Flight Dynamics II (Stability)

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Module No. # 03

Longitudinal Static Stick Fixed Stability

Lecture No. # 05

Criterion for Stability, Wing Contribution

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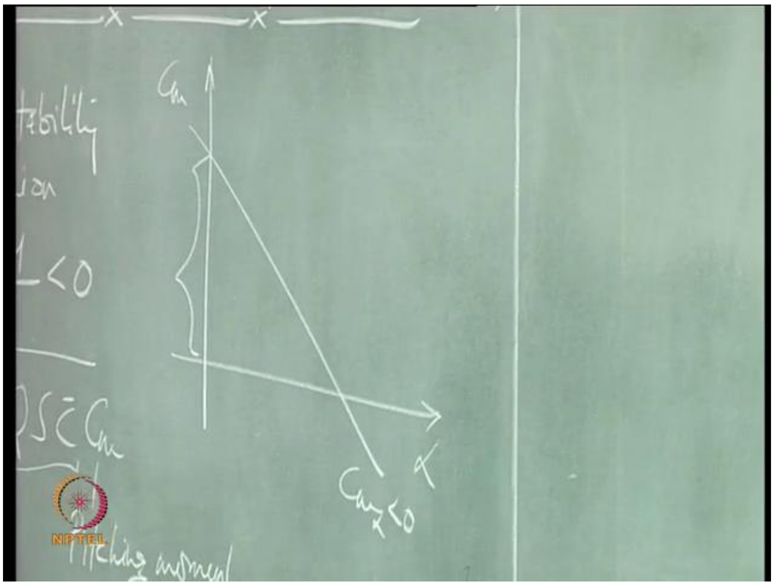

We will continue today from where we left on static stability. We were talking about longitudinal stick fixed static stability. The criteria for this is related to stability in pitch, we can also call it pitch stability. So, if you remember, we said an aircraft to be statically stable in pitch has to have Cm versus alpha curve which has a negative slope. This was the criteria for pitch stability.

So, change in pitching moment, you know, because of a change in angle of attack, positive change in angle of attack should automatically result in a negative change in pitching moment so that the tendency of the aircraft is to kill that extra angle of attack.

So, this was the criterion from which we, because this pitching moment, expression for pitching moment is $QS\bar{c}C_m$; C_m is pitching moment coefficient. We also said that this is equivalent to, everything else being constant and positive here. So, we are checking the static stability in pitch at a particular trim condition and this is the criteria. $\frac{dC_m}{d\alpha}$ should be less than 0 and this is also equivalent to saying $\frac{dC_m}{dC_L}$ is less than 0. So, in short form that is, $C_{m\alpha} < 0$.

What we said is, in order to fly the aircraft, stable and at a positive trim angle of attack, the aircraft must also have C_{m0}, which is pitching moment at $\alpha = 0$, which is positive. So, these ($C_{m0} > 0$ and $C_{m\alpha} < 0$) are the two criteria, you know, which an aircraft two features, which an aircraft must possess.

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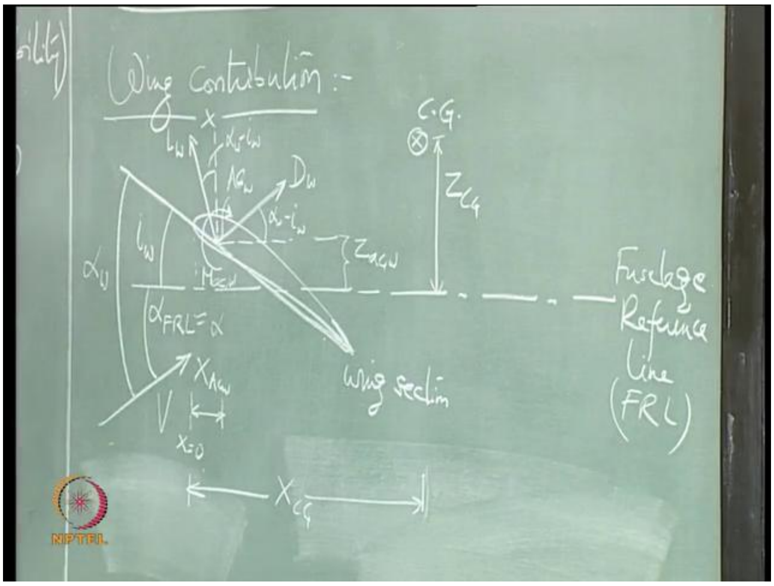

