Flight Dynamics – II

(Stability)

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Module No. # 11

Lateral – Directional Dynamic Modes

Lecture No. # 37

Stability in Steady Roll Maneuver

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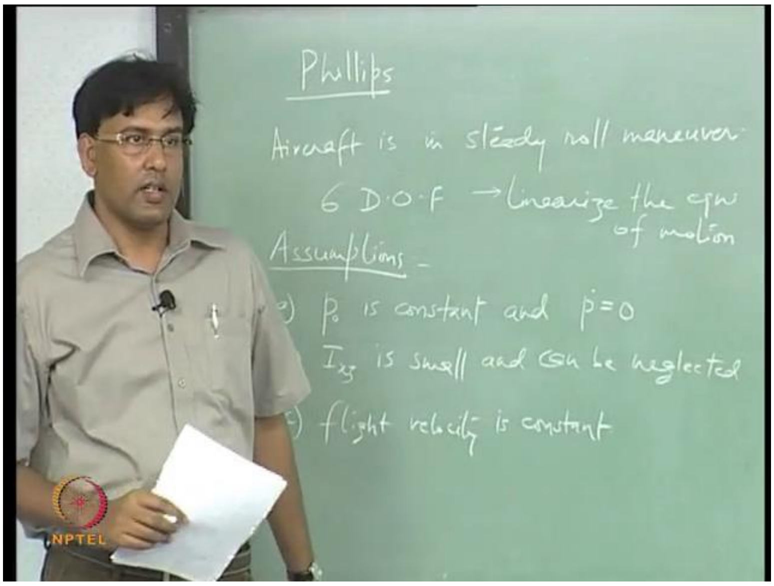

What we are trying to show there is an application of Routh's stability criteria to actual flight dynamics problem, trying to derive some analytical criteria for instability. This work was done by Phillips. So, what he wants to do is, he wants to find stability, analytical formula for stability criteria, using Routh's criteria for stability. So, what he assumes is aircraft is rolling, aircraft is in steady roll maneuver.

Here (angular) rates (p, q, r) are not 0 like the cases that we have seen earlier. So, rates are involved here and the equations of motion are coupled, we start with 6 degree of freedom equations of motion, and after making some more assumptions. For example, ...

what we will do is we will take the aircraft equations of motion and then linearise the equations .... and afterwards he made some assumptions.

So, one is this, steady rolling roll rate (p_0) is constant, $p_0 = const.$, and this is not small because we are talking about steady roll maneuvers. So, this p naught p_0 is not small, but p naught is constant, so that p dot is 0 $\dot{p} = dp/dt = 0$. b) I_{xz}, this inertia term which is cross inertia term is small and can be neglected. c), flight velocity is constant. He (Phillips) further assumes that the damping terms are 0 and some other terms as well.

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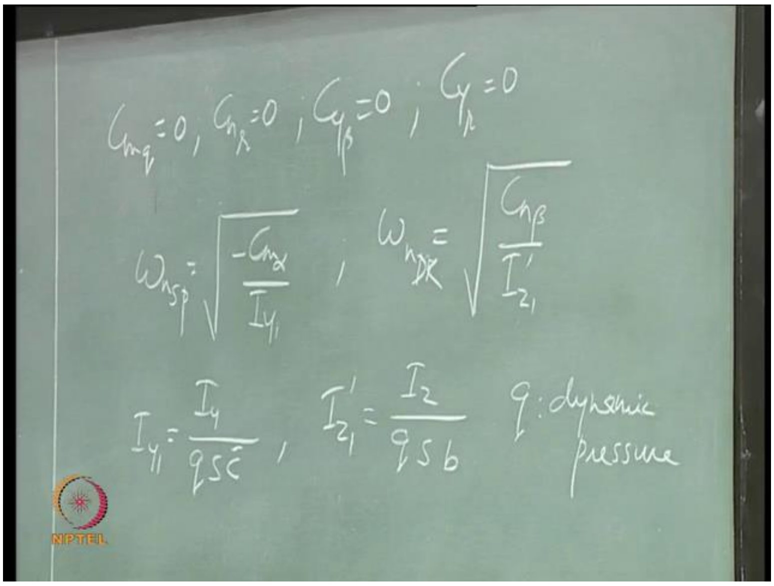

Cm q is 0 $C_{mq} = 0$. Interestingly he drops this equation, so that you will not see the rolling moment coefficients. The whole objective of doing this exercise is to only show you the usefulness of Routh's criteria. Not to go into details of this. Now he defines the two frequencies corresponding to short period mode and the Dutch roll mode in terms of whatever parameter is, parameters are left now.

