Gas Dynamics

Prof. T.M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module - 3
Lecture - 10
Isentropic Flow with Area Variation
(Relations and Solved Examples)

Hello everyone. Welcome back to our next class, where we will just go over a little bit of mathematical calculations, so that you will get comfortable solving problems in gas dynamics course. But, I will give you a separate exercise on the website later for people who are on video. As of now, we will just solve two-three problems. But, before that, there is a little bit of more discussion left.

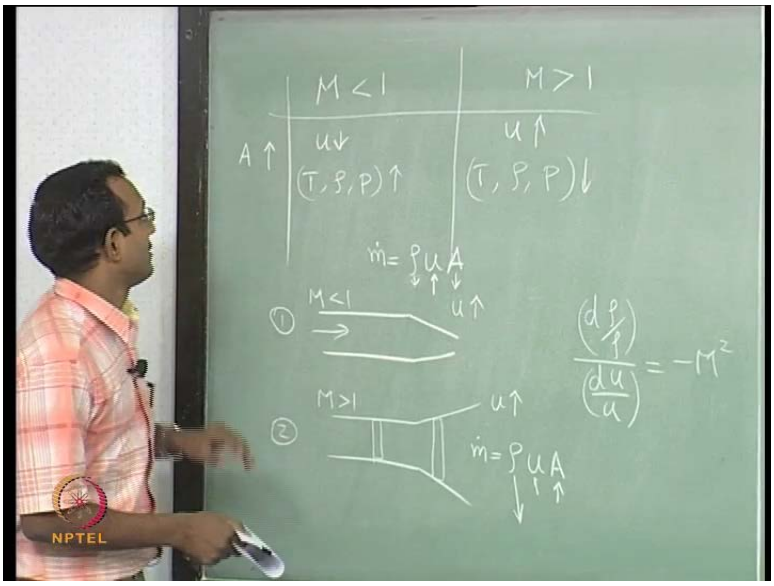
Student: ((Refer Slide Time: 00:40))

What was the question I asked last time?

Student: ((Refer Slide Time: 00:45))

I am going to discuss that question, which I left you at last time.

(Refer Slide Time: 01:01)



I basically asked… I most likely would have asked this question – M less than 1; M greater than 1. If I increase the area, velocity decreases here and velocity increases here. We have this table; I am just drawing, I am just writing part of this table, which we solved last time anyway. So, I basically asked this question – why? And, actually, the answer is on the board; anyway, I already told you that also. So, if I have a flow, that is, let us say I have a flow through a duct. And then, I am going to have two cases. In case 1, I am going to decrease the area; flow is going through this duct. So, I am decreasing the area. And, in the other case, I am increasing the area – case 2. And, let us say here this is M less than 1; M greater than 1. What will happen in each of these cases to velocity? For case 1, what happens to velocity? M less than 1; I am decreasing area, which is opposite of this; velocity will increase. In this case, M greater than 1 – I am increasing area, which is this case; velocity is increasing. In both cases, velocity is increasing.

And then, I asked you to explain why. They are having opposite behaviors. Of course, mathematically, we can explain it very easily; the expressions just give you the answers. Whatever we wrote last class, all the expressions will tell you that, this is what will happen. But, that is not physical for us. We want to have some feel for the flow. So, I will go back and write that other expression I wrote that day. This is the other critical equation, which we are interested in. I wrote this also that day.

So, now, in one case, it is low Mach number; in other case, it is supersonic, that is, very high Mach number. And, you are going to see that, density effect is much more than velocity change if Mach number is higher. That is what we are seeing. And, let us… For us to compare this flow, ideally, I have to say that, the mass flow rate is the same or something; I have to compare somehow. So, I am going to set the mass flow rates the same let us say; otherwise, I am comparing apples and oranges; nowhere to compare really. Let us say roughly, same mass flow rate; comparing just for comparison sake.

Now, what am I going to look for? I am going to say that, if there is a small change in velocity, there is a huge change in density. And, that is what is given here. And of course, you know that, they are opposite relation, which is given by this minus sign directly there. That is all obvious. Now, in here when it has subsonic flow, we are going to say something like mass flow rate is equal to rho u A; I am decreasing area; and, I am going to say density does not change so much. Why? Mach number is very small. It does not have so much effect like velocity. So, I am going to say that, the effect of density is small. So, velocity has to increase to keep the same mass flow flowing through this duct; that is, there is a particular mass flow rate and it has to flow through this area also. Even if it is lesser area, it has to flow through same mass flow, has to go through. So, this is what is happening there. If I want, I can also say that, this is increasing slightly. This is d u; it should be opposite sign; d u and d rho are opposite signs. So, it has to be slightly decreasing. As per this, Mach number is not very high.

When I go do the same thing for this case, I will again write the same expression; I have the same expression. And now, I am saying area is increasing. Now, it can do two things. And of course, we wrote expressions in terms of d rho by rho in terms of d A by A; d u by u in terms of d A by A and all that. But, I just want to see this connection first. I am going to say Mach number is high; which means for a small change in u, there is a huge change in density and they are going to go opposite direction. Overall it has to make the same mass flow; go through this whole duct. Of course, remember it is 1-D flow. So, it is going to occupy the full region with uniform velocity and all that; it is Quasi 1-D assumption everything; remember all that. In reality, the flow may separate from a wall and just get out; we will not have such things in our 1-D world.

So, now, I am going to say the flow is going to accelerate; but, density is going to drop a lot. What is really happening in here is if I want to have a physical feel for things; when the speed is so very high, the fluid does not have enough time to adjust for this area change by decreasing its velocity like what it will do in subsonic flow. Subsonic flow is slow; it has a lot of time to change things. Here it is going so fast; it has just now given the information that, something changed; and, it has to immediately change. So, what does it do? It immediately changes density; decreased density means more volume for the fluid element. It will go immediately; occupy the whole volume. Think that way; that is the physical feel I am trying to give you.

