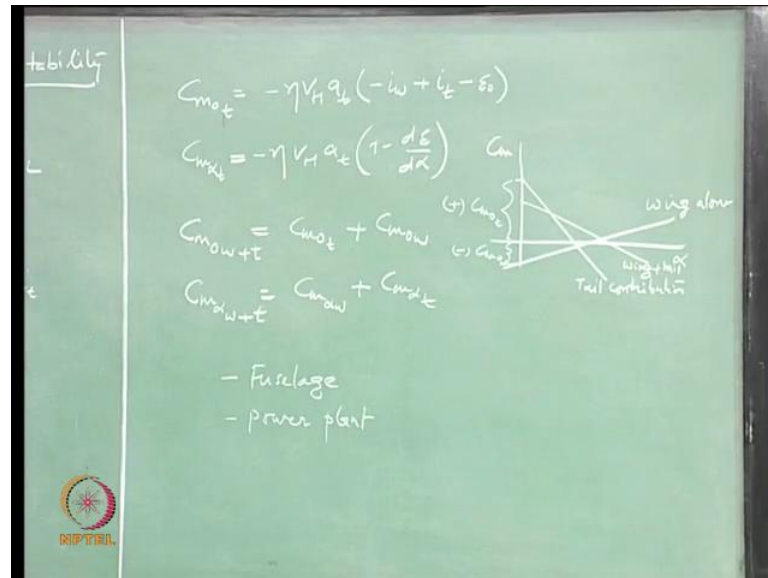




We also said that this epsilon has two components. So, downwards angle itself is changing with angle of attack of the wing plus this epsilon when the wing is at 0 angle of attack.

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So, coming down to what we achieved by adding a tail we found that the tail not only adds to the  $C_{m0}$ , which I am calling this  $C_{m0}$  due to tail  $C_{m0t}$  equal to minus eta  $V_H$  into  $a_t$  into minus  $i_w$  plus  $i_t$  minus epsilon 0  $C_{m0t} = -\eta V_H a_t (-i_w + i_t - \epsilon_0)$ . It is not only adding to  $C_{m0}$ , but also to the tail, to  $C_{m\alpha}$  of the total airplane and  $C_{m\alpha}$  due to the tail  $C_{m\alpha t}$ . Expression for that is  $C_{m\alpha t}$  equal to minus eta into  $V_H$  into  $a_t$  into one minus  $d\epsilon$  over  $d\alpha$   $C_{m\alpha t} = -\eta V_H a_t (1 - \frac{d\epsilon}{d\alpha})$ . We did those in the last class. So, now, the total  $C_{m0}$  of the, wing tail combination is going to be sum of the tail contribution and the wing contribution, and  $C_{m\alpha}$ , the new  $C_{m\alpha}$  of the wing tail combination  $C_{m\alpha w+t}$  is sum of the wing contribution and the tail contribution. So, if you want to look at it graphically, what is happening is this.

(Refer Slide Time: 7:12)

This was the wing alone contribution to the  $C_m$  alpha  $C_{m\alpha}$  curve. The tail alone contribution may be something like this, and this is, has to be positive. So, finally, for this wing tail airplane configuration, what you would get is something like this, wing plus tail. Anything else which is also important in contributing to this, to these quantities? Do you have anything else on the airplane which may contribute to pitch stability or  $C_m$  naught  $C_{m0}$ ?

We have fuselage. Anything else?

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Power plant. Fuselage contribution to  $C_m$  alpha  $C_{m\alpha}$  is usually destabilizing. I will cover that aspect in the next lecture, and power effects are actually very prominent. That also we will discuss with respect to what kind of power plant you have. You can have jet engines; you can have turboprops. So, we will look at all that in the next lectures.

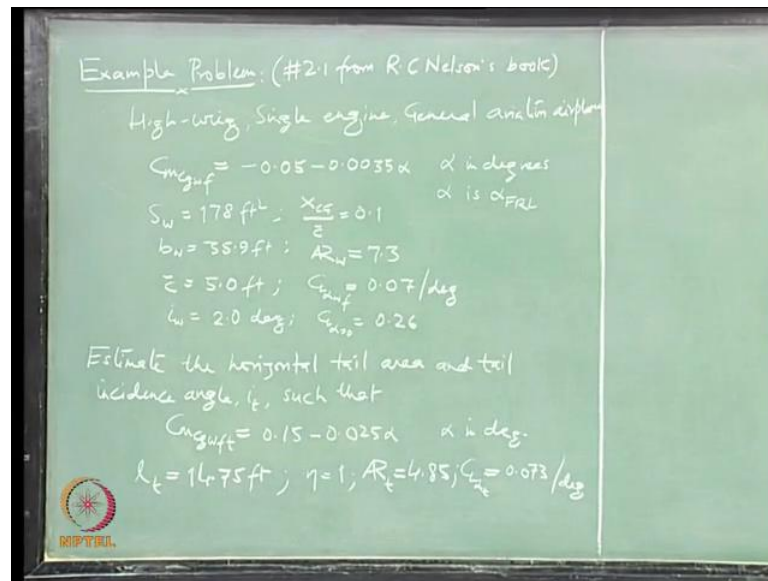
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C m.