And what we want we do now is, we want to look at how different components of aircraft contribute to these two parameters. So, we will look at contributions from different components one by one. Let us first look at what is, wing is the main component of an aircraft other than the fuselage. Let us look at; we will later on also look at the contribution coming from the fuselage. Let us first start to look at the wing contribution. We define some axis. Here, this line is the reference line which is called fuselage reference line. Our idea is to look at this derivative ($C_{m\alpha}$), sign of this derivative, due to different components. So, this line is the fuselage reference line, which you can obtain by joining the center line of fuselage different sections, and let us say this is our you know the section of the wing.

So, wing section, the chord line of this wing section is this, which is making an angle i_w with the fuselage reference line. i_w is known as wing incidence angle. All angles we are measuring with respect to the fuselage reference line for the aircraft. The angle of attack of the aircraft is with respect to this fuselage reference line which we can call α_{FRL} or we can drop this F R L and simply write this as α, which is for the whole aircraft.

So, α is the angle of attack of the aircraft which is measured with respect to the fuselage reference line, the total angle of attack of the section. V is the relative wind speed coming on to the aircraft, and the angle of attack that this wing section sees is α_w, which is equal to $\alpha + i_w$. Finally, we want to write down an expression for this moment, pitching moment M. So, we have to look at the forces and the distances, distance of the point where the forces are acting from the CG.

So, we will define CG somewhere here to start with, and we take the aerodynamic center of the wing at quarter chord in this simple formulation. So, D_w is the drag due to the wing and L_w is the lift created at the wing. In this formulation, or in general, the distances on the aircraft are measured with respect to the wing leading edge as far as the static stability analysis is concerned.

So, this point is the wing leading edge and distance of the aerodynamic center of the wing from this leading edge is X_{ACw}. You also define the distance of the CG from this, the leading edge, which is X_{CG}. Other than these two forces, if the, and we are saying that the aerodynamics center is lying at the quarter chord.

If the wing section is not symmetric, you know, wing airfoil section, then we know that the center of pressure will lie slightly off this aerodynamic center which is at the quarter chord in this. OR, Let us not assume that. So, we will not assume that this aerodynamic center of the wing is lying at the quarter chord.

For symmetric airfoil, the aerodynamic center is at the quarter chord. But for any cambered airfoil section, the distance between the aerodynamic center and the center of pressure is a non-zero value, because of which, there is a moment created about the aerodynamic center and that is M_{ACw} (or M_{acw}, AC or ac stand for aerodynamic center).

We will also look at the vertical distances. So, in this case, the vertical distance of the CG from the reference line is Z_{CG} and the height of the aerodynamic center from the fuselage reference line is Z_{acw}. Let us try to write down the expression for moment now.

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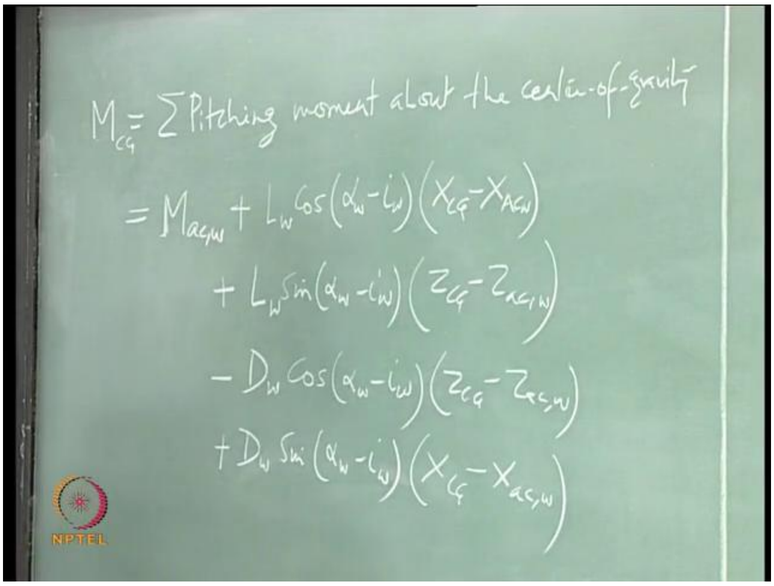

