Gas Dynamics

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Module - 4
Lecture - 11
Normal Shock Concepts, Normal Shock Relations

Hello everyone welcome back, we are going to start our Normal Shock analysis as class of today. We already gave a little bit of background last time, little more physical feel today as in, we already had this particular example of a whole stadium full of people we said everybody standing next, next to each other and suddenly there is something entering it we said. And then there will be a compression wave going through all the way; that is what we were talking about.

Now, if I say it is not just a simple compression wave, where it is a bulldozer just walking through, it does not care for people, it would not wait, it will just walk through just go straight away. Imagine something like that, what will happen will be a whole set of people who will be piled on top of each other and trying to run away from this. If I think about a set of people getting crowded in some small area, as higher pressure compared to the remaining area then suddenly just in front of the bulldozer there will be a lot of pressure, there will be a lot of crowd trying to adjust going past this bulldozer, that is what they are trying to do in this.

So, if I have something that is moving faster than of course, if the bulldozer comes extremely slow, people will just move away by the time it comes close they will wait for it to come close and then they will just walk away. We are not thinking about that kind of cases, we are thinking it is coming very fast that fluid elements cannot adjust or people standing there in front of the bulldozer cannot adjust, they do not have time to adjust, they cannot run that fast, it is coming that fast.

Think about it coming very, very fast then this set of people will be crushed with the next set of people standing there. So, in a way that people density is increasing, density is increasing or pressure is increasing the way to think about it. Now, if I think about a gas situation, again will go back to our there is a tube and we assume that we can see the molecules inside this tube and then I am having a piston and that piston is coming in fast very fast.

So, fast that molecules cannot adjust fast enough to move away from here, before the piston moved, all the particles are staying still, not moving at all, when this piston comes in suddenly there was set of molecules I have to move, they do not have time to send that information to the next. If I have such a situation, then if the piston is currently here, it is going to come in this way this particular particle does not know that there is something going on here, while this particular particle is getting crushed.

It is still getting crushed, but that information never reached here, it will reach at speed of sound, times the distance by speed of sound that much time later it will reach, but before that the piston is going to come and reach here, thus the kind of thing we are imagining. So, if that is the case then eventually there will be a point where, this guy by the time he gets the information he is also crushed that will happen that is what is happening in a gas if I move very fast in a gas, that is static. And I am moving at mach number more than, one which is speed greater than speed of sound.

If I am moving like that, then the particles in front do not know that there is something coming at that speed, till it hits it that is what will happen. Now, it could so happen that it is not a bulldozer which has a flat front, but it has a some tilted front like this, if it has some tilted front like this coming like this let us put it like this for video. It is going like this then whatever particles that are coming that are above this point will be bouncing off outward.

Of course, they are still going to get hit and after they get hit, they still have a path to go that will be along this slide, as it goes in this will be sent out like this upward, if it is below then it is going to be this surface it is going to be sent below, as it comes in it will be going down that will also happen. Whatever, I am talking about is all unsteady everything is changing in time.

What is changing in time here, the position of my bulldozer with this shape is changing in time, instead of it being a bulldozer it is easier to imagine if you want if I think about as a rocket that is going supersonic, rocket front cone may be like this or the missile front cone is like this typically, there are several missiles which are going supersonic. So, if I go supersonic then I am thinking about, what will happen in front of it that is what I am looking for, something like this will happen.

If I now, define that there is a line, that is traveling with this body let us say, this is my missile for body and this is going very fast supersonic in my still gas and as it goes there will be a line traveling with this body. On this side of the line everything is high-pressure alright these are the people that are got crushed already, the molecules that got crushed already that side of the molecules that are just trying to get to this point. So, that they will know the information, in the way this is the line which tells the particles ahead that there is something traveling at supersonic speed.

Till that point, till this line goes and touches this point say this is the point that I am imagining, till this line comes and touches this point does not know, that there was a body here that is coming in like this. Only after that it knows, it is related to your mach cone, mach angle, cone of influence and all that in a way, but it would not be the same mach angle that is the difference here. This information travels actually faster than speed of sound if it is a compression wave it increases the temperature of the gas and so it travels faster.

Will go and look at that after deriving some set of expressions here, then will go and get more physical feel after some time, but this is the idea. I already gave you this example that if it is supersonic flow it is equivalent to driving a car with front windshield not transparent. You remember that hopefully I did this first few classes when it introduced mach number, if you I did not now I am telling you, this there is a car and I am going with front windshield completely not transparent, that is I cannot see through the front I can drive only with rearview mirrors.

If I have such a situation and I go very, very fast what will happen most likely I am going to crash into something in the front, I will know that there was a bridge pillar only after I hit the pillar, till that time I will not know, that is information comes to me only after that point is crossed. That is what we understood already from supersonic flow the mach cone, I am going to say if this is the point that is sending out the disturbance. If I am going this way I will know it only after I have crossed this point, and gone there this mach wave will come and tell me the information there later.

That is equivalent to my having a rearview mirror and seeing that it is happening here, something like that just a rough estimate is the easy way to remember things. So, you can; obviously, imagine now that if I ever drive such a car with this completely not transparent in the front opaque in the front, will have a serious problem we will always end up crashing into something in the front. So, supersonic flow typically will be crashing into some particles ahead of it.

Typically, if there is a particle going at supersonic speeds into still air, then definitely there will be some region of crashing zone, crashing zone will be in front of it would not be all the way out there. As you can expect in traffic if I keep on going at the speed whether I crash into something or not I will have traffic crashing all through, but this is not all through by the time there is a feedback mechanism it will come and stop; that is unsteady gas dynamics we will go to it towards the end of the course.

But, as of now if there is a speed then there will be some distance in front of it where, there will be a line which is going to tell that there is something moving supersonic and that line is now, what we start calling shock. It, so happens that it is a set of waves that are coming from this body and telling that, this is moving at this high-speed that is the information that this set of waves are having. They are all come together to form a very strong wave that it can go faster than speed of sound in this medium.