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Yes, here, thanks.

(Refer Slide Time: 10:31)



We are going to discuss these two in detail, but in this class, we are going to do a problem, example problem. There is this high wing, single engine, general aviation airplane. This example I have taken from the book by R C Nelson. So, number, I think is, it is 2.1. What we have been given is  $C_{mCG}$  for wing fuselage combination, and that is, this alpha is alpha FRL  $\alpha_{FRL}$ . Other data which will need are these. This  $\bar{c}$  is the mean aerodynamic chord of the wing. So, wherever you see this 'w' subscript it is for the wing.  $C_{Lw+f}$  alpha for wing fuselage  $C_{Lw+f}$  combination.

Clearly the wing is positively cambered.  $C_L$  at alpha 0  $C_{L0}$  is 0.26 which is for the positively cambered wing. Now, the problem is to estimate the horizontal tail area and tail incidence angle... Now, we have seen here, we have to, or we can adjust the parameters in  $V_H$ .  $V_H$  is tail volume ratio. After we have fixed the wing, to change this  $V_H$ , we can only play with  $S_t$  or  $l_t$  and the tail setting.

Other parameters, that is, that are given are these. Assuming tail efficiency to be 1. So, from this data which is the lift curve slope of the tail, it is clear that the tail is, tail is, what?

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It is symmetric airfoil, consisting of symmetric airfoil. So, you can back calculate and you will find that this is for the symmetric airfoil. How do you decide on  $l_t$ ? Now,  $l_t$  will depend upon lot of things. First of all the tail has to produce enough moment to trim the aircraft, that is one condition, and this is also going to affect the stability through this term. And it is also going to be affected by what is the overall length of your aircraft.

Once you have placed your wing at some location, you only have, only so much space left here where you can place your tail. So, let us look at this problem. We want to estimate the horizontal tail area and tail incidence angle  $i_t$  so that final  $C_m$  CG of wing fuselage tail contribution is this. How do you go about it? All I need to do is, find out what is the tail contribution and that I can obtain by subtracting this, from this, is not it? This is for the wing fuselage tail combination; this is for wing fuselage combination.

(Refer Slide Time: 20:39)

Handwritten mathematical derivation on a chalkboard:

$$C_{m0t} = C_{m0wft} - C_{m0wf}$$

$$C_{m0t} = 0.15 - (-0.05) = 0.2$$

$$C_{m0t} = -0.025 - (-0.0035) \quad C_{L\alpha t}$$

$$= -0.0215 / \text{deg} = -\eta V_H a_t \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$\eta V_H a_t \left(1 - \frac{d\epsilon}{d\alpha}\right) = 0.0215 \times \frac{180}{\pi}$$

$$\eta V_H a_t (l_w + \epsilon_t - l_w) = 0.2$$

1 deg =  $\frac{\pi}{180}$  rad.

NPTEL logo is visible in the bottom left corner of the chalkboard image.

Now, I can find out what is  $C_m$  alpha  $C_{m\alpha}$  due to tail by subtracting 1 from 2. This is not  $C_m$  alpha  $C_{m\alpha}$  ..... So you have  $C_m$  naught tail  $C_{m0t}$  which is,  $C_{m0t} = 0.15 - (-0.02) = 0.2$  (Time: 22:15) What is  $C_m$  alpha tail  $C_{m\alpha t}$  ?

So, again, this minus this  $C_m$  alpha wft minus  $C_m$  alpha wf  $C_{m\alpha t} = -0.025 - (-0.0035) = -0.0215$  and this is per degree. So, this should be equal to minus;  $a_t$  is the lift curve slope of the tail, it is also  $C_L$  alpha tail  $C_{L\alpha t}$ , .... And before

you can do that, you have to convert this to per radian. So, you have to multiply this by a factor which is  $180/\pi$ . (Time: 25:00) So, what all you know? You know this, lift curve slope of the tail is given here. You know  $l_t$  inside this. Let us look at what all you know. So, you know this quantity, this quantity,  $l_t$ , only  $S_t$  is an unknown here. In the second equation, we do not know about this. And you also do not know about what is epsilon naught  $\epsilon_0$ ,  $i_w$  is given.