So, moment about the center of gravity, the pitching moment about center of gravity is equal to M_{acw}, which is the residual moment, and the moments coming because of the lift and drag forces. So, plus L_w into the sine component into $Z_{AC\,w}$ minus D_w *cos* i_w into Z_{CG} minus $Z_{AC\,w}$, so, this distance. Drag is acting in this direction is giving a pitching down moment. That is why this negative sign here. The cosine component of L_w is acting in the vertically up direction, it is giving a pitching up moment about Z_{CG}.

And similarly, this component, sine component of L_w is also going to give a pitch up moment. So, that is why this positive sign here and the final one is the sine component of this drag. And this is equal to, so, sum of all the moments, sum of the pitching moments due to all components of forces here is equal to M. This M is about the CG. Now, we can expand this equality relation.

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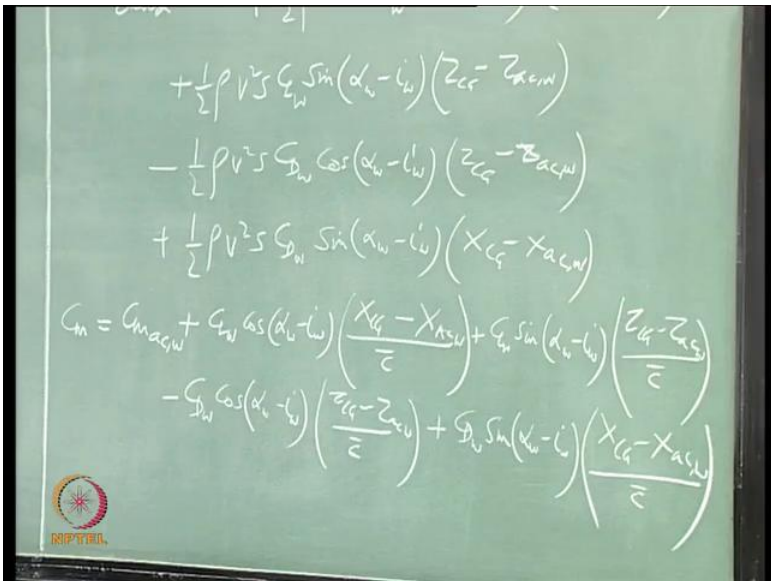
$$M_{CG} = M_{ac,w} + L_w \cos(\alpha_w - i_w)(X_{CG} - X_{ACw}) + L_w \sin(\alpha_w - i_w)(Z_{CG} - Z_{acw})$$
$$- D_w \cos(\alpha_w - i_w)(Z_{CG} - Z_{acw}) + D_w \sin(\alpha_w - i_w)(X_{CG} - X_{acw})$$



M_{CG} is half rho V squared S into c bar into C_{mCG} ($M_{CG} = \frac{1}{2}\rho V^2 S\bar{c}C_{mCG}$). We can drop the CG because we are always taking moments about the CG. So, wherever we do not need it, we will drop that CG. Here, this moment is about the aerodynamic center of the wing. So, we will keep that as C_{macw}, (ρ) rho is the (air) density, V is the speed, S is the wing planform area which is taken as a reference area. ($\bar{c}$) c bar is the mean aerodynamic chord (of the wing).

Let us try to first expand this and then look at what we are getting from here. So, L_w is half rho V squared S C_{Lw} cos α minus α_w minus i_w. So that, now with this expression, we can write down what is the expression for this pitching moment coefficients, non-dimensionalized form. So, C_m is C_{macw} plus C_{Lw} *cos*

$$C_m = C_{mac,w} + C_{Lw} \cos(\alpha_w - i_w)(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}}) + C_{Lw} \sin(\alpha_w - i_w)(\frac{Z_{CG}}{\bar{c}} - \frac{Z_{acw}}{\bar{c}})$$
$$- C_{Dw} \cos(\alpha_w - i_w)(\frac{Z_{CG}}{\bar{c}} - \frac{Z_{acw}}{\bar{c}}) + C_{Dw} \sin(\alpha_w - i_w)(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})$$

Now, this is the complete expression.