That is what has eventually happened, they have all come together to form a very strong wave. And now, that wave can go faster than speed of sound that is what is happening. So, that is the physical feel part of normal shock now, we want to derive expressions for what is the actual change caused by this shockwave, when there is a shockwave moving somewhere actually we would not do this moving shockwave business we are wanting steady calculations.

So, what we will do is, we will go sit on the wave. So, the wave is moving and I am sitting on the wave. So, with respect to me the still air is moving and it is going out this way, I am sitting on the wave always, that is the condition we are going to talk about. Remember we are doing 1D problem. So, shock does not have this tilt, it is only normal shock nothing else as of now, will go and look at tilted shock which we talked about in this particular missile fore body problem later and actually I will start showing you pictures of these things after sometime.

So, I will have something like this we are not treating this problem right now, we are considering a case where it is like a bulldozer coming in and there will be a straight flat portion moving in front of it, that is the kind of thing we are imagining right now. So, when I want to start solving this problem is there any special consideration that I have to take into account. One thing is this flow is not isentropic I know this already, alright that is the only thing, I think I will have to take into account other things are it is a flow. So, it has to obey conservation of mass, conservation of energy, conservation of momentum anything else. State equation, if you want to say c p is constant all those things we have picked such a gas. So, it has to obey p equal to rho r t and it has to say c p is constant all that, a picking such a condition. Now, will go to board and look at some expressions.

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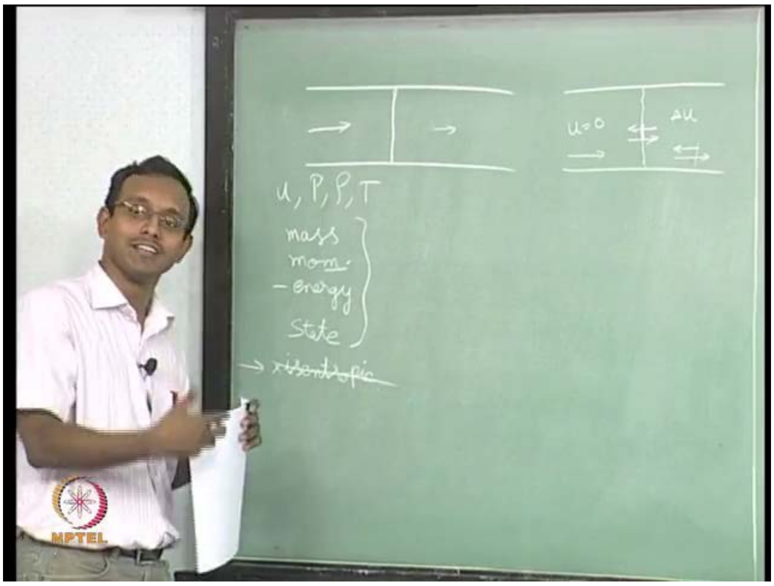

We will first draw schematic of what we want, I am imagining a stream tube, that is like this and I have 1D flow inside here and I am going to say I am having a wave, which is my shockwave and it is not moving, it is just stationery. Because, I am sitting on top of it that is the way I am looking at it, reference frame is on top of it fixed to it. So, there is slow coming in, and flow going out.

This is the condition we are having, if I want to think about the other way I will just go back to the other frame of reference and come back to this, if I have the other frame of reference where this is stationery this wave is moving here and then there is some delta u across here, this happens to be a compression wave. So, will this delta u be positive or negative, which way will the flow go here to the left or to the right. We discussed this sometime back when we introduced compression waves and expansion waves.

I am saying it is a compression wave because something is compressing let us say a piston is moving this way and because of that this wave is created, then which we will this fluid move, to the left it will move to the left. So, this is what will happen now, I am going to change reference frame from here to here where, I am going to say everything that is moving with this is at 0 velocity. So, I will add this velocity to this same velocity to this, same velocity to this, something like this.

So, overall this will have a small velocity that way, it will have a huge velocity this way, that is this particular case I have come from there to this reference frame. So, if there is a shock velocity that is coming as very high, will become lesser velocity. We already know that compression wave is what is a shock? So, it is going to have higher pressure, higher density, higher temperature all that will be higher.

Now, we want to solve for flow variables what are the variables I just now, told these are your flow variables, how many relations do we have across these three relations, we have mass, momentum, energy, what else state equation, that is p equal to rho r t anything else. Gas constant c p equal to constant your saying, what is other relation. Sometime back we listed five what was the fifth one we listed, we used isentropic relation somewhere.

These are various relations I could use, but now here I am going to be very careful not using this, but sometime back we derived equations using this and we found that energy equation is not needed or it will come out of momentum equation, state equation and isentropic relation. Now, we are being very careful not to use this one. So, we are going to use only these four. We are having four variables, we want to solve it four relations it will be solved. So, that is all we need that is why I put this up we will not use isentropic here because I know a priori that it is not isentropic flow, we will keep it like. Now, what do I expect in such a case now, I will erase this portion I just want to keep the shock actually, no I will go to the next board.