I am trying to say that, there is some particular fluid element say a disc of fluid coming through this tube; that disc suddenly wants to occupy this whole region. It will just suddenly expand and occupy this region, because that is the fastest thing it can do; it will just expand. It so happens that, it expanded too much that it has to increase velocity for it to satisfy the remaining mass flow rate. That is what is the physical feel I am trying to give you here; of course, if we go to high-temperature gas dynamics or more advanced topics, where we will explain how Mach number is related to the response time for the fluid versus what is given – the flow time – flow time versus the response time for the fluid; that is also related to Mach number. We would not deal with that here; but, let us just think about physically like this. I am just going to say if Mach number is very high, fluid does not have enough time to change to adjust to the flow situation. I am talking fluid separately, flow separately. Fluid is that particular say a bunch of air molecules together; and, flow is the particular pressure, temperature, area – conditions I am applying on that particular fluid element. So, the flow is setting suddenly that, there is a new area. And now, the fluid has to adjust to it; the fastest way it does is through this.

Why cannot it do that here? It will be doing it there. First, immediately, it does only that. If I am thinking unsteady, it is going to do that; instantaneously, it is going to do only that. Whatever it does here, it does here also. It will immediately increase density, because it is a smaller area; it will increase density compressed. But, then it finds that, conditions did not change for a long time, because Mach number is less than 1. It is like fluid finds that, there is enough time to adjust some other way. So, it now adjusts the other way. That is the other argument I can give. I cannot explain any more detail than this unless you already know something more about non-equilibrium phenomena in gases. So, this is a nice way of looking at what will happen in supersonic flow. If I suddenly change something, the first thing changes is density; and, then only it will start changing remaining things. Area increased; it has to drop density to occupy it. Now, it so happens that, it changes too much that, this will increase slightly. That is the way we are going to look at things.

Now, I hope you can explain things in CD nozzle, why a nozzle should be that way. But, there was one small gap, which we never filled; remember? The last time when we were doing this whole table, we said I have to decrease area, so that the Mach less than 1 flow will go close to Mach – 1. But, then I said if the Mach number is slightly more than 1, then I can increase area and accelerate the flow further to more supersonic. What will I do to go from say Mach number – 0.999 to 1.0001? That we are not talking here. So, there is something sitting there. There has to be some other condition, which has to do that job for us. We will go and deal with it when there is nozzle flows, which we deal with later. First, I just want to give you all the mathematical tools available; then, we will go and solve one problem at a time. So, after a few more classes, you will know all the features that are possible in flow. Today, we will pick up normal shock. And, after that, you will know everything possible. And, after that, we will just start going solving problems one after the other – flow through a duct, flow through a nozzle, flow in a jet – all variations we will do.

So, as of now, we have generated a lot of formulae for relating one variable with another variable or relating change in one variable with change in another variable; whichever form we want, we have relations for everything. We have a whole bunch of relations, which we derived in the last class and the class before. And, based on that, we created a table last class at the end of it and that looked something like this. I did not finish the full table. There is two more boxes here. Anyways now, I am thinking about if I know Mach number of the flow, I can find every other property. Not exactly; there is something I need to be given. Typically, I should be given state at one point in the flow. After that, I can find every point downstream or upstream – whatever I want.

Typically, they have to give you state at one point. State is basically they have to give you pressure and temperature; typical variables in gas dynamics are pressure and temperature. Or, they can give you stagnation temperature and stagnation pressure. More common thing to be given in isentropic flows are stagnation temperature and stagnation pressure; why? These quantities stay same all through the flow field if the flow is fully isentropic everywhere in the flow. That is the special case there. But, if I am in the lab and I am doing experiments; and, I want to find Mach number; how will I find Mach number in the flow? Give me ideas.

Student: From area ratio

From area ratio; I have not given you area ratio yet. That is a possibility. We will wait on that area ratio. Something else?

Student: Pitot tube.

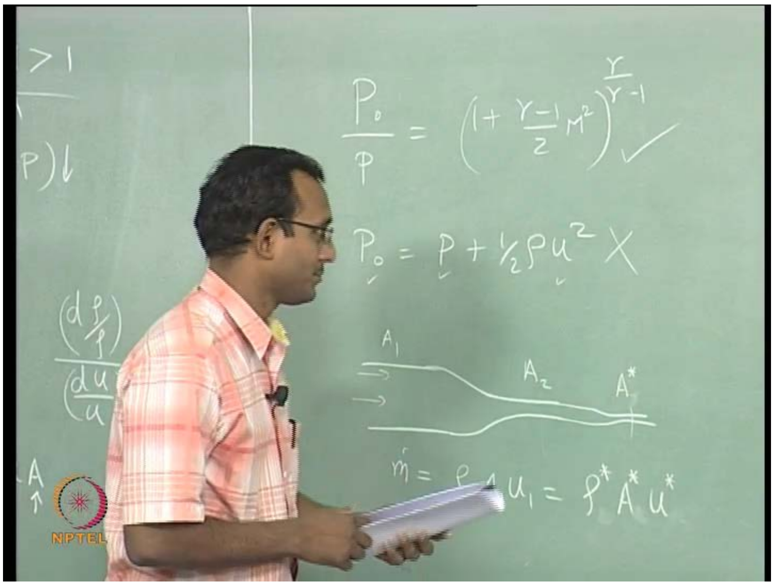
Pitot tube; pitot tube will give me what?

Student: Static tube will give static pressure.

Static tube – static pressure probe will give you static pressure; and, stagnation pressure probe or pitot tube will give you stagnation pressure. So, I can get Mach number?

Student: Yes; in pitot tube ((Refer Slide Time: 14:30).