So, omega n S P ω_{nSP} short period mode frequency, Dutch roll mode frequency given by omega n DR ω_{nDR} this square root of minus Cm alpha over Iy1 $\omega_{nSP} = \sqrt{-C_{m\alpha}/I_{y1}}$. Iy1 is Iy over q S c bar $I_{y1} = I_y / qS\bar{c}$. This q is the dynamic pressure. So, after dropping couple of equations, for example, he dropped this roll rate equation and he also drops the V dot $\dot{V}$ equation. So, we have six equations here now excluding the phi dot $\dot{\phi}$ and theta dot $\dot{\theta}$

equations. 6 equations here in terms of three velocities (u, v, w) and three rates (p, q, r) and from there he drops the first equation corresponding to this velocity. And this velocity is the u velocity, or if you write it in, write the equations in the wind axis fixed system then it is V dot $\dot{V}$.

And these equations, two equations actually he drops, and he formulates an Eigenvalue problem which is after linearising the equations of motion around the equilibrium state, which is given by this p naught constant. So, this Eigenvalue problem is this .. This lambda is the Eigenvalue.

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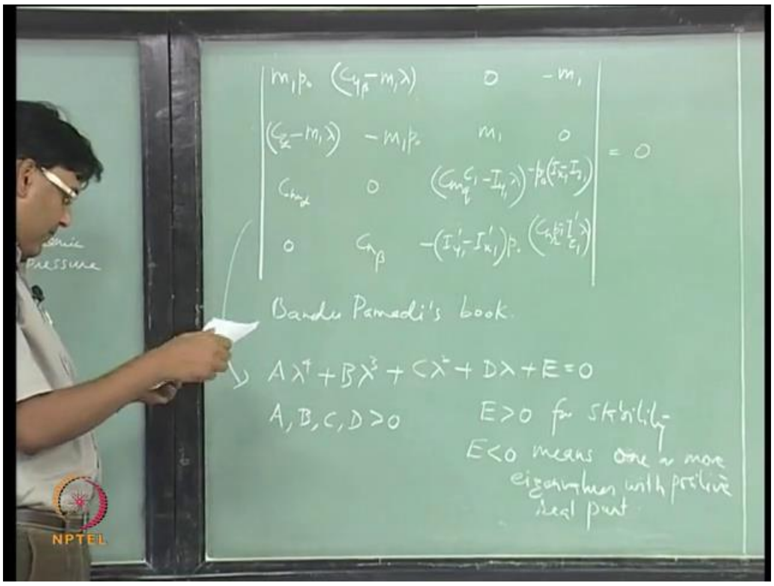

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$$\begin{vmatrix} m_1 p_0 & (C_{Y\beta} - m_1\lambda) & 0 & -m_1 \\ (C_{Z\alpha} - m_1\lambda) & -m_1 p_0 & m_1 & 0 \\ C_{m\alpha} & 0 & (C_{mq} c_1 - I_{y1}\lambda) & -p_0(I_{x1} - I_{z1}) \\ 0 & C_{n\beta} & -(I'_{y1} - I'_{x1})p_0 & (C_{nr} b_1 - I'_{z1}\lambda) \end{vmatrix} = 0$$

Let me read what I have written here. m 1 p naught. This m1 is some non dimensionalized mass, each of these terms which you do not recognize, they are written in terms of aircraft parameters that you know. So, this particular example is from Bandu Pamadi's book. You want to know more details on this you can look into the book. So, this matrix is consisting of lambda, four lambdas 1, 2, 3 and 4. So, we are going to end up with the quartic equation in lambda.

You want me to read this, m1 p naught, p naught is the steady roll rate, C y beta minus m1 into lambda 0, minus m1, Cz alpha minus m1 lambda, minus m1 p naught, m1, 0, Cm alpha, 0, Cm q c1 minus Iy1 lambda, minus p naught Ix1 minus Iz1, 0, Cn beta, minus Iy1 prime minus Ix1 prime into p naught, Cn r into b 1 minus Iz1 prime into lambda. So, what we are going to see is, we are going to see a characteristic equation which is looking something like this A into lambda raised to power 4 plus B into lambda cube plus C into lambda squared plus D into lambda plus E.

