with expansions.

So, this is one extreme of the case, we said P 2 by P 1 tending to 1, very weak compression waves. Now, we will look at strong compression waves, really strong compression waves. What will happen there? We have to do the same kind of analysis as before we are going to do the asymptotic analysis, we are going to set that P 2 by P 1 tends to infinity, very large values and we want to see what is the effect?

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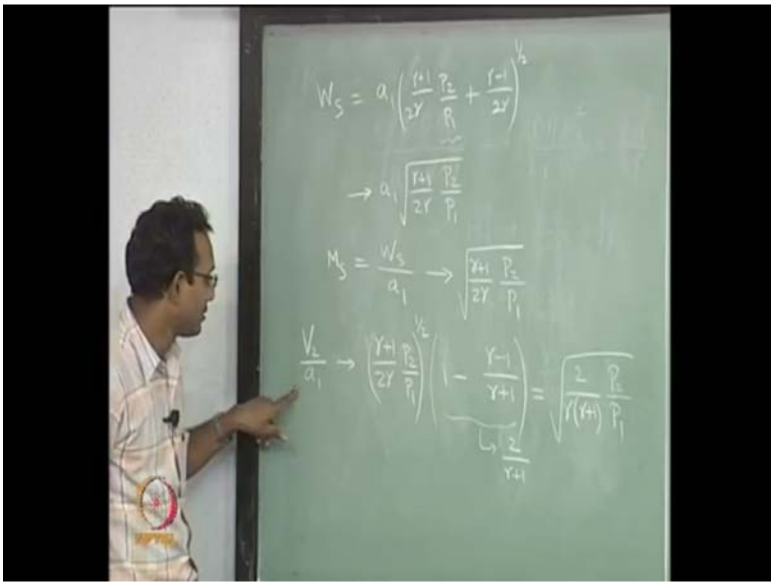

The first one we want to do is W s, I will write it again here, I have this and I am going to say this tends to infinity which means I will neglect the other term. So, I am going to get to only the first-term remaining, roughly rough estimate, I will just say it tends to that is more mathematically correct, we will just say it tends to this for large values of P 2 by P 1 which means if I keep on increasing my P 2 by P 1 my shock mock number, if I want to call it is a shock mock number, shock velocity divided by a 1, I will use it that way. Now, I will put 1 more term here, I will call the actually I will write it separately, I do not want to rewrite everything. This one tends to square root of gamma plus 1 by 2 gamma times P 2 by P 1, this is what it will come down to and we are seeing that if my compression gets stronger and stronger, my shock mock number will keep on increasing, there is no limit to this, it is not asymptotic to any values. It will keep on increasing.

Now, the next thing is, we have to do this analysis for V 2 by V 1, V 2 by a 1, I want to call this V 2 by a 1, I could do it as V 2 by a 2 also. We will currently do it as V 2 by a 1 and then we will look at a 2 by V 2 by a 2. From whatever we wrote before, I can say that I will just directly write the tending 2 thing, gamma plus 1 by 2 gamma times P 2 by P 1, this 1 just comes from W s whatever is here is exactly here. Now, the remaining term is 1 minus u 2 by u 1, that is going to tend to 1 minus gamma minus 1 by gamma plus 1, this is what it should become. So, if I set P 2 by P 1, P 1 is very large you should get to this particular formula.

Now, once I have this. Now, I just have to simplify this it is going to, this whole bracket is going to become 2 by gamma plus 1, it is going to become 2 by gamma plus 1, because this gamma will get cancelled with minus gamma there, minus gamma plus 1 is what is there. So, it will become 2 by gamma plus 1. So, I will get to this particular form. Now, we just have to write it this gamma plus 1 and square root of gamma plus 1 inside, simplify this you will get to 2 by gamma times gamma plus 1, P 2 by P 1, this is what you are getting. Again, we are seeing that the induced velocity, I am dividing by a 1, this is not like mock number behind the shock, it is not really that, I am dividing by a 1. We have to go and divide by a 2 next we will do it again, if I divided by a 1, V 2 by a 1 then, I am getting that particular value will keep on increasing forever. For a given a 1, if I keep on increasing my compression, my V 2 keeps on increasing. If I have a stronger shock, the flow induced by the stronger shock will be higher, that is what we are having.