(Refer Slide Time: 14:33)

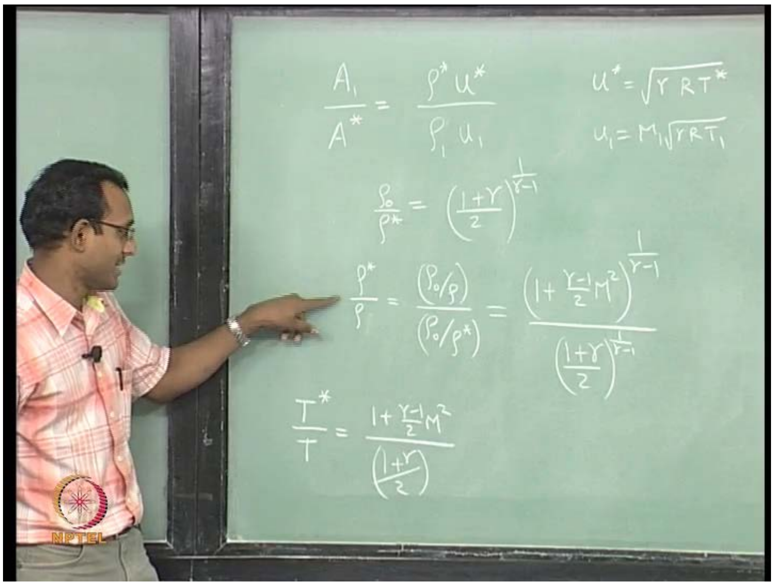


Yes; so, I am going to use P naught by P relation. I hopefully know the gas. If I do not know, I am in trouble. We should know the gas. Once I know the gas, this is just a function of Mach number. I am going to give you this number and this number. So, I can find M. So, I know Mach number at that particular point. That is one way of finding Mach number. Remember that, people do not use this in… Aerodynamists do not use this formula. What do they use? Aerodynamists – what do they use? I kept telling this so many times. So, you should know by now. Aerodynamics people typically use this using pitot probe and static probe. So, they are going to use these two numbers from there and they are trying to get velocity here. But, remember that, this is only first two terms of expansion of this; there is lot more terms available here. So, we are in compressible flow world; we will not use this formula; we will only use this formula. So, if you are given a static pressure and a stagnation pressure… I have already defined these two variables, right? I think so. So, if I am given these two variables, I am going to find Mach number from there. Even if it is very low subsonic, this formula works correctly. We can do that even if that is the case. Even if its Mach number is 0.0001, it will still work. It so happens that, this also works. If it is Mach number, 0.0001; that is all.

Now, we will pick the other method. If I want to guess based on what is the area given to the flow; one of you just told this idea also. So, if I have flow going through some area and then I suddenly change the area to something else, I know area here and here; let us say A 1 and A 2; if I know Mach number at one place, I can find the Mach number at the other place. This is the other method. This is the only thing that is left currently to be done in isentropic flows. So, we will do this next. What am I assuming? I am assuming that, these are the stream tubes. Remember we said 1-D flow, Quasi-1-D flow. We can always use stream tubes as walls. So, we are going to use that. These are just stream tubes. Maybe the whole flow may be much bigger. We are looking at this particular portion of the flow, which is having area A 1 here and it is becoming area A 2 here.

Now, I will consider a special case, where this goes to a condition, where I will put A star here. I am considering an imaginary case even if needed. I am going to consider this stream tube going to some area such that it will have a star condition – the critical condition, that is, M equal to one condition. From now on, in course, wherever I put star there, it is corresponding to M equal to 1 at that point. So, I am going to consider an imaginary portion of this flow field, where it will have A star – area as A star. And, I am having one constraint in all these. What is that? It is a stream tube. What is the constraint for a stream tube? No? No cross flow or mass flow inside remains the same. So, that is the constraint I am going to have. So, I am going to say m dot. Let us not use A 2. We will just use A 1 and A star from now on. I said star and I am putting 2. It should all be star. Rho star is the density corresponding to; at that condition, if M equal to 1 happens, what will be the density? Area is that particular area; u star is speed of sound really; velocity at M equal to 1; which will just come out to be speed of sound.

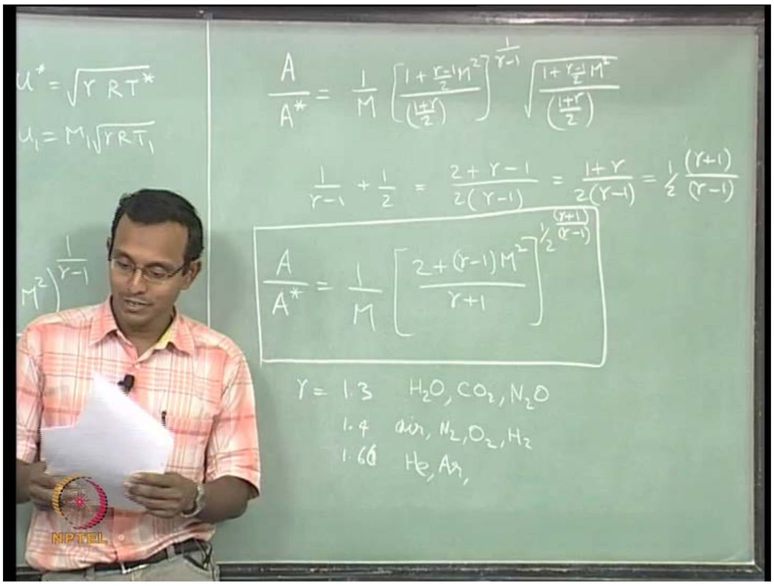
(Refer Slide Time: 19:37)



So, we just have to substitute things inside there. I want to find an expression for A 1 by A star. A 1 by A star will be rho star u star by rho 1 u 1. Now, I have to find individual terms in terms of something I know. I want everything in terms of Mach number. That is the overall idea for me. I can write u as that. And, u 1 as M 1 times square root of gamma R T 1. I can write these two. That is just direct definition of Mach number giving me the answer. Rho star from our expression; we already did this; we can write it like this. I have to put a rho naught; there is a mistake there. I will rewrite things. I will write it as rho naught by rho star. What I have done here is the same expression for rho naught by rho for M equal to 1 condition. 1 plus gamma minus 1 by 2 into 1 square to the power 1 by gamma minus 1. That is what I have here. This is the correct expression.

