Flight Dynamics II (Stability)

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Module No. # 12

Aircraft Response to External Inputs

Lecture No. # 38

Wind Effect on Aircraft Pure Plunging Motion

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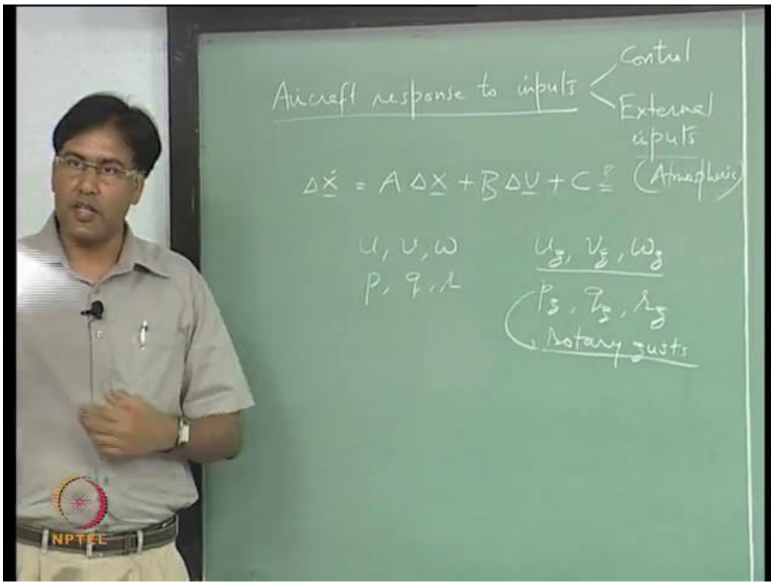

We are looking at aircraft response to inputs. Inputs are the inputs that we can control, you know changing elevator, aileron, rudder and so on and inputs that is not in our control, external inputs. .... So, we are already looked at one simple example where we talked about the response of aircraft in roll to aileron input. Not making things more complicated we will just stop at that.

Now, we will look at aircraft response to external inputs and these external inputs are coming from the wind. Wind can have any form which may be known or not known, but what we will discuss here is the form of the wind, which is known to us. So, what we are expecting is .... this extra term, accounting for the external inputs and these inputs are atmospheric inputs.... What is going to change if we have wind changing; the one thing is flying your aircraft in still air, then, what you are talking about is these velocities along X, Y, Z axes of the aircraft and the rates.

These velocities are going to enter into the aerodynamic forces and moments. So, if I talk about another situation which is not still wind, not still air, but the air has some velocity, the wind has some profile in space or in time. Now, that has to be taken into account because aerodynamic forces and moments we calculate based on the relative wind speed. If you assume that your gust, gust of wind has some profile; we will denote that speed with the subscript g.

So, wind has some velocities, it is not still air, and it can also have profile over some distance, which can be larger than the size of the aircraft. In such a case, you will also have gust which are known as rotary gust. .... Let us see how we can model the wind into our aircraft equations of motion.

Sir what about (( )).

Right.

(( ))

We will look at that, I am going to take these gusts as function of time so that you can differentiate with, as many times as you want. So, we are going to look at temporal variation and also a spatial variation both. What is it going to change, it is going to change the relative air speed coming on to the airplane.

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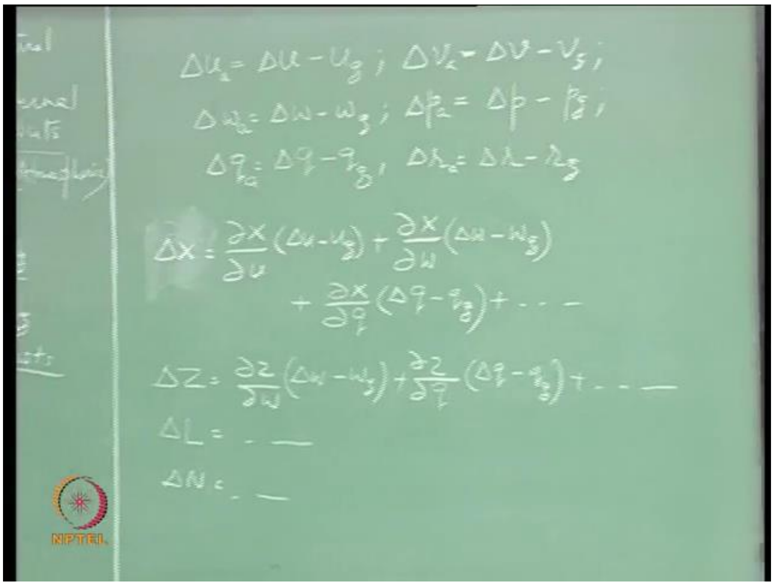