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

So, this is a quartic equation in lambda and now we want to use the Routh's criteria. Actually he uses the necessary condition only which says that all the coefficients must be of the same sign in this equation. So, it turns out that *A*, *B*, *C* and *D* are all positive. So, for stability, automatically *E* should also be positive $E > 0$, and E less than 0 $E < 0$ means, we are going to have one or more roots of, or the Eigenvalues of this equation with positive real part. But for that you need to also carry out the sufficient condition creating the Routh's array and then looking at the sign of the elements in the first column of the Routh's table. You just need to look at what this *E* is, you do not have to really bother about the other four coefficients.

$$E = I_{y1} I'_{z1} m_1^2 \left[\left(\frac{I_z - I_x}{I_y} \right) p_0^2 - \omega_{SP}^2 \right] \left[\left(\frac{I_y - I_x}{I_z} \right) p_0^2 - \omega_{DR}^2 \right]$$

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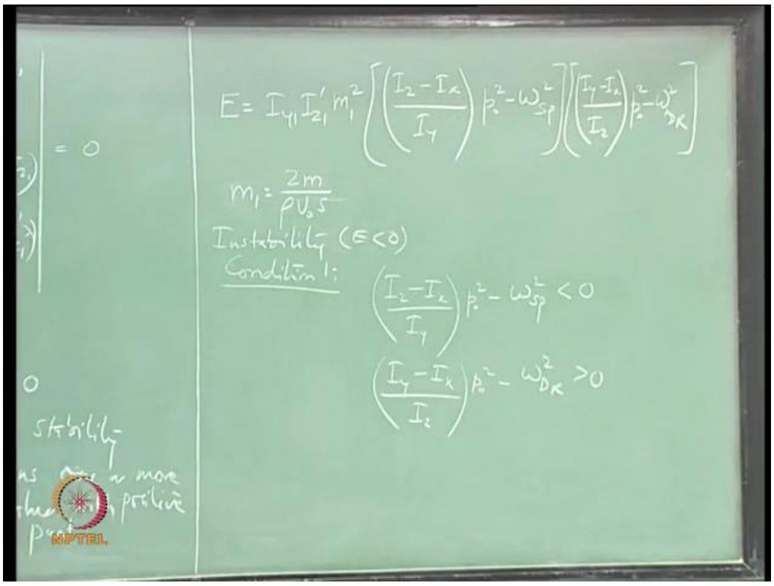

Let us look at what E is Iy1into I z1 prime into m 1 squared into Iz minus Ix over Iy into p naught squared minus omega SP squared into Iy minus Iy over Iz into p naught squared minus omega DR squared. This is what E is in terms of the aircraft parameters. Iy1, which is Iy over q s into c bar, Iz1 prime which is Iz over q s into b. (())

Yeah, I have written in there into m 1 squared, this m 1 is 2 into m over rho into U naught into S $m_1 = \frac{2m}{\rho S U_0}$. Now, use this criteria for instability or stability, stability criteria is this E should be positive and for instability E should be less than 0. So, there are two conditions now, the condition one for instability: Iz minus Ix over Iy into p naught squared minus omega SP squared less than zero $\left[\left(\frac{I_z - I_x}{I_y}\right)p_0^2 - \omega_{SP}^2\right] < 0$ and Iy minus Iy over Iz into p naught squared minus omega DR squared greater than zero $\left[\left(\frac{I_y - I_x}{I_z}\right)p_0^2 - \omega_{DR}^2\right] > 0$. So, one of these terms should be negative, one should be positive the other one should be negative. Condition two is Iz minus Ix over Iy into p naught squared minus omega SP squared greater than zero $\left[\left(\frac{I_z - I_x}{I_y}\right)p_0^2 - \omega_{SP}^2\right] > 0$ and Iy minus Iy over Iz into p naught squared minus omega DR squared less than zero $\left[\left(\frac{I_y - I_x}{I_z}\right)p_0^2 - \omega_{DR}^2\right] < 0$. So, this is condition one and condition two.

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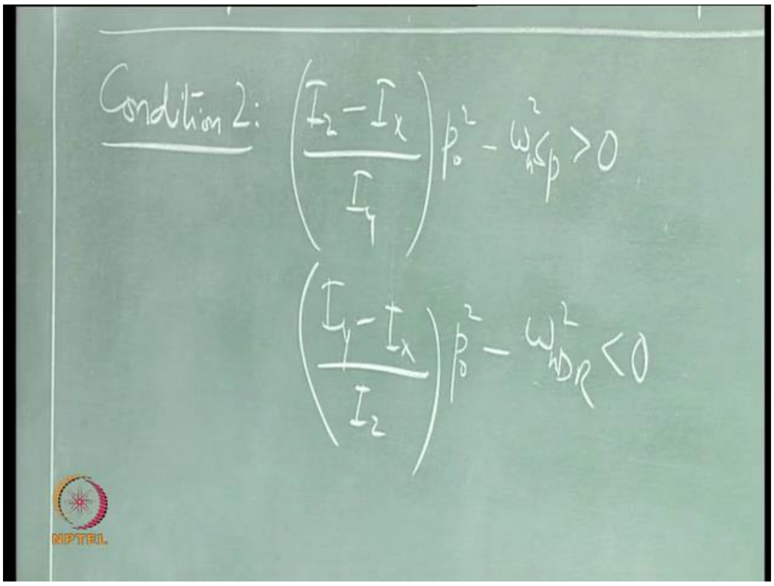

((Audio not available refer time: 16:36 to 17:22)) These are two conditions for instability and we look at the parameters that are involved. Parameters involved are inertia terms and the frequencies of short period mode and the Dutch roll mode. Rolling maneuver is considered to be a fast maneuver and the instabilities are also are due to mainly inertial coupling terms.

