Flight Dynamics – II (Stability)

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Module No. # 02

Equilibrium and Stability

Lecture No. # 04

Static vs Dynamic Stability

If you look at this picture, which I drew in the last lecture based on our concept of static stability, we said that all points on this side of the drag curve are unstable equilibrium states now, if we just go by balancing thrust equal to drag and not really caring about the other degrees of freedom in this motion.

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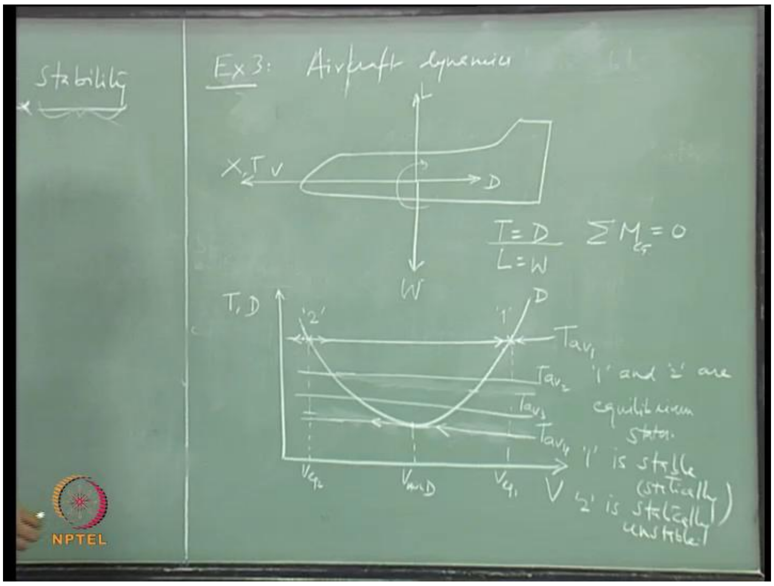

And, on the other side, what we found was that all the equilibrium states are stable. And, this we can obtain, all the states we can obtain by changing the thrust available. In reality, it is not true that all the states are unstable on this (left) side of the curve (Refer Slide Time: 01:17). The reason is that when we are talking about static stability, we are actually talking about only the system's tendency to come back to its equilibrium state; and, not really seeing if it has really come back to its equilibrium state or not.

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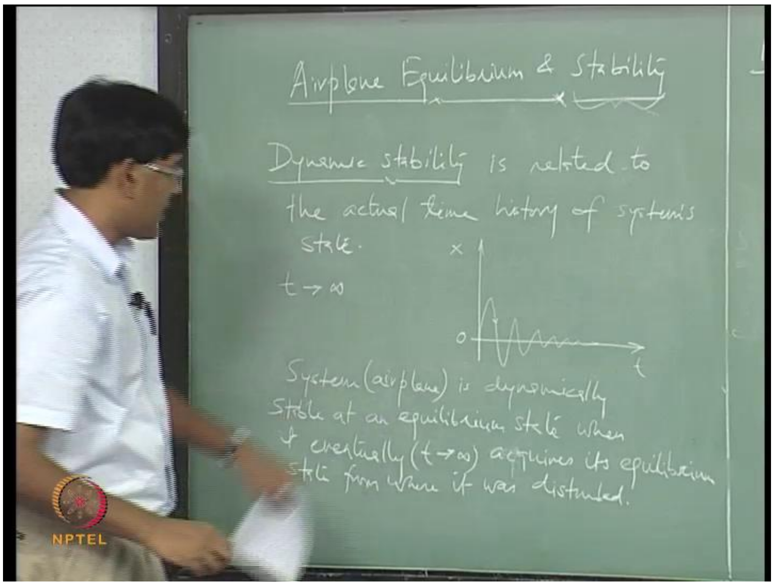

