Flight Dynamics – II

(Stability)

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Module No. # 13

Introduction to Aircraft Control Systems

Lecture No. # 40

Stability/Control Augmentation

If you remember in the last few classes we talked about flying and handling qualities of aircraft and these qualities were based upon which class of aircraft, which category of flight and the levels. And all of them were related to where the eigenvalues of the aircraft dynamic modes are located in the complex plane.

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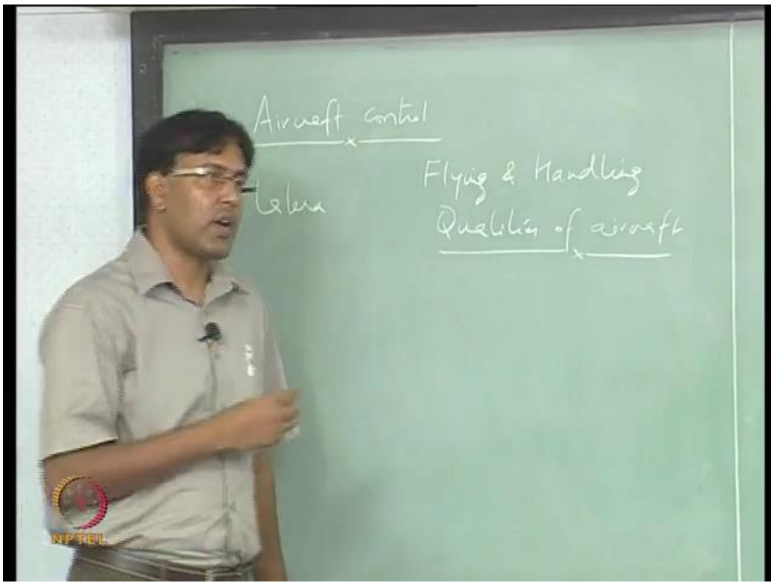

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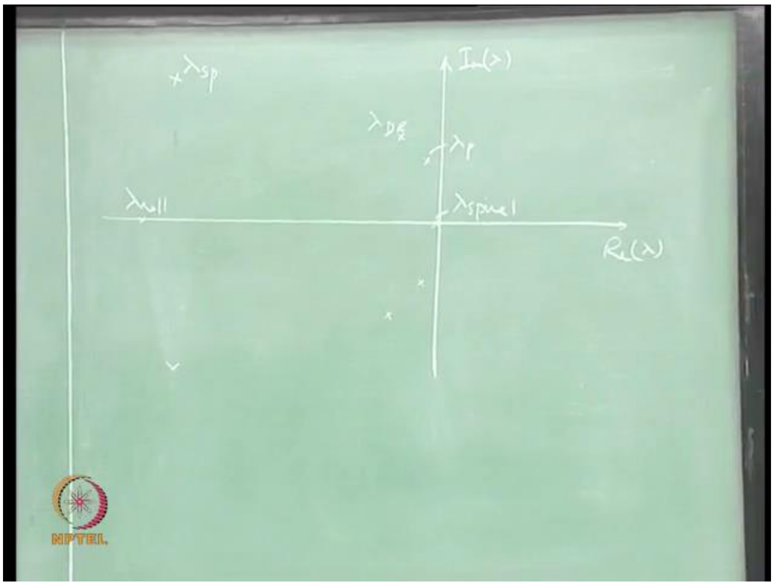

So, this (Refer to the screenshot) is how typically we saw the eigenvalues of aircraft (were) located in the complex plane. Clearly the real eigenvalues are giving the time constants and the complex conjugate eigenvalues are giving us damping and the frequency characteristics (of aircraft response to small perturbations via linearized models around specific equilibrium states). Let us say, our basic aircraft design results in giving us, in one of the flying conditions (equilibrium state), not in all, the eigenvalues at these places located in the complex plane. Let us also assume that these are not what is required (or acceptable) according to the flying and handling qualities.

Remember we said that flying and handling qualities will depend upon these (time constant, frequency and damping) parameters associated with the location of the eigenvalues of the aircraft in the complex plane. If there is no matching of what is required and what we have achieved by the design, then either we can find out the parameter - the aircraft design parameter (geometry or shape related) which can be changed so that we get the location of the eigenvalues in the complex plane as required by the flying and handling qualities. Or, we do something else. And that something else is related to the subject of aircraft control.