(Refer Slide Time: 26:12)

The image shows a chalkboard with the following handwritten equations:

$$\epsilon = \text{downwash angle} = \frac{2C_{Lw}}{\pi AR_w}$$

$$\epsilon_0 = \frac{2C_{L0w}}{\pi AR_w} = \frac{0.26 \times 2}{\pi \times 7.3} = 0.0226 \text{ rad}$$

$$\frac{d\epsilon}{d\alpha} = \frac{2C_{L\alpha w}}{\pi AR_w} = 0.35$$

In the bottom left corner of the chalkboard, there is a logo for NIPTEL.

Let us try to find out what is epsilon naught  $\epsilon_0$  and what is d epsilon over d alpha  $\frac{d\epsilon}{d\alpha}$ .

So, there is a simple relation between epsilon  $\epsilon$ , this downwash angle, and the lift coefficient of the wing. So, there is a relation which is between downwash angle lift coefficient of the wing and the aspect ratio of the wing and it is like this. epsilon equal to

CL wing over pi into aspect ratio of with wing  $\epsilon = \frac{2C_{Lw}}{\pi AR_w}$ . (Time: 26:52) We find out

what is epsilon naught  $\epsilon_0$  and what is this derivative. So, epsilon naught  $\epsilon_0$  is nothing

but  $2 C_{L0w}$  over pi into AR w  $\epsilon_0 = \frac{2C_{L0w}}{\pi AR_w}$ . The lift contribution is mainly coming from

the wing, and not, and very less coming from the fuselage. So, therefore, we can take this now as a total lift coefficient, so, in this, of the wing.

Here, what we have been given is  $C_L$  at alpha equal to 0  $\alpha = 0$  for the wing fuselage combination, but fuselage will not have. Similarly here also this  $C_L$  alpha  $C_{L\alpha}$  which is given for wing plus fuselage combination will actually represent mainly the wing contribution. So, we can find out what this epsilon naught  $\epsilon_0$  is using this expression for the  $2 C_{L0w}$  over  $\pi$  into  $AR_w$   $\epsilon_0 = \frac{2C_{L0w}}{\pi AR_w}$ .  $d$  epsilon over  $d$  alpha  $\frac{d\epsilon}{d\alpha}$  is nothing but the derivative of this with respect to alpha  $\alpha$ ,  $2 C_L$ ,  $\frac{2C_{L\alpha w}}{\pi AR_w}$  (Time: 29:30) because the downwash is mainly because of the wing. That is why we have this 'w' here.

(Time: 29:35)

Now, this is, this is very close to this quantity what is given for the wing fuselage combination, and using that, you can find out what is  $d$  epsilon over  $d$  alpha and I am just writing it down here. You can find that is correct, yeah, that you have to do anyway.

(Refer Slide Time: 30:15)

Handwritten mathematical derivation on a green chalkboard:

$$C_{m0t} = 0.15 - (-0.05) = 0.2$$

$$C_{m\alpha t} = -0.025 - (-0.0035) \quad C_{L\alpha t}$$

$$= -0.0215 / \text{deg} = -\eta V_H a_t \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

③ —  $\eta V_H a_t \left(1 - \frac{d\epsilon}{d\alpha}\right) = 0.0215 \times \frac{180}{\pi}$

④ —  $\eta V_H a_t (i_t + \epsilon_t - i_{\alpha}) = 0.2$

Unknowns:  $S_t$  &  $i_t$

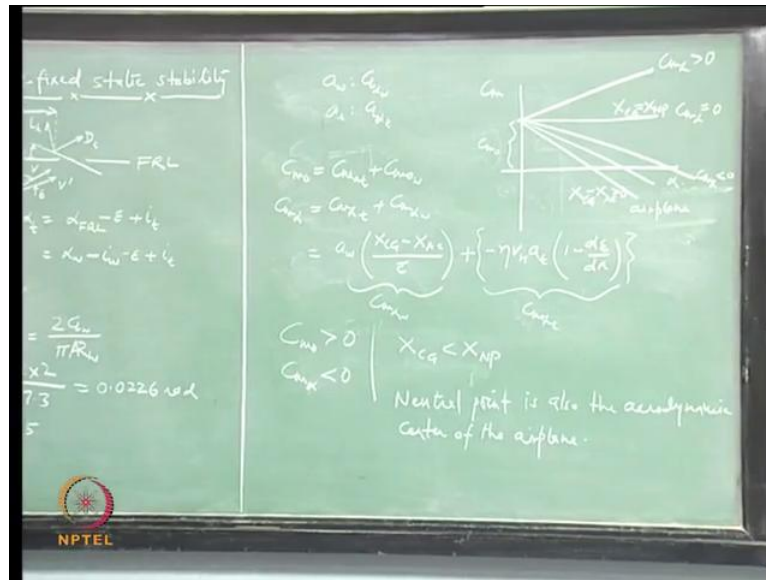
③ & ④  $\Rightarrow S_t = 27.3 \text{ ft}^2$