This we have talked about previously, whenever you have an aircraft which is fuselage heavy, I mean all the masses are concentrated towards the fuselage, there you will have instability because of the inertia terms. So, rightly he arrived at an expression which is related to the inertia of the aircraft, and he found some stability boundaries using this criteria. So, what we have done is, we have used Routh's necessary condition for stability and trying to find out the stability boundaries in terms of these parameters.

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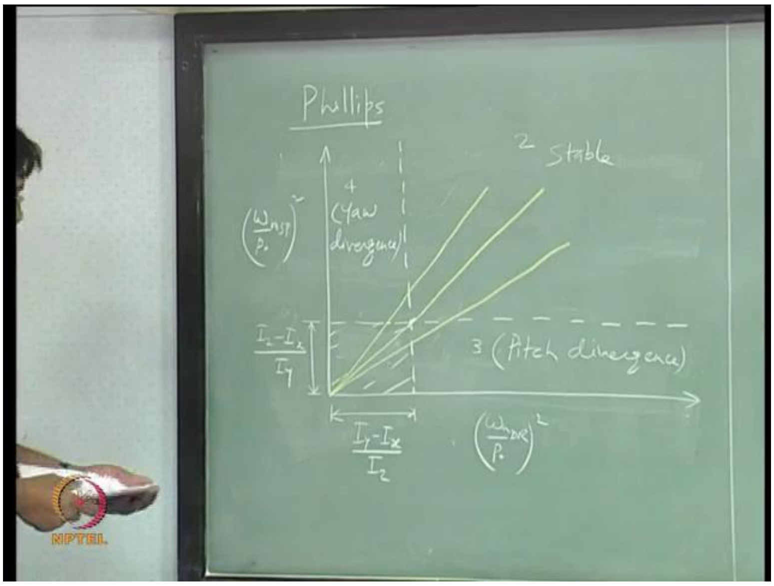

((No audio 18:48 to 19:12)) So, this boundary is defined in terms of this ratio, ratio of short period frequency over the steady roll rate squared $(\omega_{nSP} / p_0)^2$ and Dutch roll frequency over steady roll rate squared of that, $(\omega_{nDR} / p_0)^2$, and using the conditions that we have, based on the inertia terms and these frequencies ((no audio)). .....

So, now, you go back and look at those conditions and you will find out that, in this particular maneuver, steady rolling maneuver, this is a region where, you can have stable flying conditions. This region is also stable. Region one and two are stable regions. Three and four are unstable flight regions and here you will see what is known as pitch divergence. In this region four, you see what is known as yaw divergence. So, this is the point which separates out whether you are going to see yaw divergence or the pitch divergence. If you draw a line through this region one and passing through this critical points which is lying on the boundary of the two divergent regions. If you look at any other line, they are passing through these divergent regions, and this line is indicating increasing roll rate.

So, starting from here, 0 roll rate, in this region the flight is, maneuver is stable. If you are passing through this, then you are going to see in this part pitch divergence, and in this part, you are going to see yaw divergence and again in this part where the roll rate is quite high, its stable, but under some conditions. You could actually infer lot of things just by using the Routh's stability criteria in terms of aircraft parameters. So, this is one of the famous examples of use of Routh's criteria to flight dynamics problems.

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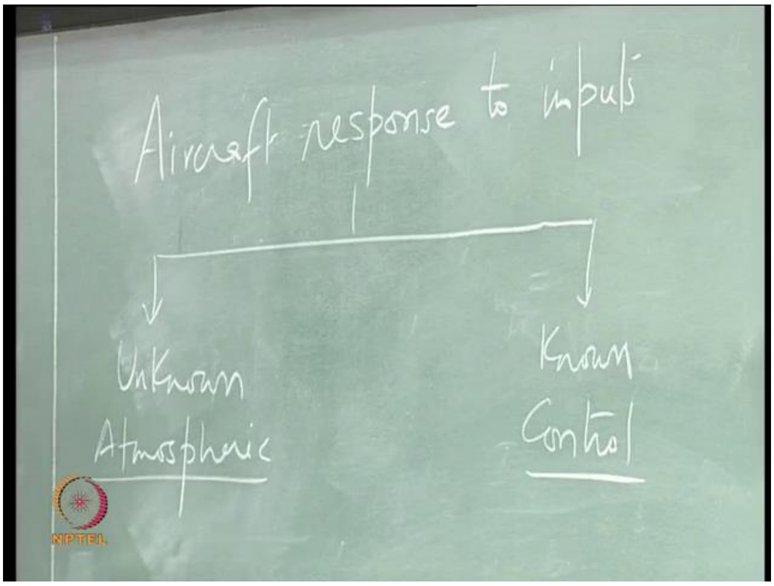