There is another notion of stability, which is dynamic stability, is related to the actual time history of system's state. What we do in this case, we look at the response at time t going to infinity and see if the system has really come back to its original equilibrium state or not. Here x equal to 0 is the equilibrium state. And, because of the external disturbances or perturbation, this is the response in x from this equilibrium condition and we look at the time history. If the transients have died down and the system has achieved this state again, x equal to 0, then we say that the system or our airplane is dynamically stable at an equilibrium state, when it eventually – t going to infinity, acquires its equilibrium state from where it was disturbed. So, really it looks like, that the system for which I have written this state x and plotted it against time, it really looks like that the system is having a tendency to go back to its equilibrium condition. What it is telling is that system is statically stable at this equilibrium condition. And, since the transients have also died out in time and system is able to come back to this 0 equilibrium state, it is also dynamically stable, from this definition (x tends to zero as time t increases) (Refer Slide Time: 05:44). So, here statically stable does mean dynamically stable equilibrium condition. What actually you need to do when you want to look at the dynamic stability? You need to find the solutions of system's equations of motion when it is disturbed from the equilibrium condition.

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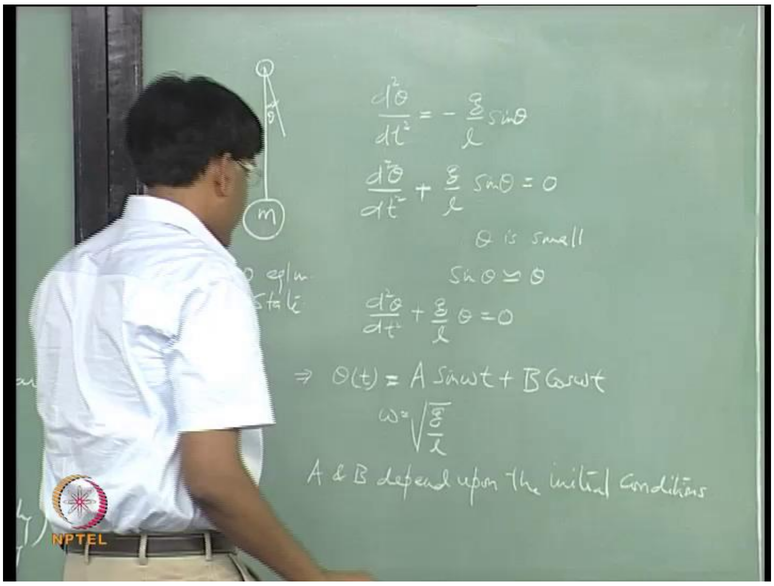

Let us look at a simple example – again, the example of this pendulum. We already wrote equation of motion for this pendulum, $\frac{d^2\theta}{dt^2}+\frac{g}{l}\sin\theta=0$. Physically we already see that the system is statically stable at this equilibrium position. And, we want to see that mathematically also this position (Refer Slide Time: 07:19) is statically stable; that is what we said.

Now, we want to do the same analysis through this equation of motion (Refer Slide Time: 07:32). Clearly, when we are talking about the stability, we are talking about small motion around this equilibrium state. We are actually talking about θ, which is small. Remember, θ is a small change in state (angular displacement around equilibrium state) of the pendulum, because of a small disturbance, this is when we try to associate this with stability. So, θ is small, $\sin\theta$, this is a non-linear equation of motion, which is not really easy to solve. We want to look at simpler ways of looking at stability without looking at the actual time response for this equation (Refer Slide Time: 08:44). $\sin\theta$ is θ. And, when θ is small, we can make this approximation ($\sin\theta \approx \theta$) and the equation simplifies to this linear equation ($\frac{d^2\theta}{dt^2}+\frac{g}{l}\theta=0$). And, this simplification has been done around $\theta=0$ equilibrium state.

Now, solution for this we know. (Refer Slide Time: 09:27) We can write the (general) solution for this equation in terms of parameters, $\theta(t) = A\sin\omega t + B\cos\omega t$; ω is the natural frequency; and, A and B depend upon the initial conditions.