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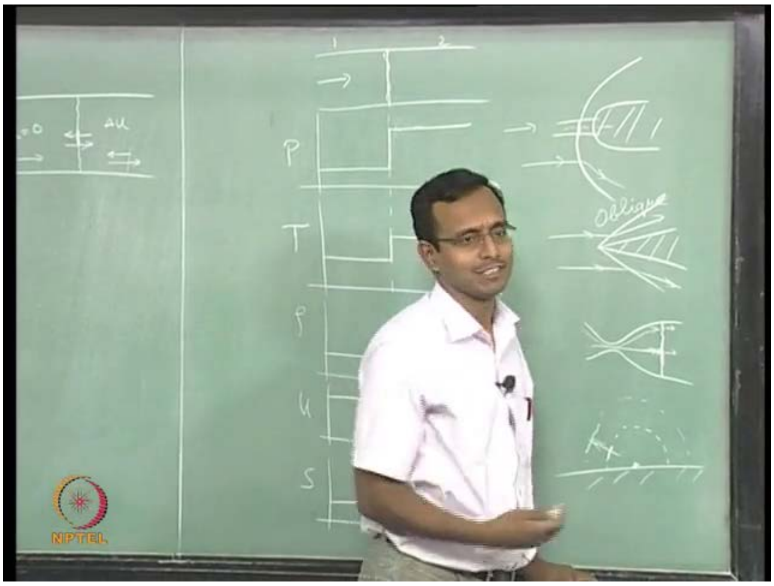

This is my shock 1 and 2 are my states and I just want to put what will my property look like across all this, am just going to put a dotted line here, saying this is our location I am going to look for various variables, pressure, temperature, density, velocity and entropy. I want to look at what all are going to happen in all these waves, if I look at pressure we know it is a compression wave.

So, it is going to be some low-pressure, going into a higher pressure, in reality it may not be going like this, but we are going to assume to be this, in reality may be its going to go like this will be a slide tilt like this probably. It may even be doing something like this, if you know something more about high-temperature gas dynamics I will tell that it will do something like this, but we are going to assume that it is a simple step change, that is the assumption used in our gas dynamics course.

Will assume that it is just this temperature going to rise, density it is of course going to rise, velocity we know that it is going to decrease, entropy what will happen to entropy. It has to increase it cannot decrease, if it decreases then this particular problem will not occur right. So, entropy if at all it changes it will increase. So, entropy will increase this is what we know about this. I am assuming that it is adiabatic flow, if it is non-adiabatic I can have a situation where entropy can decrease.

Because, then I have to think about overall universe entropy should increase, may be it is like a refrigerator problem where I am decreasing the temperature of objects inside, but I am heating the whole room. So, overall entropy increases, entropy inside the fridge will decrease, something like that can happen. So, entropy of the universe should increase I am thinking it is an adiabatic system, isolated system, if that is the case entropy of that system should increase.

That is the condition am using hear and I am thinking about this problem, there is no heat transfer across this streamlines, that is my assumption. Now, I could have lot of possibilities in real flows there will be, so many possibilities will just give you, if I already showed you some examples with hand, just put it out here. There could be a case where there is a body like this, it is going supersonic flow is going like this, with respect to this body flow is coming in there will be a shock like this.

That is till this line everywhere this side it is all same pressure as this, after this the pressure is different, only after this fluid knows that something like this body is sitting here, and then it turns around and goes that is the special line shock. Till that point it will not know, this will happen only in supersonic flows, in subsonic flows such thing will not happen. Another case could be a very nice slender thin sharp object, then it may have some that line very close to this object and a straight-line, if this is a straight-line this is a straight-line.

Something like that and this flow will turn like this, this is also a possibility and there are special cases where I have some flow through a nozzle and there could be a shock sitting here, where flow comes here and then goes past with the lesser velocity, varied goes straight and goes past with a lesser velocity something like that. You could have various cases will go look at individual problems later as of now, I just want to tell that there are various forms of shocks possible, this is what is called normal shock, normal to the streamline it. So, happens that in this flow the centre most line has a normal shock.

That set of streamlines if I consider, this stream tube is just having a normal shock inside that region alone is a normal shock. If I think about this one it is a slant shock, it is called the oblique shock. We could have a oblique shock system, oblique shock this is a normal shock that is perpendicular to the flow direction, and this overall thing if there is something like this that is called the bow shock, looking like a bow fine, so it is a bow shock.

I could say that bow shock is actually a shock where it is not a straight-line it is actually a curved shock, curved shock has it is own effect we will look at something like this later. It will start having more problems, other than this is all steady problems, if I have a blast wave some bomb exploding, I have a ground and I exploded a bomb here. There will be a shock travelling outward from that centre. This is a case where there is a moving shock, shock is moving as it goes out and as it moves it is compressing the air.

So, when the shock is here, inside this everywhere it is high-pressure, outside is atmospheric pressure, when the shock goes here this full region is higher pressure and outside is atmospheric pressure. Like that it keeps going it is compressing the gas as it goes out, this is a moving shock one example of moving shock I could have, so many other examples we will just pick one and live with it.

We will go look at unsteady problems only later towards the end of the course, as of now we will just deal with these kind of shocks of which first thing we will deal with only normal shock. So, we want to derive stuff because 1D assumption is very easy to work with if it is normal shock. So, we want to solve for flow variables and we want to use mass momentum energy and state equations only those.

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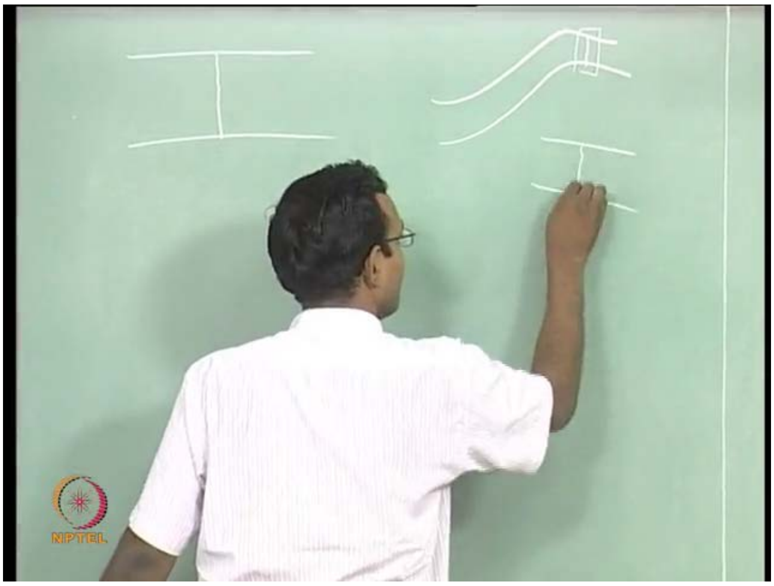

So, we have to pick the correct control volume. I am going to say my stream tube I am considering, is constant area. I need not say this maybe I have a case where I have some duct like this and then there is shock sitting her, where area is decreasing, but locally if I zoom in this region, that will look like this and I said shock is very, very, very thin that is our assumption, in our plots we said go back here.