We are starting a new chapter today. Next three lectures will be on this chapter ((no audio)) ..... Aircraft response to various inputs. Some inputs can be inputs which are unknown, some inputs are known. What are unknown inputs? Atmospheric inputs! Many times you will not have any idea about what kind of wind inputs you are going to see and at times also the pilot input.

Because each one of us will have different reflex systems, the way we respond or the way we move our muscles, it will vary from pilot to pilot, isn't it? So, that can also come under this category, we will not have an exact idea of what kind of inputs actually can come from the pilot himself. But we do not worry about that right now. We take the atmospheric inputs as unknown inputs and known inputs are the control inputs.

I know what kind of input I am going to give to my control surfaces, for example, step input, or you can move your control surfaces in step, ramp or you can give a pulse, doublet and so on, and we want to look at how the aircraft is going to respond to those inputs. So, the first case that I am going to take is of the rolling motion and this we have discussed in detail.

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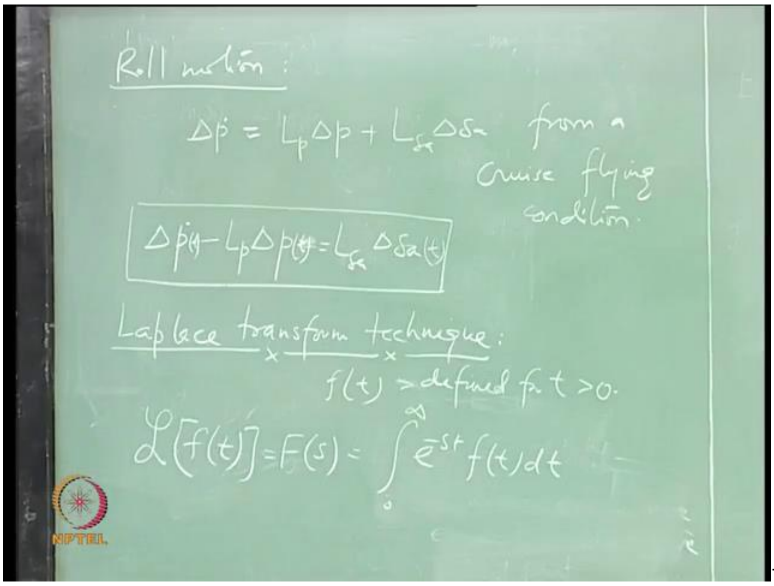

((No audio)) If you look at the roll motion of aircraft, the equation for a pure rolling motion is given by delta p dot, this is a linearized equation about the flying condition that we have, cruise condition, from that we give an aileron input, so that we can get into a roll Delta p dot equal to L p into Delta p plus L delta a Delta delta a $\Delta\dot{p} = L_p \Delta p + L_{\delta a} \Delta \delta a$. This is from a cruise. Now, what we are looking at is the aircraft response in roll to aileron inputs, and this input is not 0. We are not talking about free response any more, we are talking about the forced response. How do you solve this? One of the popular techniques to solve such linear equations of motion is Laplace technique.

So, you can use Laplace transform technique here, it works only for the linear systems. This is a linear system ((no audio)) .... So, if you have any signal, just any signal f (t) which is defined for t greater than zero, then, you can find Laplace transform of that, which is written as F of s, s is a complex quantity. Now, use this, apply it here (to the Delta p dot equation), and you will find, there are standard tables given in terms of, what signal, corresponding to what signal what Laplace form you can get.

Table is given, it is also easy to find the Laplace transform using this integral, it is not so difficult. So, let me write down this equation in terms of, Laplace form of this equation ((no audio)) And this you can do only when this Delta p Δp is starting from 0 initial condition, ... ((no audio)) which we will have in our case. We are looking at roll dynamics starting from a level cruise condition.