$i_t = -2.7 \text{ deg}$

Now we have two equations and two unknowns -  $S_t$  and  $i_t$ . Everything else is known. Using 3 and 4, you can find out the tail, horizontal tail area which is  $S_t$  equal to 27.3 ft square. Now this tail setting has to be negative if you want to achieve  $C_{mCG}$  wing fuselage tail combination to be this, is it clear? So, what we have found is, from the given requirement on the aircraft in terms of the  $C_{mCG}$  wing fuselage tail contribution and

this  $C_{mCG}$  wing fuselage contribution, the information on how to size your tail. There will be other requirements also which will talk about later when we come to talk about longitudinal controls. But let us look at something else now.

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Let say this is the  $C_m$  versus alpha  $\alpha$  curve for your airplane. Will this curve depend upon something? How will the slope change? So, let us write down the expression for  $C_m$  naught  $C_{m0}$  again. This is  $C_m$  naught due to tail  $C_{m0t}$  plus  $C_m$  naught due to wing  $C_{m0w}$ , and  $C_m$  alpha  $C_{m\alpha}$ , we are still talking about wing and tail combination.

$C_m$  alpha  $C_{m\alpha}$  is  $C_m$  alpha tail  $C_{m\alpha t}$  plus  $C_m$  alpha wing  $C_{m\alpha w}$ . If I want to write the expression for this, what is  $C_m$  alpha due to wing  $C_{m\alpha w}$ ? It is  $a_w$  which is  $C_L$  alpha  $w$   $C_{L\alpha w}$ . This is  $C_m$  alpha due to wing  $C_{m\alpha w}$  plus  $C_m$  alpha due to tail  $C_{m\alpha t}$ . So, now, we have wing plus tail, in place on the airplane, and, let us not bother about the powerplant and the fuselage. What else can change this profile? Only wing plus tail configuration, we are not yet talking about controls. Just look at this formula. What is it that may change in a flight?

(( ))

C g can travel. This can change and that will have an effect on the slope of this curve, is not it? So, if you talk about this slope, what does it mean? Where is CG lying? If we



have fixed the tail, nothing else we are going to change here. We arrived at this slope using the fact that CG is lying after the aerodynamics center of the wing. So, I can say that ... (Time: 37:45) Now, let us start moving this CG, CG is of course, going to have an effect on also the tail arm length. This  $l_t$  is also going to change if you start changing the CG, not only this distance  $X_{CG}$  minus  $X_{ac}$ , but this  $l_t$  is also going to change because of the CG travel.

So, let us look at this curve, what will happen when the CG travels? The slope will start changing. Which side you think it is going to be more stable? Let us say I have this CG here right now. I move to, move the CG closer to the tail. You will have lesser stability because, because the wing is having a destabilizing effect and that destabilizing effect will be more if I move the CG forward towards the tail.

So, there will be a point. This curve will start now changing the slope. This is more stable, everybody agrees with this. This is more stable than this, and this will be more stable than this. You are changing CG, and now, you see that the slope of this curve is changing. So, there will be one point at which this curve will look like this, it will become flat.

And what is the, what is the  $C_{m\alpha}$  for this flat curve? zero. So,  $C_{m\alpha}$  becomes zero at this point, and this I have achieved only by moving the CG. Now, it is a fact that CG is going to move because we have fuel on the aircraft which is going to be depleting and you can also, there are also passengers moving on the airplane, but we will not bother about that, but one thing is sure that fuel is going to get depleted, and what else?

You can also have something on the aircraft which we might want to release, missiles or you can drop tanks. So, you have fuel tanks, when the fuel is over, you want to drop the tank. So, that is also going to change the CG. So, there is clearly one point, and that is the location of the CG at which the  $C_{m\alpha}$  becomes 0. You know, what is that point called? Neutral point of the airplane. So, CG location where  $C_{m\alpha}$  become 0 is called the neutral point of the aircraft. If you move your CG beyond this point, then what happens?