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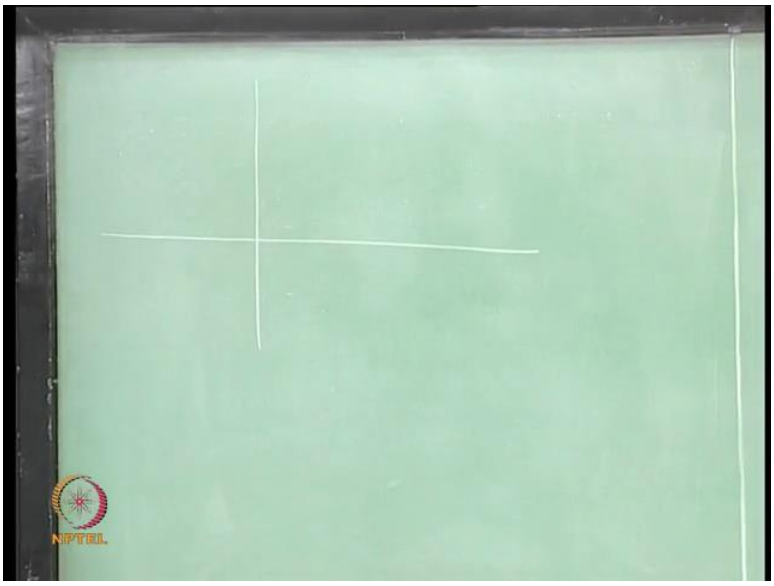

What we are looking for is, we are looking for artificially changing the dynamic response of aircraft (in) small perturbation motion, and also when we are trying to execute maneuvers, there also we want the aircraft to follow certain desired dynamic behavior, which can be achieved by the aircraft control systems.

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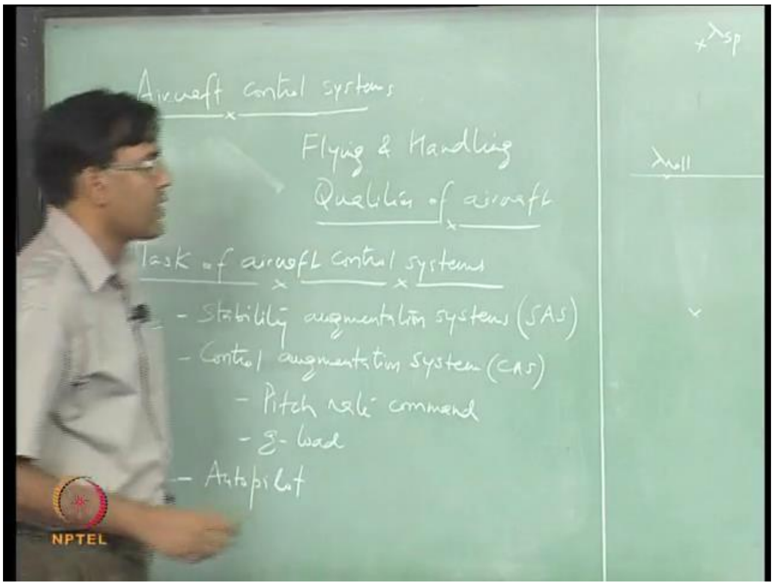

Typically, the task of aircraft control system will fall into 3 different categories; (first) one is stability augmentation. Control system which results in providing artificial stability to the aircraft are known as stability augmentation systems. In short form it is also written as SAS.

The other type of control task is to augment the control system itself and those control systems are called control augmentation systems (CAS). So, task of this control system can be, for example, commanding the pitch rate, or commanding, for example, the acceleration. There are other class of control systems which are designed for pilot free flights, wherein, pilot does not have to do any work. He just fixes (sets) some number/s (for aircraft states) or attitude of the aircraft, and then, he can leave his hands off the control surfaces or sticks, and automatic control systems take care of the desired dynamic behavior of aircraft. In this category we have control systems that are known as autopilot control systems. (Refer Slide Time: 07:12)

Auto-pilot control systems are those control systems where pilot can fix the desired attitude and the path for the aircraft and leaves his hands off. The control system takes care of the aircraft.