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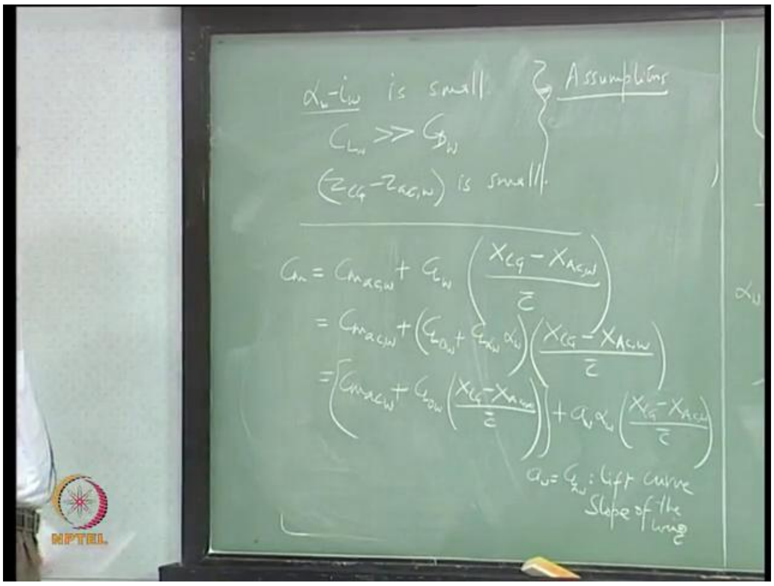

If we have all the distances which are significant and all the numbers which are significant, we have to include them, but here, for simplifications, let us assume that the angles, this angle $(\alpha_w - i_w)$, is small which is perfectly valid approximation or assumption, because this we, we are talking about the pre-stall conditions of flight. So, α is a small angle, and if you use this assumption, then let us also make another assumption and, that is, let us assume that the lift created at the wing is much larger than the drag $(L_w >> D_w)$, which is also a valid assumption.

Another assumption that one can have, only to look at the simpler form, is that, this distance $(Z_{CG} - Z_{acw})$ is small. But in general, if these are not valid, then we have to retain all the terms as they are in that equation. So, let us make these assumptions. So, these are the assumptions that we are making to simplify that expression for C_m.

So, because of my assumption, this $(\cos(\alpha_w - i_w))$ is equal to 1. This $(\sin(\alpha_w - i_w))$ is a small number multiplied with the small number $((Z_{CG} - Z_{acw})/\bar{c})$. So, this (product) is further small which can be neglected. We are assuming that this angle is small and this distance is also small so that this whole term is much smaller as compared to other terms. So, we can neglect this term. Similarly here, we are assuming that the C_{Dw} is very small and this distance $((Z_{CG} - Z_{acw})/\bar{c})$ is also small then we can, so, small compared to the largest number here. Similarly, C_{Dw} and this angle $(\alpha_w - i_w)$ both are small. So, we can also neglect this term.

So, what we get is C_m now, after using these assumptions. $C_m = C_{mac,w} + C_{Lw}(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})$.

Lets also try to include the effect of camber. So, this is $C_{Lw} = C_{L0w} + C_{L\alpha w}\alpha_w$. Now, there is one term which is depending upon the angle of attack, the other one which is not depending upon the angle of attack. So, let us try to separate them, which is, a_w is the lift curve slope of the wing into $\alpha_w(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})$.

$$C_m = C_{mac,w} + (C_{L0w} + C_{L\alpha w}\alpha_w)(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}}) = \underbrace{C_{mac,w} + C_{L0w}(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})}_{C_{m0}}$$

$$+ \underbrace{C_{L\alpha w}(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})}_{C_{m\alpha}}\alpha_w$$

This is the expression for the total pitching moment coefficient, and what we want to do is - we want to see if they are satisfying these two criteria ($C_{m0} > 0$ and $C_{m\alpha} < 0$) or not. So, let us try to write down expressions for C_{m0} and $C_{m\alpha}$ from here.