Now, I can have a similar expression for rho 1 for that particular Mach number. Let us say I would not write that separately; I will just write… I want rho star by rho 1. So, I will write that. Actually, I will just drop this subscript 1 from now on; I will just drop the subscript, because 1 is the only other number that is around. We will just drop the subscript from now on; we will just write what is happening. This could be written as rho naught by rho divided by rho naught by rho star. I am assuming isentropic flow. So, the rho naught for both the cases cannot change; P naught cannot change; T naught cannot change. So, rho naught cannot change. Now, I have an expression for this. It should be M 1; I just made it M. We just drop the subscript. And, a denominator should actually be star condition as M equal to 1; like that expression here. That will just come out to be 1 plus gamma by 2 to the power 1 by gamma minus 1. This is this expression rho star by rho 1 or rho star by rho. I also have u star and u expressions. I will put all of them together. Actually I just want to write one more step. Then, I will put everything together. If I put u star by u, I will have a square root of T star by T. So, I will write T star by T. This will again be a similar formula to the power 1; that is it. So, that is going to be 1 plus gamma minus 1 by 2 into M square divided by 1 plus gamma by 2. It will be this expression; where, I have assumed that T naught is constant. Isentropic flow is still assumed. So, it is going to be coming out to be this. In a similar form like this, I can derive this also. Not very difficult to derive; I will just leave you for you to derive it.

(Refer Slide Time: 23:43)



Now, I am writing it as A by A star instead of A 1 by A star. This will come out to be 1 by M into 1 plus gamma minus 1 by 2 into M square divided by 1 plus gamma by 2 whole to the power 1 by gamma minus 1 into square root of 1 plus gamma plus 1 by 2 into M square divided by 1 plus gamma by 2. It will come out to be this expression; where, this part is coming from u star by u; right? u star by u. This is coming from u star by u. And, this part is coming from rho star by rho. And, this 1 by M is coming from u star by u actually. That is the whole expression you have. Now, if we look at the term inside the square bracket and inside the square root, they are exactly the same. So, I can simplify this further. Multiplication will be in… power will just add up.

So, if I look at just the powers… I will like to write it like this – half times gamma plus 1 by gamma minus 1. That will be the power. I just added this and the half. The square root will be the half. So, I will get a very nice simple expression. And, I am going to simplify this a little bit; multiply numerator and denominator by 2. This is the expression in terms of Mach number for A by A star; that is, I am talking about area currently for that Mach number divided by the Mach 1 area, is given by this relation. That is what we have. And, it has just a function of Mach number and gamma.

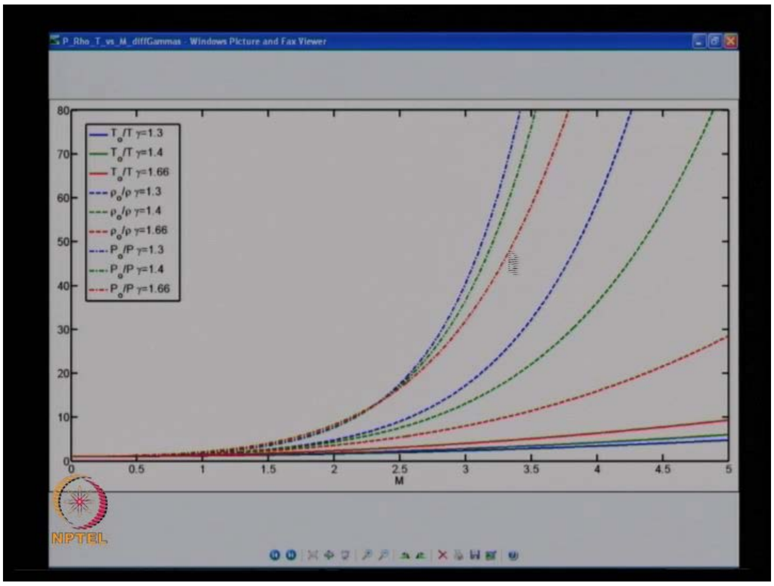
So, basically, we have every variable some way or the other related to Mach number and gamma; nothing else. We have expressions for everything like this. It so happens that, if I know Mach number at that particular point and I know P naught and T naught, I can find any property I want for the flow field assuming the flow is isentropic. I am keeping on saying this, because we are going to switch to non-isentropic flows immediately after this. And, it so happens that, people have created compressible flows tables. There are so many variable avail… variety of books available; which will all give you a list of these functions; values at specific values for Mach number and gamma.

Typically, there are tables for gamma equal to 1.3, 1.4 and 1.67 or 1.66 bar. That is the kind of numbers you have – 5 by 3. What are special about this? If it is inert gases like helium, argon, etcetera; inert gases are monatomic gases like say hydrogen atom; they will all follow this – gamma equal to 1.66. For the air, all the diatomic molecules – they will be using gamma equal to 1.4 in simple gas dynamics by the way. Even H 2 will have the same thing. They will all have this gamma equal to 1.4. If I have a slightly more complex molecules say H 2 O, CO 2, N 2 O, etcetera; then, they will not exactly be 1.3, but they will be close to 1.3. Say for H 2 O, I think it is 1.31; while CO 2 is something like 1.29, something. It will roughly work. 1.3 will work close enough. So, people have listed these three very common stuffs.