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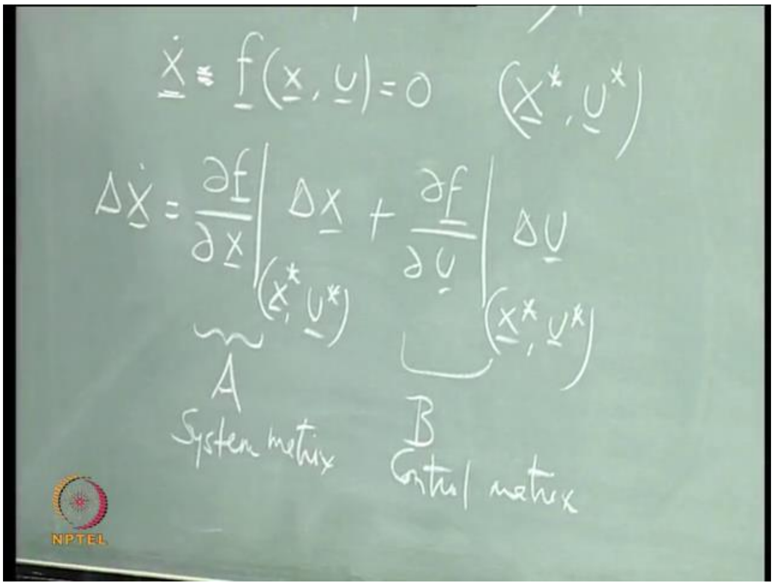

In this class, we will first look at the stability augmentation system. As the name suggests, this particular type of control system is for (artificially) augmenting the stability of aircraft. What essentially it does is, it helps in locating the eigenvalues suitably using a feedback mechanism. (In order to understand that) Let us go back to our equations of motions where we had complete aircraft equations of motions written in compact form as $\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{U})$. (Refer Slide Time: 09:40). Linear control systems are actually designed around particular equilibrium flying conditions and those flying conditions are obtained by setting this (time) derivative of the aircraft state variables to 0 ($\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{U}) = 0$). So, we have equilibrium conditions which are denoted by ($\underline{x}^*, \underline{U}^*$). The linear model around the equilibrium point is developed using a small perturbation theory and what it gives is:

$$\Delta \dot{\underline{x}} = \underbrace{\left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{(\underline{x}^*, \underline{U}^*)}}_{A} \Delta \underline{x} + \underbrace{\left.\frac{\partial \underline{f}}{\partial \underline{U}}\right|_{(\underline{x}^*, \underline{U}^*)}}_{B} \Delta \underline{U}\,. \qquad \text{(Refer Slide Time: 10:53).}$$

Now, this matrix (A) is a matrix of constant numbers at this equilibrium flying condition and that we call this a system matrix A. And the matrix (B) is also evaluated at this equilibrium point ($\underline{x}^*, \underline{U}^*$) and is called the control matrix. So, A is system matrix and B is control matrix.

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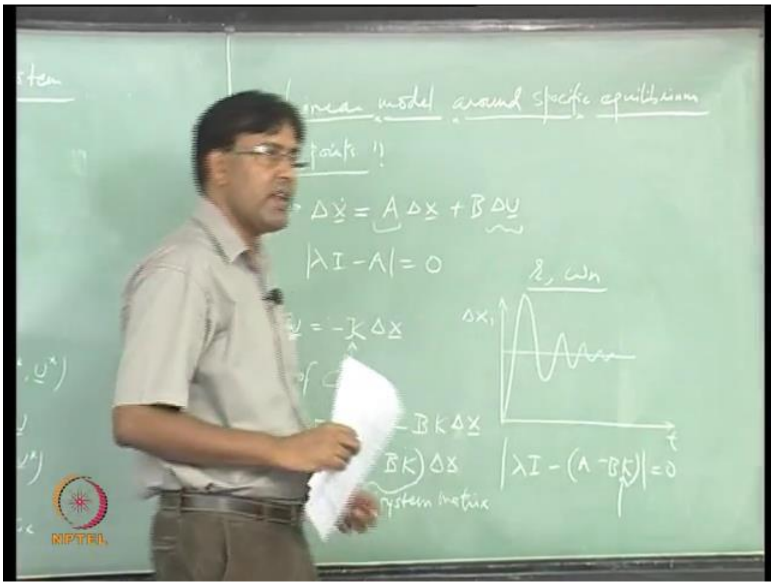