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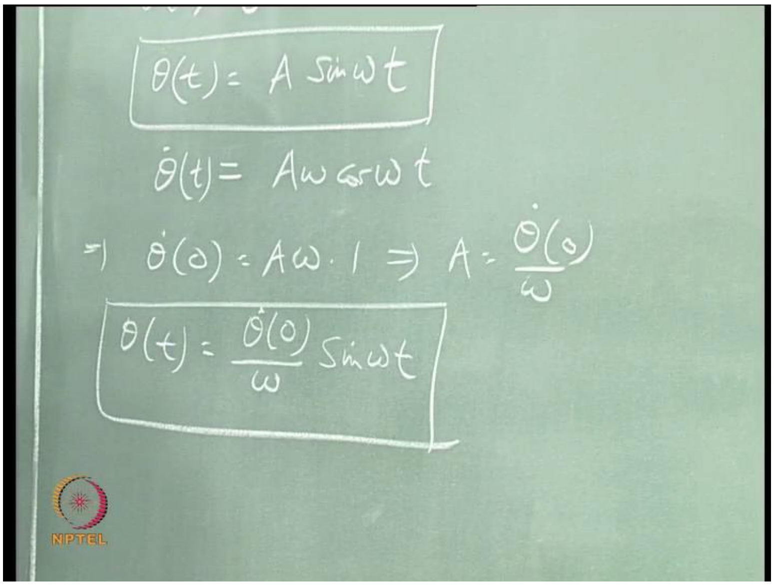

If we have an initial condition, which looks like, theta at time t equal to 0 is 0 (Refer Slide Time: 10:43) and the velocity ($\dot{\theta}$) is non-zero, then you can also write this $\theta(t)$ as ($\theta(t) = A\sin\omega t$), or lets do one more step. When θ is 0, $\theta(t=0) = 0$, what we get from here, from this solution is A into 0 plus B into 1 and that gives us B equal to 0. So, we have solution for this initial condition as $\theta(t) = A\sin\omega t$; where A is given by this initial condition (Refer Slide Time: 12:13). And, what we get is $\theta(t) = \dfrac{\dot{\theta}(0)}{\omega}\sin\omega t$. Clearly depending upon the impulse that you will give to this mass to make it move away from this equilibrium condition, will result in the amplitude ($\dfrac{\dot{\theta}(0)}{\omega}$) of the motion.

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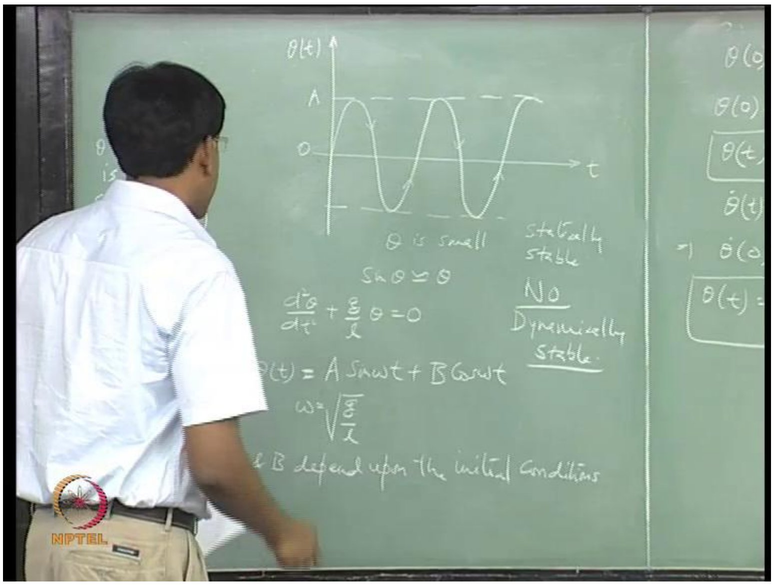