So, if delta u Δu, I am again talking about the linear motion about an equilibrium state, we have this linear model; here we are trying to model the wind. So, absolute Δu_a is now going to be Δu - u_g. I am assuming that, I am putting a minus sign here because this u I am taking in the direction of the X axis, this u. It is in the same direction, so relative air speed is low, if the wind is in the same direction. Similarly, you can also write other components of velocity; this is what now the aircraft is going to see .... And the rates are also going to be different and I will tell you where this change in roll rate due to the wind is coming, where this is coming from, I will tell you shortly. ...

What is it going to change? It is going to change the stability derivatives. In here you have the stability derivative, which are playing role in describing the behavior of the aircraft about the equilibrium state, is not it? So, let us look at how the stability derivatives are going to change. Now, this Δu here is the new Δu .... ((no audio)) anything else also which changes this force can be included. For example, if you have X changing due to the pitch rate, then you have to include this q, change in pitch rate in your model and so on.

$$\Delta X = \left(\frac{\partial X}{\partial u}\right)(\Delta u - u_g) + \left(\frac{\partial X}{\partial w}\right)(\Delta w - w_g) + \left(\frac{\partial X}{\partial q}\right)(\Delta q - q_g) + ...$$

This we have to do for every derivative …. and so on. You can write other terms also in this fashion. Now, the question is where this rotary gust is coming from, you understand from where this wind profile is coming. The wind has some velocity, and this rotary gust is coming from that wind profile. Let us go back to the previous lectures where I was talking about how a pitch rate can introduce a velocity in the vertical direction.

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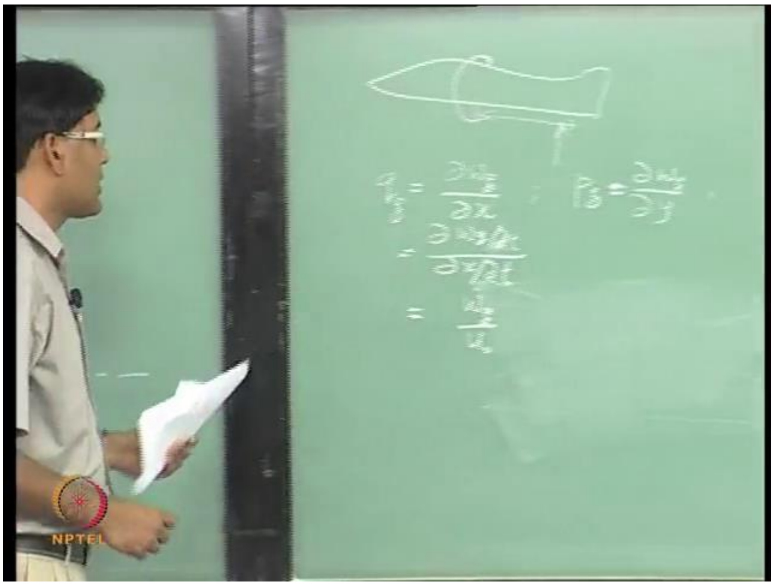

So, we had this aircraft, which is pitching, and then it can introduce this Δw because it (tail) is lying away from the CG. Going by the same argument you can write this q g as w g dot over u naught $q_g = \frac{\partial w_g}{\partial x} = \frac{\partial w_g / \partial t}{\partial x / \partial t} = \frac{\dot{w}_g}{u_0}$. This is the X direction. If you have wind changing along the X direction of the aircraft, then it can give rise to this pitch rate, which is rotary gust. Similarly, you can have wind profile along the span of the wing.