So, we have our linear aircraft model around equilibrium point ($\underline{x}^*, \underline{U}^*$) (Refer Slide Time: 12:15).

$$\Delta \underline{\dot{x}} = A\Delta \underline{x} + B\Delta \underline{U}$$

Aircraft response in perturbed variables is governed by the eigenvalues (λ) of the matrix A which we obtain by solving

$$\det(\lambda I - A) = 0 \quad \text{OR} \quad |\lambda I - A| = 0 \quad \text{where } I \text{ is the identity matrix.}$$

the equation of characteristics. These eigenvalues are located in some fashion in the complex plane. What we essentially want (from our Stability Augmentation System) is, we want to move these eigenvalues in the complex plane so that we can finally achieve what is desired for the aircraft by the flying and handling qualities in terms of time constants or the damping ratio and frequency characteristics.

And that, of course, has to be done through this $\Delta \underline{U}$. There is one case when this $\Delta \underline{U}$ is 0. Aircraft is at some equilibrium condition, it is flying, and whenever small disturbances are hitting the aircraft; aircraft is showing some response and that can be stable; stable, yet not so desired. (damping ratio ζ_n) zeta n and (natural frequency ω_n) omega n, for example, may not be what we want. One of the responses, let us say, in some variable x_1 or the perturbed variable is looking something like this (refer to the sketch on blackboard) (Refer Slide Time: 14:23). This is the natural response. What we are intending to do through the stability augmentation system is to modify this response as desired. There are many ways of doing that; linear control theory or classical control theory is full of techniques which help you to define this $\Delta \underline{U}$ in some fashion, for example, $\Delta \underline{U} = K\Delta \underline{x}$. (Refer Slide Time: 15:16). So, here this $\Delta \underline{U}$ is modified you know according to the change in state variables, perturbed variable (through) using this constant K. This K is called gain, and it can be a matrix, because if you want to control response of aircraft using different variables, then this K is not just one gain, but matrix of gains or a vector of gains. Let us see what it does to this linear model. When we incorporate this $\Delta \underline{U}$ in the linear model, it changes the system matrix through the control input.

So, the new system matrix, when we talk about including this (control; SAS) law in the linear model, the new system matrix for this model becomes $(A - BK)$. Now, if we want to look at the behavior of aircraft, including this control system, then we have to solve for the eigenvalues of this equation: $\det(\lambda I - (A - BK)) = 0$. (Refer Slide Time: 17:50).

Clearly, inclusion of this matrix or the vector of gains K, modifies the eigenvalues or when you change the gains, moves the eigenvalues to a location which is desired and the that value of the vector gains can be used to design the feedback control laws. Let us look at one example here.

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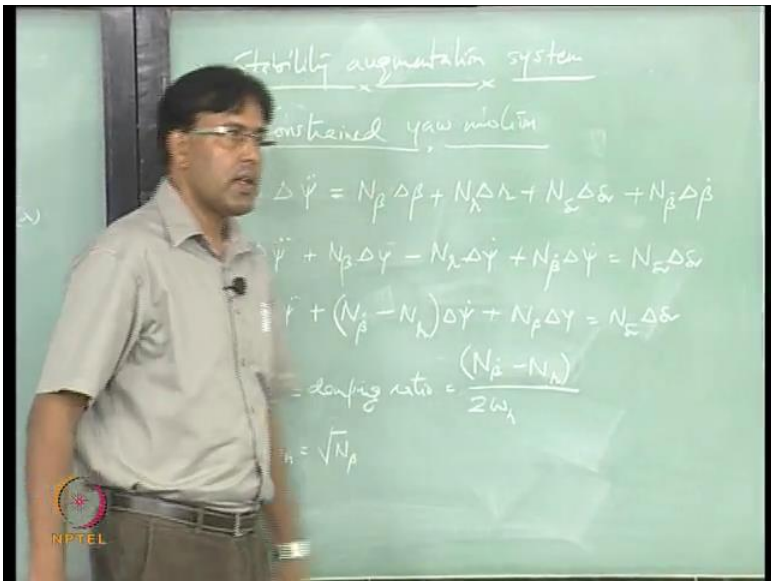