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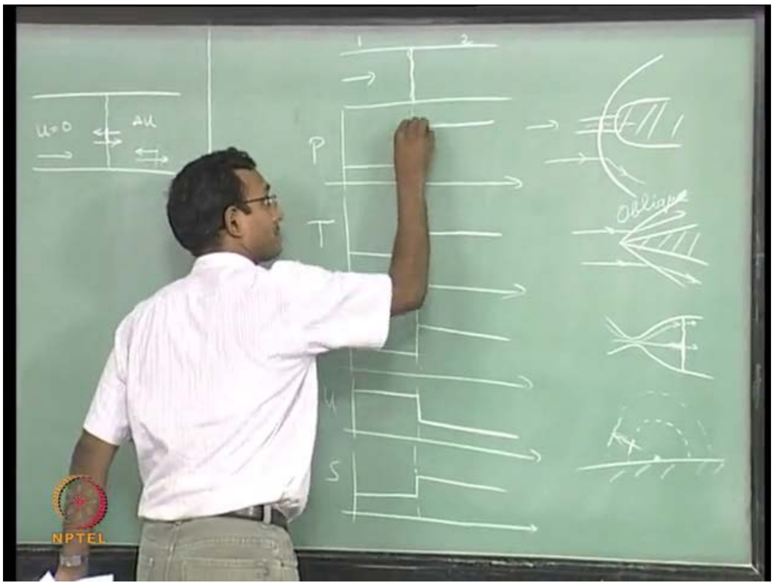

In our plot I am saying that it is a sudden change which means it is extremely thin even if I zoom in this region and expand this scale, it will still look like this that is my assumption. So, when I have such an assumption.

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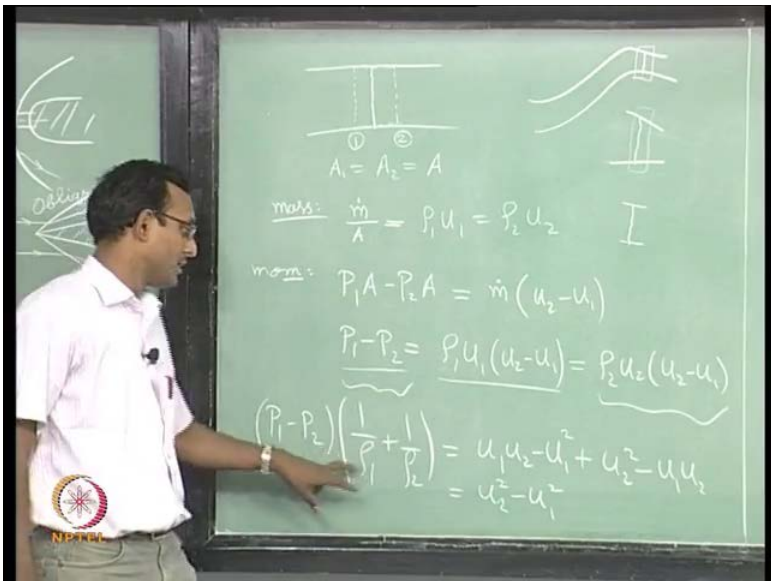

I can go back there and say my shock is a very, very thin line, that if I zoom in let us say I will take one step back and zoom in after that. If I just zoom in that region it may be looking like this and a shock hear, if I zoom in further it will look more like something like this, if I zoom in even further it will eventually look like this picture because I am assuming shock as infinite decimally thin it is a special case.

In real life shock is not that. So, there will be some effect of change in area, we would not talk about that in our course, we want to understand to some extent not to complete detail our course has some scope and we would not go past that. So, we are going to say that it we are assuming constant area duct and I am going to pick a control volume around the stream tube across the shock. This is my control volume I am going to pick and this is my state 1 and this is state 2 it is just gone past this it is the only thing that happened.

And I am going to say A 1 equal to A 2. Now, if I go to mass equation it is becoming simpler, let us say this is equal to A. So, my mass equation m dot by A is a constant now. Because, A is also constant I divided by A, I am going to get this. I am trying to link all the variables across 1 and 2 that is the goal. So, we will have something like this, next one momentum equation. Of course, we derived momentum equation for straight duct already, then I have that f x will become 0, we said that already right.

So, it will just be P 1 A minus P 2 A equal to I could write it as rho u square A or I will use this kind of explanation and I will put them m dot times u 2 minus u 1 I could write it whichever way I want, both are right. I could have written this actual form as rho 2 u 2 square times A minus rho 1 u 1 square times A where I have. Pulled out a rho 2 u 2 and a rho 1 u 1 and I know these 2 are equal then that is equal to m dot by A. So, I could write it like this, I missed A no I did not miss any A is still inside this m dot, everything is fine.

Now, we will divide this whole thing by A and m dot by A can be replaced by either this or this, I have a choice this is like algebraic manipulation trick nothing more. So, I will write it as P 1 minus P 2 is equal to I could write it like this or it is also equal to rho 2 u 2 times u 2 minus u that is an algebraic trick. So, now I am having some relations across these, what will this become now, I am going to have one expression if I pick these 2, I will have another expression if I pick these 2 I have 2 relations for P 1 minus P 2 that is what I am having.

I am going to do something special I am going to take P 1 minus P 2 divided by rho 1 from this underlined thing and P 1 minus P 2 divided by rho 2 from this flower bracket underlined thing. And then I am going to add those 2 what will I get, I will have this plus this in there. When I do that actually, I will write one more step and then it will be easy for you P 1 minus P 2 times 1 by rho 1 plus 1 by rho 2 I am basically, having 2 equations here this and this is equal to that I am going to add those 2 equations, I do that I am going to have from u 1 u 2 minus u 1 square plus u 2 square minus u 1 u 2 this is what I will have, which will simplify to u 2 square minus u 1 square.