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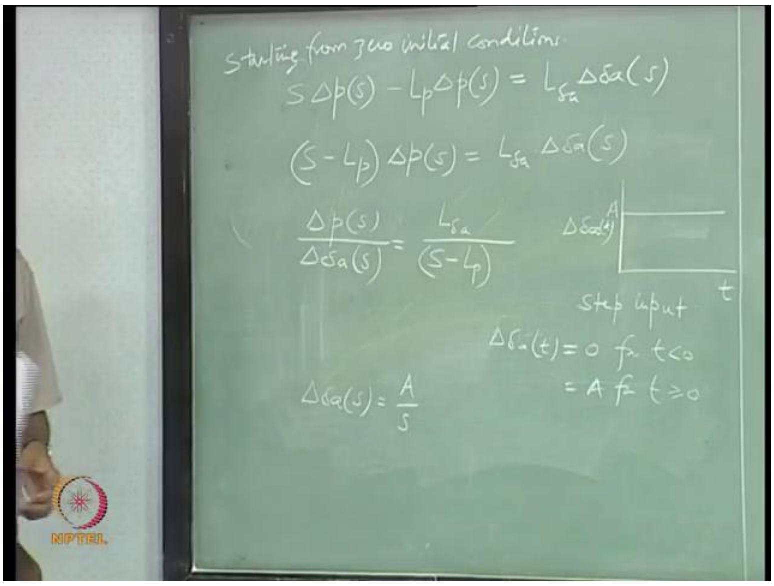

p is 0 there and Delta p Δp is definitely 0. Delta p Δp is the disturbance in p. s minus L p into Delta p(s) $\Delta p(s)$ equal to L delta a. Relation between the roll rate and aileron deflection in s-domain is this Delta p of s $\Delta p(s)$ over Delta delta of s $\Delta \delta a(s)$ equal to L delta a over s minus L p from this equation. This is d Delta p over d t $\Delta \dot{p} = d\Delta p/dt$. Now, you can use this formula to convert it to Laplace form. So, for 0 initial conditions, the Delta p naught is 0 $\Delta p_0 = 0$ this is what you will get.

What I want to know is the response with respect to aileron input, what kind of input? We can take any input, of any form. Let us assume that this Delta delta a $\Delta \delta a$ is a step input ... and let us say this (A) is the amplitude of the step input. So, the function can be written in this form Delta delta a $\Delta \delta a$ equal to zero for t less than zero and equal to A for t greater than or equal to zero. Now, I want to find out the Laplace transform of this signal as well. So that I can write everything in terms of s. You can again use this to find out the corresponding function in the s-domain and you will find that for this function, it is A over s. What I am interested in is, finding out what is the variation of this Delta p is in time.

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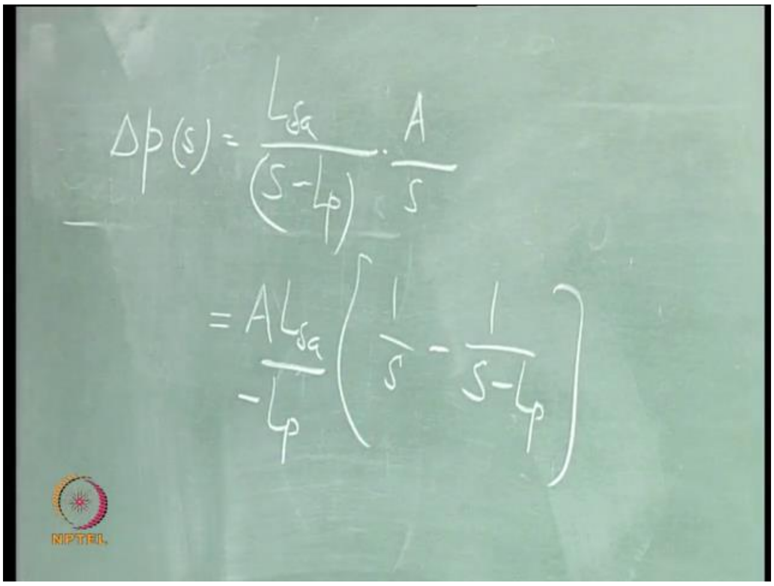

((No audio)) ....You can use partial fraction method and split this into two parts, and now, what you do is, you take a inverse Laplace. Inverse Laplace of this returns you the original signal. So, first of all what we did, we took Laplace of this signal, we found this function F in s-domain. Later on, we can take inverse Laplace of that function and find out what this original signal is. This is one of the easiest methods of solving a linear differential equation, wherever you can use this.