So, wind changing along Y, vertical gust changing along Y and that can give you rotary gust in roll, everybody gets this? Look at this airplane from, you are looking at it from the front and you have a profile which is changing along the span of the wing. Along the span of the wing this w g is not constant, if it is constant then it will not give any roll. But if it is some arbitrary profile, this is going to give rise to the roll motion. So, because of w, vertical wind changing along the span of the wing, we get this p_g term. $p_g = \frac{\partial w_g}{\partial y}$. Is it also going to give you yaw?

(())

Not really! This vertical wind is only going to give you the roll and the pitch. Now, this can also be written in terms of .... now, this is what you were talking about. This is what you talked about. Now, if we include these terms, extra terms in the aircraft linearized model about the equilibrium state, then what you are going to see is this extra term.

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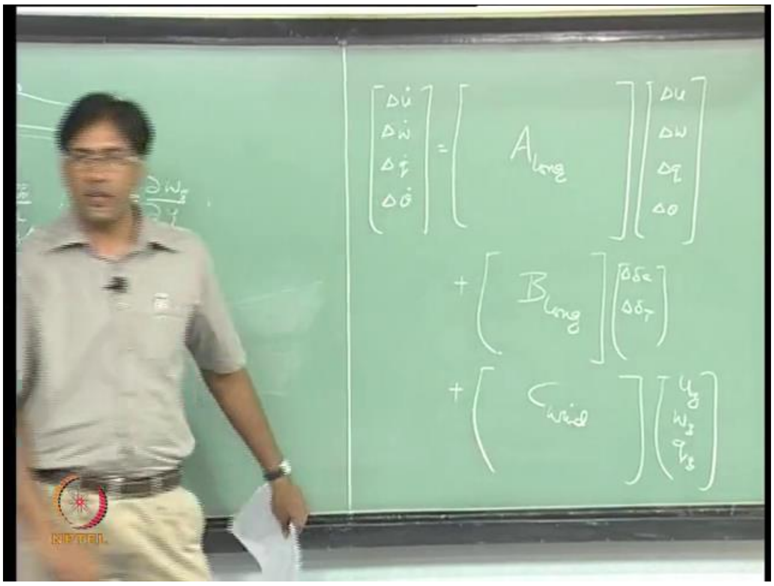

Let me just write down the equations for the perturbed longitudinal motion. .... ((no audio)) This is what you are going to get after you substitute these derivatives in the original equation. This is my original *A* matrix, which was obtained by linearizing aircraft equations of motion around the equilibrium state and this is my original *B* matrix, and this comes because of the wind. Again, if you want to model all of them together, then doing anything analytically is not possible here. So, what we are going to do is, we are going to take one simple case and see how you can model the wind into your equations of motion and study the aircraft response to the gust.

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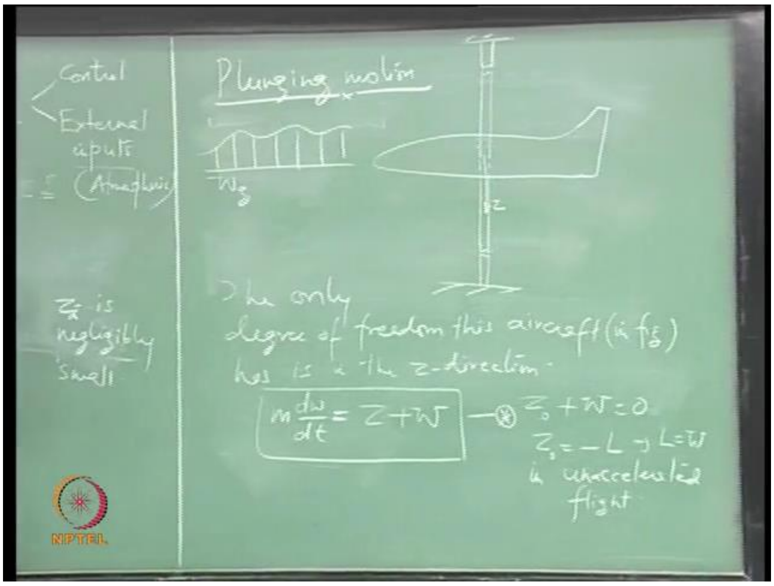