If I want to plot this response, it is going to look like this. Now, the question is how do we infer about the stability from here? Clearly, this system is, this mass *m* is statically stable about this equilibrium position, which is $\theta = 0$. Statically stable, because it is coming, it has a tendency to come back to this equilibrium state. So, it is statically stable. Is it dynamically stable – this equilibrium condition? No. $\theta = 0$ equilibrium condition for this pendulum is not dynamically stable, because, a dynamically stable response will look something like this (converging to the equilibrium condition $\theta = 0$) (Refer Slide Time: 15:00). The transients decaying in time, so that system has finally reached this position of equilibrium in time when t goes to infinity (represents a dynamically stable equilibrium state) mathematically. So, it ($\theta = 0$ equilibrium state) is not dynamically stable, but it is statically stable. And, stability actually has to be with respect to the dynamics. What you can infer (conclude) from here is that statically stable does not always imply that the system is also dynamically stable. This is clear from this exercise.

While the other way round its true. When a system is dynamically stable it is also statically stable. But, this (concept of static stability) is also useful. Because the problem is, when you want to look at the dynamically stable or dynamic stability of an equilibrium state of the system or the airplane, what you need to do is, you need to incorporate all the parameters, use all the equations, and solve all the equations together to find the solutions with respect to time; and then, look at the time history and see if the transients are decaying in time or not. So, clearly there is an advantage of using static stability criteria. There we just have to look at how the restoring forces are behaving with respect to the change in state of the system. And, we can get some information about the stability of the system. But, in this case, we have to look at (deal with) the equations of motion, which is more difficult. And, if you want to talk about aircraft, an aircraft designer will usually have lot of difficulty generating all the data possible at the preliminary design stage itself, so that he can carry out the dynamic stability analysis. So, what he would rather prefer is, to look at to start with, once he has come down to some design, meeting the mission requirements or the performance requirements. Then, what he would be interested in knowing about stability is, some information. It is not the complete information, because it may not be possible to do that, because aircraft is having 6 degree of freedom; three in translation and three in rotation. And, it has lot many parameters, which are difficult to generate at that preliminary stage. So, there are established criteria (related to stability) to be followed by aircraft designers.

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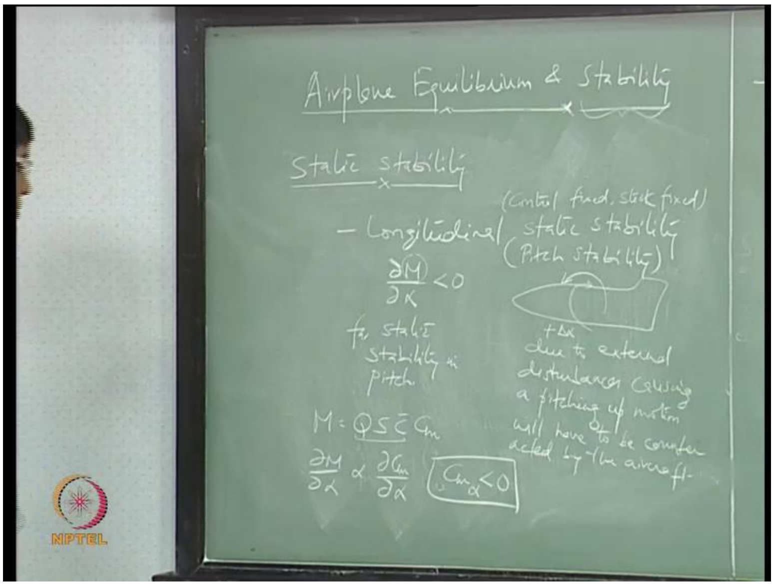

What they would start with is, looking at static stability first. They would want to get first hand information about if the aircraft really has tendency to restore itself to the equilibrium state (design operating flight conditions) or not. And then, based on that, he can go on building the aircraft. He can include many more design parameters in the aircraft model and finally come to a stage, where the aircraft model or the mathematical model, the ordinary differential equation (model) is complete. And then, he can go on doing the dynamic stability analysis. And, dynamic stability is also related to static stability. Its (except) only in some cases, which are critical cases; otherwise it is possible to infer something about dynamic stability by looking at static stability. So, there are different stability criteria that an aircraft designer will start with and they are related to different flight conditions. The static stability criteria, which are basically guidelines for aircraft designers, are based on its (aircraft) rotational motion about the CG in different planes.