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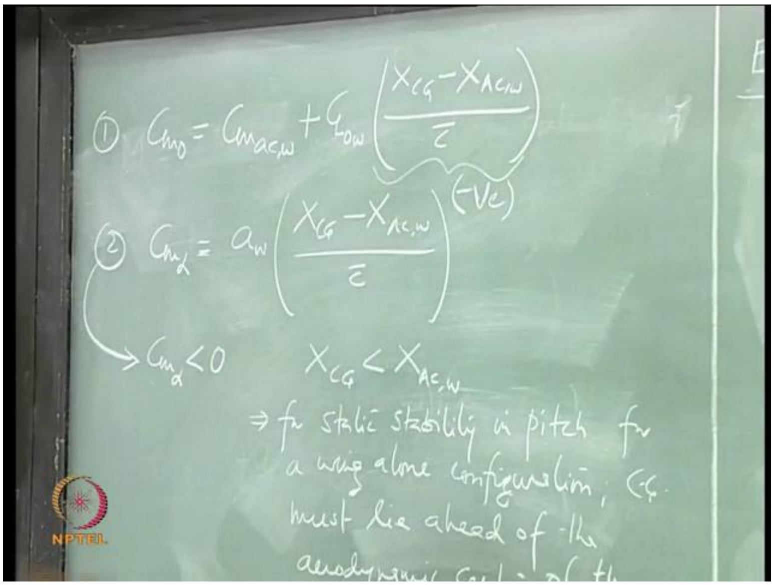
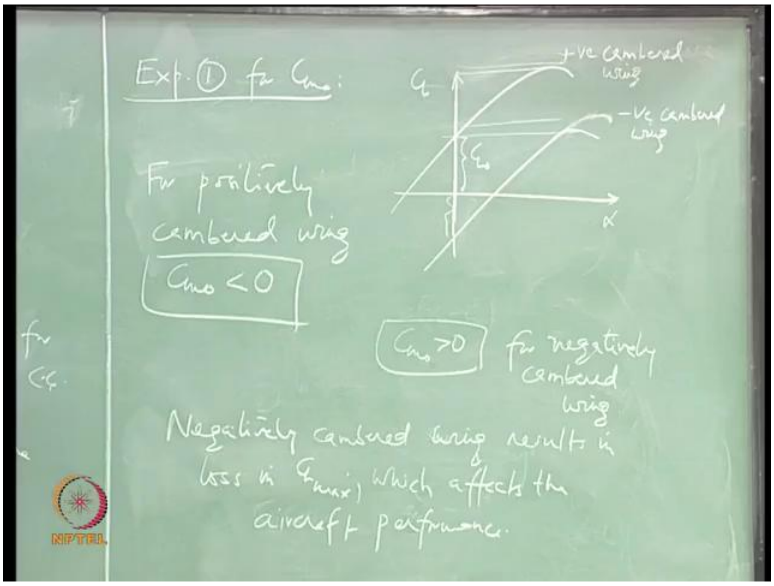


The expression for C_{m0} is $C_{mac,w} + C_{L0w}(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})$ and $C_{m\alpha}$ is now we take the derivative of this expression first; C_m with respect to the angle of attack. What we get is this

$a_w(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACw}}{\bar{c}})$; $(a_w = C_{L\alpha w})$. So, clearly for $C_{m\alpha} < 0$ for this kind of configuration where we have nothing else on the aircraft but the wing, X_{CG} must lie ahead of the X_{ACw}; the location of the aerodynamic center. So, CG must lie before the aerodynamic center for stability. What it says is, for static stability in pitch for a wing-alone configuration, CG must lie ahead of the aerodynamic center of the wing, this is for stability. Now, let us look at what is happening to C_{m0}. So, the expression 2 results in this conclusion. (Refer Slide Time: 32:42)



Now, expression 1 for C_{m0}, let us look at that. So, this quantity $(C_{m\alpha})$ has to be negative because we have to ensure stability. So this is negative and what we want is C_{m0} must be positive. So, C_{m0} positive is the condition to fly the aircraft at positive angle of attack and which we would want for several reasons, because negative angle of attack is going to result in negative lift (for symmetric airfoil).