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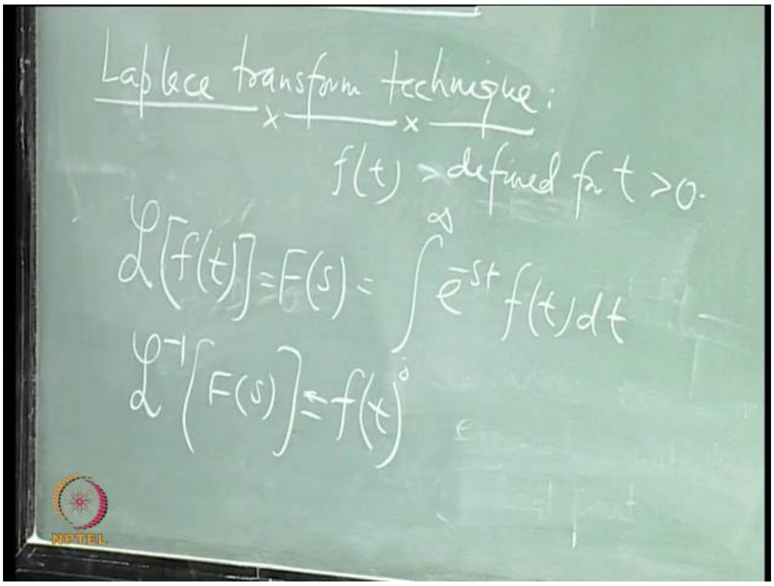

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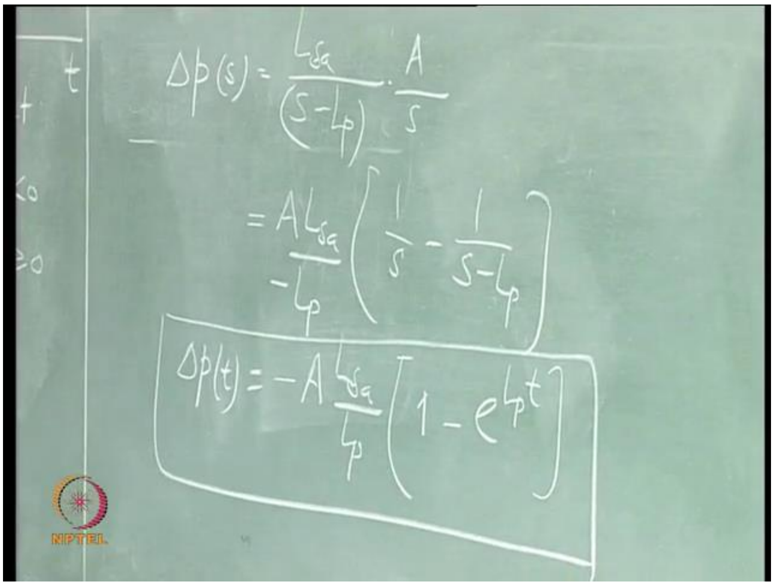

So now if I do that I can find out what delta p is in terms of time. Delta p t equal to minus A into L delta over L p into 1 minus e raised to L p into t ((audio not available refer time: 37:43 to 38:13)) $\Delta p(t) = -A\frac{L_{\delta a}}{L_p}\left[1-e^{L_p t}\right]$. Clearly it depends upon this Lp, Lp also gives us the time constant τ , and time constant we have defined to be this tau equal to minus of one over Lp $\tau = -1/L_p$. Lp is usually negative, Lp is negative. So, the time constant is positive. What it tells me is that, in the steady state, Delta p Δp is going to approach some value which is not 0, is not it? For the unit step deflection in aileron, step input in aileron, with amplitude A.

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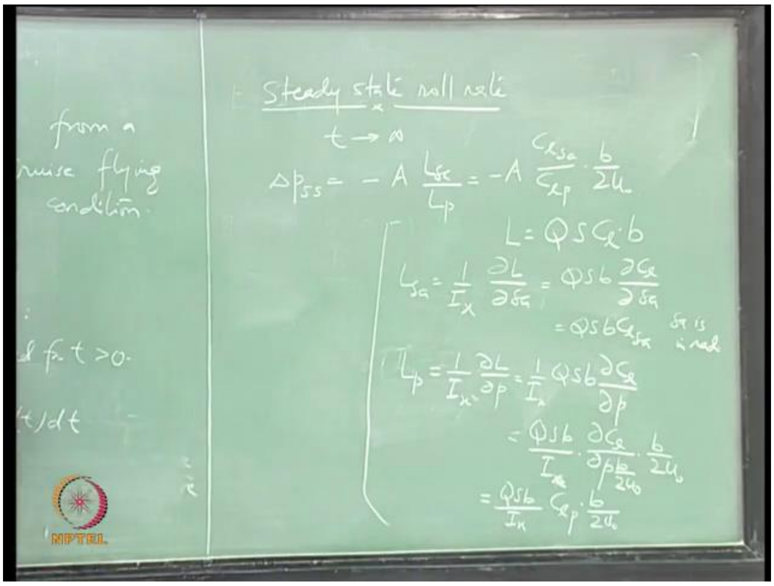