Why are we interested in these gases – H 2 O, C O 2, N 2 O? Where do we see them in real life? We breathe carbon dioxide, but that is not the main thing. Gas dynamics – high-speed flows; where, we will see carbon dioxide, water, etcetera. When you burn a fuel – combustion products; fuel and oxidizers will burn to give CO 2 and H 2 O typically, except if your fuel is H 2. If it is H 2, then you will still produce H 2 O. If you are in a space shuttle main engine, then it is just hydrogen and oxygen burning. If it is just hydrogen and oxygen burning, it is going to produce only water vapor. Then, if you want to do calculations for that particular flow, I need to use gamma 1.3. It may not be very well valid; I would not talk about that point right now. For simple cases, it will most likely be valid – gamma equal to 1.3. If I have very high temperatures, then none of these things are valid. Whatever I have tabulated here – all these will not be valid for very high temperatures – say temperatures above 2000 kelvin. Most of these will not be valid for temperatures above 2000 kelvin. For air, temperature above 2000 kelvin, gamma will not be 1.4; it will be less than 1.4. And, depending on the temperature, it will keep changing; it will go up and down if I keep on increasing temperature continuously.

There is a lot more that will happen for air, because there are reactions that are happening inside. We do not need to know for this particular course; we are going to assume that, they are non-reacting gases, perfectly ideal gases; C P is constant; calorically perfect gas. We are going to assume all kinds of things in this particular course. I am just telling you if you are calculating stuff for temperatures of the order of 6000 kelvin, 5000 kelvin; if you are assuming gamma equal to 1.4 and doing this calculation, you may be having some errors if you go and see actual experiments. Or, you designed a nozzle for the rocket and then you use gamma equal to 1.4 there; most likely it will be wrong. You just have to know that. You should go and use different gammas depending on the temperature. We would not go into more details there.

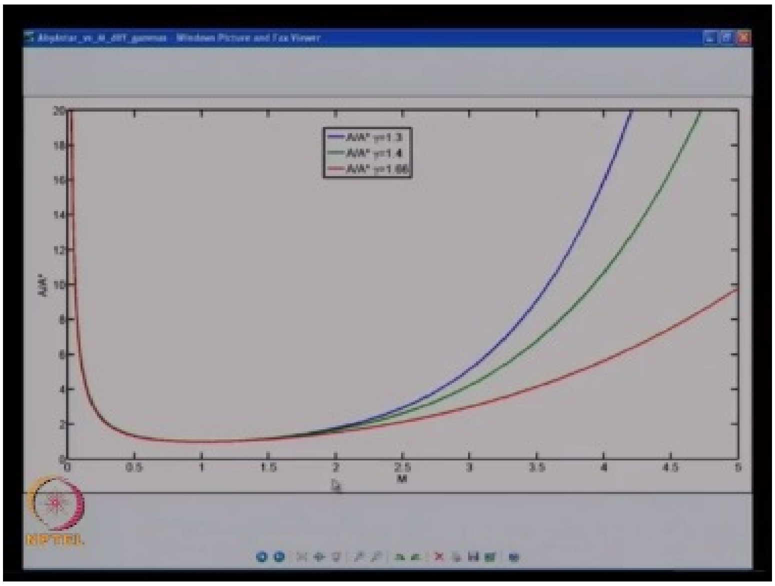
(Refer Slide Time: 31:27)



Now, before we go look at values of these functions, let us just go look to a screen and look at the plots of P by P naught. Actually I am plotting inverse of it; P naught by P, rho naught by rho, T naught by T, etcetera for different gamma values. We already know what gases have these gammas. Anyways if I pick T naught by T function, that is, here; if we see that for three different gamma values, always the function is going up; which means temperature is always dropping while the flow is accelerated from Mach number 0 to Mach number 5. Actually it keeps on increasing; we do not want to show more than this value. And, one more thing; if the gas is less compressible, it seems like the temperature increases more. T naught by T increases more; which means temperature drops a lot; T drops a lot from T naught. That is why T naught by T is increasing faster.

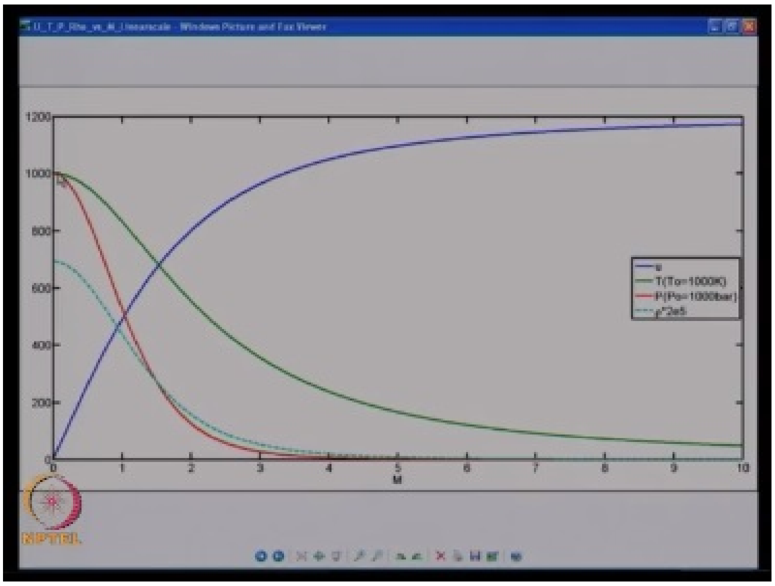
When we go look at pressure plot; this is the pressure plot. What we are seeing here is again pressure drops very fast. In fact, it drops the fastest out of P, rho and T. And, the most compressible gas has the most change in pressure. For a given Mach number, most compressible gas gets maximum change in pressure when we are accelerating to a particular Mach number. And, density is somewhere in between this; it is supposed to be matching P equal to rho R T anyway. It is supposed to be somewhere in the middle; if I multiply density and temperature, I should get this plot. That is what you will get. And, there is a glitch here; in somewhere in the middle if you look here, this is opposite; the trend is opposite for around Mach 1.5 to 2, 2.3 something. That is because of the temperature having opposite trend compared to density. And, because of that, you are going to have that glitch for some time; and, after that, it goes behaves like this for higher Mach numbers.