In all this analysis we have assumed V 1 equal to 0. If V 1 is not 0, really nothing goes wrong, we just have to keep track of what is the value of u 1 all the time. We will go and do a numerical example next class where we will see what if V 1 is not equal to 0. Then we just have to keep track of it, the values will be much higher in that is all will happen nothing special. Now, we will look at I want to make this become M 2 as in mock number. So, I have to look at V1 by a 2. So, I have to multiply this with a1 by a 2. So, let us find a 1 by a 2.

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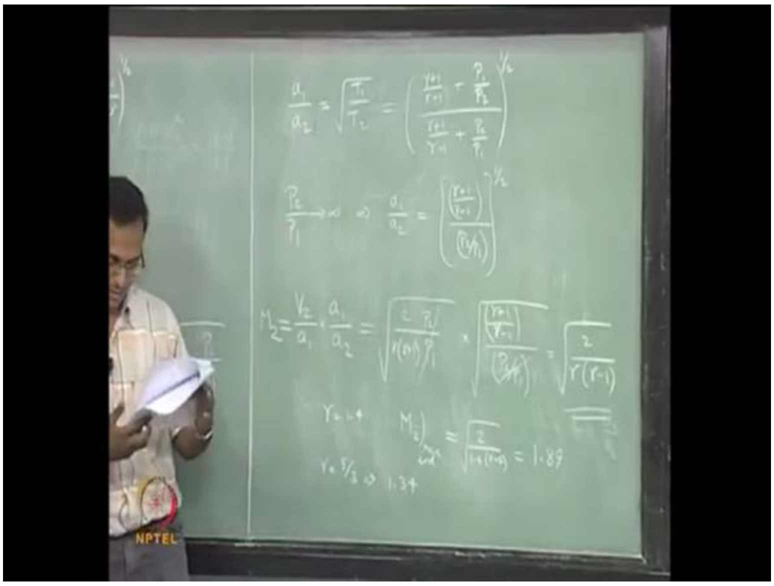

I will go to the next page, a 1 by a 2 it is the same gas, same gamma, same R, I will just remove it. So, it will just become square root of T 1 by T 2. I do not need to do much about this. Now, we have to write this T 1 by T 2 in terms of P 2 by P 1 because, we are writing everything in terms of P 2 by P 1. T 1 by T 2 is whatever we wrote before for T 2 by T 1, we have right it inverted. So, it is going to be, this is the only place where it is different, it will be P 1 by P 2 divided by gamma plus 1 by gamma minus 1, plus P 2 by P 1, whole power half, this is what you are going to get.

So, for large values of P 2 by P 1 similar analysis, we just want to see what happens when P2 by P1 tends to infinity, a 1 by a 2 will become, I am saying this is very high, this I will neglect, I will have on the denominator only P 2 by P 1, in the numerator this is very small 1 by infinity very small. So, I will have only this term. So, I am going to get gamma plus 1 by gamma minus 1 divided by P 2 by P 1 whole power half. Now, I want to go and write it with the V 2 by a 1, already we have V 2 by a 1 into a 1 by a 2 is my M 2, which is my mock number of the flow induced due to the shock, that is this value.

So, if I take the previous formulae which is here this one and multiply with whatever we derived just now I will take this particular of course, remember that this is only for very large values of P 2 by P 1 multiplied by a 1 by a 2 which is here for large values, again I will put it as square root of. So, we are finding that P 2 by P 1 gets cancelled which means irrespective of the strength, if the large value if this very strong shock, it goes to one particular final mock number, that is why I am doing the all this analysis if it does not go to any asymptotic I would not even be dealing with such a thing, anyways we will simplify this further, inside the square root there is gamma plus 1 which I can cancel.

So, it will come down to square root of 2 by this gamma minus 1 will go to the denominator, there is a gamma already in the denominator. This is the expression you will get finally for I have to a closed bracket square root of 2 by gamma times gamma minus 1. If I say my gamma is 1.4, if I say I am having air kind of gas, diatomic molecules then M 2 max I have to also say induced, I will keep on using this induced this is a induced flow mock number, maximum possible is going to be equal to what is this square root of 2 by 1.4 times 0.4. I am just substituting gamma equal to 1.4 there and that comes out to be 1.89.

If I want to create a supersonic flow by sending a shockwave into still medium, I can maximum achieve only 1.89, that is what this is telling you finally. Irrespective of whatever be the initial gas temperature initial whatever be the gas present I can maximum reach only this, if my gamma for the gas happens to be 1.4, the same thing if it is 5 by 3, monotonic gases this is going to give I do not want to write the whole thing again, I will just write the final value that comes out to be 1.34, If it is a less compressible gas, I cannot induce very high mock number, it is going to go only slowly that is what we are seeing here.