Let us say we have this model mounted in the wind tunnel and we are looking at the motion of aircraft only in the z (vertical) direction. This is z direction and there is some mechanism which allows it to move in the vertical direction, but that is only degree of freedom it has, the only degree of freedom. This motion is called plunging motion. The only degree of freedom ... this aircraft has is in the vertical motion. And you have some (wind) profile which is in the vertical direction. We look at some cases, to start with, let us assume that this is the case, what is it going to change? It is going to change the angle of attack because this velocity is in the vertical direction.

Let us try to look at the motion of the aircraft along z direction. Writing down the equations for the motion .... it is m into dw over dt, this w is the velocity in the z direction and that is going to be equal to, this acceleration is coming because of the aerodynamic forces and the weight of the aircraft. $m\frac{dw}{dt} = Z + W$. Let us also assume that we study the motion, perturbed motion about a steady state condition which is this ..... Aircraft is perfectly balanced with the lift, weight is perfectly balanced with the lift in the steady state condition. This Z_0 is minus of L.... Now, let us take a perturbation from this condition.

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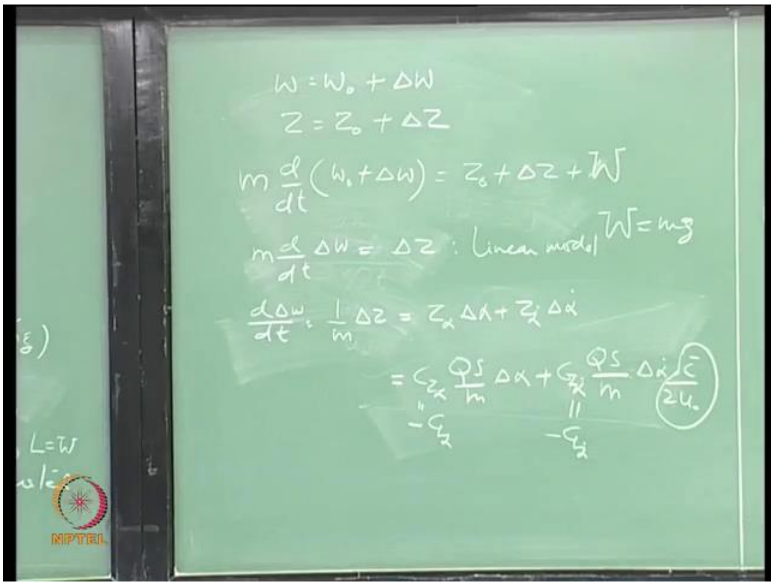

So, we will assume that the equilibrium vertical speed is w_0 and then add Δw to that .... and this is going to change the force in the vertical direction. This Δw is going to change this force and that is how you get the motion from the steady state condition. Substitute these in this equation $m\dfrac{d(w_0+\Delta w)}{dt}=(Z_0+\Delta Z)+W$ Small w's are the vertical speeds and this capital W is the weight. What we get from here is $m\dfrac{d\Delta w}{dt}=\Delta Z$. This is the linear model and I am looking at the linear motion of aircraft around equilibrium condition.

Now, this force is going to change because of a steady part which is coming from change in angle of attack and there is another part which is because of change in rate of angle of attack. So, we can write this also ... as this. $\Delta Z = Z_{\alpha}\Delta\alpha + Z_{\dot\alpha}\Delta\dot\alpha$. In terms of the coefficients, it is going to look ..... something like this. We have to non-dimensionalize this coefficient. We are using this factor to non-dimensionalize this number, which otherwise will have a unit because of this alpha dot $\dot\alpha$. This ($C_{Z\alpha}$) is also equal to minus of $C_{L\alpha}$ and $C_{Z\dot\alpha}=-C_{L\dot\alpha}$. Now, what is this delta alpha $\Delta\alpha$? This $\Delta\alpha$ is absolute $\Delta\alpha$. So how we are going to write it in terms of, including the gust.