Basically, I am having P 2 P 1 minus P 2 times 1 by rho 1 plus 1 by rho 2 is equal to u 2 square minus u 1 square. This is coming from both mass equation and momentum equation together I have one relation between P rho and u across 1 and 2 and that is coming from, mass equation and momentum equation both together. Now, we will start using energy equation. We said it is an adiabatic system.

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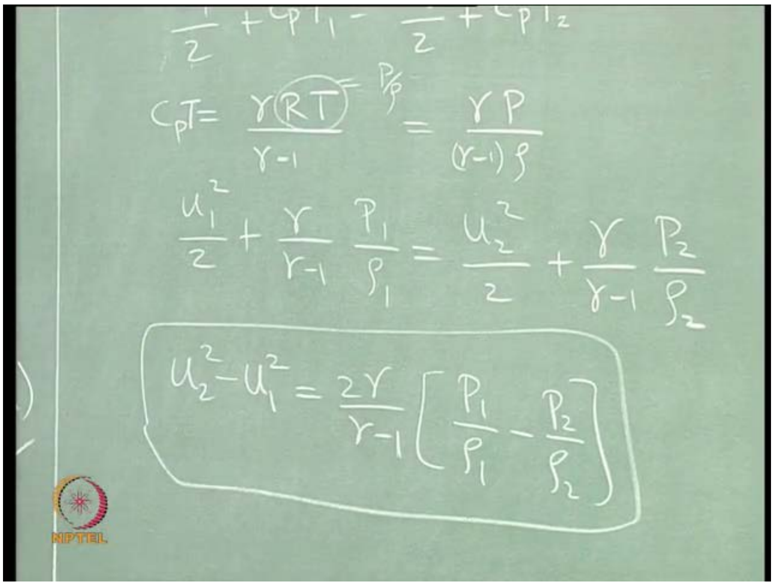

So, I am going to say h naught equal to constant that is my energy equation. Remember that we are not using isentropic, it is just adiabatic only if the flow is reversible also, it will become isentropic, we are keeping it just adiabatic, no heat transfer across my stream tube or my control volume. So, my expression will become something, like this and I want to write this C p in terms of gamma r T and convert it to mach number that is the overall goal.

Actually, not mach number in this case we want to eliminate temperature variable. So, I have to get it to gamma P by rho then temperature will be eliminated in terms of pressure. We will do that my writing C p I will just put C p T here, as of now will become gamma R T by gamma r minus 1 and my I could think of this as A square and put gamma p by rho or I could just say R T is equal to p by rho p equal to rho R t. So, I am using state equation immediately there.

So, this becomes gamma P by gamma minus 1 times rho this is my C p T. So, I am going to write this in there. So, I have eliminated variable temperature, temperature variable if I want I can find from pressure and density variable. So, I write written some expressions like this. So, my expression if I substitute this inside there, I am going to have I will have some expression like that. And I am going to rearrange this such that I will get u 2 square minus u 1 square. This will become 2 gamma by gamma minus 1 u 2 square minus u 1 square I have taken this velocity there and that pressure here. So, p 2 by rho 2 will have negative sign. So, I have one expression here and if I go to the previous board I have another expression here.

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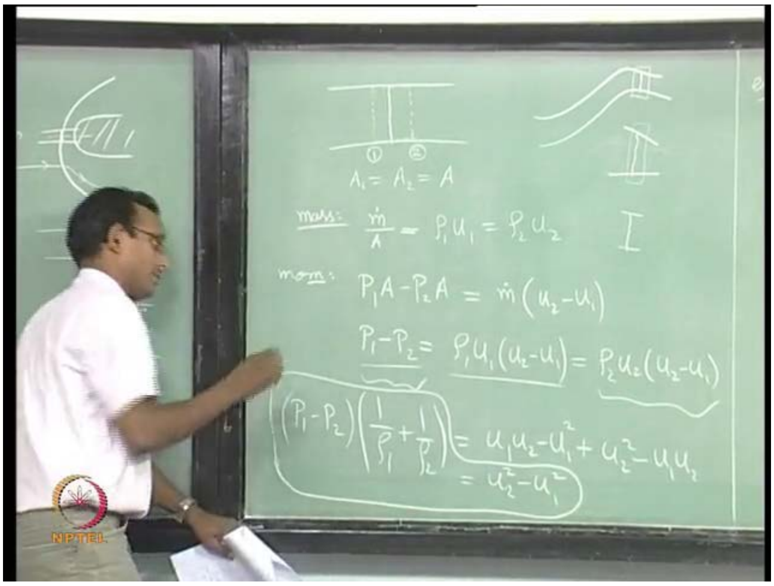

Both are for u 2 square minus u 1 square. Now, if I use this and that both together then I am at a situation where I eliminate variables velocity also. So, that is what we will do next. So, we will equate these two. So, I have a relation between P 1 P 2 and rho 1 rho 2.