((No audio)) .... So, this will become 0, when t goes to infinity and what you have is this Delta pss equal to minus A into L delta a over Lp $\Delta p_{ss} = -A\frac{L_{\delta a}}{L_p}$. What is L delta a? L is Q S into rolling moment coefficient into span of the wing, $L = QSC_l b$, and how we get L delta ($L_{\delta a}$) is this one over Ix into partial derivative of L with respect to delta a $L_{\delta a} = \frac{1}{I_x}\frac{\partial L}{\partial \delta a}$, where delta a δa is in radians because I am writing this as a non-dimensional unit. So, delta a δa here is in radian .... Lp is one over Ix into partial derivative of L with respect to p $L_p = \frac{1}{I_x}\frac{\partial L}{\partial p}$, p has a unit know which involves second. I have to somehow non-dimensionalise this, and we have done that earlier Lp equal to QSb over Ix into Clp into b over 2 u naught $L_p = \frac{QSb}{I_x}C_{lp}\frac{b}{2u_0}$, ((No audio)) this Cl p C_{lp} has no unit.

So, whenever you have been given aircraft data you have to look at if it has unit or not. If it has no unit, then you have to take care of this part. .... So, this steady roll rate in terms of the coefficients is Delta pss equal to minus A into Cl delta a over Clp into b over 2 u naught given as this. $\Delta p_{ss} = -A\frac{C_{l\delta a}}{C_{lp}}.\frac{b}{2u_0}$. And many times you will size your aileron based on Delta p s s into 2 u 0 over b $\left(\Delta p_{ss} . \frac{2u_0}{b}\right)$. So aileron is supposed to give you roll rate and you will size, size your aileron by looking at this parameter. It is roughly of the order of 0.07 to 0.09, $\left(\Delta p_{ss} . \frac{2u_0}{b}\right) \approx 0.07 - 0.09$.

Now, let us do an example problem. What you have to remember is when we are talking about this Delta p Δp in time, the time response of the roll rate, then you have to find out this L_p and not the C_{lp}. This formula is for actually the steady roll rate. If you want to find out how this Delta p varies in time, $\Delta p(t)$, then you also have to find out what this L_p is. So, let us look at one example problem, and this is for the aircraft F104 A. What we want to find out is the roll response to a 5 degree step change in aileron deflection.

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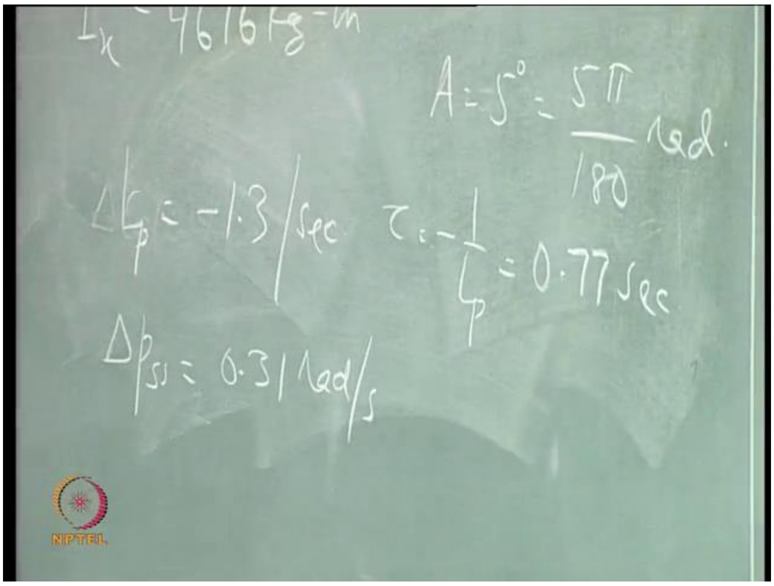

Trim speed is given .... ((no audio)) This is a solved example in Nelson. What other things are given are: this $C_{lp} = -0.285$. Cl delta a $C_{l\delta a}$ is 0.039. *S* is 18 meter square and *b* is 6.7 meter and, *Ix* is 4676 kg-m square. So, *A* is 5 degrees; *A* is the amplitude of the step input in aileron. So, *A* is 5 degrees which is 5 into pi over 180 radians, and L_p you can find out using that formula, ... it turns out to be minus 1.3 per second.

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So, Delta p s s Δp_{ss} is 0.31 radian per second using the formula, $\Delta p_{ss} = -A\frac{C_{l\delta a}}{C_{lp}}.\frac{b}{2u_0}$, and so you know the time constant from here, tau equal to minus 1 over Lp $\tau = -1/L_p$, which is 0.77 second. This is Delta p s s Δp_{ss} for 5 degree aileron deflection as step, and ….. response is going to converge to this steady state roll rate. So, this is how roughly it will look like (Refer to the plot). Any question? So, we can stop, if you do not have any question.