$$\Delta Z = C_{Z\alpha}\frac{QS}{m}\Delta\alpha + C_{Z\dot\alpha}\frac{QS}{m}\Delta\dot\alpha.\frac{\bar c}{2u_0}$$

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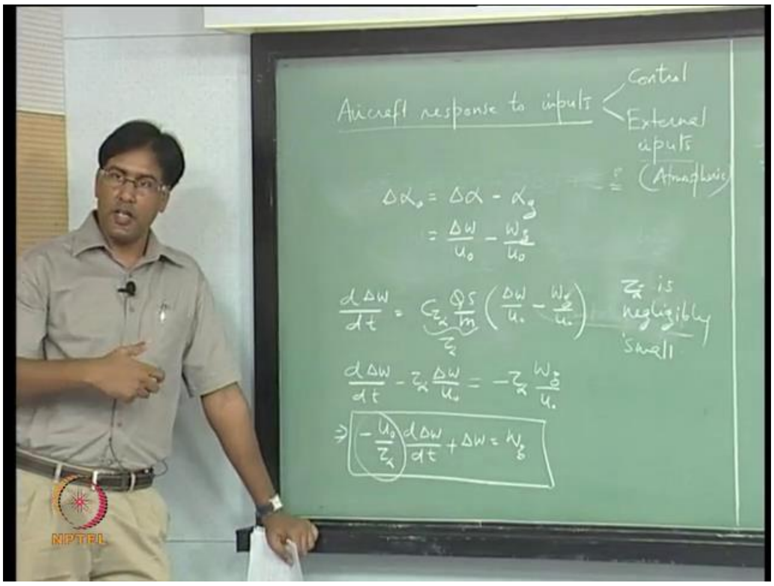

So that $\Delta\alpha$ is $\Delta\alpha$ relative which is equal to $\Delta\alpha$ earlier, and including the alpha coming from the gust. ... $\Delta\alpha_a = \Delta\alpha - \alpha_g = \dfrac{\Delta w}{u_0} - \dfrac{w_g}{u_0}$. This is how we have written small angles of attack in term of the vertical velocity. Now go back to that equation, and assume that this term is small, so that we can further reduce our model to a simpler one, $\dfrac{d\Delta w}{dt} = C_{Z\alpha}\dfrac{QS}{m}\left(\dfrac{\Delta w}{u_0} - \dfrac{w_g}{u_0}\right)$. You can also go back and just write this $C_{Z\alpha}\dfrac{QS}{m}$ as Z_α. So what you have is $\dfrac{d\Delta w}{dt} - Z_\alpha\dfrac{\Delta w}{u_0} = -Z_\alpha\dfrac{w_g}{u_0} \Rightarrow -\dfrac{u_0}{Z_\alpha}\dfrac{d\Delta w}{dt} + \Delta w = w_g$. Remember what we are assuming here is, that this $Z_{\dot{\alpha}}$ is negligibly small and we can ignore the second term.

Clearly this term is appearing as an input term which is not under your control. We can only define some particular variation and look at the dynamics of aircraft in the vertical direction, in the plunging motion. If we do not have any knowledge of this w_g, then you have to go for, for example, turbulent models where you have to go for higher level mathematical techniques to find out the response. But here we will look at some simpler models for this wind and see the response of aircraft to that.

Call this, this (u_0/Z_α) is like a time constant. If you set this (w_g) equal to 0, this is giving you the free response. w_g here is appearing as an input which is almost same as the control input, in case you have the knowledge of this. Its acting as an input. So, call this (u_0/Z_α)

tau which is the time constant of the motion .... And try to solve this equation. We already solved one such equation in the last class.