We talked about constrained yaw motion of aircraft in one of the lectures about an equilibrium (flight) condition. And the equation - linear equations for that particular case, aircraft had only one degree of freedom that was in yaw and the equation of motion in the perturbed variables around the equilibrium state turned out to be

$$\Delta\ddot{\psi} = N_\beta \Delta\beta + N_r \Delta r + N_{\delta r} \Delta\delta r + N_{\dot{\beta}} \Delta\dot{\beta}$$

Substituting $\Delta\beta = -\Delta\psi; \Delta r = \Delta\dot{\psi}$ in the above equation one can get

$$\Delta\ddot{\psi} + N_\beta \Delta\psi - N_r \Delta\dot{\psi} + N_{\dot{\beta}} \Delta\dot{\psi} = N_{\delta r} \Delta\delta r$$

and $$\Delta\ddot{\psi} + (N_{\dot{\beta}} - N_r)\Delta\dot{\psi} + N_\beta \Delta\psi = N_{\delta r} \Delta\delta r$$

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What we could find from here was the natural response of the aircraft to disturbance and that was by solving a quadratic equation corresponding to this (above) equation with $\Delta\delta r = 0$.

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Of course, if we do not change this $\Delta\delta r$ from the steady condition, then what we are looking at is the free response and that is given by the solution of this (above equation) set to 0 with $\Delta\delta r = 0$. Then it becomes an eigenvalue problem. So, the eigenvalue problem corresponding to this system when $\Delta\delta r$ is 0 will result in giving us the damping ratio and the frequency, which is, $\omega_n = \sqrt{N_\beta}$ and damping ratio is: $\zeta = \dfrac{(N_{\dot\beta} - N_r)}{2\omega_n}$.

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What we want to do in this case is, we want to design a stability augmentation system which will enhance the damping of aircraft in yaw motion that will also have an effect on the frequency. So, let us see what happens if we change the rudder deflection around the fixed equilibrium condition.

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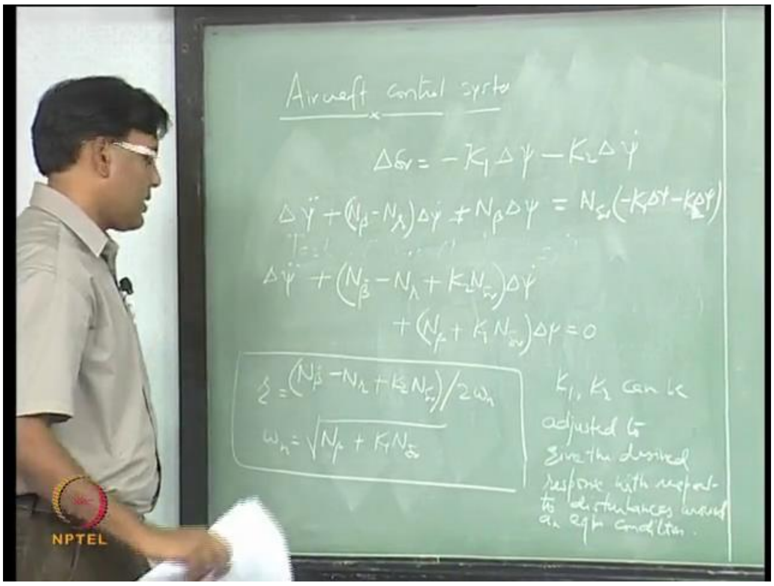

This $\Delta\delta r$ now can be deflected around the trimmed condition according to the feedback law $\Delta\delta r = -K\Delta\psi$. What it does is, it reads the $\Delta\psi$ from the equilibrium condition, multiplies it by this factor K. This factor K is actually sitting as controller (amplifier or attenuator). So, it reads this $\Delta\psi$, multiplies it by this factor K and deflects the rudder according to this formula, $\Delta\delta r = -K\Delta\psi$. What this does to the equations is

$$\Delta\ddot{\psi} + (N_{\dot{\beta}} - N_r)\Delta\dot{\psi} + N_\beta \Delta\psi = N_{\delta r}(-K\Delta\psi)$$

Which is $\Delta\ddot{\psi} + (N_{\dot{\beta}} - N_r)\Delta\dot{\psi} + (N_\beta + KN_{\delta r})\Delta\psi = 0$

(Refer Slide Time: 25:09). What we achieve by deflecting the rudder in such a fashion ($\Delta\delta r = -K\Delta\psi$) is changing this stiffness coefficient N_β to ($N_\beta + KN_{\delta r}$).