First of that is longitudinal static stability. And, this is related to this derivative ($\frac{\partial M}{\partial \alpha}$) (Refer Slide Time: 22:29). M here is the pitching moment. What this means is, whenever there is a change in angle of attack (α) because of gust – gust can change the angle of attack (α) – and there is a change in angle of attack in the positive direction, what is happening to this pitching moment to ensure static stability? It is not difficult to see that whenever there is a positive change in angle of attack, which is also associated with the pitching up motion of the aircraft, if this pitching moment should be automatically generated, not that we are looking at the control side. We are not looking at controls; stability is the inherent property of the aircraft. As soon as there is a disturbance and that disturbance is creating a pitching up motion of the aircraft, alpha is changing in the positive direction. Whether the aircraft is able to restore itself or not. So, aircraft is pitching up when there is a positive change in angle of attack. Aircraft is pitching up. Whether the aircraft will naturally try to generate a pitching moment in the direction to decrease the $\Delta\alpha$ or not, that is related to the static stability of the aircraft in this case – longitudinal static stability. So, first, longitudinal static stability or the pitch stability; and, this is control fixed. So, control stick is fixed in this case.

We will later on look at what happens when the pilot leaves the stick free at the trim (equilibrium flight condition) point. But, here let us first begin with looking at the control fixed or stick fixed condition at the trim point. So, this $\Delta\alpha$ that is due to external disturbances causing a pitching up motion will have to be counteracted by the aircraft. So, aircraft has to kill this change in positive $\Delta\alpha$, which is coming because of the external disturbances. It has to automatically generate a pitching moment in the other direction, so that this $\Delta\alpha$ is killed. So, what you see here for, so this will be (Refer Slide Time: 26:28) a statically stable condition. So, aircraft will have automatic tendency, natural tendency to generate a restoring moment, which will try to kill this $\Delta\alpha$. And, that is when we will say that aircraft is having static stability properties. So, writing it in mathematical terms, this derivative ($\frac{\partial M}{\partial \alpha}$) (Refer Slide Time: 26:57) must be less than 0 for static stability in pitch; M is the pitching moment. M is $QS\bar{c}C_m$; C_m is the pitching moment coefficient; these three quantities ($QS\bar{c}$) remaining constant. You can also write this derivative as $\frac{dC_m}{d\alpha}$. And, sometimes, it is also written as $C_{m\alpha}$, should be less than 0 for pitch stability.

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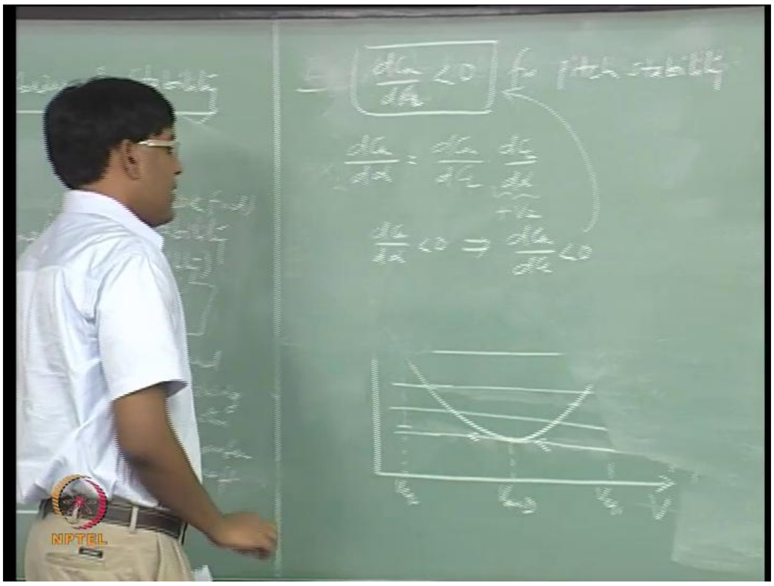