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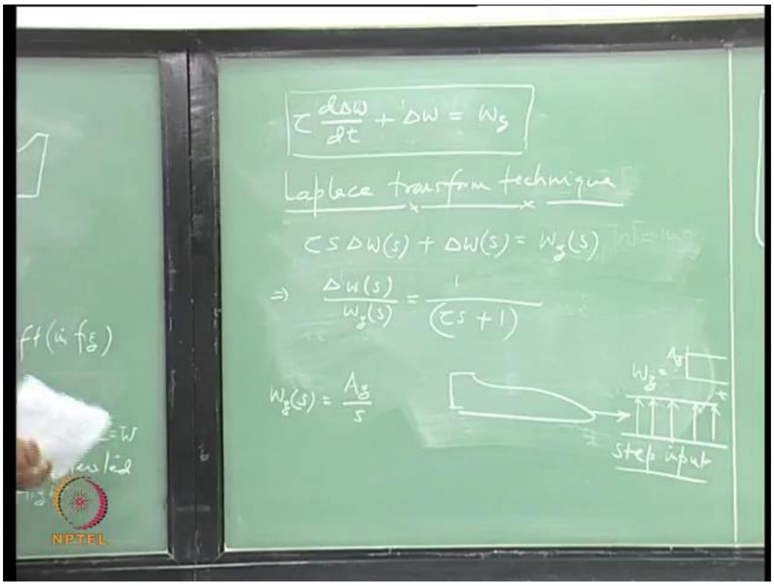

We used what is known as Laplace transform technique. We will apply that to this problem as well. .... The advantage here is that it converts a differential equation into an algebraic equation and you are more at ease solving algebraic equation than solving a differential equation. This is one big advantage of using this technique and it is applicable only to linear models. Now, let us look at a profile of this w_g and it is looking something like this (step with amplitude A_g). So, your aircraft is going in this direction and suddenly it sees wind in the vertical direction.

It is like step input. Let us say that this amplitude of this step is A_g. What is $w_g(s)$ for this step input? You have to take the Laplace transform of this time variation. So, this is a step or .... vertical gust uniform is going to look like a step input and unit step input, if you take the Laplace transform of that, it is $1/s$ multiplied by this amplitude, $w_g(s)=A_g/s$. Now, let us try to solve that equation, write it in time domain.

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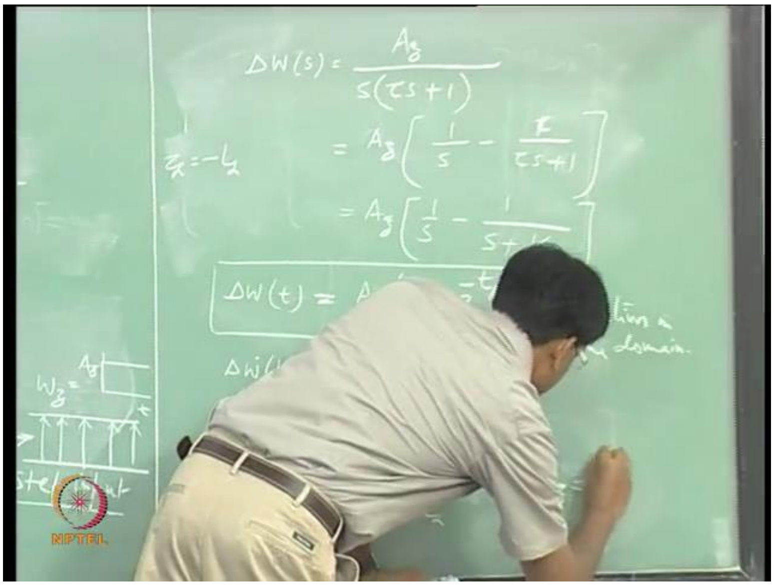

So, take inverse Laplace transform of this $\Delta w(s) = A_g\left[\frac{1}{s} - \frac{1}{s+1/\tau}\right]$ now and you have found your solution in time domain. $\Delta w(t) = A_g\left[1 - e^{-t/\tau}\right]$. You can also calculate the normal acceleration form this by differentiating it with this respect to time and that is going to be $\Delta\dot{w}(t) = \left(A_g/\tau\right)e^{-t/\tau}$. That is the normal acceleration of the aircraft because of the gust. When is it going to be maximum? It is going to be 0 at steady state, but it is going to be maximum at t equal to 0, $\Delta\dot{w}(0) = \left(A_g/\tau\right)$. This tau which is the time constant is u_0/Z_α. Z_α is nothing but $-L_\alpha$, Z is in the negative direction, this direction is the vertically down, this is the positive direction. So, lift is acting in the negative z direction according to the choice of axis system that we have. So, Z_α is $-L_\alpha$.