(Refer Slide Time: 33:23)



If I look at A by A star as a function of Mach number, this is what we derived just now. What we will see here will be… A by A star looks like a u kind of shape curve with minimum at M equal to 1. At M equal to 1, A equal to A star. So, A by A star is 1. And, what we are going to mainly note from this is for a given A by A star, there is a subsonic solution and a supersonic solution. So, when you are looking at tables, you have to know already what Mach number you want; subsonic or supersonic. Based on that, it will change.

(Refer Slide Time: 33:54)

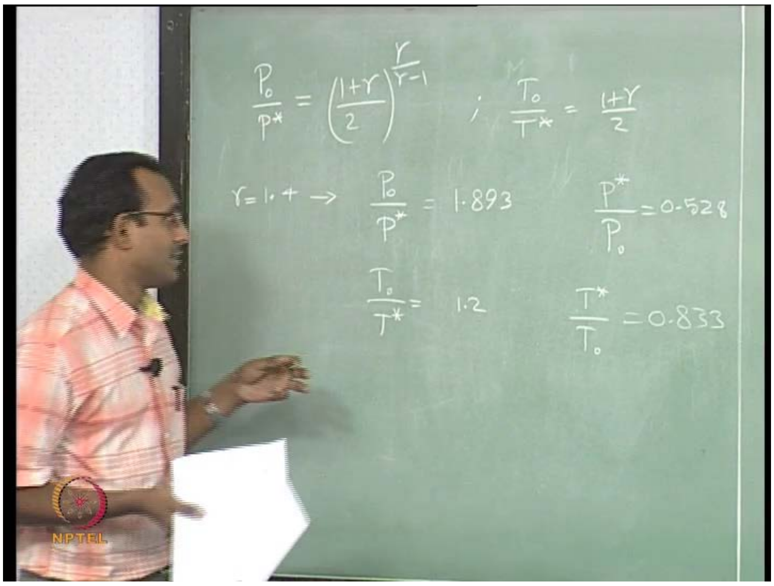


Now, we will go look at some values. I am going to start with T naught and P naught of thousand kelvin and thousand bar; and, I am accelerating the flow from 0 to M equal to 10. That is what we are showing here. What we are seeing is actual pressure values. Temperature – I am showing here temperature is dropping continuously; which is what we expect anyway. Pressure is dropping continuously. It looks like it is asymptotic to 0. We will look at that later. Density is plotted here. Since density value is very low, I multiplied it with a huge number, so that it looks nice on this plot. That is this curve. Again it looks like it is going to asymptote to 0; we will look at that again. Velocity is increasing continuously. It also seems to asymptote.

(Refer Slide Time: 34:35)

We will go look at the same plot in log scale. What we are seeing here is the velocity is really going to an asymptote while temperature, pressure, density – all of them are continuously dropping, they are not reaching an asymptote; which is what we want to see. Of course, velocity will have to asymptote to the v max value, which we saw already related to T naught value. It will be related to T naught converted all to v square by 2. That will be the v max you will get here.

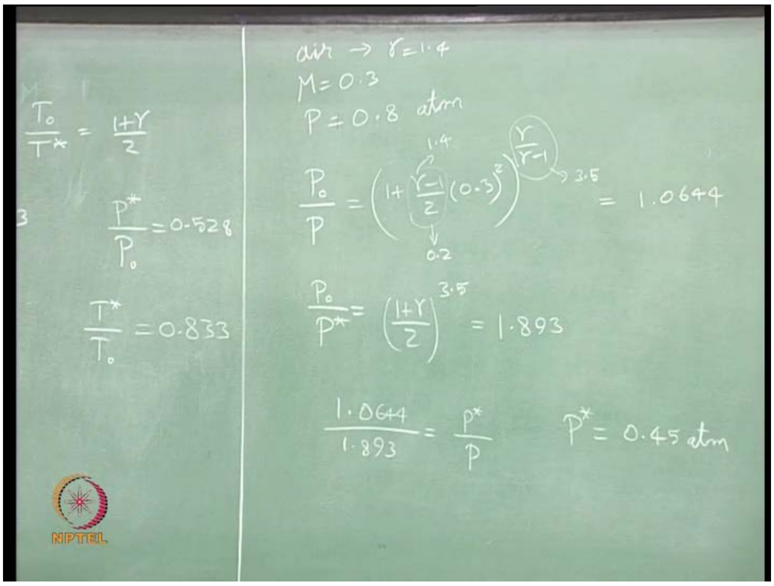
(Refer Slide Time: 35:13)



Now, let us go to the board and look at the actual numbers for some of special cases. So, some special cases. It is just special case of the formulae, which we already derived. It is good to remember this number – P naught by P star; that is, stagnation to static ratio at M equal to 1. I will put the other one also. T naught by T is this for gamma equal to 1.4; why? Most often we are dealing with flow of air. And, we already said that, air is gamma equal to 1.4. So, you are going to just use that. T star; thank you. So, that is T star. So, for gamma equal to 1.4, P naught by P star is 0.528. It is a nice number to remember. Maybe sometimes you may need to remember… I think I made a mistake somewhere. This is not 0.528; it cannot be less than 1; 1.893 and 0.528 is the reciprocal of that. Remember both of these numbers – one is reciprocal of the other. Sometimes this will be useful; sometimes that will be useful. Just for quick check; nice numbers to remember; 0.528 is easier to remember I think than 1.893; it is up to you.

And, if I think about T naught by T star, T naught by T star for air is 1.2; very easy to remember. The other one is not very easy to remember. So, I will typically not remember T star by T naught; I will just remember this. Reciprocal of this is not a simple number that is why. 0.833 – we will get it to be these. Of course, if you go look up at a particular table for compressible flows for gamma equal to 1.4 at M equal to 1 line, you will see these numbers exactly. Now, that is the next thing. Basically, all these tables are available. And, if you want to do calculations fast, all you have to do is just go pick up a compressible flows table; look up isentropic flow properties. There are so many properties in that tables book. We are going to look for isentropic flows properties. Typically, each book will give you for several gamma values – at least three gamma values – gamma equal to 1.3, 1.4, and 1.66. So, for your problem, you have to pick the correct page; otherwise, you will make a lot of mistakes. Of course, you can always remember all the formulae and use that; that is more difficult; calculation of numbers is more difficult; instead, just go look up numbers like this and just use it directly is easier. So, you have to look for correct gamma value isentropic tables as of now. You can use only that table right now.