If it is less compressible gas, it will have only lesser velocity, lesser mock number, I am not talking about velocity here. Velocity is not this simple, velocity is more complex than this. It is not very easy, we will go look at what the plots look like, I just want to write one more expression will start the next class with all the plots. I will just write an expression for this whole thing, V 2 by a 1 multiplied by a 1 by a 2, the full expression not just the large P 2 by P 1 value, just write the full thing and probably that, with that will close today will deal with how the function looks like on plot next time.

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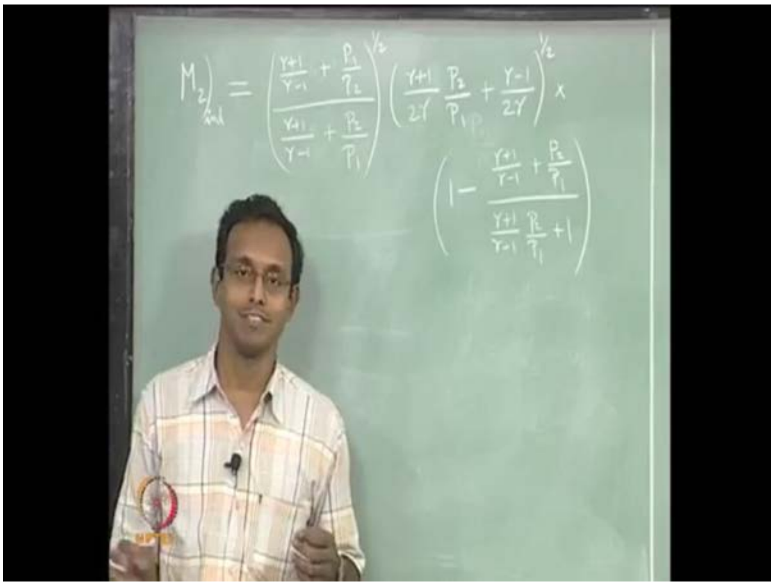

So, I am going to call it M 2 induced, this is induced in V 1 equal to 0 gas, remember that, if V 1 is not equal to 0, all the formula here are all wrong. I have to keep track of V 1, we have made V1 equal to 0. I will get to something like, multiplied by, I am folding and writing here. So, I am multiplying this with gamma plus 1 by gamma minus 1 plus P

2 by P 1 divided by gamma plus 1, gamma minus 1, P2 by P1 plus 1, this is my whole expression.

Now, I am going to observe something. I am going to say when my P 2 by P 1 is close to 1, my V 2 was 0, which is what we saw first. Alright V2 is 0, which is if it is a weak compression wave it does not induce any velocity that is the first thing we said which means, my M 2 induced must be 0 and we saw that, if it is very very strong shock, infinitely strong shock, then it is going to induced some finite mock numbers finally, which is supersonic we saw for gamma equal to 1.4, it was 1.89 value. If that is a case, there was a point where it should have crossed from subsonic to supersonic. There must be some P 2 by P 1 at which it will cross from subsonic to supersonic, that is if I want to create a supersonic flow behind my shock, my compression ratio for that shock should at least be some value.

How will I find that I have to set this equal to 1, and find the P 2 by P 1 for that. I tried solving that from here, it is very complex because it is a P 1 by P2, P2 by P1 to the power half again to the power half here, P 2 by P 1, P 1 by P2, P2 by P1 everywhere, not easy to solve. So, I reserved at to graphical way of doing things, this is the old-style longtime back when mathematics was to complex, they just went to plots, easier just go 1 direction, now find out the inverse easy to do that is what people followed. I will just follow that, simple enough for our course it is nice we do not need to inverse this, it is not going to be much use anyway. So, I just want to say that if I create a strong enough shock wave I maybe creating supersonic flow, most likely I will be creating subsonic flow behind.

We will look at all this next time with plots of these expressions, every expression I have put up here for moving shock. We will look at plots of it and then we will see where it goes, it looks like have already crossed my time. We will look at the plots and then we will start inferring items from there and will say what does a moving shock do to a gas that is stationary, after that we will go to numerical examples where we will first solve a simple problems of shocks inflow that is whatever we said we will do later and then we will go to moving shocks where we will say, shock is moving into stationary gas, after that we will say shock is moving into moving gas, moving one way and moving the other way and we will see what the effects are. We will do all this hopefully next class if not we will go to one more class, any questions? See you people next time.