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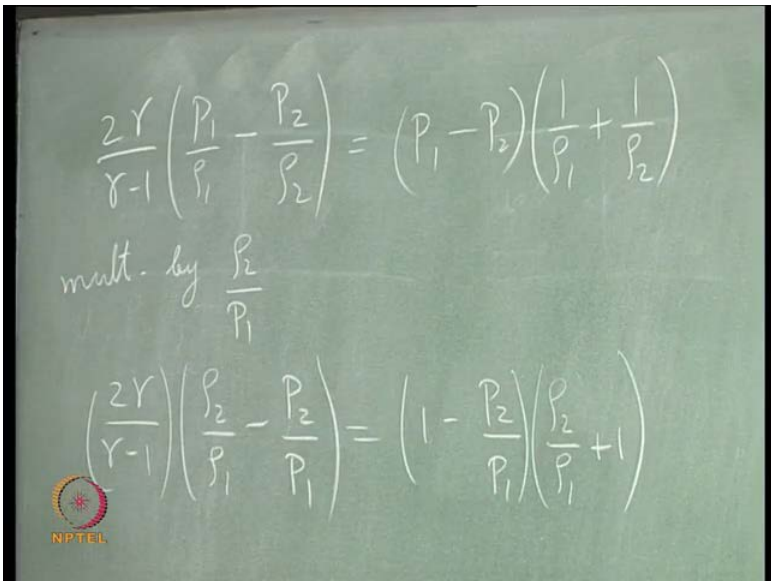

So, I will get now, for this manipulation algebraically I am going to multiply by rho 2 by P 1 I multiply by rho 2 by P 1 I will have rho 2 by rho 1 and P 1 by P 2 by P 1 that kind of expressions. So, we want to multiply that wave. So, when I do that I will get to this form. Now, all you have to do is find P 2 by P 1 in terms of rho 2 by rho 1. So, I have linked one variable in terms of other variable, once I know these variables I can go and substitute this in terms of u 2 square minus u 1 square then I will know velocity variable and then I will go and use C p T equal to gamma by gamma minus 1 times P by rho I will get the other expression temperature variable. So, every variable can be linked in terms of these expressions. So, if I find what this P 2 by P 1 is basically, I have to just multiply it out then collect all the P 2 by P 1 terms group them together to one side and I will get to this expression.

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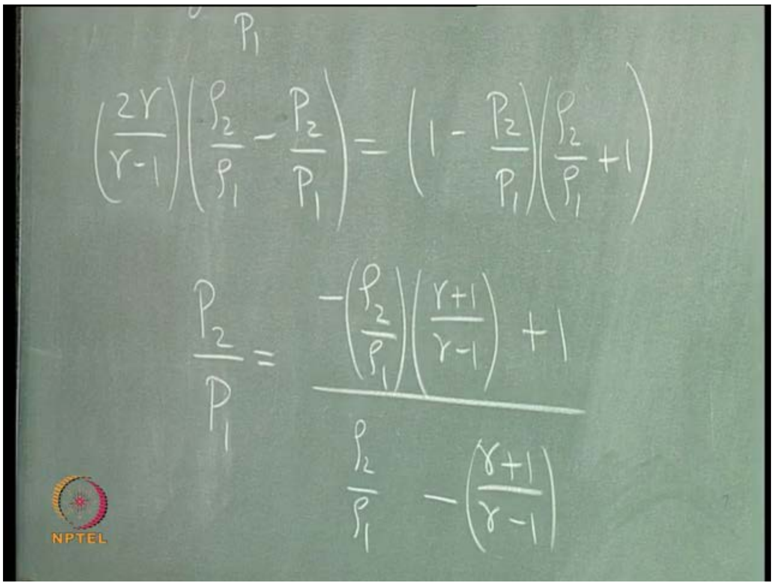

I am skipping one step here, you can rearrange this to get to this form I have it written here, but I do not want to write it on the board. So, I have this form this is typically better written in some other forms I will write that other form.

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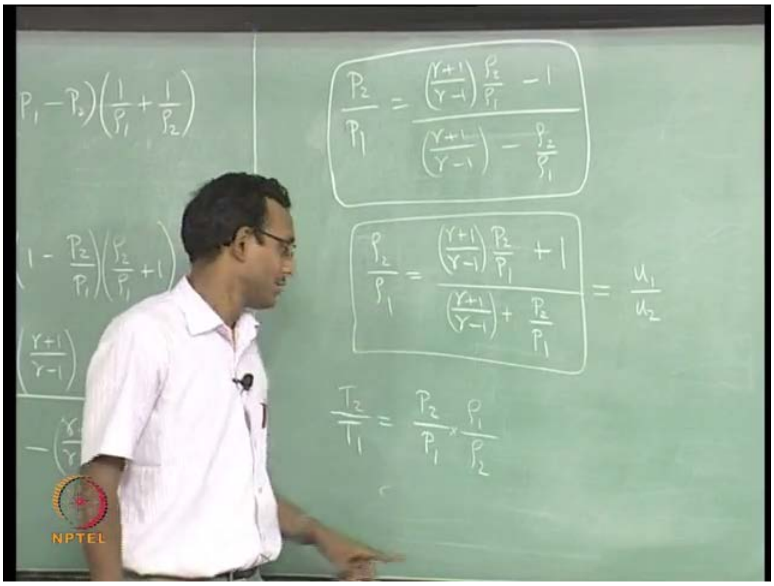

This is one relation between density ratio and pressure ratio, if I invert this I will get the opposite relation rho 2 by rho 1 in terms of P 2 by P 1 you can go and derive it I will just put the final form here, it looks similar except for this plus sign here. It, so happens that from mass equation I can directly link, we know that rho 2 u 2 equal to rho 1 u 1. So, rho 2 by rho 1 is u 1 by u 2. So, I can say that this is also equal to u 1 by u 2 is also there.

So, I am getting velocity ratio also, in terms of pressure ratio. Pressure ratio in terms of density ratio, every ratio is given in terms of some other ratio that is what we have done till. Now, if I want to find temperature ratio what should I do, just gas log is simple enough. So, T 2 by T 1 P equal to rho r T or p by rho is r T. So, I am going to get P 2 by P 1 divided by rho. So, it will be rho 1 by rho 2. So, this is my expression. So, if I want to find, I can find T 2 by T 1 in terms of P 2 by P 1 if I substitute rho 1 by rho 2 in terms of P 2 by P 1 if I use this expression inside here, then I will have T 2 by T 1 in terms of P 2 by P 1.