(Refer Slide Time: 38:44)



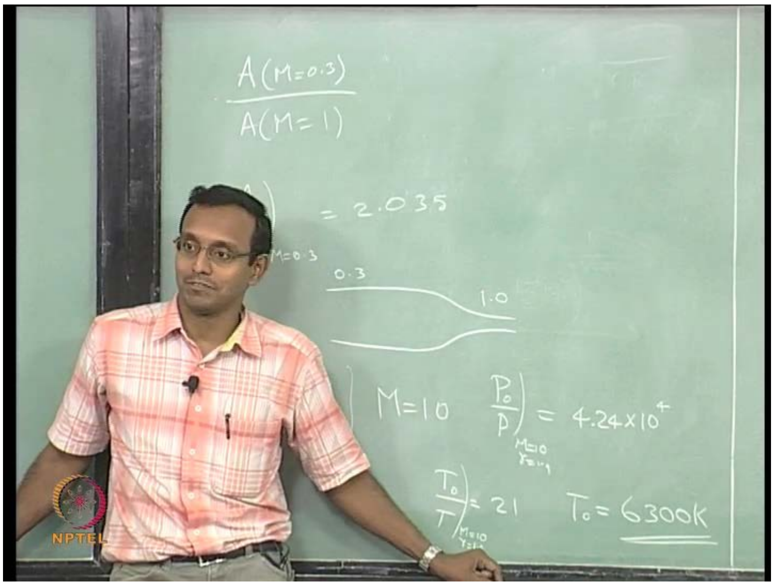
Let us pick an example – numerical example. So, I am going to say I have a flow of Mach number 0.3; and, pressure is given to be 0.8 atmospheres. Now, I am saying this is the static pressure by the way. Now, I am saying if I want to accelerate this flow to Mach number 1, what will be the pressure? That is the question. What will be the pressure if I accelerate this air? I have to tell you what the fluid is; fluid is air. So, air means gamma equal to 1.4. So, I am having incoming Mach number as this at this pressure. I want to accelerate this flow to M equal to 1. So, I want to find that value. How will I find it? Just there are direct tables available, which is faster. I will tell you from expressions; you can even use table; where, here I am going to put gamma equal to 1.4. And, this will come out to be 3.5. Gamma by gamma minus 1 for gamma equal to 1.4 will come out to be 3.5; just remember this quick; easy to calculate. And, gamma minus 1 by 2 will come out to be what? 0.2. Easy to remember numbers.

Now, you just have to substitute this; you will get a number. That happens to be 1.0644. I just calculated this. Ideally, you can get this number directly from tables. If you go look for Mach number 0.3, gamma equal to 1.4; look at the column P 0 by P. You will get to this. There are some books, which give you reciprocal of this – P by P 0 as the table columns. Be careful about it. If this number is more than 1, then you are looking at stagnation by static. If it is less than 1, you are looking at static by stagnation. Be careful. Now, you wanted M equal to 1; that is, we wanted the star condition – P naught by P star. This… I will just put 3.5 directly there; you know what it is supposed to be; gamma by gamma minus 1.

And then, what is this value? 1.893; we just wrote it in the other board. So, I have this; simple enough. I do not even need to find the P 0 value. If you want, you can find it right now. We are given the P; you want to find the P 0; how? This ratio is equal to this. So, I can find P 0 from there. I can use that P 0 here and I can find P star by using this number. That is one way of doing it.

We will just take ratio of these two; that is faster. So, I finally want P star in the numerator. So, I will take this divided by this. So, 1.0644 divided by 1.893 is equal to P star by P. So, from here I can take P here, which is 0.8 atmospheres; I can get a P star. This is simple enough way to solve. And, the answer happens to be 0.45 atmospheres. Is the answer correct? Just a simple crosscheck. Mach number is increasing from 0.3 to 1; star condition is M equal to 1. Mach number increased; pressure decreases. When I accelerate, the fluid expands. Remember that, we did this in the PV diagram, TS diagram, everywhere. So, when I accelerate the flow, fluid expands. That is why it seems to be going in the correct direction. So, everything is matching.

(Refer Slide Time: 43:50)



We will pick another example. It is not really a separate example; it is a continuation. Let us say I take this flow and I want to find the ratio of stream tube areas between this point and the star point; simple enough, right? Or, will that be just directly A by A star for that particular Mach number 0.3, because it is a star condition. I could have asked you a more complex question. Currently, I just want to pick a simple example – A at M equal to 0.3 divided by A at M equal to 1. This is what we wanted. This is just simply A by A star calculated at M equal to 0.3.

So, instead of going and calculating that whole formula we derived today, I could just go and look up table for gamma equal to 1.4, Mach number 0.3; the tables will directly give you the answer – 2.035. So, from Mach number 0.3 to 1, the area is going down by a factor of 2; the new area is half of the previous area. That is what is happening roughly. So, whatever picture I drew before of something like this, it is going something like this. If here it is 0.3 Mach number, here it becomes 1.0; something like this – half this area is what is finally left there. And, all the fluid is going inside this; it is a stream tube. We are trying to find stream tube cross-section area. That is what we found.

I will just give you one more silly example. Find the stagnation condition. This is more application kind of problem. Say I have a rocket launch; and, it is happening at P equal to 1 atmosphere of course; and, temperature of that particular day is 300 kelvin; I pick this for easy calculation. And, I want to find if the rocket is travelling in this particular gas air at Mach 10, what will be the stagnation conditions experienced by this Mach 10? So, you are given this plus M equal to 10. Simple enough problem again. I just have to find P 0 by P at Mach 10 and gamma equal to 1.4. I just need to find P 0 by P for M equal to 10, gamma equal to 1.4. That number happens to be 4.24 into 10 power 4. So, what is the stagnation pressure?