The slope will look something like this. So,  $C_m$  vs  $\alpha$  curve is positive. Clearly we have, we also ensure that the CG is lying ahead of the neutral point. So, that will be another criteria. We already have two conditions that have to be met.  $C_{m0}$  should be positive and this slope should be negative. On top of that, you should have your CG ahead of the neutral point (which is also essentially related to  $C_m$  vs  $\alpha$ ).

So,  $X_{CG}$  must be less than  $X_{NP}$ . From the definition of this neutral point, you know  $C_m$  vs  $\alpha$  has to be 0. You want to give it any other name? What we call that point about which there is no change in pitching moment with respect to change in angle of attack?

(()).

There will not change on aerodynamic.

Aerodynamic center. So, neutral point location on the airplane is also the aerodynamic center for the whole airplane. (Time: 44:33) So, how do you, how do you find the expression for  $X_{NP}$  here? Set this  $C_m$  vs  $\alpha$  to 0 and that is the point where this CG becomes  $Np$ .

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Stick-fixed neutral point

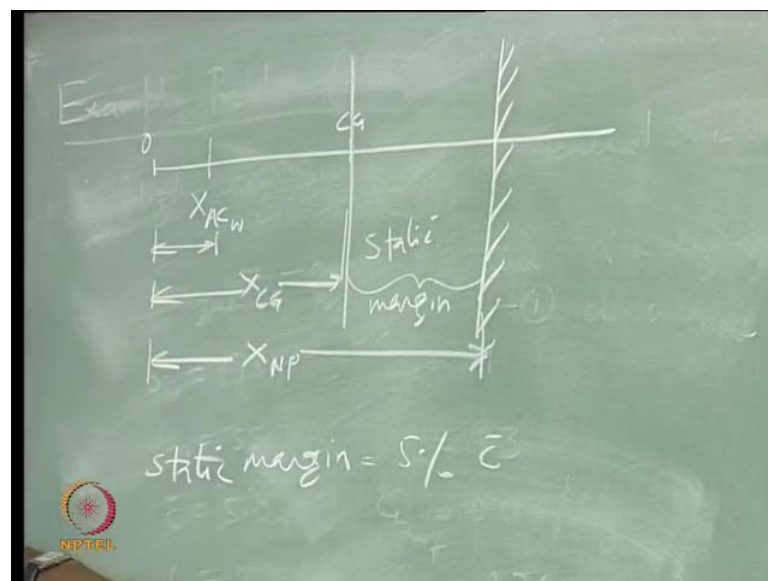
$$0 = a_w \left( \frac{X_{NP} - X_{Ac}}{\bar{c}} \right) - \eta \frac{V}{V_H} \frac{q}{a_w} \left( 1 - \frac{d\varepsilon}{d\alpha} \right)$$

$$\frac{X_{NP}}{\bar{c}} = \frac{X_{Ac}}{\bar{c}} + \eta \frac{V}{V_H} \frac{q}{a_w} \left( 1 - \frac{d\varepsilon}{d\alpha} \right)$$

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So, let us try to get the expression, and this is for stick fixed case, we are still looking at stick fixed static stability. And that is when we will call this neutral point, stick fixed neutral point. So, this  $C_m$  alpha  $C_{m\alpha}$  is 0 and that is the point when CG becomes, CG becomes, the location of the neutral point when  $C_m$  alpha  $C_{m\alpha}$  becomes 0. So, you have, XNP over c bar equal to XAC over c bar plus eta into VH into a t over a w into one minus d epsilon over d alpha  $\frac{X_{NP}}{\bar{c}} = \frac{X_{AC}}{\bar{c}} + \eta V_H \frac{a_t}{a_w} \left(1 - \frac{d\epsilon}{d\alpha}\right)$  right?

(Refer Slide Time: 46:30)



So, we have found .... So, this location is fixed -  $X_{AC}$  of the wing, and we are measuring all the distances from the wing leading edge. We will call this as 0 and this is your  $X_{ACw}$ . So, let us draw a line here which is the location of the neutral point on the airplane.

So, this is like a stability boundary. If your CG is lying ahead of this, then the aircraft is statically stable in pitch. If CG goes behind this, then what happens? The aircraft will become unstable in pitch. So, we have to ensure that the aircraft is having a CG lying ahead of the neutral point all the time.

That's where there is a requirement for this distance which is called static margin. So, every airplane, conventional airplane should have this static margin which is 5 percent of the mean aerodynamic chord of the wing. So, when you are designing your aircraft, you should have static margin which is 5 percent of  $\bar{c}$ . We will stop at this point.