So, in all this I still cannot find the absolute values all I can tell is if P 2 by P 1 is something or if rho 2 by rho 1 is something across my shock, then I know my P 2 by P 1 if my rho 2 by rho 1 is known then I know my u 1 by u 2 or in a way I know u 2 by u 1. And if I know P 2 by P 1 and rho 2 by rho 1 I can find T 2 by T 1 that is all we have written, we still do not know absolutely what will happen. All we know is just this could be the jump, relative jumps before we go to the next part I just want to point out one thing I have it here right now.

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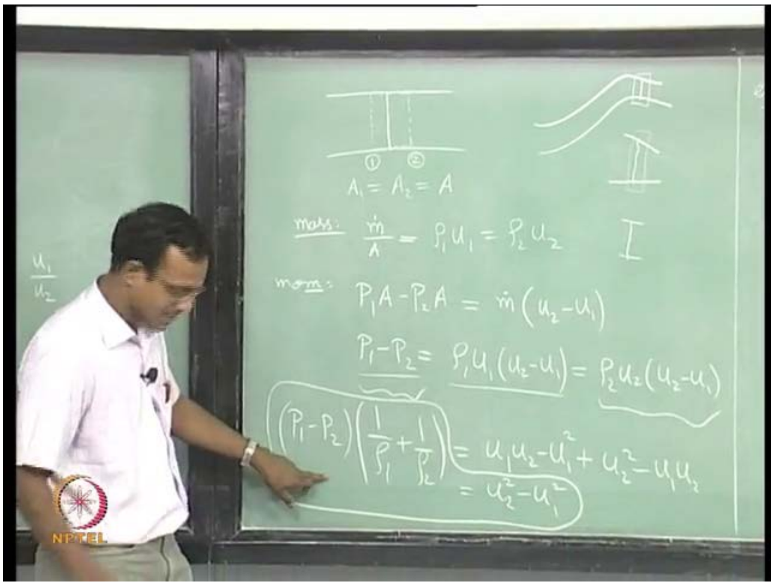

So, I will just go back to this page itself I have this expression here. I could solve this expression just by putting u 2 equal to u 1 P 1 equal to P 2 then also this expression is satisfied. If I say that what will happen my P 2 by P 1 is 1 rho 2 by u 2 by u 1 is 1 I will go back here.

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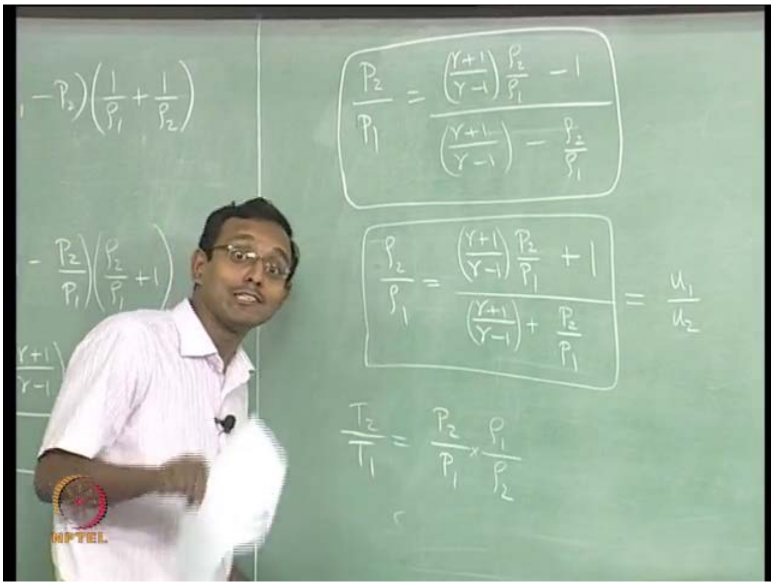

If I put my P 2 by rho 2 by rho 1 is 1 or P 2 by P 1 we said right. So, P 2 by p 1 is one I say. So, I will have gamma by gamma plus 1 by gamma minus 1 times 1 plus 1 I will again get this plus 1 I will get rho 2 by rho 1 is 1 velocity is 1 no change in velocity, no change in pressure, no change in density, no change in temperature, everything will become 1 that is also a solution to this system, remember that that is also a solution to the system it, so happens that we did not write any exact solution to the system.

There is a possibility that if I put P 2 by P 1 equal to 1 I will get u 2 by u 1 equal to 1 T 2 by T 1 equal to 1 rho 2 by rho 1 equal to 1, if everything is 1 no property changed really, but we said shock is the condition of sudden change. So, we are not looking for that particular solution remember that, but that solution is also part of this solution, it is also hidden inside here. So, now you want to find the actual solution, all we know is just if I know one ratio I can know everything else, but we do not even know any answer yet. So, how we will look at things in terms of Mach number, then if I give a mach number you can find every other property that is the hope. So, we will go look at things in terms of Mach number.

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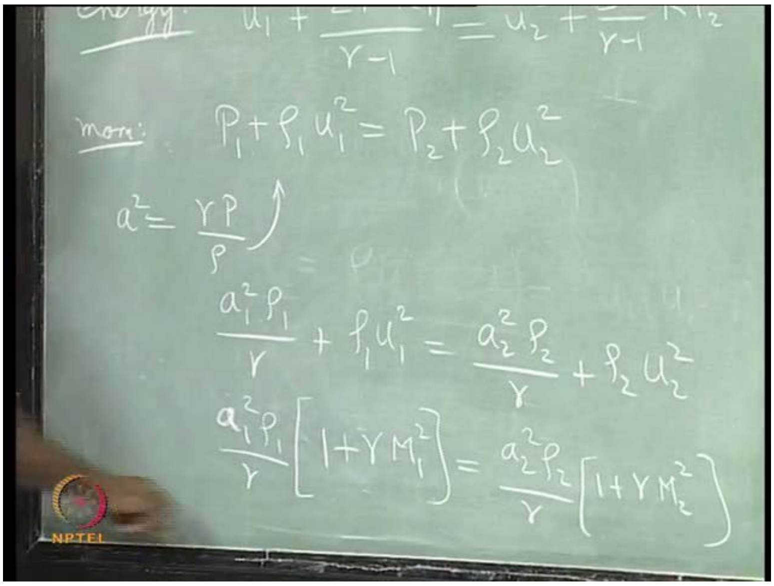

So, I will start with energy equation again, but now I will write it in terms of mach number. So, I am going to write it in a different form. So, I have written this form and momentum equation for constant area duct we know is P plus rho u square is a constant. This is another form of momentum equation, remember that this is valid only for constant area I talked about this that time also, when we derived this expression also it is valid only for constant area and in our problem, we have already assumed is to be constant area we said A 2 equal to A 1.