Many times you will also see $C_{m\alpha}$, $\frac{dC_m}{d\alpha}$, or, the stability criteria is also written in terms of $\frac{dC_m}{dC_L}$ for pitch stability. And, this clearly comes from here (Refer Slide Time: 28:30). So, you can write $\frac{dC_m}{d\alpha}$ as $\frac{dC_m}{dC_L}.\frac{dC_L}{d\alpha}$. This ($\frac{dC_L}{d\alpha}$) is positive. So, $\frac{dC_m}{d\alpha} < 0$ automatically also implies $\frac{dC_m}{dC_L} < 0$ (Refer Slide Time: 29:08).

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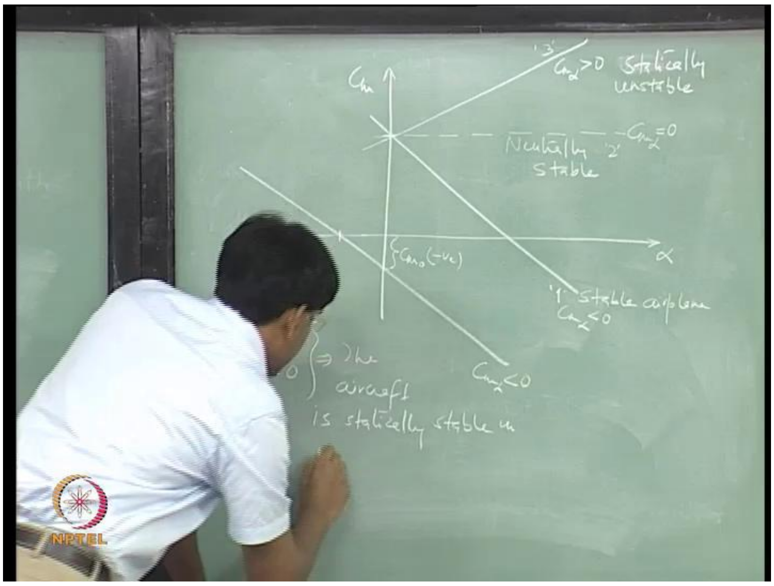

Let us look at this graphically. Let us plot C_m versus C_L. Remember, all the moments are (Refer Slide Time: 29:36) about the central gravity of the airplane. So, clearly, if we want our aircraft or airplane to possess stability in pitch, we should have a C_m versus α (or C_L) curve, which looks like this. This is a stable case. So, an aircraft having C_m-α curve, which is having a negative slope, is possessing static stability in pitch. Aircraft having $\frac{dC_m}{d\alpha} = 0$ is a neutrally stable aircraft. This aircraft '3' is having a positive $C_{m\alpha}$ slope. And therefore, we say that its a statically unstable airplane.

The other thing, that also aircraft designer has to keep in mind is not only about satisfying just this criteria; you can satisfy this criteria (Refer Slide Time: 31:41) even when you have the C_m versus α profile like this. So, here clearly, you have $C_{m\alpha} < 0$. But, what is happening in this case is this curve is cutting the alpha axis at negative angle of attack. So, this – if the aircraft is possessing you know stability, but is having a slope like this, what is resulting in, is this C_{m0}, which is the C_m at $\alpha = 0$, which is negative. C_{m0} negative automatically mean, C_{m0} negative and $C_{m\alpha} < 0$; so, both of them negative means that the aircraft is statically stable in pitch, but it can trim only at negative angle of attack. So, $C_{m0} < 0$ and $C_{m\alpha} < 0$ is telling me that the aircraft, possessing these two (Refer Slide Time: 33:14) characteristics, is statically stable in pitch, but can trim, now, trim condition is defined by this equation ($C_m = 0$) (Refer Slide Time: 33:39). We are looking at the moment equation; then, we have to satisfy $C_m = 0$, C_{mCG} equal to 0 for the trim condition. So, here is where the C_m becomes 0 and that is giving me α, which is negative. So, the aircraft possessing these two characteristics, is statically stable in pitch, but can trim only at negative α. Now, clearly, we do not want to trim our aircraft at negative angle of attack, because that is very unrealistic, because if you look at the lift versus angle of attack curve, with angle of attack, lift increases. And, that is what we want to maximize. So, we would never want to actually fly negative angles of attack.