Essentially, what it is going to do is, it is only going to change the natural frequency of the response (of aircraft in pure yaw motion under consideration). In case we also want to change the damping ratio, what we can do is, (we can define a new) through this feedback mechanism $\Delta\delta r = -K_1\Delta\psi - K_2\Delta\dot{\psi}$. We not only read the change in the yaw angle from the reference condition, but also we can modify our $\Delta\delta r$ based on the rates. Then, we can achieve a change, in not only the frequency characteristic, but also the damping characteristic.

$$\Delta\ddot{\psi} + (N_{\dot{\beta}} - N_r)\Delta\dot{\psi} + N_\beta \Delta\psi = N_{\delta r}(-K_1\Delta\psi - K_2\Delta\dot{\psi})$$

So, this is how it happens. Let us look at how it is changing the frequency and damping both together.

$$\Delta\ddot{\psi} + (N_{\dot{\beta}} - N_r + K_2 N_{\delta r})\Delta\dot{\psi} + (N_\beta + K_1 N_{\delta r})\Delta\psi = 0$$

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So, now the new damping ratio - the expression for new damping ratio and the frequency are

$$\zeta = \frac{(N_{\dot{\beta}} - N_r + K_2 N_{\delta r})}{2\omega_n}; \qquad \omega_n = \sqrt{N_\beta + K_1 N_{\delta r}}\,.$$

these (Refer Slide Time: 27:57). Clearly deflecting the rudder around the equilibrium point or around the equilibrium value of δr corresponding to the particular trimmed (equilibrium) condition, if we deflect rudder in this fashion then we can modify the damping ratio and the frequency characteristic and the location of eigenvalues in the complex plane. And that is done through these two gains K_1 and K_2 which can be adjusted.

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So, K_1 and K_2 can be adjusted to give the desired response with respect to the disturbances from the equilibrium condition. This is how typically a yaw damper will work.

Let us say, you want to modify the short period dynamics.

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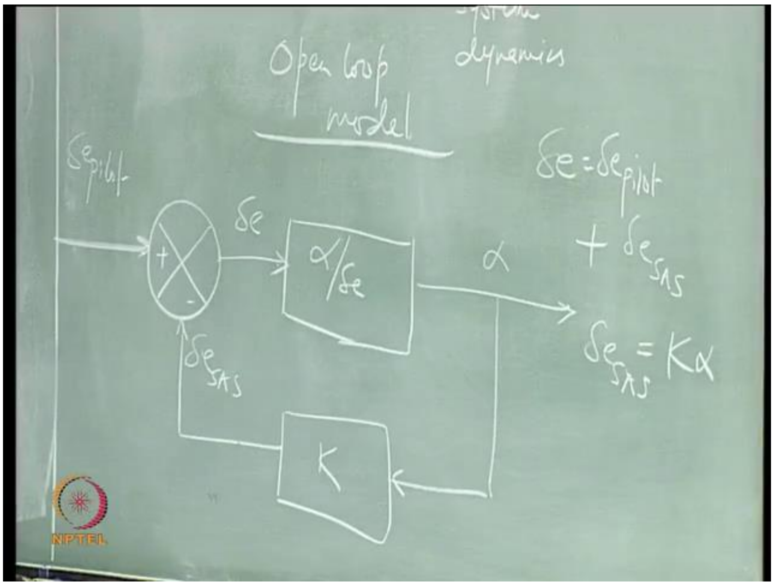

We want to modify the short period response of aircraft using stability augmentation system. What we have is, some response, which is given by $\alpha / \delta e$. To input in elevator aircraft is giving some response in angle of attack and we want to modify this response using stability augmentation system.

So, in general, if you have an open loop and the pilot is using the stick which is giving the deflection in elevator, the response is angle of attack which is governed by this ratio which is called also *transfer function* (Refer Slide Time: 31:52). Or, called also system dynamics, which is alpha dynamics in this case.