Student: 4.24 into 10 power 4

4.24 into 10 power 4 multiplied by P, which is 1 atmosphere. So, it is… P 0 is this much atmospheres. That is extremely high. It is like 40,000 atmospheres pressure. That is what it has to be withstanding. But, of course, remember the rocket is going to be a very very slow accelerating object. It is going to pick up very slowly; it will just rise up over 20 seconds from the launch pad. By the time it becomes Mach 10, it will not be at this pressure. Pressure will be like one-third of atmosphere or something. So, they do not need to design for this really; by the time it reaches Mach 10, the static pressure at that upper atmosphere is lesser. So, they do not really need to design for 42,000 atmospheres; they will design for something lesser.

Anyways, now you want T naught. Again for M equal to 10, gamma equal to 1.4. I just have to calculate this number also. That happens to be 21. So, T naught will be easy to calculate; 6300 kelvin. So, it is experiencing such high temperatures. Remember in front of the rocket, if the temperature is this high; I already told you something; gamma we assumed to be 1.4; it will change if the temperature is above 2000 kelvin. Here it is 6300 kelvin. Ideally, I cannot directly use this number and calculate stuff. If I change the gamma, this number will drop; if I decrease gamma, this number will decrease. So, I do not need to really design for this high temperature. I need to design for slightly lesser temperature; life is nice for us. Gamma changing is actually helping for us. We do not need to design for such high pressures; we do not need to design for such high temperatures; everything will be easier for us. But, in our scope of this course, we would not deal with that gamma as a constant for us. That is the only thing we will deal with.

And, at this point, I am going to close discussion on isentropic flows; we will deal with non-isentropic flows next and we have to derive equations again. Next two classes, we will again be deriving expressions. But, soon after deriving normal shock expressions, there is nothing more to derive; it is all using those expressions, physical feel pictures, movies, whatever. So, just go over this for a few more classes; next class and the next class will be the most boring classes, because you have to derive big big big expressions. I do not like deriving expressions as much, but I like algebra by the way. Anyways it is nice; but, unless I have physical feel, it is no fun. Anyways, for the people in video, I will give separate exercise put up on the course website; I think it can be done; just next to the video, there will be a link I believe; it can be done. So, I will do it over some period of time.

So, the next thing – non-isentropic flows. I can start right now; there is some two minutes. So, we will just start now and then we will reiterate it next class. So, there are so many non-isentropic processes that are possible. Simplest – in gas dynamics or high-speed flows, is what they call a shock or very common. When we think about high speed flows, people immediately think shocks, because that is the most cool thing to see or whatever; very nice to see.

Other things that can be non-isentropic; of course, if you go and ask a thermodynamics guy non-isentropic, they are going to tell immediately heat transfer. Heat transfer is a non-isentropic process. And of course, we saw that expression. In the first three classes, we were doing thermodynamics; where, we also saw that if there is heat transfer, there is going to be non-isentropic process. Other than that, there could be other irreversibilities like friction. Whichever direction I push some object against the table let us say; whichever direction I go, I have to spend energy. So, it is not really reversible. If I push it this way and push it back, I spent energy going and coming back. So, I wasted energy. So, that is like a non-reversible process. So, there are different ways we can have non-reversible processes.

Other than that, there could be chemical reaction. Molecules may want to react and form some other new compound. Say I have N 2 and O 2; they may want to form nitric oxide NO. Or, N 2 O or N O 2; so many varieties possible. Or, if the temperature is very high, they may just dissociate to form n atoms. Or, if it is even higher, it may even get ionized; n atoms may become n plus and electron if the temperature is very very high. There are so many other non-isentropic processes possible. We will not deal with all of them. We are going to say first that, for our world, we are still sticking to no reacting flows; which means gamma will not change much; will keep the gamma same; we will deal with first non-isentropic process to be shocks. And, there are so many varieties of shocks. We will look at normal shock, oblique shock, bow shock, moving shocks; and, moving normal shock, moving oblique shock, moving bow shock; so much of variety there. We can deal with all of them. In fact, I planned to deal with more of unsteady flows. So, there will be moving oblique shocks and moving normal shocks, everything; which is not typically taught in all the old books.

Other than that, I just want to make you go do one experiment today at home. If there is tap water flowing; keep it in such that, it is flowing like nice cylindrical; it should not be having this waviness in it; nice clean laminar flow. And, I will take a metal plate; whatever; your mess plate, whatever plate; take a smooth plate. Put it in flow at an angle and put some blockage here. Flow water will come here; flow along this as a very thin liquid layer; it will flow like that. And then, if you put a blockage, there will be a sudden change in thickness; as it flows, it is thin – probably of the order of 2-3 millimetres thickness. Suddenly, it will become 5 mm or 6 mm thickness. And then, it will flow around my hand; something like that will happen. You can go and do this experiment; very easy to do. You would have seen this so many times if you are ever washing dishes; you can see it very often.

Or, another example is – if you put a plate straight on the ground and put tap water at the center, it will be a very thin layer going concentric out; suddenly, the thickness of the layer will increase and form a big circle, which is higher thickness; and then, it will flow with the higher thickness. This particular concept is called hydraulic jump in liquids. Hydraulic jump is the equivalent of the gas dynamic shocks in liquids. The difference is in us – in our case, it is all speed of sound as in sound waves that matter. For that particular problem, it is gravity waves that matter. That is the only difference. It has been shown that, if I use gamma equal to 2 for the gas, I will be able to describe hydraulic jump. All the flow phenomena can be matched exactly. But, this is an aside; I do not need to really include this in gas dynamics; but, I just wanted to say that, there are other phenomena, which can be linked to this. Next class, we will start dealing with normal shocks. Any questions you guys have? So, see you people next class.