So, let us look at now what is happening here. This term is negative. Now, if I have a positively cambered wing, then, a positively cambered wing will result in C_{L0} which is positive. C_{L0} positive, so this quantity is positive for a positively cambered wing which generally we would want to have. These are resulting in C_{m0} which is negative, from this term. For positively cambered airfoil, this term is also negative. So, for positively, I can say from here that, for positively cambered wing, C_{m0} is negative.

But if we look at a negatively cambered wing, then this term is negative $C_{L0\mathrm{w}}$. Positively cambered wing sections and this is negatively cambered wing sections; C_{L0} is negative here. So, if we use a negative camber for the wing, then we can make this C_{m0} positive. Now, this is working to our advantage, but disadvantage here is; so, we are, by choosing a negatively cambered wing, you know we are satisfying the conditions for flying positive trim angle of attack and also we are satisfying the conditions for static stability in pitch. But you see, this portion of, ofcourse there is a loss of lift over all and also this portion of the angle of attack for which the lift coefficient is negative, negatively cambered wing results in loss in $C_{L\max}$ which is we can see from here. So, in the case of positively cambered wing, $C_{L\max}$ is at this point, and for the negatively cambered wing, the $C_{L\max}$ is far below. So, there is a loss in $C_{L\max}$ which will affect the performance of the aircraft. In general, if we do not use anything else, for example, we will add some more components here. In a normal aircraft, we will see there, components like tail attached to a wing or tail, tail surface usually on the aircraft behind the wing.

For this configuration, the wing alone flying configuration of aircraft, you have to have negatively cambered wing for this C_{m0} to be positive, and we are seeing here that if we use that, then what we see is the drop in $C_{L\max}$. This drop in $C_{L\max}$ can be reduced if we use, what is known as, an airfoil section which is having a reflex trailing edge.

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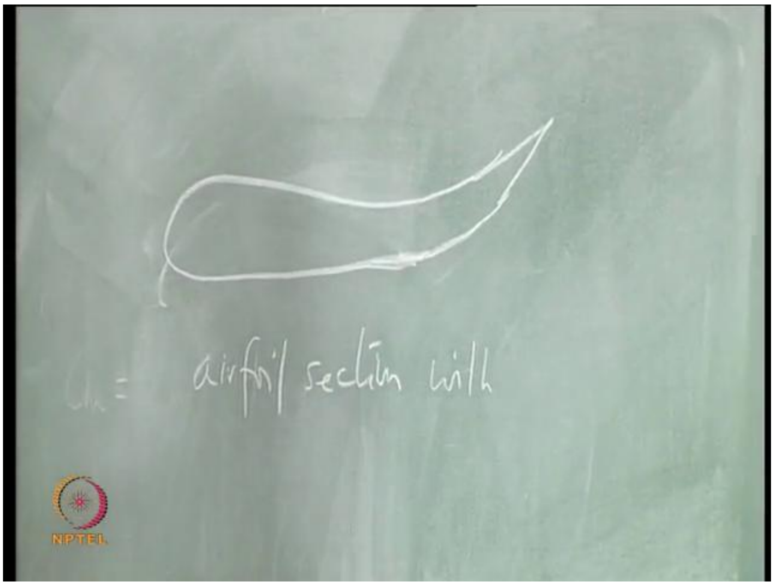

So, in this case, the airfoil or wing section is not negatively cambered for all chord portion, it is only negatively cambered for a small portion of the wing and it looks something like, something like this. So, this is the, wing section with reflex trailing edge. This results in less drop in $C_{L\max}$ and it also gives us C_{m0} which is positive. Normal aircraft will come equipped with horizontal tail, which will provide extra stability to the aircraft in pitch, and we are going to look at the contribution to C_{m0}, C_{m0} and $C_{m\alpha}$, coming from the tail in the next class.