So, pilot is giving a stick command and there is some response in angle of attack and that response will depend upon the aircraft parameters. What we achieve by stability augmentation system is, artificially changing the dynamic response of aircraft around normal (equilibrium flight) condition.

32:50 (Refer to the sketch)

So, this is an open loop model. Now if you want to see how stability augmentation system will work which is a feedback system. This is (Refer Slide Time: 33:06) δe which is given by the pilot and because of this feedback loop, the stability augmentation system results in changing this pilot input by this amount ($K\Delta\alpha$). What δe we see is now, δe, which is sum of δe_{SAS} coming through the stability augmentation system and the pilot input δe_{pilot}. And, in this case, this δe_{SAS} is $K\Delta\alpha$. Let us look at what is happening here. So, we have the equations for the pitch rate dynamics which is given by

$$\Delta\dot{q} = M_{\alpha}\Delta\alpha + M_{q}\Delta q + M_{\delta e}\Delta\delta e$$

Since Δ 's are perturbance from reference flight condition, the above equation can be equivalently also written as

$$\dot{q} = M_{\alpha}\alpha + M_{q}q + M_{\delta e}\delta e$$

Assuming reference condition to be zero.

(Refer Slide Time: 35:15). So, this is again a constrained situation.

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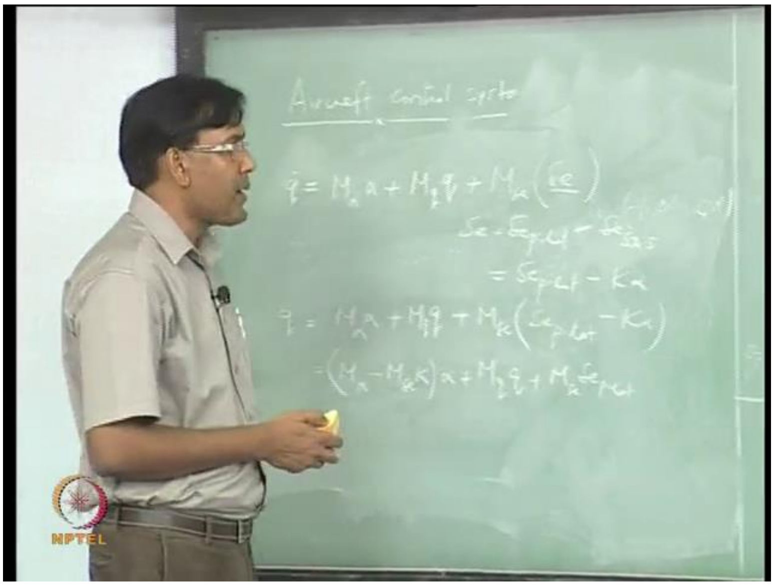

This δe here is, $\delta e_{pilot} - \delta e_{SAS}$ and which is also $\delta e_{pilot} - K\alpha$. What it does is, it modifies

$$\dot{q} = M_{\alpha}\alpha + M_{q}q + M_{\delta e}(\delta e_{pilot} - K\alpha)$$

this is,

$$\dot{q} = (M_{\alpha} - M_{\delta e}K)\alpha + M_{q}q + M_{\delta e}\delta e_{pilot}$$

Now, because of this stability augmentation system in place, this quantity $(-M_{\alpha})$ which is representing the frequency of the short period motion is automatically modified. And so therefore, it has an effect upon on the dynamic response and the stability characteristic of the aircraft in short period motion.

In general, a typical stability augmentation system will not look so simple, but it will be involving lot of other elements. To give you a very brief introduction to the technique which is used to design stability augmentation system using one of the techniques called Laplace transform technique which is applicable for linear system models, I have shown there is