So, I can use that expression here right now. Now, I want to substitute things together. So, if I look at this expression I can say that my gamma R T 1 will be P 1 by rho 1 sorry gamma P by rho 1 and if I use a square equal to gamma P by rho which is just a form of state equation in a way right, it is also having a square equal to gamma R T definition along with that this is having state equation a square equal to gamma R T with state equation together gives you this I am introducing another variable.

I want to apply this to these 2 expressions, but first let us say I will put it into this equation, supporting to this equation I will pull out some terms. So, that I will get Mach number. So, it is going to look like, what I have done is a square rho by gamma is my pressure I am substituting for pressure in terms of density and a square, that is what I have done here that is my P 1. Now, I will pull out some common factors I want to pull out this whole thing out, as common from this whole thing.

If I do that it will look like a 1 square rho 1 by gamma multiplied by 1 plus rho 1 could be taken out naturally, but the other things multiply and divided by a 1 square by gamma. So, I will get gamma M 1 square and this is equal to similar thing here a 2 square rho 2 by gamma times 1 plus gamma M 2 square I will get to this form.

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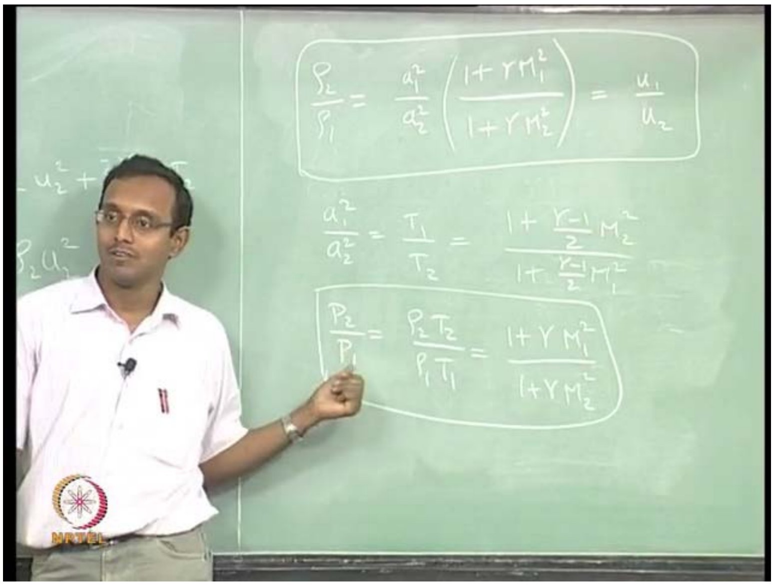

From here, I want to write rho 2 by rho 1 if I rearrange these terms, such that it is looking like rho 2 by rho 1 then I am going to have a 1 square by a 2 square, some gamma gets cancelled there into 1 plus and gamma M 1 square by 1 plus gamma by M 2 square it. So, happens that this is always equal to u 1 by u 2. So, I have really solved the problem yet, but it has come to a point where, if I give M 1 and M 2 I can get the variables, that is where I am how this is going to be T 1 by T 2 is going to be gamma r T 1 gamma r T 2 gamma r will get cancelled this is actually T 1 by T 2.

So, if I am given that set of ratios I can get to the answer, directly from here I can write a 1 square by a 2 square is equal to T 1 by T 2 from here, I will take the T 1 by T 2 to this side. So, I will have P 2 by P 1 equal to rho 2 T 2 by rho 1 T 1 is equal to I could write like this also. I still have not solved the problem really, have just written one in terms of another that is all I have done till I still did not solve the problem. If I am given m one by M 2 or M 2 by M 1 then I can solve this problem that is all I have been given till now.

Till now I am just re manipulating things. So, that everything looks like if I know one variable ratio I can get every other variable ratios. Once I know this P 2 by P 1 I can go and put it in all the previous expressions we did today, and then I will get rho 2 by rho 1 T 2 by T 1 whatever else I want u 2 by u 1 everything I will get. Of course, I can even substitute in this expression, they are all the same just rewritten in a different form.

And of course, you know this T 1 by T 2 can be written as this is M 2 will come there 1 plus gamma minus 1 by 2 M 2 square by 1 plus gamma minus 1 by 2 M 1 square this is also true, right T 1 by T 2 in terms of M 2 and M 1 can be written like this. There are, so many expressions that you can get this is one very nice expression to remember, remove the middle 1 we do not need to remember this part, but P 2 by P 1 in terms of 1 plus gamma M square by 1 plus gamma M square.

Remember that this is 2 by 1 and this is 1 on the numerator 2 on the denominator, that is the one which will give you high number P 2 by P 1 should be more than 1 you will get it to be this. So, now we still have not solved the problem, but we are close, there is just one set of relations left I want to eliminate every variable keep only M 2 and M 1 in my expression that is the only thing left, but it is going to take a lot of time doing that, we still when we derived this way we did not use energy equation yet.

So, I have to use energy equation, momentum equation, mass equation altogether eliminate every variable keep only M 2 in terms of M 1, when we do that then we will be able to solve the problem, if I am given a M I can use, so that expression which we are going to solve tomorrow next class and from that I will get M 2. Now, once I know M 1 and M 2 I can get a P 2 by P 1, once I know one ratio I can get every other ratio. So, I can solve the full problem. We will pick up from this point next class. So, we will stop here.