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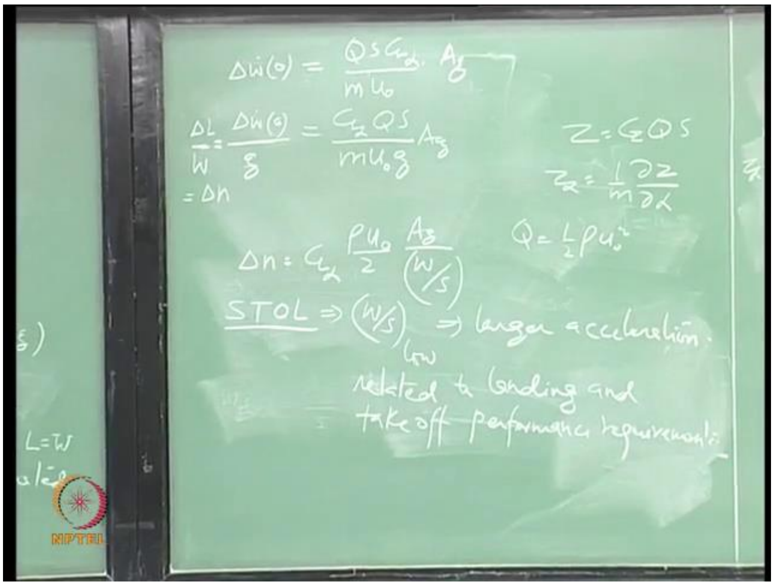

So, let us expand that QSCL alpha over u 0 into the amplitude of the gust and there is one m also here, $\Delta\dot{w}(0) = \dfrac{QSC_{L\alpha}}{mu_0}.A_g$. Whenever we write Z_α, we are taking m as the factor. You are dividing, this Z is C_zQS. When we write Z_α we write it like this $Z_\alpha = \dfrac{1}{m}\dfrac{\partial Z}{\partial \alpha}$. This is where this m comes from. .... We can find out the relative magnitude of this acceleration with respect to g ..... And this is nothing but, $\Delta L/W$, can you see that? If you just balance these two, what you are going to get is $m\Delta\dot{w}$ and that is coming from the change in lift.

So, corresponding to this, the load factor, there is change in load factor which is Δn. L/W is the load factor and the change in lift is going to cause a change in load factor, which is giving the acceleration. This load factor is going to be a function of, change in load factor is a function of $C_{L\alpha}$, on expanding this Q, which is half rho u 0 square $\frac{1}{2}\rho u_0^2$. So, this acceleration is definitely, from this formula you can see that, is going to depend upon how your wing loading (W/S) is. High wing loading means low acceleration because of the vertical gust, low wing loading high acceleration at any particular speed, this is the equilibrium speed; forward velocity.

It is going to depend upon what is the wing loading, where do you see the wing loading low? Which aircraft will have low wing loading?

Transport aircraft.

Aircraft which needs short take-off and landing requirements, they are usually having low wing loading. ... Short take-off and landing requirements results in low wing loading or you can make statements other way round as well. In such a case you are going to see larger acceleration because of the wind. .... The aircraft with, so this is clearly related to the landing and takeoff performance of the aircraft, landing and takeoff performance requirements of the aircraft. So, short takeoff and landing aircraft designed for this case we will have low wing loading and in such a case, you will see larger acceleration due to vertical gust.

This is one useful inference we can make from this analysis and this is for a very simple case, when you are only looking at the plunging motion. What we will do in the next class is, we will look at another (gust) profile again in time, but the wind may be varying also you know spatially, it is not just a temporal variation. So, how you are going to model that is, in your aircraft equations, is also something that we will talk about in the next class, we can stop at this point.