$$\Delta\underline{\dot{x}} = A\Delta\underline{x} + B\Delta\underline{U}$$

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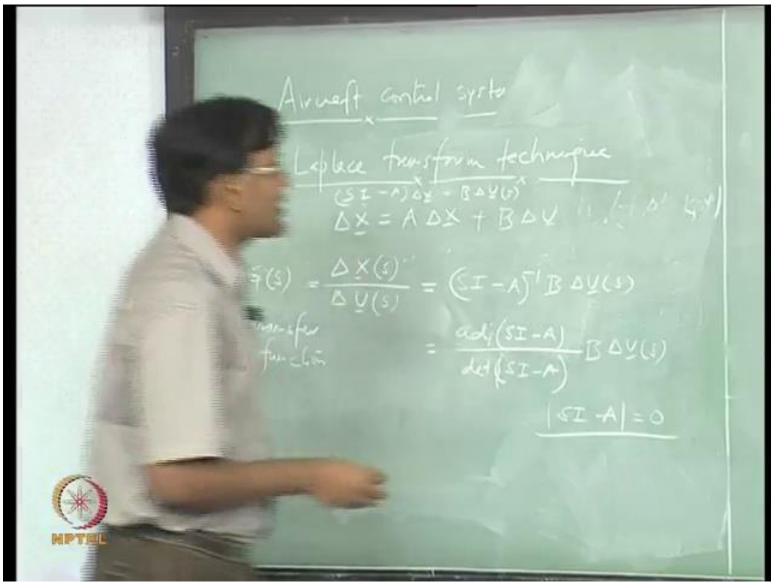

So, clearly if we want to transform these equations in algebraic form, we can use Laplace transform technique and it results in giving us the relation between the input and output which is in this case

$$G(s) = \frac{\Delta \underline{X}(s)}{\Delta \underline{U}(s)} = (sI - A)^{-1} B = \frac{adj(sI - A)}{\det(sI - A)} B$$

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We know that the open loop response is governed by the solution of $\det(sI - A) = 0$. And this function ($G(s)$) which relates the input and the output is called the transfer function.

So, this $\alpha / \delta e$ in this case is a transfer function for the open loop model and the response is governed by the solution of this $\det(sI - A) = 0$ in s; s is also similar to eigenvalues. It is here called characteristic values (poles). Now, let us see what happens when we close the loop. So, $G(s)$ is the open loop transfer function and the response is $C(s)$. (Refer Slide Time: 41:30).

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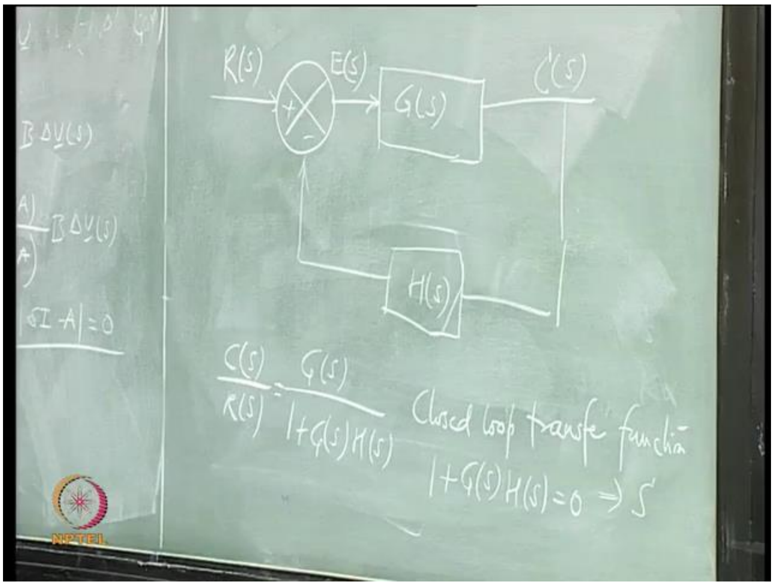

So, there is some response output $C(s)$, corresponding to this input $R(s)$. So, in our case, it is $\underline{X}$ and $\underline{U}$; the usual notation is to use C and R for the output and input, respectively. So, what happens when we close the loop is, it changes the input characteristics, depending upon the error that is fed back through a feedback element.

New input output relation with this feedback system in place is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

(Refer Slide Time: 42:59) and this (above equation) is closed loop transfer function. When this happens, clearly the denominator here is modified and this is. Solution of

$1 + G(s)H(s) = 0$; this is what governs the (closed-loop or feedback control modified) dynamics.

So, now if we want to look at the behavior that will be governed by the solution of this equation ($1 + G(s)H(s) = 0$) which is the characteristic equation. And the solution of which is s, and s is nothing but the poles or the eigenvalues, or the closed loop poles and eigenvalues of the system.