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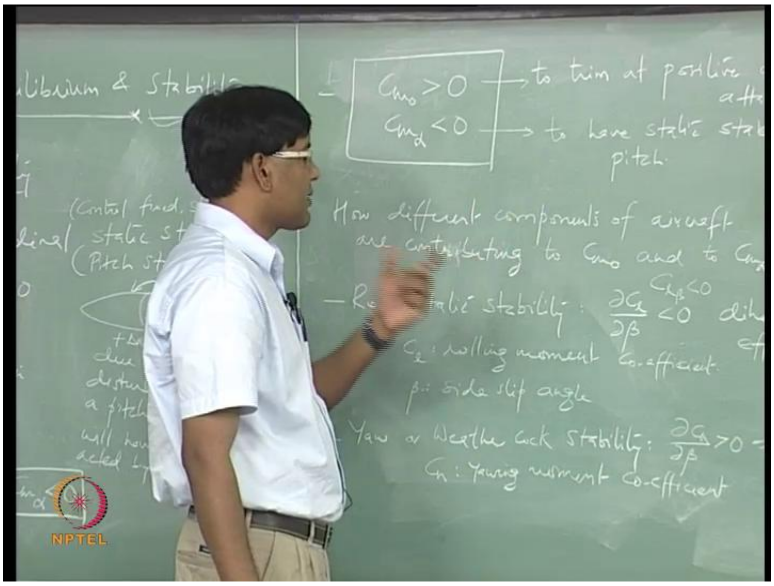

So, two things that that an aircraft designer will keep in mind as far as this pitch, this C_m - α curve is concerned. One thing is, the aircraft must have C_{m0}, which is positive; and, $C_{m\alpha}$, which is negative. So, this (C_{m0} positive) is to trim at positive angles of attack; and, $C_{m\alpha} < 0$ to have static stability in pitch.

In the coming lectures, what we will see is how different components of aircraft are contributing to C_{m0} and to $C_{m\alpha}$. This is what we will look at in the next few lectures. But, before that, let me just complete this. So, this is the criteria for stability in pitch (Refer Slide Time: 37:24). There are criteria for stability in roll, and that criteria is

$\frac{\partial C_l}{\partial \beta} < 0$ – this. So, change in the rolling moment coefficient with respect to change in slideslip angle must be negative. This is also known as dihedral effect. We are going to see this again. So, this C_l is coming from the rolling moment. So, this is another (Refer Slide Time: 38:38) static stability criteria that the aircraft designer will have in mind. And the third one is, so, you can see that clearly, we are talking about the stability with respect to the angles α and β. Those are the angles, which are defining the orientation of the wind with respect to the aircraft. So, we are looking at disturbances in α and β, how they are going to change the moment characteristics.

The third one is (Refer Slide Time: 39:32) the stability of motion in yaw. And, criterion for this is, $\frac{\partial C_n}{\partial \beta}$ should have a positive sign. So, this derivative is also written as $C_{n\beta}$; C_n here is the yawing moment coefficient. As far as the static stability part is concerned, we are going to look at these three derivatives (Refer Slide Time: 40:47) ($C_{m\alpha}, C_{l\beta}, C_{n\beta}$), and, what are the contributions of each of the aircraft components (to the stability derivatives)? Components mounted on aircraft – either mounted or constituting the aircraft; how they are contributing to each of these terms: $C_{m\alpha}$, $C_{l\beta}$ and $C_{n\beta}$. This is what we are going to see in the following classes.